

Course Overview

PreCalculus with Limits, Level 1

Ms Laskaris

Course Description:

This is an advanced pre-calculus course in which students study related quantities using the concept of mathematical functions (e.g. polynomial, trigonometric, exponential, logarithmic, absolute value, greatest integer, rational, and piece-wise) using verbal, numerical, graphical, and analytical methods of representing functions.

Prerequisites:

Students are expected to be **strong independent learners** with a special ability and interest in math. IAG 4 level 1 with **"B" or better** or IAG 4 level 2 with teacher recommendation and curriculum coordinator approval.

Course Content (by chapter)

- 1 Functions and Their Graphs
- 2 Polynomial and Rational Functions/Complex Numbers
- 3 Exponential and Logarithmic Functions
- 4 Graphs and Inverses of Trigonometric Functions * Ind. Study *
- 5 Trigonometric Identities and Equations * Ind. Study *
- 6 Applications of Trigonometric Functions *Ind. Study*
- 6 Vectors
- 8 Sequences and Series
- 9 Topics in Analytic Geometry
- 10 Analytic Geometry in Three Dimensions
- 11 Limits (if time allows)

Textbook

0618 39480X

Required Materials:

Pencil or pen

Textbook

Notebook - Large 3-ring binder

Loose leaf paper and graph paper

TI-83/84 Graphing calculator- This will be an invaluable tool for this course.

Please see me if there is a financial concern about purchasing a calculator.

Grading:

Your grades are based on total point system. Averages are calculated by dividing your total points earned by the total number of possible points. Letter grades will be assigned in accordance with school guidelines. Your grade will be determined based on assessments in the following areas:

- Tests and Quizzes (approximately 75%)
- Homework and class work
- Independent study

Course grade	Marking period 1 - 40%
	Marking period 2 - 40%
	Final Exam - 20%

It is your responsibility to make up all missed work. It is crucial that you do not fall behind in this course. Should you need to make up an assessment or need extra help, I am usually available after school until at least 3:00pm. If you need extra help, please see me ASAP to schedule a meeting time.

If you have any questions or concerns please feel free to contact me via email at klaskaris@havsd.net or voice mail at (610)853-5900 ext. 2747.

Parent/Guardian

Signature _____

(indicates that this course overview has been reviewed)

This should be kept in your notebook at all times!!

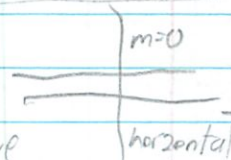
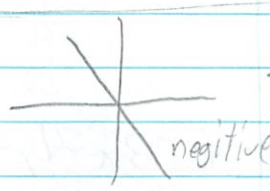
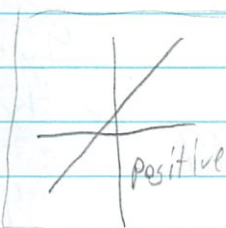
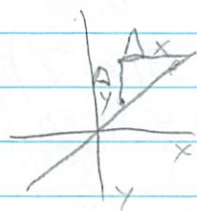
1.1 Lines in the Plane

9/5

to find slopes of lines + write linear equations

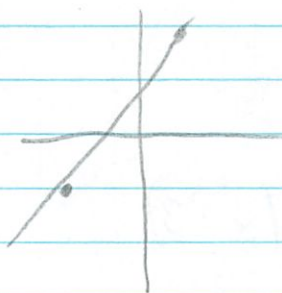
- point-slope forms
- y-intercept forms
- parallel + perpendicular lines

$$\text{slope} = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} \quad \text{where } x_1 \neq x_2$$



pl

10. Draw a line connecting the points + find slope
 $(-3, -2), (1, 6)$



$$\frac{6 - (-2)}{1 - (-3)} = \frac{8}{4} = 2 = m$$

Point-slope form of a line

$$y - y_1 = m(x - x_1)$$

Ex $(1, -2) \quad m = 3 \rightarrow y - (-2) = 3(x - 1)$
 $y + 2 = 3x - 3$

$$y = 3x - 5 \text{ is y-intercept form}$$

General form / standard form

$$Ax + By + C = 0$$

A positive

example from front

$$0 = 3x - y - 5$$

p12

28.

Find general form

$$(-2, -5) \quad m = \frac{3}{4}$$

→

$$y - y_1 = m(x - x_1)$$

$$y - (-5) = \frac{3}{4}(x - (-2))$$

$$y + 5 = \frac{3}{4}(x + 2)$$

$$4y + 20 = 3(x + 2)$$

$$4y + 20 = 3x + 6$$

$$0 = 3x - 4y - 14$$

A pos

$$\frac{4y}{4} = \frac{3x - 14}{4}$$

$$y = \frac{3}{4}x - \frac{7}{2}$$

30.

$$(10, -4) \quad m = \text{undefined}$$

$$x = -10$$

$$x + 10 = 0$$

26.

$$(-3, 6) \quad m = -2$$

$$y - 6 = -2(x + 3)$$

$$y = -2x$$

$$y - 6 = -2x - 6$$

-6 cancel out

$$-2x + 6 - y + 6 = 0$$

$$2x - y + 12 = 0$$

$$2x + y = 0$$

32.

$$(2.3, -8.5) \quad m = 0$$

$$y = -8.5$$

$$0x + y + 8.5 = 0$$

don't mention

Library of Functions

Linear Functions

$$f(x) = mx + b$$

$m > 0$ ($m \oplus$)	$m < 0$ ($m \ominus$)
Domain $\mathbb{R}$ $(-\infty, \infty)$	Domain $\mathbb{R}$ $(-\infty, \infty)$
Range $\mathbb{R}$ $(-\infty, \infty)$	Range $\mathbb{R}$ $(-\infty, \infty)$
x intercept $(-\frac{b}{m}, 0)$	x intercept $(\frac{b}{m}, 0)$
y intercept $(0, b)$	y intercept $(0, b)$

Constant Function $m = 0$
Horizontal line

slope y-intercept form of a line

$$y = \underset{\text{slope}}{m}x + b \quad b = \text{y-intercept value}$$

ex1

$$\begin{array}{r} x + y = 2 \\ -x \quad -x \\ \hline y = -x + 2 \end{array}$$

slope = -1
y-intercept = 2 $\rightarrow (0, 2)$

ex2

$$y = 2$$

slope = 0
y-intercept = $(0, 2)$

p12

Find slope + y-intercept

46.

$$3x + 4y = 1$$

$$\begin{array}{r} -3x \\ -3x \end{array}$$

$$\frac{4y}{4} = \frac{-3x + 1}{4}$$

$$y = -\frac{3}{4}x + \frac{1}{4}$$

$$\text{slope} = -\frac{3}{4}$$

$$\text{y-intercept pt} = (0, \frac{1}{4})$$

48.

$$y = 12$$

$$\text{slope} = 0$$

$$\text{y-intercept pt} = (0, 12)$$

Recap

General form

$$Ax + By + C = 0$$

slope-intercept

$$y = mx + b$$

pt-slope form

$$y - y_1 = m(x - x_1)$$

vertical

$$x = 0$$

horizontal

$$y = 0$$

Parallel lines

$$m_1 = m_2 \quad (\text{same slope})$$

$$m_1 = -\frac{1}{m_2}$$

ex Find slope-intercept^{form} for line that goes through $(2, -1)$ + parallel

$$2x - 3y = 5$$

$$\begin{array}{r} -2x \\ -2x \end{array}$$

$$\frac{-3y}{-3} = \frac{5 - 2x}{-3}$$

$$y = -\frac{2}{3}x - \frac{5}{3}$$

$$y + 1 = \frac{2}{3}(x - 2)$$

$$\frac{3y + 3}{-3} = \frac{x - 2}{-3}$$

$$\frac{3y}{3} = \frac{x - 5}{3}$$

$$y = \frac{x}{3} - \frac{5}{3}$$

Perpendicular

$$m_1 = -\frac{1}{m_2} \text{ negative reciprocal}$$

(1)
ex2 find perp. line for previous problem

$$y+1 = \frac{-1}{\frac{2}{3}} (x-2) \quad \frac{-1}{\frac{2}{3}} = -\frac{3}{2}$$

$$y+1 = -\frac{3}{2}(x-2)$$

$$2y+2 = -3(x-2)$$

$$2y+2 = -3x+6$$

$$\frac{2y}{2} = \frac{-3x+4}{2}$$

$$y = -\frac{3}{2}x+2$$

p12

59. Get line equation + find $\perp$ line

$$x+4=6$$

$$x=4$$

$$m=0$$

$$y-2 = 0 \quad (3, -2)$$

$$y-2 = 0 \quad (x-3)$$

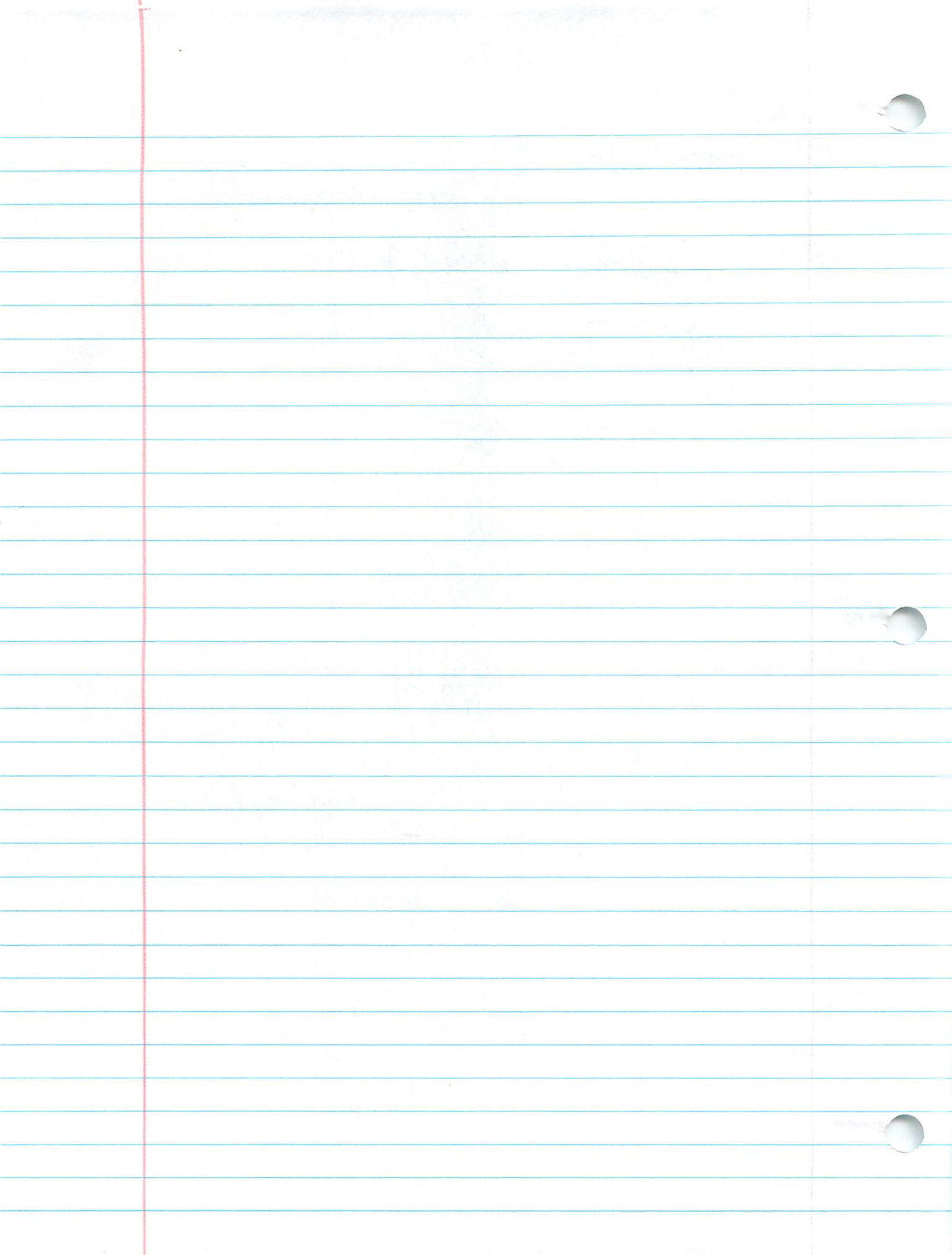
$$y+2 = 0$$

$$y = -2$$

↓ duh - stupid mistake

$$y = -2 \quad \parallel$$

↓ is undefined



1.1 Lines in the Plane

Homework

9/5

p 11-14

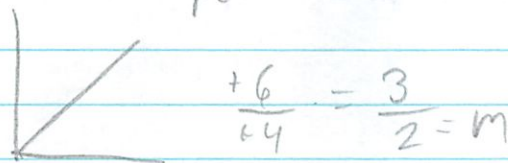
Identify line

1. $m = \frac{2}{3} \rightarrow L_2$

$m = \text{undefined} \rightarrow L_3$

$m = -2 \rightarrow L_1$

5. Estimate Slope



9. Plot + find slope

$(-6, -1) (-6, 4) \rightarrow \frac{4 - (-1)}{-6 - (-6)} = \frac{5}{0} = \text{undefined}$



Find 3 additional points

11. $(2, 1) m = 0$



$(5, 1)$

$x = \text{constant}$

$(-5, 1)$

$(0, 1)$

13. $(1, 5) m = \text{undefined}$

$(1, 5)$

$x = \text{constant}$

$(1, -5)$

$(1, 0)$

17. $(7, -2) m = \frac{1}{2}$

$(9, -1)$

$(11, 0)$

$(13, 1)$

Find slope + y-intercept form

19. $5x - y + 3 = 0$
 $\quad \quad \quad +y \quad \quad +y$
 $y = 5x + 3$

slope = 5
 y-int, y.t. (0, 3)

21. $5x - 2 = 0$
 $\quad +2 \quad +2$
 $5x = 2$
 $\quad \quad \quad \frac{5}{5} \quad \frac{2}{5}$
 $x = \frac{2}{5}$

slope = $\frac{2}{5}$
 y-int = (0, 0)

30. Find general form
 (-10, 4) m = undefined

$x = -10$
 $x + 10 = 0$

32. (2, 3, -8, 5) m = 0

$y = -8.5$
 $y + 8.5 = 0$

Find slope-intercept form
 35. (-8, 1) (-8, 7)

$\frac{7-1}{-8--8} = \frac{6}{0}$ $x = -8$

~~38.~~ $(-\frac{1}{10}, -\frac{3}{5}) (\frac{2}{10}, -\frac{9}{5})$

$\frac{-\frac{9}{5} - (-\frac{3}{5})}{\frac{2}{10} - (-\frac{1}{10})} = \frac{-\frac{6}{5}}{\frac{3}{10}} = -\frac{6}{5} = m$

$-\frac{9}{5} = -\frac{6}{5} (\frac{2}{10}) + b$

$-\frac{9}{5} = -\frac{12}{25} + b$

$\frac{72}{25} + \frac{12}{25}$

$b = -\frac{72}{25} + \frac{1}{10}$

???

$y = -\frac{6}{5}x + \frac{17}{25}$

Convert to lowest common denominator

43. Jeweler made 28,500 in 2000) 1/yr
 32,900 in 2002
 ↑ in 2006
 $32,900 - 28,500 = 4,400$ in 2 years
 $4,400 \times 2 = 8,800$
 \$ 41,700 in 2006

46. Determine slope + y-intercept

$$3x + 4y = 1$$

$$-3x \quad -3x$$

$$\frac{4y}{4} = \frac{-3x + 1}{4}$$

$$y = -\frac{3}{4}x + \frac{1}{4}$$

$$\text{slope} = -\frac{3}{4}$$

$$y\text{-int pt} = (0, \frac{1}{4})$$

48. $y = 12$

$$\text{slope} = 0$$

$$y\text{-int pt} = (0, 12)$$

51. || or $\perp$ or neither?

$$(0, -1)(5, 9) \quad (0, 3)(4, 1)$$

$$\frac{9 - (-1)}{5 - 0}$$

$$\frac{10}{5} = 2$$

$$\frac{3 - 1}{4 - 0}$$

$$\frac{2}{4} = \frac{1}{2}$$

$\perp$

53.

$$(3, 6)(-6, 0) \quad (0, -1)(5, \frac{7}{3})$$

$$\frac{6 - 0}{-6 - 3}$$

$$\frac{6}{-9} = -\frac{2}{3}$$

$$\frac{-1 - \frac{7}{3}}{5 - 0}$$

$$\frac{-\frac{10}{3}}{5}$$

neither

Find (slope-int., || + $\perp$ at point
 $(-\frac{2}{3}, \frac{7}{8})$

57,

$$3x + 4y = 7$$

$$\begin{array}{r} -3x \\ 4y = -3x + 7 \end{array}$$

$$y = -\frac{3}{4}x + \frac{7}{4}$$

$$y - \frac{7}{8} = -\frac{3}{4}\left(x - -\frac{2}{3}\right)$$

$$y = \frac{7}{8} = -\frac{6}{12} - \frac{3}{4}x$$

$$8y - 7 = -6x - 4$$

$$y = -\frac{3}{4}x + \frac{3}{8}$$

do LCM

$\perp$

$$\frac{-1}{-\frac{3}{4}} \rightarrow -\frac{1}{1} \cdot \frac{4}{-3} \rightarrow \frac{4}{3}$$

$$y - \frac{7}{8} = \frac{4}{3}\left(x - -\frac{2}{3}\right)$$

$$\begin{array}{r} 72y - 63 = \\ 96x + 64 \end{array}$$

$$\begin{array}{r} y - \frac{7}{8} = \frac{4}{3}x + \frac{8}{3} \\ + \frac{7}{8} \quad + \frac{7}{8} \\ y = \frac{4}{3}x + \frac{85}{24} \end{array}$$

58,

$$6x + 2y = 9$$

$$\begin{array}{r} -6x \\ 2y = -6x + 9 \end{array}$$

$$y = -3x + 4.5$$

$$(-3.9, -1.4)$$

$$y - (-1.4) = -3(x - (-3.9))$$

$$\begin{array}{r} y + 1.4 = -3x - 11.7 \\ -1.4 \quad -1.4 \end{array}$$

$$y = -3x - 13.1$$

$\perp$

$$y - (-1.4) = \frac{1}{3}(x - (-3.9))$$

$$\begin{array}{r} y + 1.4 = \frac{1}{3}x + 1.3 \\ -1.4 \quad -1.4 \end{array}$$

$$y = \frac{1}{3}x - 1$$

59,

$$x - 4 = 0$$

$$+4 \quad +4$$

$$x = 4$$

$$m = \text{undefined}$$

||

⊥

$$y - (3, 2) = 0(x - 3)$$

$$y - 2 = 0$$

$$x = 3$$

$$y = 2$$

⊥

$$(-4, 1)$$

$$y - 1 = 0(x - (-4))$$

$$y - 1 = 0$$

$$y = 1$$

⊥

$$x = -4$$

60,

$$y + 2 = 0$$

$$-2 \quad -2$$

$$y = -2$$

$$m = 0$$

||

⊥

82,

Thing costs 25,000, After 10 years value = 2,000

$$\frac{25000 - 2000}{10} = \frac{23000}{10} = \$2300/\text{year loss}$$

$$a) y = -2300x + 25,000$$

t	0	1	2	3	4	5
V	25000	22700	20400	18100	15800	13500

t	6	7	8	9	10
V	11200	8900	6600	4300	2000

c) ✓

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1.2 Introduction to Functions

9/6

pairs of quantities which are related in some way
kind of rule = relation

ex

$$I = P \cdot r \rightarrow I = 1000r$$

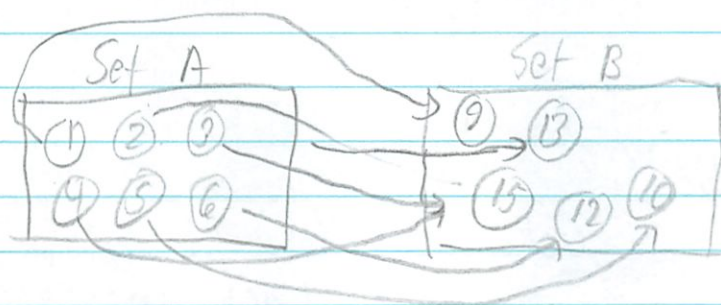
ex2

$$\text{Area of circle} \rightarrow \pi r^2$$

) relation

If a relation matches each item from 1 set with exactly one item from a different set - Then it is called a function
one-to-one correspondence

ex



function

Domain
(Input)

Range
(Output)

$$\{(1, 9), (2, 13), (3, 15), (4, 15), (5, 12), (6, 10)\}$$

Function - a relation that assigns to each element, x , exactly one element y

Characteristics

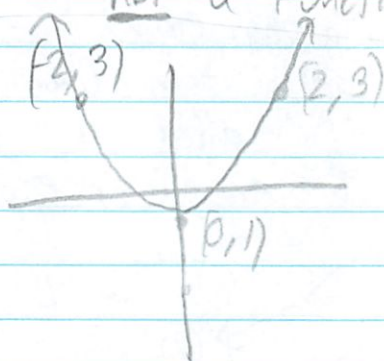
1. each element of "A" must be matched w/ element "B"
2. Some elements of "B" may not be matched to any elements of "A"



3. Two or more elements of "A" may be matched with the same element in "B"
4. An element of "A" (domain) can not be matched w/ 2 elements of B

In	2	2	3	4	5
Out	11	10	8	5	1

↑ not a function - 2 defined 2x

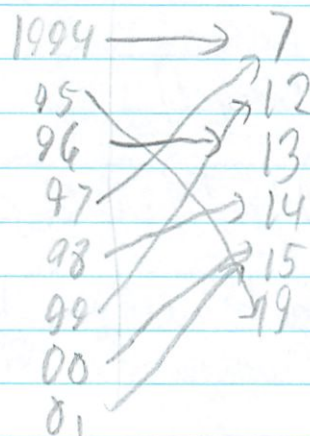


← function

p 24
4.

Function?

In Out



Yes, It is a function,
Each in corresponds
to an out.

Which set

10.

$A = \{a, b, c\}$ $B = \{0, 1, 2, 3\}$

a) $(a, 1)$ $(c, 2)$ $(c, 3)$ $(b, 3)$ $\in c$ defined 2x

b) $(a, 1)$ $(b, 2)$ $(c, 3)$ $\in \checkmark$

c) $(1, a)$ $(0, a)$ $(2, c)$ $(3, b)$ $\in$ not from A to B

d) $(c, 0)$ $(b, 0)$ $(a, 3)$ $\in \checkmark$

Functions can be represented

1. verbally (word problem)
2. Numerically (ordered pairs)
3. Graphically
4. Algebraically

x = input
independent
domain

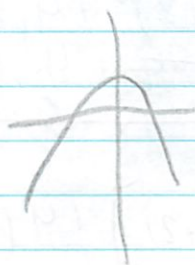
y = output
dependent
range

Which represents a function of x ?

a) $x^2 + y = 1$
 $-x^2$ $-x^2$

$y = -x^2 + 1$

quadratic

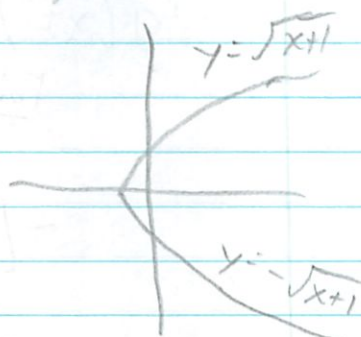


function

b) $-x + y^2 = 1$
 $+x$ $+x$

$y^2 = x + 1$

$y = \pm\sqrt{x+1}$



not function ← vertical
line test

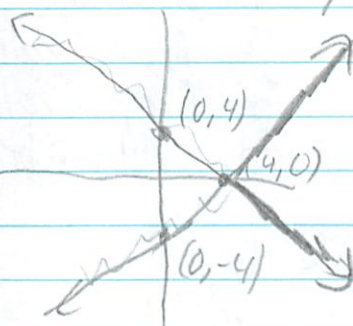


p25

Does equation represent $y = f(x)$

22. $|y| = 4 - x \rightarrow y = 4 - x$ and $y = -(4 - x)$
 $y = -4 + x$

No, not a function



$\hookrightarrow$ x-intercept
is the same

 $f(x)$ = function notationEvaluating a function

$$g(x) = -x^2 + 4x + 1$$

$$g(2) = -(2)^2 + 4(2) + 1$$

$$-4 + 8 + 1$$

$$\textcircled{5}$$

$$g(x+2) = -(x+2)^2 + 4(x+2) + 1$$

Fail $\swarrow$

$$\cancel{-x^2 - 4} + 4x + 8 + 1$$

$$\textcircled{-x^2 + 4x + 5}$$

$$-[(x+2)(x+2)] + 4(x+2) + 1$$

$$-(x^2 + 4x + 4) + 4x + 8 + 1$$

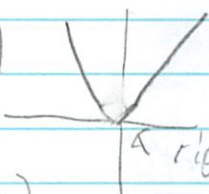
$$-x^2 - 4x - 4 + 4x + 8 + 1$$

$$\textcircled{-x^2 + 5}$$

Piecewise Function

a function which is defined by 2 or more equations over a specified domain

ex1

$$f(x) = |x|$$


right angle

$$\begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Domain

Range

$\mathbb{R}$

$y \geq 0$

$(-\infty, \infty)$

$[0, \infty)$

Calculus
piece

Intercept $(0,0)$

Decreasing: $(-\infty, 0)$

Increasing: $[0, \infty)$

ex2

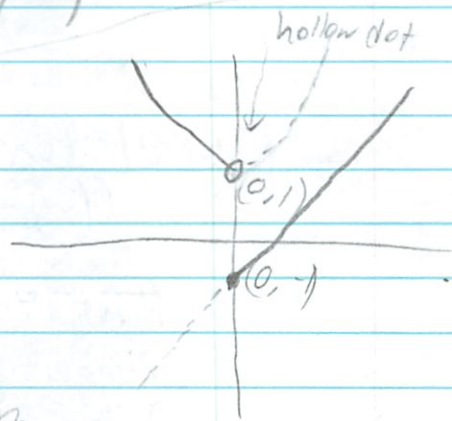
Eval. when $x = -1$, $x = 0$

$$f(x) = \begin{cases} x^2 + 1, & x < 0 \\ x - 1, & x \geq 0 \end{cases}$$

$$f(-1) = (-1)^2 + 1$$

$$f(0) = (0) - 1$$

use the
function in the range



25

Evaluate

38.

$$f(x) = \begin{cases} x^2 + 2, & x \leq 1 \\ 2x^2 + 2, & x > 1 \end{cases}$$

$$a) f(-2) = (-2)^2 + 2$$

$$4 + 2$$

$$\textcircled{6}$$

$$b) f(1) = (1)^2 + 2$$

$$\textcircled{3}$$

$$c) f(2) = 2(2)^2 + 2$$

$$2(4) + 2$$

$$\textcircled{10}$$

Domain of functions

Set of all real numbers for which all real numbers are defined

$$f(x) = \frac{1}{x^2 - 4}$$

 $x \neq 2, -2 \downarrow$ write

$$D = \{x : x \neq 2, -2\}$$

such that

$$f(x) = \sqrt{x}$$

 $x \neq \ominus \downarrow$ write

$$D = \{x : x \geq 0\}$$

Interval of domain is $[0, \infty)$

Radical function

$$f(x) = \sqrt{x}$$

Domain: $[0, \infty)$

Range: $[0, \infty)$

Intercept $(0, 0)$

Increasing $[0, \infty)$



More

More Domain

ex

$$f: \{(-3,0), (-1,4), (0,2), (2,2), (4,-1)\}$$

↗

$$\text{Domain} = \{-3, -1, 0, 2, 4\}$$

ex2

$$y(x) = -3x^2 + 4x + 5$$

$$\text{Domain} = \mathbb{R} \text{ (all reals)}$$
$$\rightarrow \{x \mid x \in \mathbb{R}\}$$

exists

$$h(x) = \frac{1}{x+5} \quad \text{Domain} = \{x \mid x \neq -5\}$$

$$V = \frac{4}{3} \pi r^3 \quad \text{Domain} = \{r \mid r > 0\}$$

$$k(x) = \sqrt{4-3x} \rightarrow 4-3x=0$$

$$x = \frac{4}{3}$$

$$\text{Domain} = \{x \mid x \leq \frac{4}{3}\}$$

Finding Domain + Range

$$f(x) = \sqrt{9-x^2}$$

$$y = \sqrt{9-x^2}$$

$$y^2 = 9-x^2$$

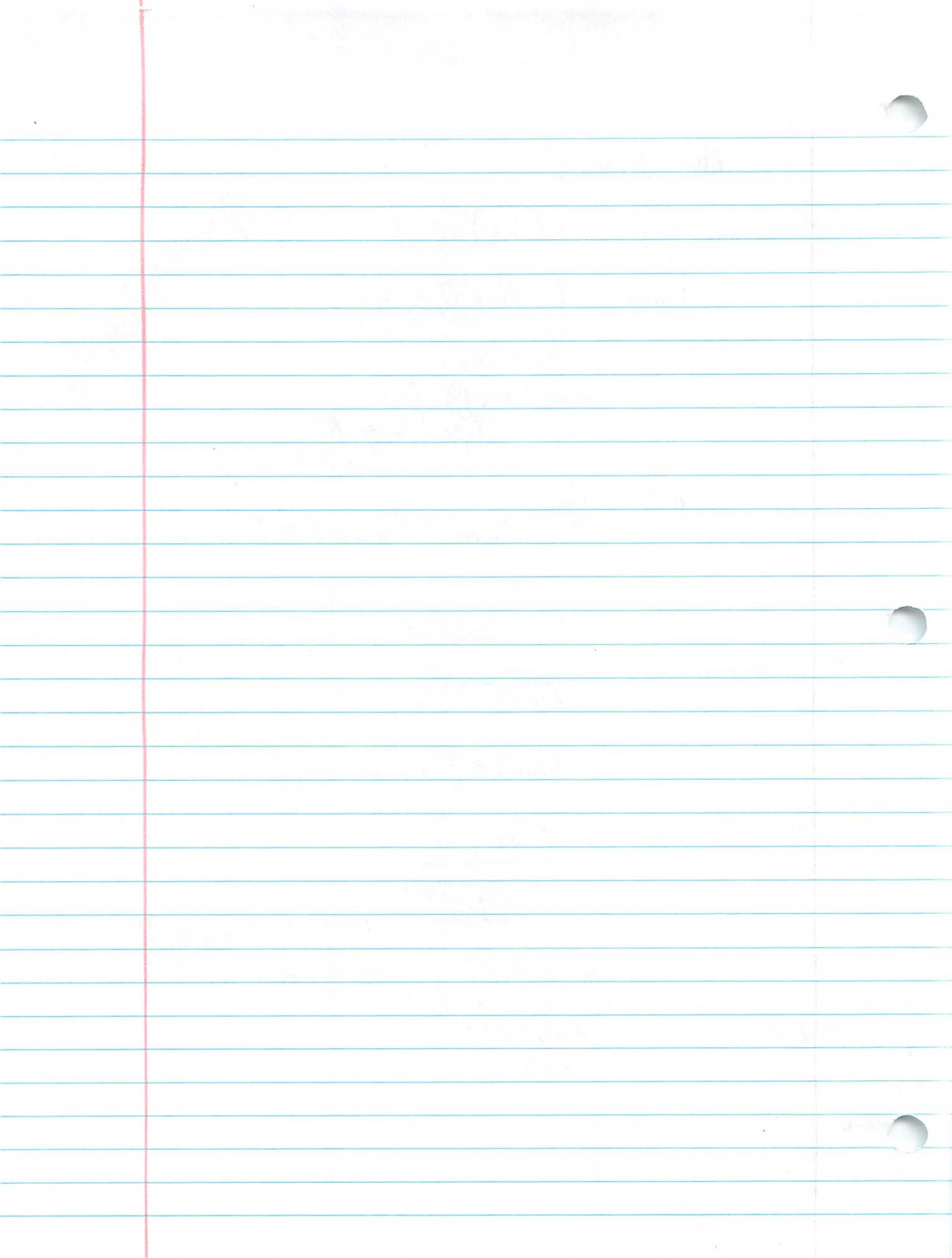
$$x^2 + y^2 = 9$$

$$x^2 + y^2 = r^2$$

$$r = 3$$

$$D = \{x \mid -3 \leq x \leq 3\}$$

$$R = \{y \mid 0 \leq y \leq 3\}$$



1.2 Functions H/W

9/6

p24

Function?

3. National → Cubs
→ Pirates
→ Dodgers
American → Orioles
→ Yankees
→ Twins

No, not a function
but would be
the other way
around

(x can't have multiple y's)

7.

In	10	7	4	7	10
Out	3	6	9	12	15

No 7 can't be defined 2x

10. $A = \{a, b, c\} \rightarrow B = \{0, 1, 2, 3\}$

- a) $\{(a, 1), (c, 2), (c, 3), (b, 3)\}$ - No - c defined 2x
b) $\{(a, 1), (b, 2), (c, 3)\}$ - Yes
c) $\{(1, a), (0, a), (2, c), (3, b)\}$ - No - it's $A \rightarrow B$
d) $\{(c, 0), (b, 0), (a, 3)\}$

15. $x^2 + 7 = -1$
 $-x^2$ $-x^2$
 $y = -x^2 - 1$



Yes, quadratics always
are functions

16. $y = \sqrt{x+5}$



Yes

21. $y = |4-x| \rightarrow y = 4-x$



range: $x \geq 0$

$y = -4+x$
 $x \geq 0$

Yes

Evaluate

28. $g(x) = 7-3x$

a) $g(0)$

$g = 7-3(0)$
 $g = 7-0$
 $g = 7$

b) $g(\frac{2}{3}) \rightarrow 7-3(\frac{2}{3})$

$\frac{2}{3} - \frac{2}{3}$
 $g = 0$

c) $g(s+2) \rightarrow 7-3(s+2)$

$7-3s-6$
 $g = 1-3s$

33. $q(x) = \frac{1}{x^2-9}$ a) $q(0) = \frac{1}{0^2-9}$ b) $q(3) = \frac{1}{3^2-9}$ c) $q(y+3) = \frac{1}{(y+3)^2-9}$

$\frac{1}{-9}$
 $\frac{-1}{9}$

$\frac{1}{0}$
 undefined

foil $\rightarrow \frac{1}{y^2+6y+9-9}$
 $\frac{1}{y^2+6y}$

35. $f(x) = \frac{|x|}{x}$ a) $f(2) = \frac{2}{2}$ b) $f(-2) = \frac{|-2|}{-2}$ c) $f(x^2) = \frac{|x^2|}{x^2}$

$\frac{2}{2}$ $\frac{-2}{-2}$ $\frac{2}{2}$ $\frac{-2}{-2}$

$\textcircled{1}$ $\textcircled{-1}$ $\textcircled{1}$ $\textcircled{-1}$

$\textcircled{1}$ $\textcircled{-1}$

38. $f(x) = \begin{cases} x^2+2, & x \leq 1 \\ 2x^2+2, & x > 1 \end{cases}$

a) $f(-2) = x^2+2$
 $(-2)^2+2$
 $4+2$
 $\textcircled{6}$

b) $f(1) = x^2+2$
 $(1)^2+2$
 $\textcircled{3}$

c) $2x^2+2$
 $2(2)^2+2$
 $\textcircled{10}$

26
45. Find all x so $f(x) = 0$

$f(x) = \frac{3x-4}{5} \rightarrow 0 = \frac{3x-4}{5}$

$0 = 3x - 4$
 $4 = 3x$
 $\frac{4}{3} = \frac{3x}{3}$
 $x = \frac{4}{3}$

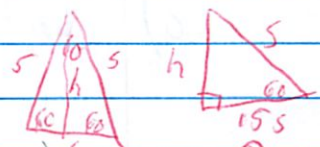
Find domain

49. $f(x) = 5x^2 + 2x - 1$ $D: (x)$ or $(x: \mathbb{R})$

53. $f(x) = \sqrt[3]{x-4}$ $D: (x) \rightarrow \{x: x \in \mathbb{R}\}$

55. $g(x) = \frac{1}{x} - \frac{3}{x+2}$ $D: (x: \mathbb{R} \neq -2, 0)$

$\nearrow x \neq 0$ $\nearrow x \neq -2$ $\rightarrow$ combine $A = \frac{\sqrt{3}}{4} s^2$



68. Write A triangle as f(sides) $\rightarrow$ area $\Delta (\frac{1}{2}bh)$ $\leftarrow$ take for gk

70. Box made



height	Volume (cm ³)
1	484
2	800
3	972
4	1024
5	980
6	864

base $h = \sqrt{3} \cdot \frac{1}{2} s$
 $A = \frac{1}{2} s \sqrt{3} \cdot \frac{1}{2} s$
 $A = \frac{1}{4} \cdot \sqrt{3} s^2$

b) on calc

c) Formula (from calc)

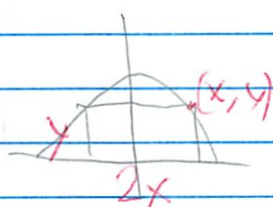
$4x^3 - 96x^2 + 576x$

a) Max amon: (4, 1024)

c) Domain $\{x: \mathbb{R} \geq 0\}$ $\leftarrow$ should give end of problem
 $0 < x < 12$ $\leftarrow$ in the problem yes

$(24-2x)(24-2x)x$
 $\rightarrow$ do it algebraically

72. Rectangle bound by $y = \sqrt{36-x^2}$, Write $A = f(x)$ + Domain



$A = (2x) \cdot y$ $A = 2x \sqrt{36-x^2}$

$D: \{x: x \in \mathbb{R} \geq 0\}$ $\leftarrow$ find on calc

$0 < x < 6$

73. $\uparrow$ girth = 108"

$\uparrow y$ $\uparrow 4x$ $V = x^2(108-4x)$

$108 = y + 4x$
 $-y - y$

a) Function $\rightarrow V = x^2 y$

b) Domain $\rightarrow \{x: 0 < x < 27\}$ $\leftarrow$ graph look at

c) Graph

d) Max

$x = 18$ $y = 36$

$\hookrightarrow 18^2 36 = 11664$

$108 - y = 4x$
 $\frac{108-y}{4} = x$

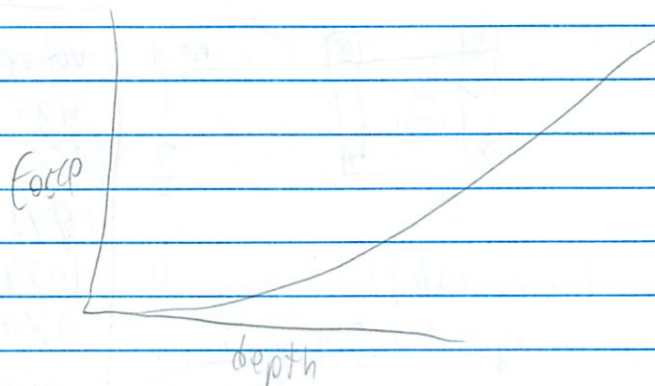
$(27 - \frac{y}{4} = x)$ max to find x

81. $F(y) = 149,76 \sqrt{10} y^{5/2}$
 $F = \text{force}$
 $y = \text{water in feed}$

a) Chart:

y	5	10	20	30	40
$F(y)$	26440	149570	846096	2331565	4786248

b) Graph:



c) $Y?$ when $t(y) = 1,000,000$

21,382683 ft

1.2 Difference Quotients

9/6

$$\frac{f(x+h)-f(x)}{h} \quad h \neq 0$$

ex

For $f(x) = x^2 - 4x + 7$, find

← sub in →

just write

$$\frac{(x+h)^2 - 4(x+h) + 7 - [x^2 - 4x + 7]}{h}$$

$$\frac{x^2 + 2xh + h^2 - 4x - 4h + 7 - x^2 + 4x - 7}{h}$$

$$\frac{2xh + h^2 - 4h}{h}$$

$$2x + h - 4$$

p29

84. $g(x) = 3x - 1$

$$\frac{3(x+h) - 1 - [3x - 1]}{h}$$

$$\frac{3x + 3h - 1 - 3x + 1}{h}$$

$$\frac{3h}{h}$$

(3)

3+h

85. $f(x) = x^2 - x + 1$

where
 $\frac{f(2+h) - f(2)}{h}$

$$\frac{(2+h)^2 - (2+h) + 1 - [2^2 - (2) + 1]}{h}$$

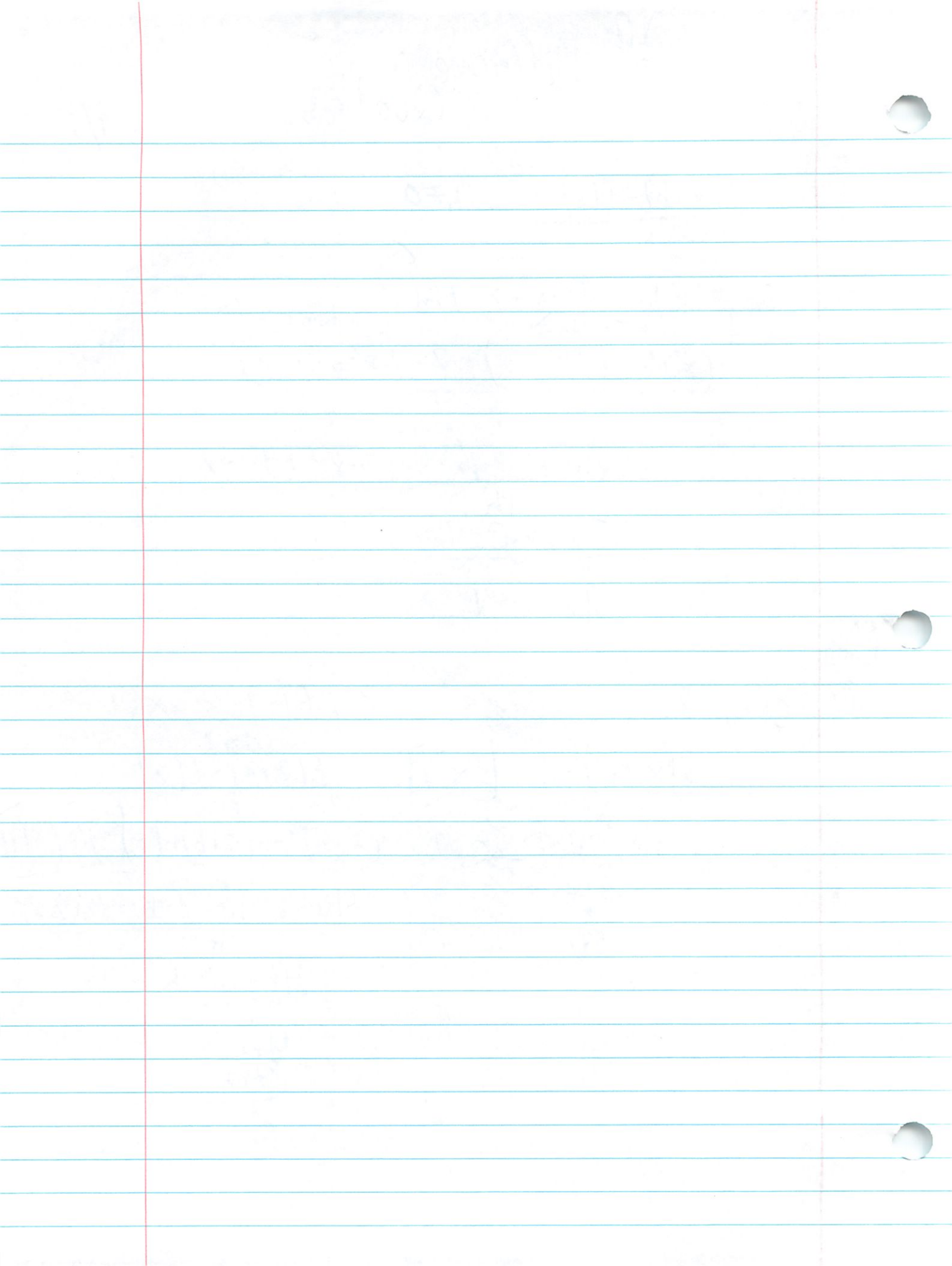
$$\frac{4 + 4h + h^2 - 2 - h + 1 - 4 + 2 - 1}{h}$$

$$\frac{4h + h^2 - h}{h}$$

$$4 + h - 1$$

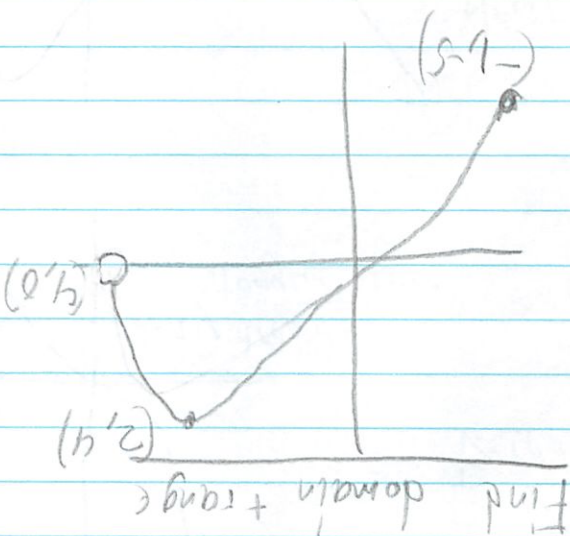
$$(3+h)$$

(✓)



u3 More Graphs of Functions

9/6



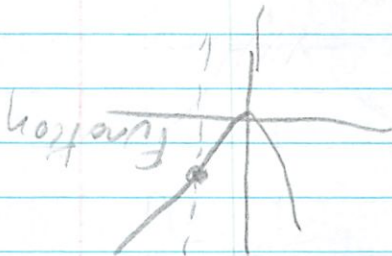
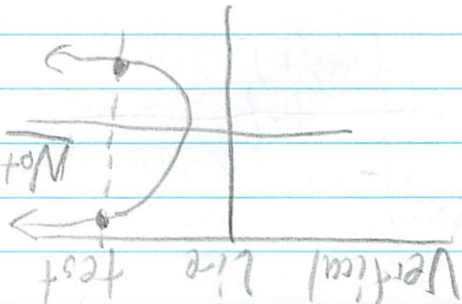
Find domain + range

Domain: $\{x: -1 \leq x < 4\}$
 Range: $\{y: -5 \leq y \leq 4\}$
 $f(-1) = -5$
 $f(2) = 4$

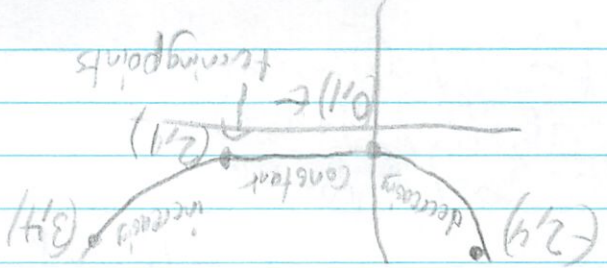
D: $\{x: x \geq 2\}$
 R: $\{y: y \geq 0\}$
 use "f"

D: $\{x: -2 \leq x \leq 2\}$
 R: $\{y: 0 \leq y \leq 2\}$

$h(t) = \sqrt{4-t^2}$
 $y^2 = 4 - t^2$
 $t^2 + t^2 = 4$
 also



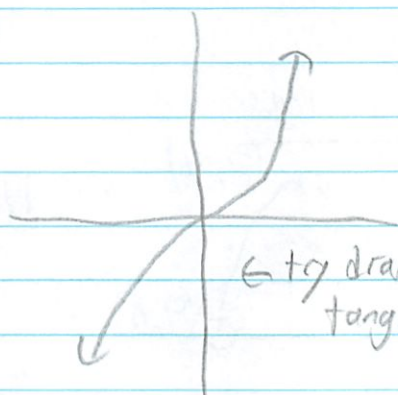
Increasing + decreasing functions



Decreasing: $(-2, 4)$
 constant: $(2, 1)$
 increasing: $(2, 3)$
 turning points

ex

$$f(x) = x^3$$

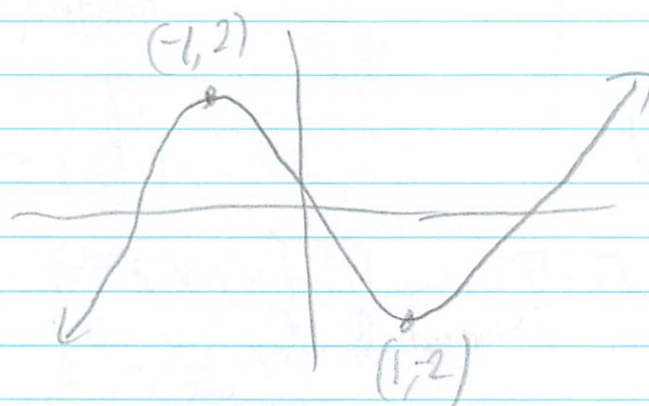


x values

increasing $(-\infty, \infty)$

← try drawing tangent lines

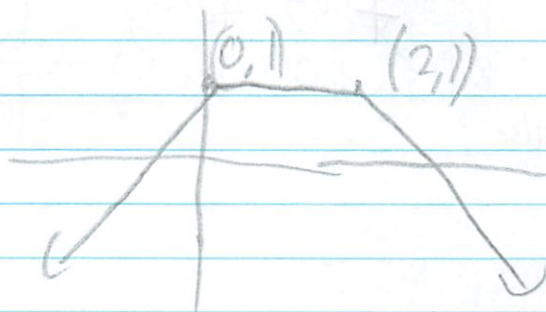
ex2



decreasing $(-1, 1)$

increasing $(-\infty, -1)$
 $(1, \infty)$

ex3

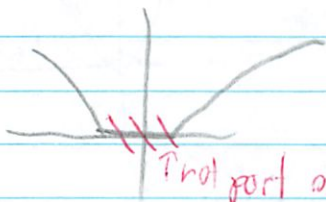


increasing $(-\infty, 0)$
constant $(0, 2)$
decreasing $(2, \infty)$

p39

20.

$$\sqrt{x^2 - 1}$$



not part of graph

u thought error

decreasing $(-\infty, -1)$
~~constant $(-1, 1)$~~
increase $(1, \infty)$

28.

$$f(x) = -|x+4| - |x+1|$$



increasing $(-\infty, -4)$
constant $(-4, -1)$
decreasing $(-1, \infty)$

1.3 Graphs of Functions

H/W

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p29

86. Find difference quotient
 $x^3 + x$ with $\frac{f(x+h) - f(x)}{h}$

$$\frac{(x+h)^3 + (x+h) - [x^3 + x]}{h}$$

$(x+h)^2$
 square out
 then do it
 again

$$\frac{(x+h)(x+h)(x+h) + (x+h) - x^3 - x}{h}$$

$$\rightarrow \frac{1st \quad 2nd \quad h}{3(x^2 + xh + x^2 + xh + xh + h^2 + xh + h^2) + x + h - x^3 - x}$$

$$\frac{6x^2 + 6h^2 + 12xh + x + h - x^3 - x}{h}$$

$$\frac{(x^2 + 2xh + h^2)(x+h) + (x+h) - x^3 - x}{h}$$

$$\frac{x^3 + 2x^2h + h^2x + x^2h + 2xh^2 + h^3 + x + h - x^3 - x}{h}$$

$$\frac{3xh^2 + 3x^2h + h + h^3}{h}$$

$$\frac{3xh + 3x^2 + 1 + h^2}{h}$$

38. Find domain + range + $f(0)$

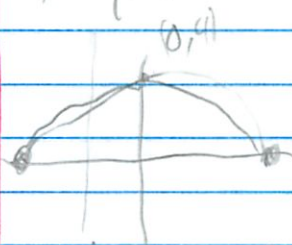
1.



$$D: \{x: x \in \mathbb{R}\} \quad f(0) = (0, 1)$$

$$R: \{y: y \leq 1\}$$

3.

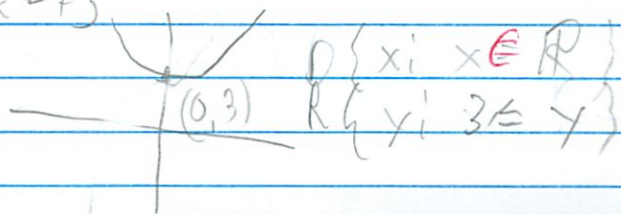


$$D: \{x: -4 \leq x \leq 4\} \quad f(0) = (0, 4)$$

$$R: \{y: 0 \leq y \leq 4\}$$

5.

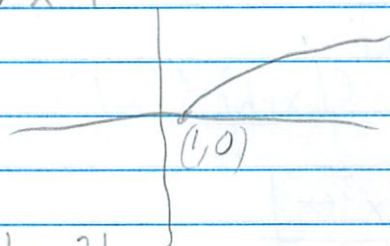
$$f(x) = 2x^2 + 3$$



$$D: \{x: x \in \mathbb{R}\}$$

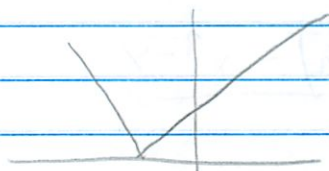
$$R: \{y: 3 \leq y\}$$

7. $f(x) = \sqrt{x-1}$



$D: \{x; x \geq 1\}$
 $R: \{y; y \geq 0\}$

9. $f(x) = |x+3|$

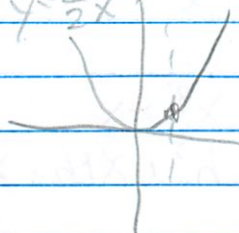


$D: \{x; x \in \mathbb{R}\}$
 $R: \{y; y \geq 0\}$

Vertical Line Test: Is y a $f(x)$?

11.

$y = \frac{1}{2}x^2$



Yes parabolas are functions

13.

$x - y^2 = 1$



No, the vertical line touches 2x
 To know algebraically - try and solve for y

15.

$x^2 = 2xy - 1$



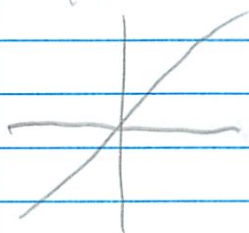
Yes, the vertical line only touches 1 place

32

17.

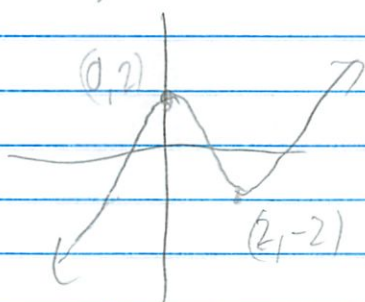
Increasing, Decreasing, Constant?

$f(x) = \frac{3}{2}x$



x values
 Increasing $(-\infty, \infty)$

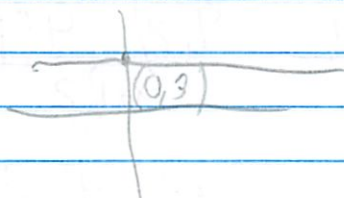
19. $f(x) = x^3 - 3x^2 + 2$



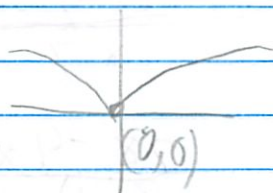
Increasing $(-\infty, 0)$
 Decreasing $(0, 2)$
 Increasing $(2, \infty)$

21. $f(x) = 3$

Constant $(-\infty, \infty)$

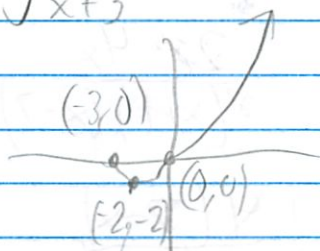


23. $f(x) = x^{2/3}$



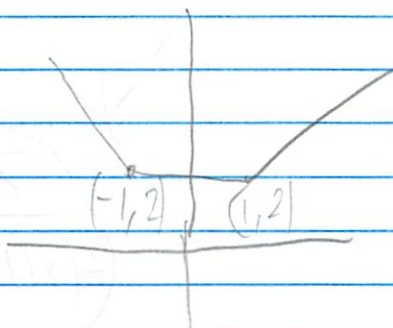
Decreasing $(-\infty, 0)$
 Increasing $(0, \infty)$

25. $f(x) = x\sqrt{x+3}$



Decreasing $(-3, -2)$
 Increasing $(-2, \infty)$

27. $f(x) = |x+1| + |x-1|$



Decreasing $(-\infty, -1)$
 Constant $(-1, 1)$
 Increasing $(1, \infty)$

29

88. $f(x) = \frac{4}{x+1}$, $\frac{f(x) - f(7)}{x-7}$

$$\frac{\frac{4}{x+1} - \frac{4}{7+1}}{x-7}$$

Common denominator $\rightarrow \frac{4}{x+1} - \frac{4}{8}$ $\cdot 4 \rightarrow \frac{4(8) - 4(1)}{(x+1)8}$

$$\frac{4(x-7)}{(x+1)(x-7)} = \frac{x-7}{2(x-7)}$$

$$\frac{32 - 4x - 4}{(x+1)8}$$

$$\frac{4x-28}{x^2-6x-7} = \frac{x-7}{2x-14}$$

$$\frac{28-4x}{(x+1)8} = \frac{1}{(x-7)}$$

$$\frac{28-4x}{8(x+1)(x-7)}$$

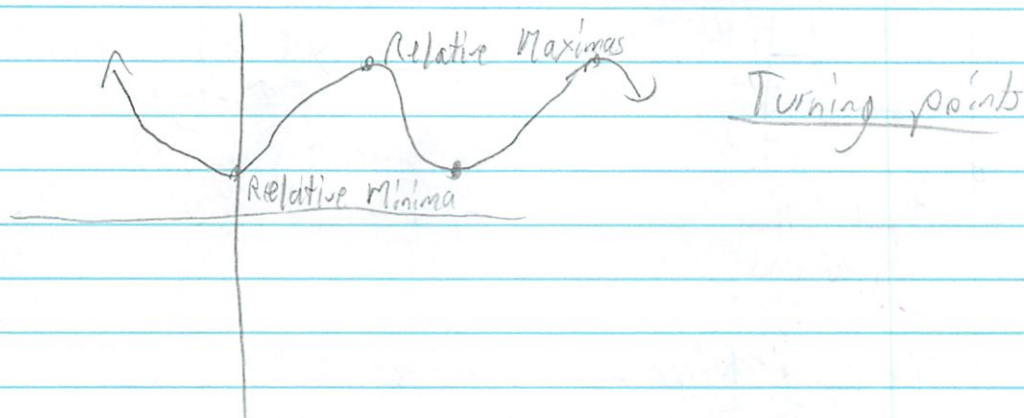
$$\frac{1 \cdot 4(7-x)}{2 \cdot 8 (x+1)(x-7)}$$

$$\frac{7-x}{2(x+1)(x-7)}$$

1.3 Graphs of Functions

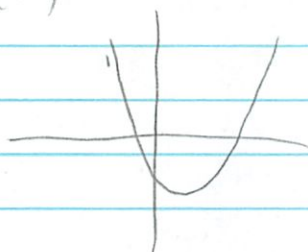
9/10

Relative Maxima + Relative Minima ↑ plural form →



ex1

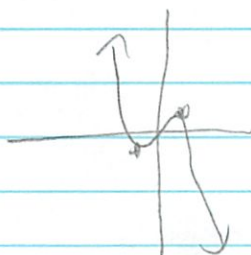
$$f(x) = 3x^2 - 4x - 2$$



Relative minimum = $(\frac{2}{3}, -\frac{10}{3})$

ex2

$$f(x) = -x^3 + x$$



Relative maximum $(\frac{1}{\sqrt{3}}, \frac{2\sqrt{3}}{27})$
" minimum $(-\frac{1}{\sqrt{3}}, -\frac{2\sqrt{3}}{27})$

ex3

Graph step functions
(greatest integers function)

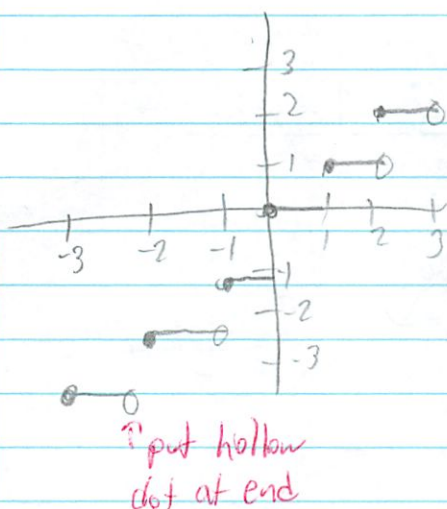
$f(x) = [x]$ Defined as the greatest integer $(\leq) x$

x	$[x]$
-3	-3
-2.7	-3
-2.1	-3

x	$[x]$
-2	-2
-1.5	-2
-1	-1

x	$[x]$
-1.5	-1
0	0
1.5	1

→



jumping to the integers

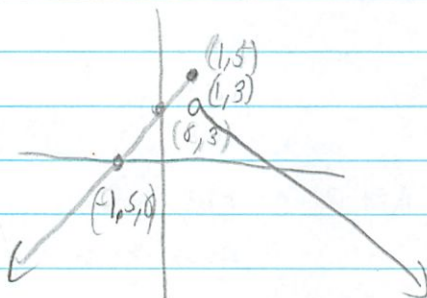
$$D\{x; x \in \mathbb{R}\}$$

$$R\{y; y \in \mathbb{Z}\}$$

x-intercept = $[0, 1)$
y-intercept $(0, 0)$

Graphing Piecewise Functions

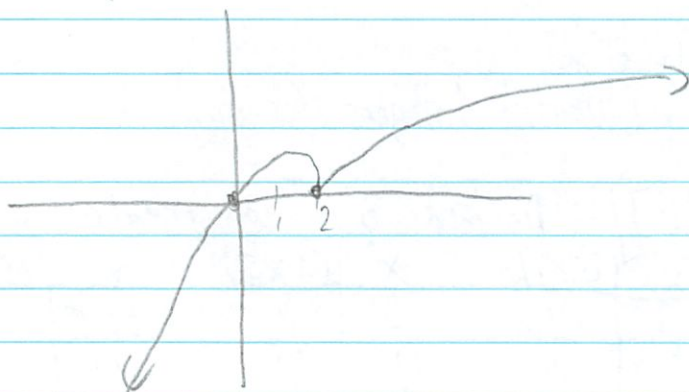
$$f(x) = \begin{cases} 2x+3, & x \leq 1 \\ -x+4, & x \geq 1 \end{cases}$$



239
44.

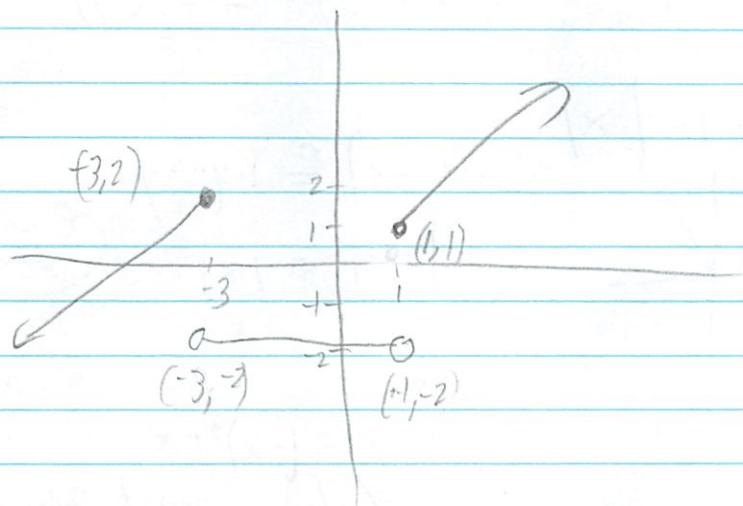
Graph

$$f(x) = \begin{cases} 1 - (x-1)^2, & x \leq 2 \\ \sqrt{x-2}, & x \geq 2 \end{cases}$$

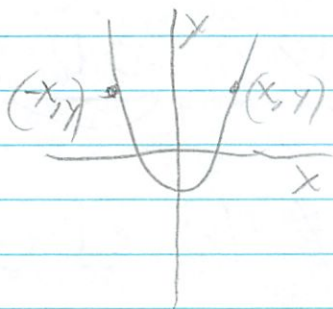


39

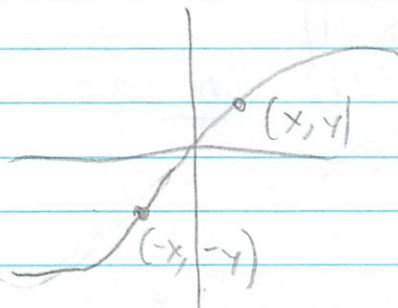
46 $g(x) = \begin{cases} x+5, & x < -3 \\ -2, & -3 < x < 1 \\ 5x-4, & x \geq 1 \end{cases}$



Symmetry of Functions
even + odd functions



Symmetrical to y-axis = even function



Symmetrical to the origin = odd function



Algebraic tests for even or odd functions

Even function $\rightarrow f(-x) = f(x)$

Odd function $\rightarrow f(-x) = -(f(x))$

ex1

$$f(x) = |x|$$



$$f(x) \stackrel{?}{=} f(-x)$$

$$|-x| = |x|$$

even

ex2

$$g(x) = x^3 - x$$

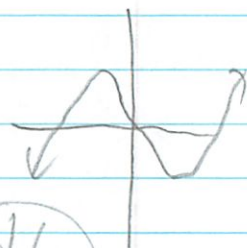
$$g(-x) = (-x)^3 + x$$

$\neq f(x)$ not even

$$(-x)^3 + x \stackrel{?}{=} -(x^3 - x)$$

$$-x^3 + x$$

odd



ex3

$$f(x) = x^3 - 1$$

$$f(-x) = (-x)^3 - 1$$

$$= -x^3 - 1 \neq f(x) \quad \text{not even}$$

$$-x^3 - 1 \stackrel{?}{=} -(x^3 - 1)$$

$$-x^3 - 1 \stackrel{?}{=} -x^3 + 1$$

not odd

neither

p39

Even, odd, neither?

54 52.

$$h(x) = x^3 - 5$$

$$f(-x) = (-x)^3 - 5$$

$$= -x^3 - 5 \neq f(x) \quad \text{not even}$$

$$-x^3 - 5 \stackrel{?}{=} -(x^3 - 5)$$

$$-x^3 - 5 \stackrel{?}{=} -x^3 + 5 \quad \text{not odd}$$

neither

p39

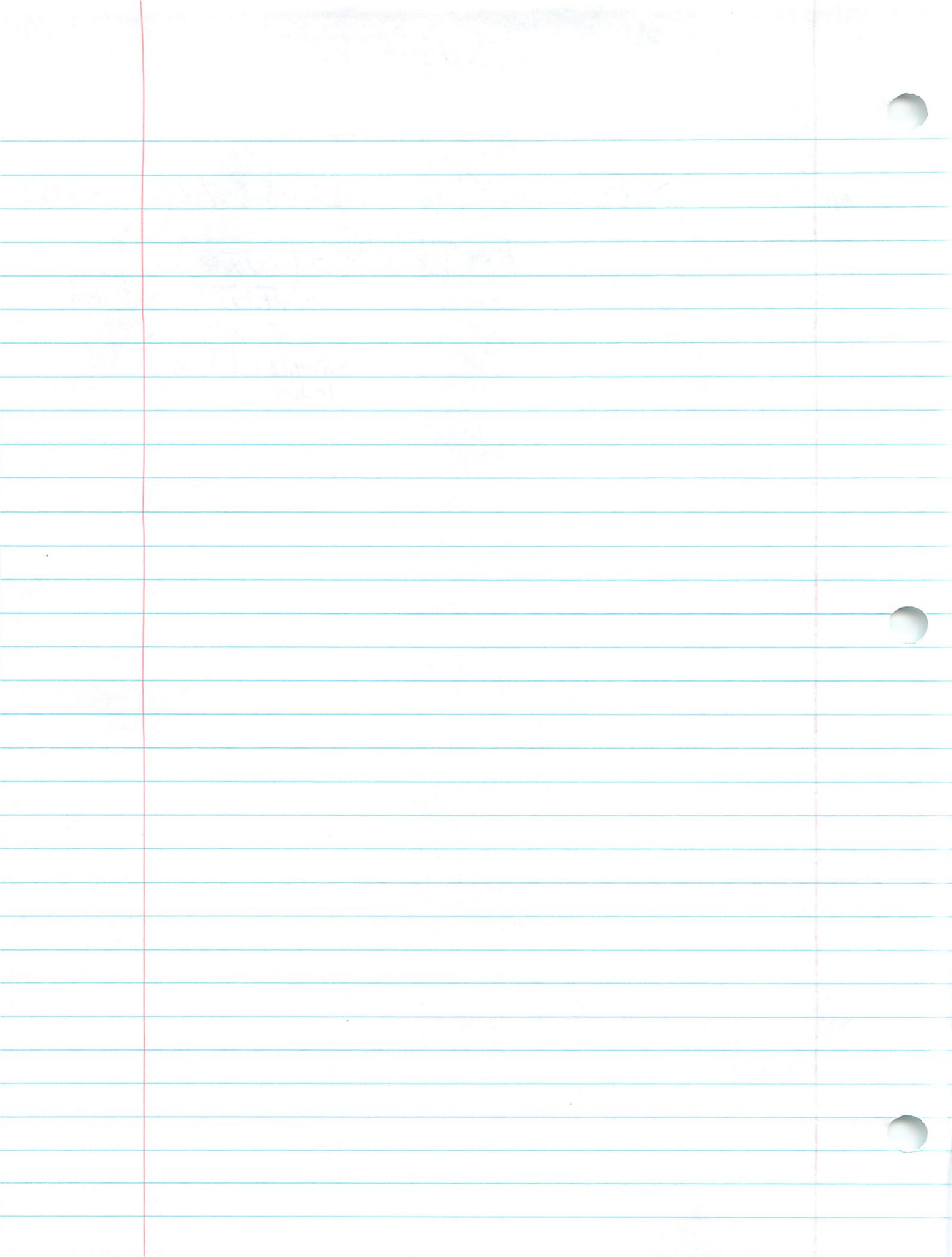
59. $f(x) = x\sqrt{x+5}$ $f(-x) = (-x)\sqrt{(-x)+5} \neq f(x)$ not even

$$\begin{aligned} f(x)\sqrt{(-x)+5} &= -(x\sqrt{x+5}) \\ &= -x\sqrt{x+5} \end{aligned}$$

not odd

don't distribute to here

neither



1.3 Graphs of Functions

H/W

9/10

p39
33.

Relative minimum?

$$h(x) = (x-1)\sqrt{x}$$

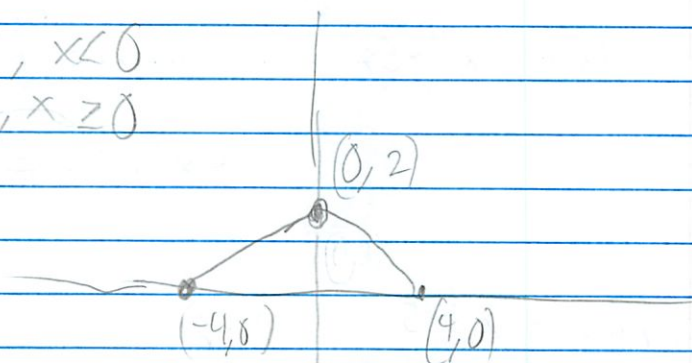


Relative minimum: $(0,0)$

Sketch

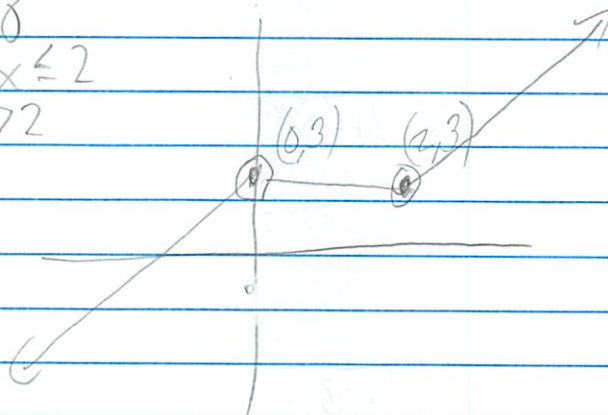
43.

$$f(x) = \begin{cases} \sqrt{4+x}, & x < 0 \\ \sqrt{4-x}, & x \geq 0 \end{cases}$$



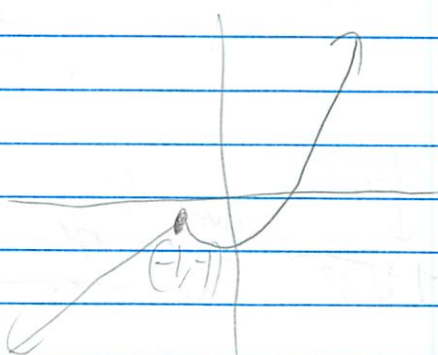
45.

$$f(x) = \begin{cases} x+3, & x \leq 0 \\ 3, & 0 < x \leq 2 \\ 2x-1, & x > 2 \end{cases}$$



47.

$$f(x) = \begin{cases} 2x+1, & x \leq -1 \\ x^2-2, & x > -1 \end{cases}$$



odd, even or neither

49. $f(t) = t^2 + 2t - 3$ $f(-t) = (-t)^2 + 2(-t) - 3$
 $\downarrow$ $t^2 - 2t - 3$ not even neither
 $(-t)^2 + 2(-t) - 3 \stackrel{?}{=} -(t^2 + 2t - 3)$
 $\stackrel{?}{=} -t^2 - 2t + 3$ not odd

50. $f(x) = x^6 - 2x^2 + 3$ $f(-x) = (-x)^6 - 2(-x)^2 + 3$
 $x^6 - 2x^2 + 3$ even

51. $g(x) = x^3 - 5x$ $g(-x) = (-x)^3 - 5(-x)$
 $\downarrow$ $-x^3 + 5x$ not even
 $-x^3 + 5x \stackrel{?}{=} -(x^3 - 5x)$
 $-x^3 + 5x$ odd

52. $h(x) = x^3 - 5$ $h(-x) = (-x)^3 - 5$
 $\downarrow$ $-x^3 - 5$ not even
 $-x^3 - 5 \stackrel{?}{=} -(x^3 - 5)$ neither
 $-x^3 + 5$ not odd

53. $f(x) = x\sqrt{1-x^2}$ $f(-x) = (-x)\sqrt{1-(-x)^2}$
 $\downarrow$ $-x\sqrt{1-x^2}$ not even
 $-x\sqrt{1-x^2} \stackrel{?}{=} -(x\sqrt{1-x^2})$
 $-x\sqrt{1-x^2}$ odd

54. $f(x) = x\sqrt{x+5}$ $f(-x) = (-x)\sqrt{-x+5}$
 $\downarrow$ $-x\sqrt{-x+5}$ not even
 $-x\sqrt{-x+5} \stackrel{?}{=} -(x\sqrt{x+5})$ neither
 $-x\sqrt{x+5}$ not odd

55. $g(s) = 4s^{2/3}$ $g(-s) = 4(-s)^{2/3}$
 $\downarrow$ $4s^{2/3}$ convert
 $4\sqrt[3]{s^2}$ $\downarrow$ $4(-s)^{2/3}$ not even $\rightarrow 4\sqrt[3]{s^2}$
 $4\sqrt[3]{s^2} \stackrel{?}{=} -(4\sqrt[3]{s^2})$ neither

p39

56

$$f(s) = 4s^{2/3}$$

↓ convert
 $4\sqrt[3]{s^2}$

$$f(-s) = 4(-s)^{2/3}$$

$$-f(s) = -4s^{2/3}$$

$$\rightarrow 4\sqrt[3]{(-s)^2} \rightarrow \sqrt[3]{(-s)^2} = \text{! unreal}$$

not even

neither

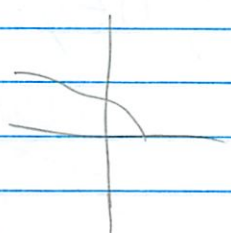
~~odd~~

not

p40

69.

$$A(x) = \sqrt{1-x}$$

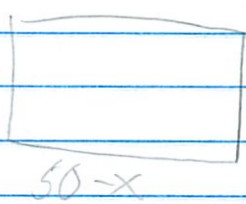


neither

79.

$$P_{\square} = 100m$$

a) $A = x(50-x)$



b) Graph



c) Maximum

$$x = 25m \text{ length}$$

$$y = 625m^2 \text{ area}$$

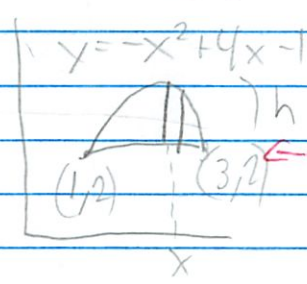
83

Write $h_{\square}(x)$

Subtract 2 boundaries (top - bottom)

$$y = (-x^2 + 4x - 1) - 2$$

$$-x^2 + 4x - 3$$



$$y = 2$$



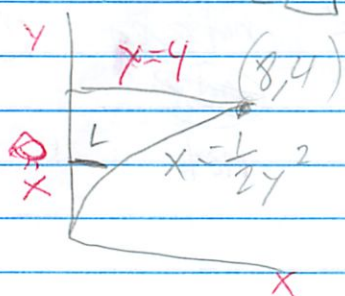
Restriction

$$h = -x^2 + 4x - 3, 1 \leq x \leq 3$$

* Formula for getting height at each x *

85.

Write $L(x)$



Right boundary - left boundary

$$x = \left(\frac{1}{2}y^2\right) - 0$$

↓ restriction

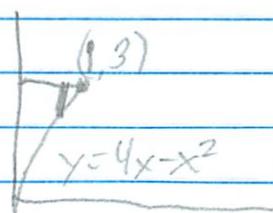
$$L = \frac{1}{2}y^2, \quad 0 \leq y \leq 4$$

p40

84.

$$h(x) = f(x)$$

top - bottom



$$y = 3 - (4x - x^2)$$

$$y = 3 - 4x + x^2$$

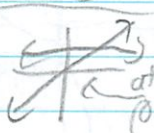
$$y = x^2 - 4x + 3$$

$$0 \leq x \leq 1$$

1.4 Shifting, Reflecting, Stretching Graphs

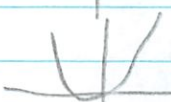
9/11

Families of functions

- linear  $f(x) = c$ $\hookrightarrow$ constant function
 $f(x) = x$ $\hookrightarrow$ identity function

- absolute value  $f(x) = |x|$

- square root  $f(x) = \sqrt{x}$

- quadratic  $f(x) = x^2$

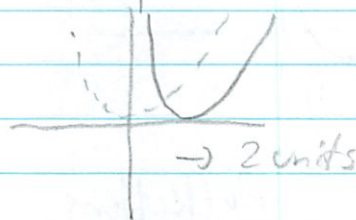
- cubic  $f(x) = x^3$

Vertical + Horizontal Shifts (Transformation)

$$h(x) = x^2 + 2$$



$$g(x) = (x-2)^2$$



1. Vertical shift $\uparrow$

$$h(x) = f(x) + c$$

2. Vertical shift $\downarrow$

$$h(x) = f(x) - c$$

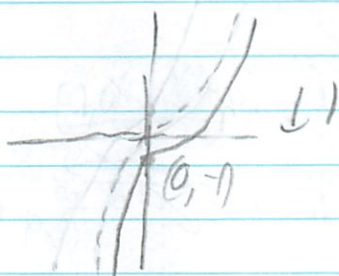
3. Shift $\rightarrow$

$$h(x) = f(x - c)$$

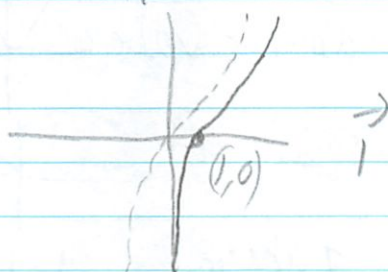
4. Shift $\leftarrow$

$$h(x) = f(x + c)$$

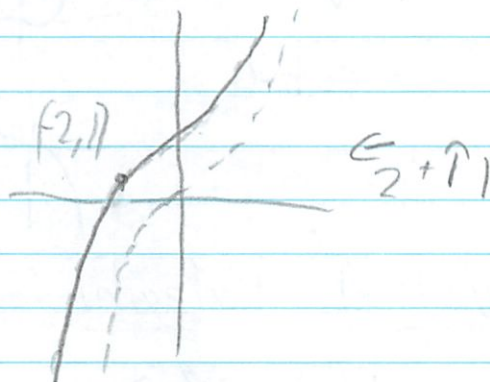
P1 $g(x) = x^3 - 1$



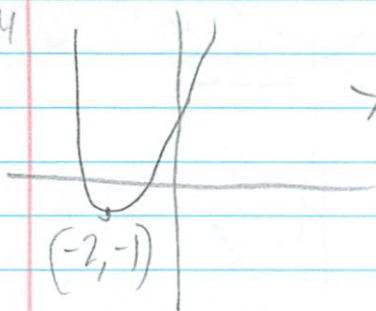
P2 $h(x) = (x-1)^3$



P3 $k(x) = (x+2)^3 + 1$



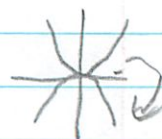
P4



$y = (x+2)^2 - 1$

Reflections

around x or y axis



$y = x^2 \rightarrow$ reflected around x axis $\rightarrow y = -x^2$

1. Reflection around x axis

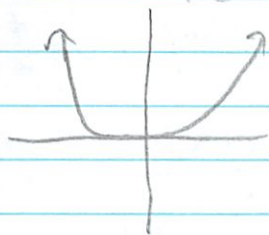
$h(x) = -f(x)$

2. Reflection around y axis

$h(x) = f(-x)$



$f(x) = x^4 \rightarrow$ From the quadratic family



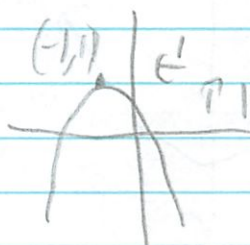
p48
22

Write equation



$y = |x|$ - absolute value
 $y = |x - 4| - 3$

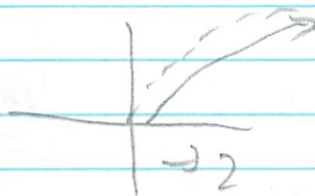
24



$y = x^2 \rightarrow$ quadratic
 $y = (x + 1)^2 + 1$

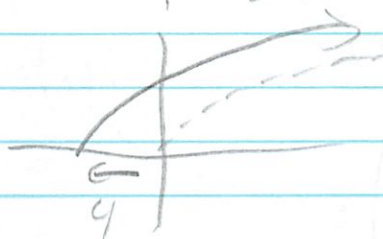
29

$y = \sqrt{x - 2}$

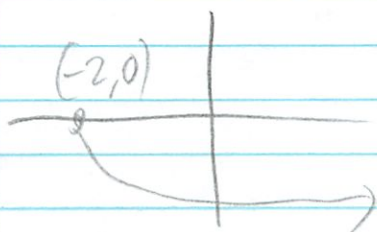


30

$y = \sqrt{x + 4}$



p1



$y = -\sqrt{x + 2}$



Rigid Transformation

- Don't Δ shape, just location

Non Rigid Transformation

- Cause a distortion of the graph

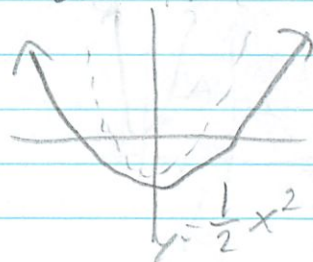
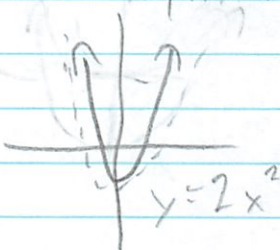
Vertical stretches + Vertical shrinks

$c > 1$ (thinner)

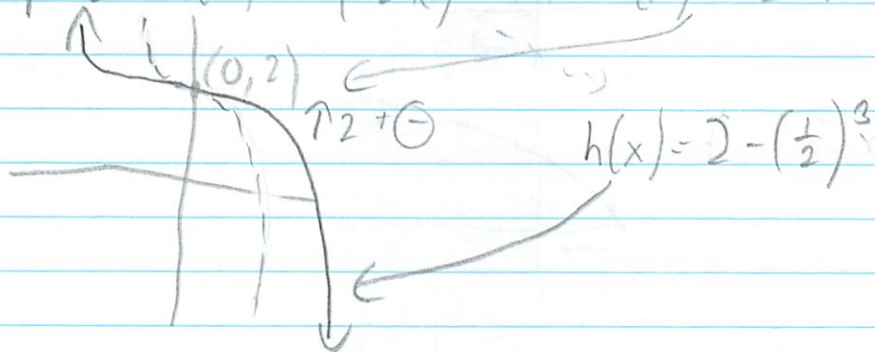
$0 < c < 1$ (fatter)

$$y = c \cdot f(x)$$

$$\text{--- } y = x^2$$



ex Compare $h(x) = f\left(\frac{1}{2}x\right)$ with $f(x) = 2 - x^3$



1.4 Shifting, Reflecting, & Sketching Graphs

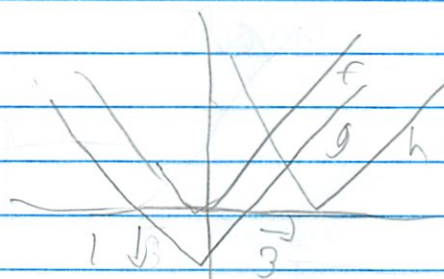
H/W

9/11

p48
9.

Sketch on same graph

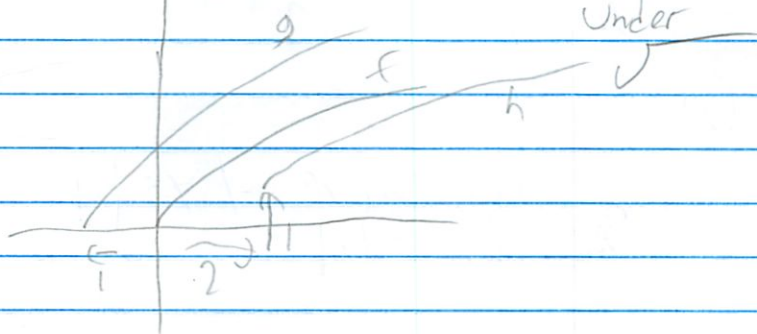
$$\begin{aligned} f(x) &= |x| \\ g(x) &= |x| - 1 \\ h(x) &= |x - 3| \end{aligned}$$



⊖ →
⊕ ∈
except
under

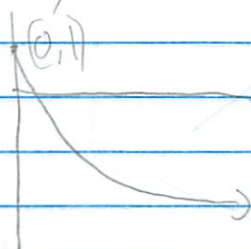
11.

$$\begin{aligned} f(x) &= \sqrt{x} \\ g(x) &= \sqrt{x+1} \\ h(x) &= \sqrt{x-2} + 1 \end{aligned}$$



p49
19.

Identify transformation

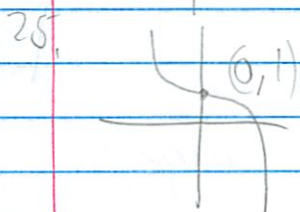


$$y = -\sqrt{x+1} + 1$$

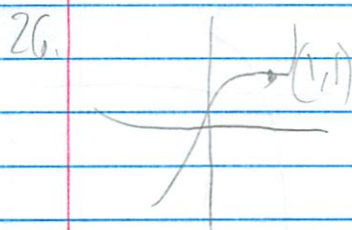
Ⓟ



$$y = -\sqrt{x+3}$$



$$y = -x^3 + 1$$



$$y = (x-1)^3 + 1$$

Graph +

Compare with $y = \sqrt{x}$

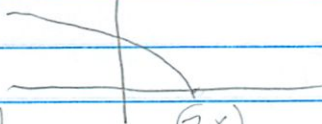
31,

$$y = \sqrt{2x}$$



32,

$$y = \sqrt{-x+3}$$

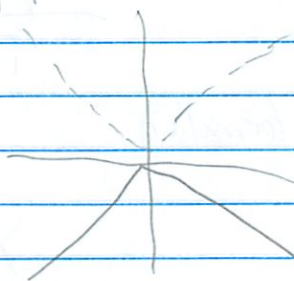


(J)

35,

Graph + Compare w/ $y = |x|$
 $y = -|x|$

(3,0)



38,

$$y = |\frac{1}{2}x|$$



39,

Graph + Compare w/ $y =$
 $y = 4 - x^3$

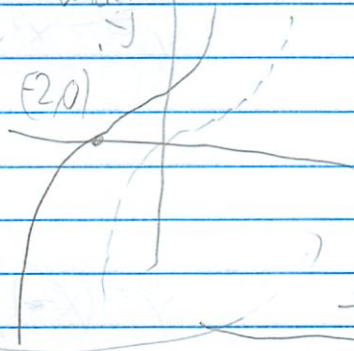


41,

$$h(x) = \frac{1}{4}(x+2)^3$$

wider

(2,0)

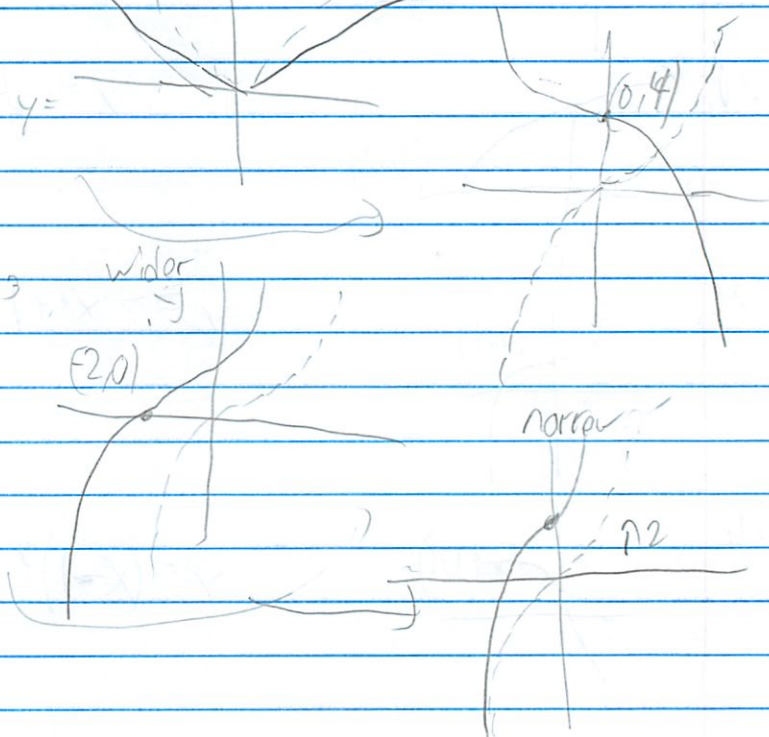


43,

$$p(x) = (\frac{1}{3}x)^3 + 2$$

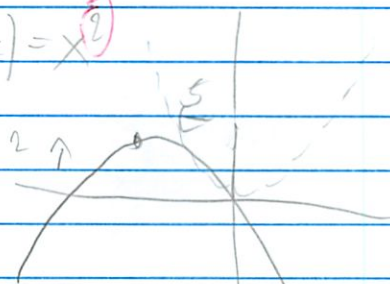
narrower

+2



51. $g(x) = 2 - (x+5)^2 \rightarrow f(x) = x^2$

$g(x) = 2 - f(x+5)$



53. $g(x) = 3 + 2(x-4)^2 \rightarrow$

$f(x) = x^2$
 $g(x) = 3 + f(x-4)$



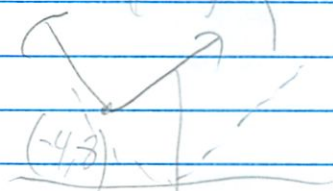
58. $g(x) = -(x+3)^3 - 10$

$f(x) = x^3$
 $g(x) = -f(x+3) - 10$



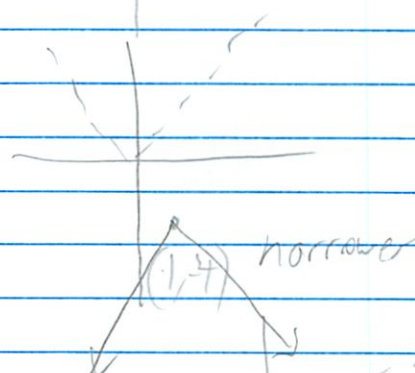
59. $g(x) = |x+4| + 8$

$f(x) = |x|$
 $g(x) = f(x+4) + 8$



61. $g(x) = -2|x-1| - 4$

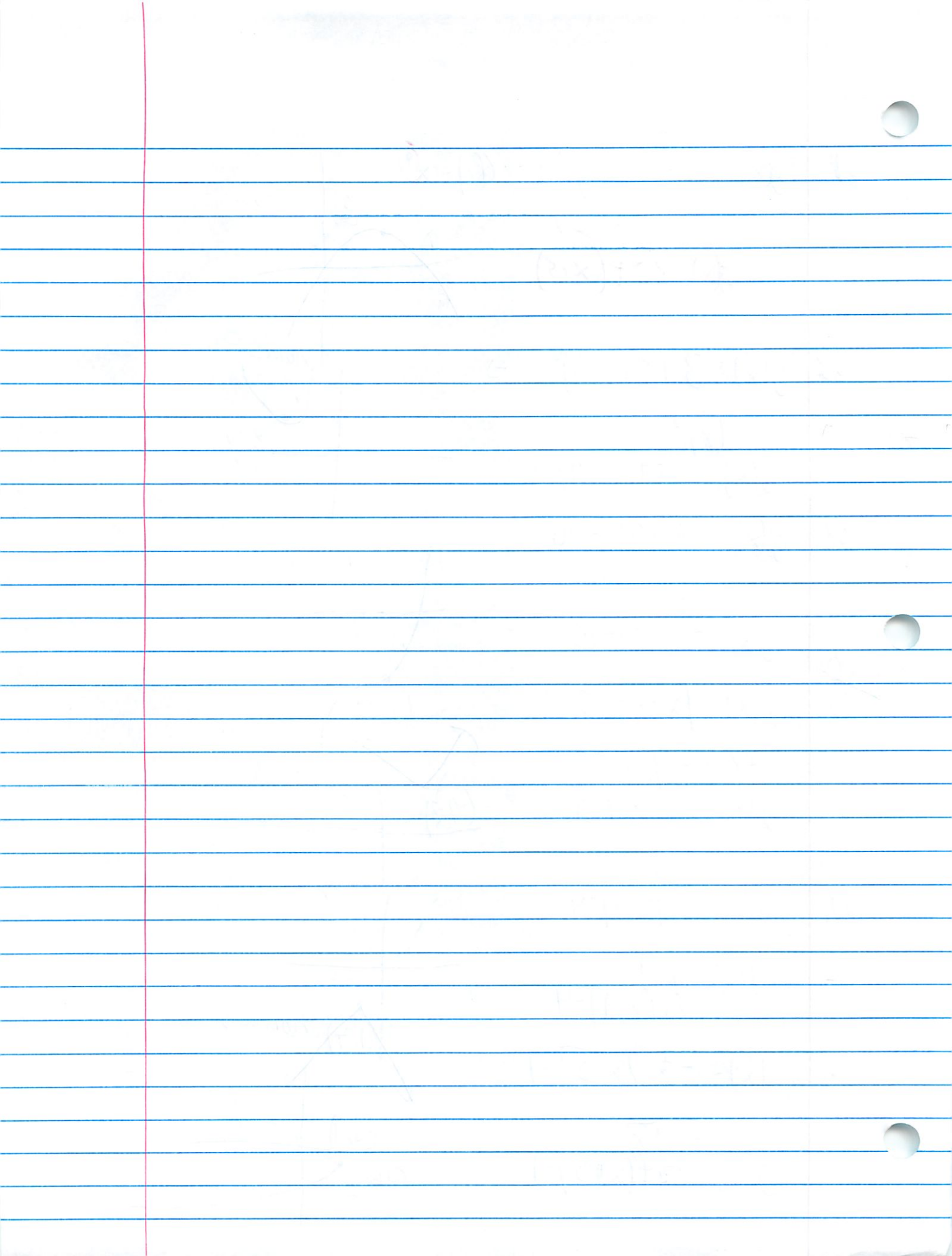
$f(x) = |x|$
 $g(x) = -2f(x-1) - 4$



63. $g(x) = -\frac{1}{2}\sqrt{x+3} - 1$

$f(x) = \sqrt{x}$
 $g(x) = -\frac{1}{2}f(x+3) - 1$





Domain of $f \circ g$: $D_f = \{x; x \geq 0\}$
 $D_g = \{x; x \in \mathbb{R}\}$

look at $D_{f \circ g} = \{x; x \geq 1\}$
 resulting function

ex3 $f(x) = x+2$ Find g of f , eval. at 2

$g(x) = 4-x^2$

$g(f(x)) = g(x+2) = 4 - (x+2)^2$

$4 - ([2]+2)^2$

$4 - 16$

(-12)

ex4 $f(x) = \frac{1}{x}$ $D = \{x; x \neq 0\}$

$g(x) = x+1$ $D = \{x; x \in \mathbb{R}\}$

Domain $f \circ g$?

$f(g(x)) = f(x+1) = \frac{1}{x+1}$ $D: \{x; x \neq -1\}$

ex5 $f(x) = x^2 - 9$ $D = \{x; x \in \mathbb{R}\}$

$g(x) = \sqrt{9-x^2}$ $D = \{x; -3 \leq x \leq 3\}$

$f \circ g = f(\sqrt{9-x^2}) \rightarrow \frac{(\sqrt{9-x^2})^2 - 9}{9-x^2-9}$ $D_{f \circ g} \{x; -3 \leq x \leq 3\}$
 $-x^2$

ex6 $f(x) = \sqrt{x+4}$ $D: \{x; x \geq -4\}$

$g(x) = x^2$ $D: \{x; x \in \mathbb{R}\}$ $g \circ f = \sqrt{x+4}^2$

$(x+4)$

$f \circ g = \sqrt{(x^2)+4}$

$D: \{x; x \geq -4\}$

$D: \{x; x \in \mathbb{R}\}$

ex7 $f \circ g(x) = g \circ f(x)$ $f(x) = 2x+3$ $g(x) = \frac{1}{2}(x-3)$ $\downarrow$ inverse

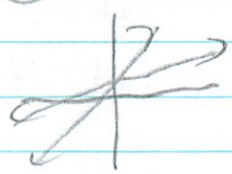
$$f \circ g = 2\left(\frac{1}{2}(x-3)\right) + 3 \quad g \circ f = \frac{1}{2}(2x+3-3)$$

(X)

(X)

f & g are
inverses of
each other

identity
function



1.5 Combinations of Functions

9/12

2 functions can be combined to create new functions

$$f(x) = 2x - 3 \quad g(x) = x^2 - 1$$

$$f(x) + g(x) = (2x - 3) + (x^2 - 1)$$
$$x^2 + 2x - 4$$

$$f(x) - g(x) = (2x - 3) - (x^2 - 1)$$
$$-x^2 + 2x - 2$$

$$f(x) \cdot g(x) = (2x - 3)(x^2 - 1) \quad \text{Foil}$$
$$2x^3 - 3x^2 - 2x + 3$$

$$\frac{f(x)}{g(x)} = \frac{2x - 3}{x^2 - 1} \rightarrow x \neq -1, 1$$

Domain of arithmetic combination of a functions is all real numbers that are common to both $f+g$.

$$f(x) \rightarrow D: \{x: x \in \mathbb{R}\}$$
$$g(x) \rightarrow D: \{x: x \in \mathbb{R}\} \quad \text{equivalent not necessarily this}$$

ex) $f(x) = x^2 + 1 + g(x) = x - 4$

$$x=3: \quad \frac{(3)^2 + 1}{10} + \frac{(3 - 4)}{-1}$$

$$D: \{f+g(x): f+g(x) \in \mathbb{R}\}$$

(9)

$$f-g(x) \quad (0)^2 + 1 - [(0) - 4]$$
$$x=0: \quad 1 - -4$$

(5)

ex2 $\rightarrow f(x) = \frac{1}{x}$ $g(x) = \sqrt{x}$
 $D = \{x: x \neq 0\}$ $D = \{x: x \geq 0\}$

Domain of $(f+g)(x) \rightarrow \frac{1}{x} + \sqrt{x}$
 $= \{x: x > 0\}$ or $(0, \infty)$

ex3 Finding the quotient

$f(x) = \sqrt{x}$ $g(x) = \sqrt{4-x^2}$
 $D = \{x: x \geq 0\}$ $D = \{x: -2 \leq x \leq 2\}$

Can graph to ✓

Domain of $\frac{f}{g}(x)$
 $\{x: 0 \leq x < 2\}$

$\frac{\sqrt{x}}{\sqrt{4-x^2}}$

walk
cause denom
to be 0

$\frac{g}{f}(x)$

$\{x: 0 < x \leq 2\}$

$\frac{\sqrt{4-x^2}}{\sqrt{x}}$

$\sqrt{x} \in \text{can't be 0}$

Composition of Functions

Plug 1 function in for another

ex1

$f(x) = x^2$ $g(x) = x+1$

$f(g(x)) = f(x+1) = (x+1)^2$
 $x^2 + 2x + 1$
 $f \circ g = f \text{ of } g$

ex2

find $f \circ g(x)$ for $f(x) = \sqrt{x}$
 $g(x) = x-1$

$f(g(x)) = f(x-1) = \sqrt{x-1}$ Find $f \circ g(2)$
 $\rightarrow \sqrt{2-1} \rightarrow \sqrt{1} \rightarrow 1$

$f \circ g(0) \rightarrow \sqrt{0-1} \rightarrow \sqrt{-1} \rightarrow \text{unreal}$

Decomposition of Functions

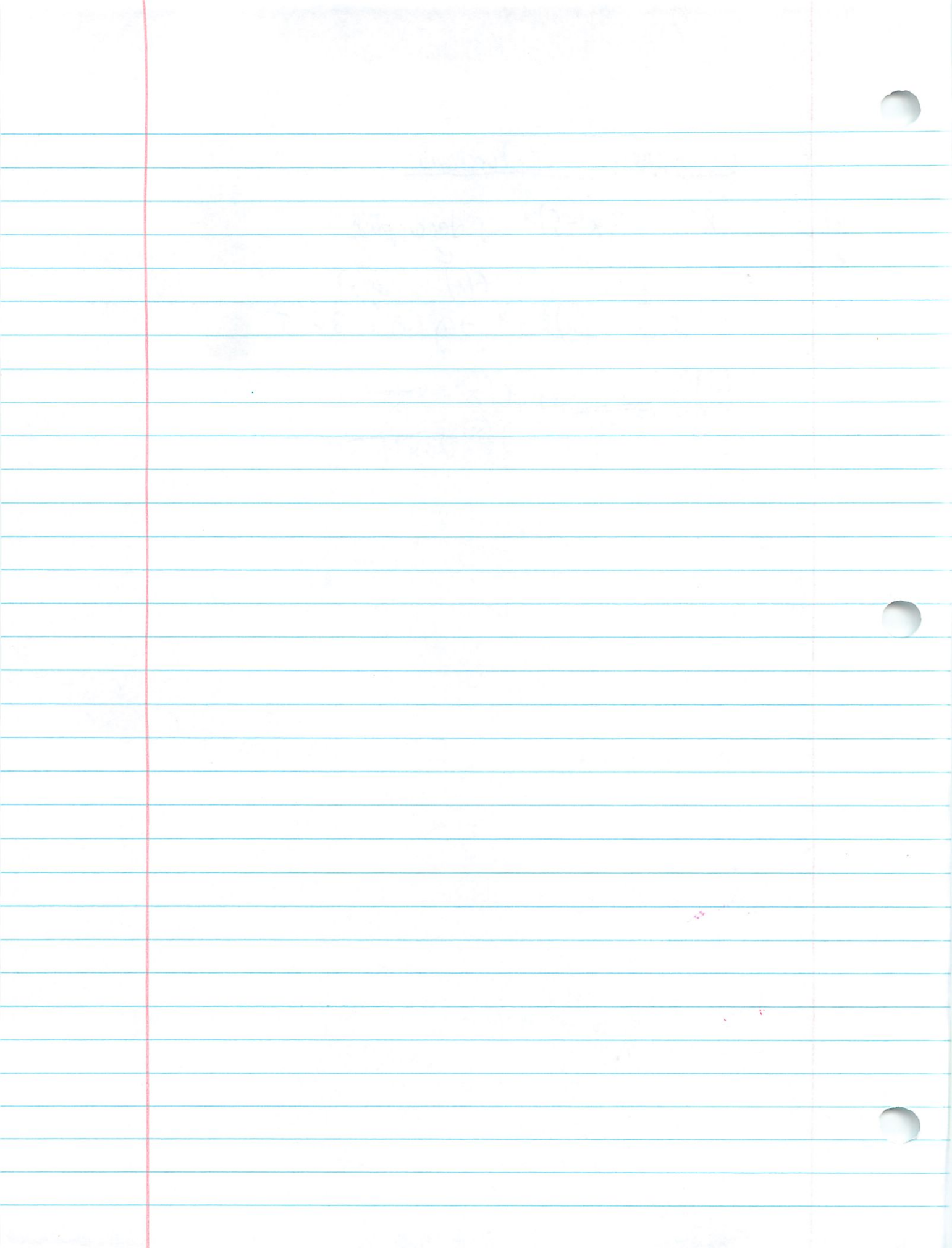
ex1

$$h(x) = (3x-5)^3 \rightarrow \text{decompose}$$

$$\begin{array}{ccc} & \nwarrow & \downarrow \\ & f(x) & g(x) \\ \text{say } f(x) = x^3 & \rightarrow & g(x) = 3x-5 \end{array}$$

ex2

$$h(x) = \frac{1}{\sqrt{3x+1}} \rightarrow \begin{array}{l} f(x) = \frac{1}{x} \\ g(x) = \sqrt{3x+1} \end{array}$$



1.5 Combinations of Functions

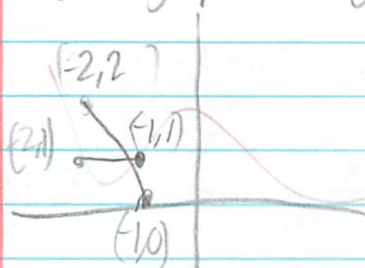
H/W

9/12

P 58

2.

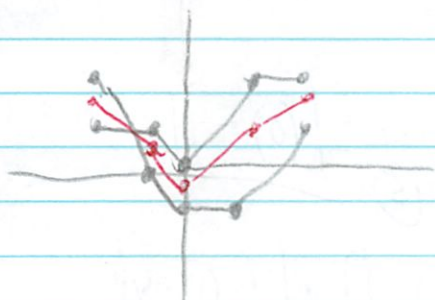
Use graphs to graph $f+g(x)$



$$\frac{2-0}{-2-1} = \frac{2}{-1} = -2$$

average visually

2.



9. Find $(f+g)$, $(f-g)$, (fg) (f/g) & Domain?

$$f(x) = x^2 + 5$$

$$g(x) = \sqrt{1-x}$$

$$f+g = (x^2+5) + (\sqrt{1-x})$$

$$f-g = (x^2+5) - (\sqrt{1-x})$$

$$fg = (x^2+5)(\sqrt{1-x})$$

~~x^2+5~~ leave it as is

$$\frac{f}{g} = \frac{x^2+5}{\sqrt{1-x}} \quad x > 1$$

10.

$$f(x) = \sqrt{x^2-4}$$

$$g(x) = \frac{x^2}{x^2+1}$$

$$f+g = \sqrt{x^2-4} + \frac{x^2}{x^2+1}$$

$$f-g = \sqrt{x^2-4} - \frac{x^2}{x^2+1}$$

$$fg = \sqrt{x^2-4} \cdot \frac{x^2}{x^2+1}$$

$$\frac{f}{g} = \frac{\sqrt{x^2-4}}{\frac{x^2}{x^2+1}} = \frac{\sqrt{x^2-4} \cdot x^2+1}{x^2}$$

Find domain of each input

Domain

look what's common
 $\{x \mid x \neq 0, x \geq 2 \text{ and } x \leq 2\}$

$$\frac{(x^2+1)\sqrt{x^2-4}}{x^2} \quad x \in \mathbb{R}$$

12. $f(x) = \frac{x}{x+1}$

$f+g = x^3 + \frac{x}{x+1}$

$g(x) = x^3$

$f-g = \frac{x}{x+1} - x^3$

$f \cdot g = \frac{x}{x+1} \cdot \frac{x^3}{1} \rightarrow \frac{x^4}{x+1}$

Domain

$D = \{x: x \neq -1\}$

$f/g = \frac{\frac{x}{x+1}}{x^3} = \frac{x}{x+1} \cdot \frac{1}{x^3} = \frac{x}{x^3(x+1)}$

15. $f(x) = x^2 + 1$ $f \circ g(0) = (0)^2 + 1 = [(0) - 4]$
 $g(x) = x - 4$ (5)

18. " $f \circ g(-6) = [(-6) + 1] \cdot [(-6) - 4]$
 $-5 \cdot -10 = (+50)$

p52

36. $f(x) = \sqrt[3]{x-1}$
 $g(x) = x^3 + 1$

$f \circ g = \sqrt[3]{(x^3+1)-1} = \sqrt[3]{x^3}$ $g \circ f = (\sqrt[3]{x-1})^3 + 1$
 $(\times)$ $(\times)$

$f \circ g(0) = \sqrt[3]{(0^3+1)-1} = \sqrt[3]{0} = 0$
 $(\odot)$

39. $f(x) = \sqrt{x+4}$
 $g(x) = x^2$

$f \circ g = \sqrt{(x^2)+4}$

$g \circ f = (\sqrt{x+4})^2$
 $(x+4)$

$D = \{x: x \geq -2\}$

$D = \{x: x \geq -4\}$

$\sqrt{0}$ is allowed

not equal



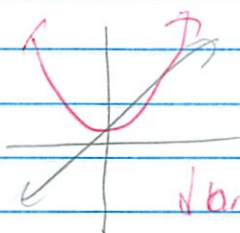
Domain

p59

43. $f(x) = x^{2/3}$
 $g(x) = x^6$

$f \circ g = (x^6)^{2/3}$
 $D_f\{x: x \in \mathbb{R}\}$

$g \circ f = (x^{2/3})^6$
 $D_g\{x: x \in \mathbb{R}\}$



Are they same

Don't test w/ $1=x$ when dealing w/ exponents

47. $f(x) = \sqrt{x+6}$
 $g(x) = x^2 - 5$

$f \circ g = \sqrt{(x^2 - 5) + 6}$
 $\sqrt{x^2 + 1}$

$g \circ f = (\sqrt{x+6})^2 - 5$
 $x+6-5$
 $(x+1)$

$\sqrt{x^2 + 1} \neq x+1$
 not the same

$D_g\{x: x \geq -6\}$

In	fog	gof
-2	2.2	1
-1	1.4	0
0	1	1
1	1.4	2
2	2.2	3

55. Find 2 functions so that $f \circ g(x) = h(x)$
 $h(x) = (2x+1)^2$

$f(x) = x^2$
 $g(x) = 2x+1$

57. $h(x) = \frac{1}{x+2}$

$f(x) = \frac{1}{x}$
 $g(x) = x+2$

62. $h(x) = (x+3)^{3/2} + 4(x+3)^{1/2}$
 $f(x) = x^{3/2} + 4x^{1/2}$
 $g(x) = x+3$

Find the domain of f , g , $f \circ g$

65. $f(x) = \sqrt{x+3}$
 $g(x) = \sqrt{x}$

$D_f = \{x: x \geq -3\}$
 $D_g = \{x: x \geq 0\}$

$f \circ g = \sqrt{\sqrt{x}+3}$

$D_{f \circ g} = \{x: x \geq 0\}$

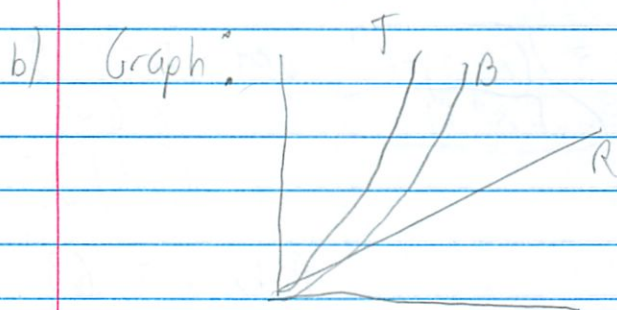
back
of
book

p59

67. $f(x) = \frac{1}{x}$ $D_f = \{x \mid x \neq 0\}$ $g(x) = x+3$ $D_g = \{x \mid x \neq -3\}$ $D_{f \circ g} = \{x \mid x \neq -3\}$

73. $R(x) = \frac{3}{4}x$ $\leftarrow$ reaction
 $B(x) = \frac{1}{15}x^2$ $\leftarrow$ braking

a) Total stopping time $\frac{3}{4}x + \frac{1}{15}x^2$



c) In the beginning, it's the R line, but afterwards it's the B-line
 $0 \leq x < 11,3$ $11,3 < x$

77. $r(t) = .6t$ Find $(A_{\text{or}})(t) = \pi (.6t)^2$
 $A(r) = \pi r^2$ $\hookrightarrow \pi .36t^2$

This is the area of the outer ripple after t seconds

1.6 Inverse Functions

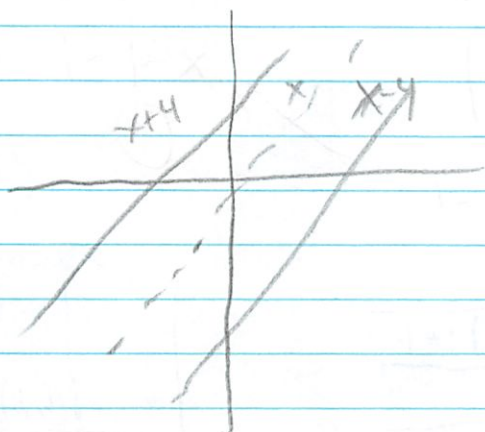
9/14

$$f(x) = x + 4 = (1, 5), (2, 6), (3, 7), (4, 8)$$

$$f^{-1}(x) = x - 4 = (5, 1), (6, 2), (7, 3), (8, 4)$$

switch ordered pairs

(domain + range switch)



Prove w/ Composition

$$f(f^{-1}(x)) = (x - 4) + 4$$

$$f^{-1}(f(x)) = (x + 4) - 4$$

identity function

ex1

Find the f^{-1} of $f(x) = 4x \rightarrow f^{-1}(x) = \frac{x}{4}$

$$f(f^{-1}(x)) = 4\left(\frac{x}{4}\right) = x$$

$$f^{-1}(f(x)) = \frac{(4x)}{4} = x$$

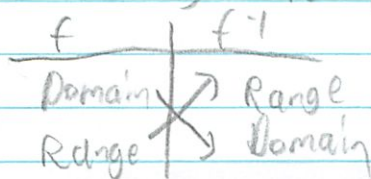
ex2

Find the f^{-1} of $f(x) = x^5 \rightarrow f^{-1}(x) = x^{\frac{1}{5}}$ or $\sqrt[5]{x}$

$$f(f^{-1}(x)) = (\sqrt[5]{x})^5 = x$$

$$f^{-1}(f(x)) = \sqrt[5]{(x^5)} = x$$

Inverse function = If $f(g(x)) + g(f(x))$ are equal,
Then $f + g$ are inverse functions of each other



p1 $f(x) = 2x^3 - 1$ $g(x) = \sqrt[3]{\frac{x+1}{2}}$

$$f(g(x)) = 2 \left(\sqrt[3]{\frac{x-1}{2}} \right)^3 - 1$$

$$2 \left(\frac{x-1}{2} \right) - 1$$

$$2 \left(\frac{x-1}{2} \right) - 1$$

$$x-1-1$$

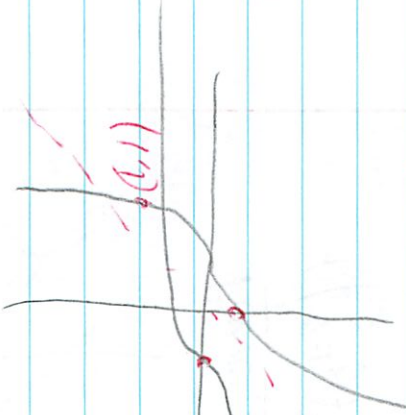
(X)

$$g(f(x)) = \sqrt[3]{\frac{2x^3-1+1}{2}}$$

$$\sqrt[3]{\frac{2x^3}{2}}$$

$$\sqrt[3]{x^3}$$

(X)



look at points to see reflection

p69

12. $f(x) = \frac{x^3}{2}$ $g(x) = \sqrt[3]{2x}$

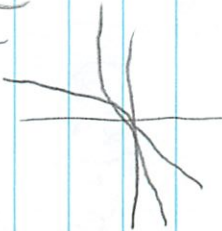
$$f(g(x)) = \frac{(\sqrt[3]{2x})^3}{2}$$

$$\frac{\sqrt[3]{2x}}{2}$$

(X)

$$g(f(x)) = \sqrt[3]{2 \left(\frac{x^3}{2} \right)}$$

(X)



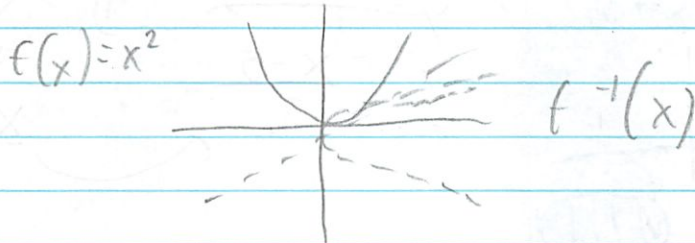
18 $f(x) = 9 - x^2$ $x \geq 0$ $g(x) = \sqrt{9-x}$

$$f(g(x)) = 9 - (\sqrt{9-x})^2$$

$$g(f(x)) = \sqrt{9 - (9 - x^2)} = \sqrt{9 - 9 + x^2} = \sqrt{x^2} = x$$

Inverse

* A function has an inverse that is a function if and only if it is one-to-one.



* horizontal line test
- intersect original function in only 1 place



$y = x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4

some output

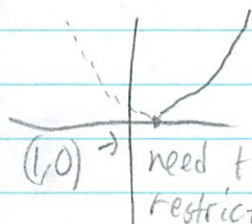
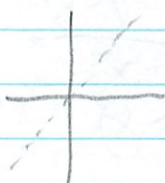
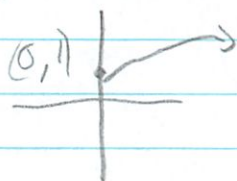
* not one to one

Functions which pass - line test

Increase over entire domain
 (or Decrease) - linear
 - cubic
 - $\sqrt{\quad}$

ex3

$$y = \sqrt{x+1} \rightarrow x = \sqrt{y+1} \rightarrow \sqrt{y} = x-1$$



$$y = (x-1)^2$$

$x \geq 1$

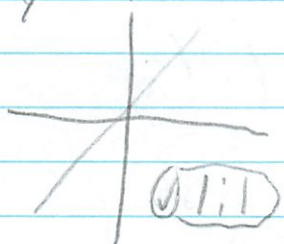
need to restrict - so don't have overlap

p76

44

Find Inverse

$$y = 3x + 5$$



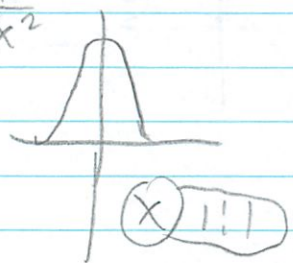
to find

$$y = 3x + 5 \rightarrow x = \frac{y-5}{3}$$

$$x-5 = \frac{3y}{3}$$

45

$$\frac{4}{x^2}$$



Inverse not a function

p4

$$f(x) = x^3 - 4$$



$$x = y^3 - 4$$

$$x+4 = y^3$$

$$\sqrt[3]{x+4} = y \quad \checkmark$$

p5

$$f(x) = \sqrt{2x-3}$$

$$x = \sqrt{2y-3}$$

$$x^2 = 2y - 3$$

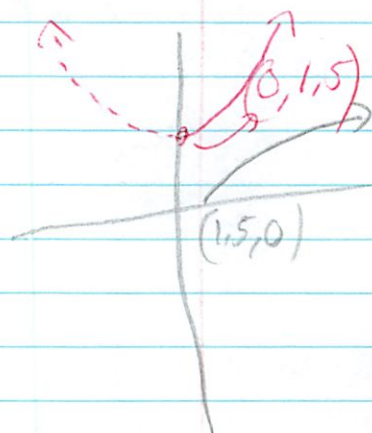
$$\frac{x^2 + 3}{2} = \frac{2y}{2}$$

$$\frac{x^2 + 3}{2} = y$$

$$x \geq 0$$

$f(x)$	$f^{-1}(x)$
$D: [\frac{3}{2}, \infty)$	$D: [0, \infty)$
$R: (0, \infty)$	$R: [\frac{3}{2}, \infty)$

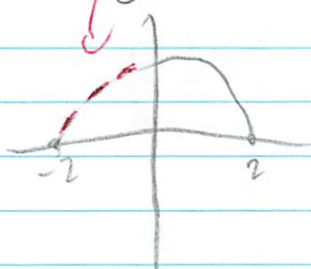
Domain Range
Switch



p6

$$f(x) = \sqrt{4-x^2}$$

$$0 \leq x \leq 2$$



$$x = \sqrt{4-y^2}$$

$$x^2 = 4 - y^2$$

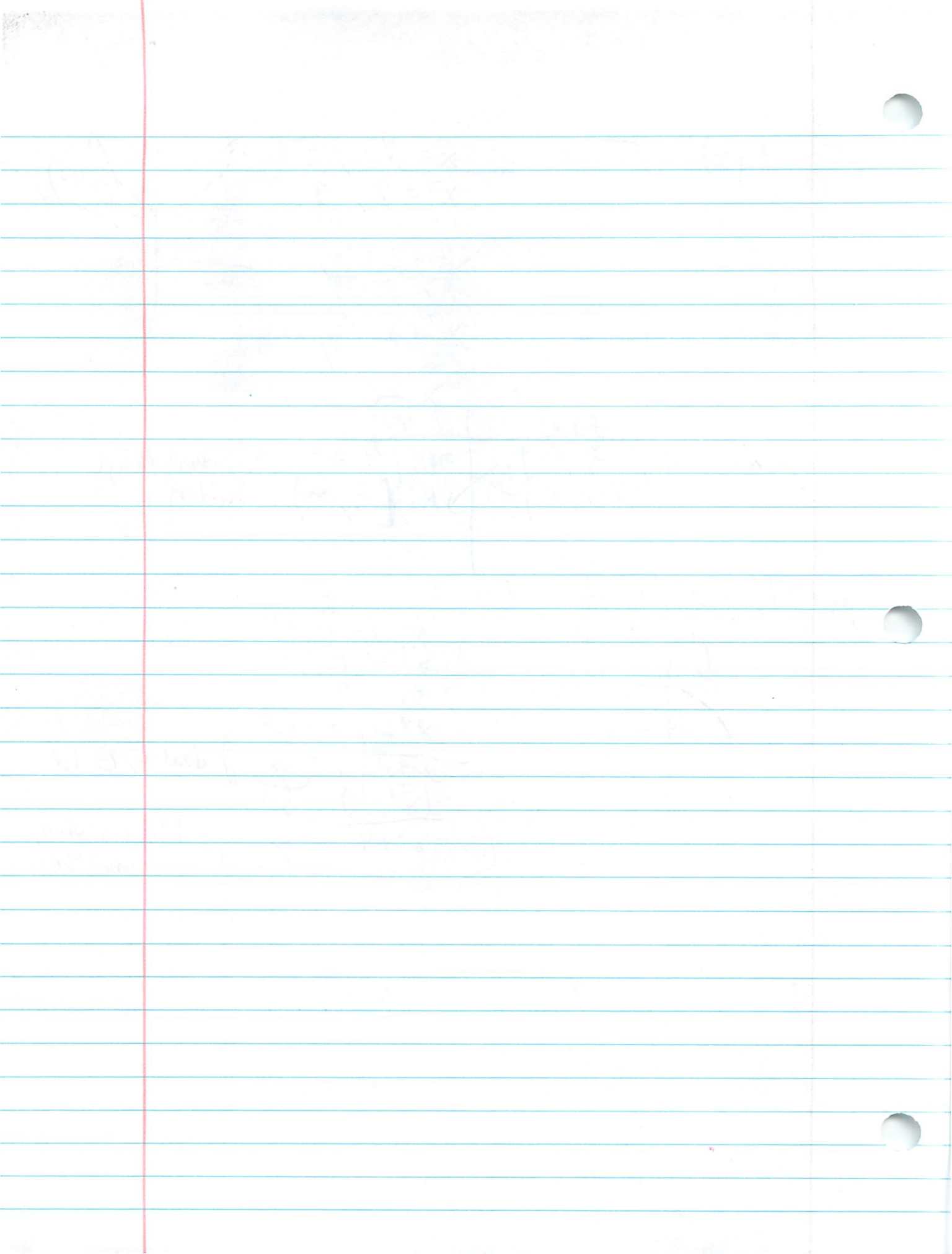
$$x^2 - 4 = -y^2$$

$$-(x^2 - 4) = y^2$$

$$\sqrt{-x^2 + 4} = y$$

deal w/ $\ominus$ 1st

It's its own
inverse



1.6 Inverse Functions

9/14

p89

Find Inverse + Verify

6. $f(x) = \frac{x-1}{4} \rightarrow x = \frac{y-1}{4}$

$$4x = \frac{y-1}{1} \quad \frac{(4x-1)+1}{4} \quad 4\left(\frac{x-1}{4}\right)+1$$

$$4x = y-1 \quad \frac{4x}{4} \quad x-1+1$$

$$(y = 4x+1) \quad (x) \quad (x)$$

7. $f(x) = \sqrt[3]{x} \rightarrow x = \sqrt[3]{y}$

$$x^3 = y \quad (\sqrt[3]{x})^3 \quad \sqrt[3]{x^3}$$

Find + Verify w/ table

9. $f(x) = -\frac{7}{2}x - 3$

$$x = -\frac{2}{7}y - 3$$

$$g(x) = -\frac{2x+6}{7}$$

$$x+3 = -\frac{2}{7}y$$

$$-\frac{2x+6}{7} = y$$

x	f(x)	g(x)
-2	4	-.29
-1	.5	-.57
0	-3	-.85
1	-6.5	-1.14
2	-10	-1.4

how table works

13. $f(x) = -\sqrt{x-8}$

$$g(x) = 8+x^2$$

$$x \leq 0$$

$$x = -\sqrt{y-8}$$

$$-x = \sqrt{y-8}$$

$$(-x)^2 = y-8$$

$$x^2 + 8 = y$$

$$(-x)^2 + 8 = y$$

x	f(x)	g(x)
8	0	72
9	-1	89
10	-1.4	108
11	-1.7	129

check

graphically for domain

could also do fog = g of ?

$$fog = -\sqrt{(8+x^2)-8}$$

$$-\sqrt{x^2}$$

$$-x$$

$$gof = 8 + (\sqrt{x-8})^2$$

$$8 + (x-8)$$

$$x$$

$$\sqrt{x^2}$$

$$|x|$$

$$y_3 = fog$$

$$y_4 = gof$$

make (in calc)

$$-(-x) = x$$

$$17, f(x) = \sqrt[3]{3x-10}$$

$$g(x) = \frac{x^3+10}{3}$$

✓||

$$-x = \frac{y^3+10}{3}$$

$$3x = y^3+10$$

$$3x-10 = y^3$$

$$\sqrt[3]{3x-10} = y$$

Skipping
Table

$$18, f(x) = 1-x^3$$

$$g(x) = \sqrt[3]{1-x}$$

✓||

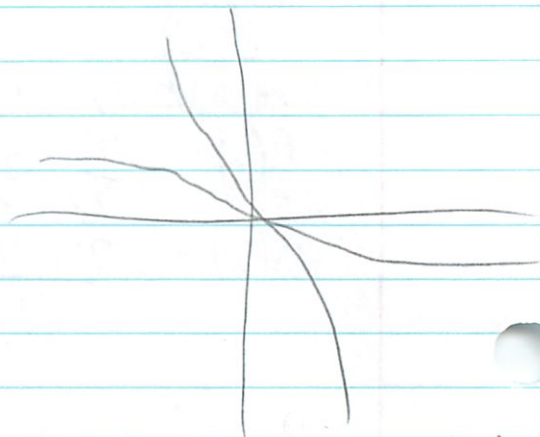
$$x = 1-y^3$$

$$-1 \quad -1$$

$$x-1 = -y^3$$

$$-x+1 = y^3$$

$$\sqrt[3]{-x+1}$$



$$20, f(x) = \frac{1}{1+x} \quad x \geq 0$$

$$g(x) = \frac{1-x}{x} \quad 0 < x \leq 1$$

$$f \circ g = \frac{1}{1 + \left(\frac{1-x}{x}\right)}$$

$$g \circ f = \frac{1 - \left(\frac{1}{1+x}\right)}{\frac{1}{1+x}}$$

get
common
denominator
as
x=1

$$\frac{1}{x+1-x}$$

$$\frac{1}{\frac{1}{x}} = \textcircled{x}$$

$$\frac{1}{1 + \left(\frac{1-x}{x}\right)}$$

$$\frac{1 - \left(\frac{1}{1+x}\right)}{\frac{1}{1+x}}$$

$$\frac{(1+x)-1}{1+x}$$

$$\rightarrow \frac{x}{1+x}$$

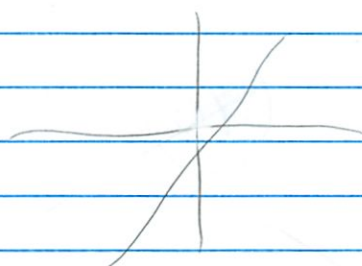
$$\frac{1}{1+x}$$

⊗

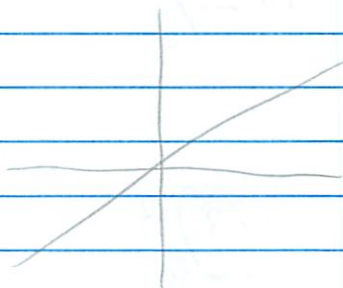
p69

21,

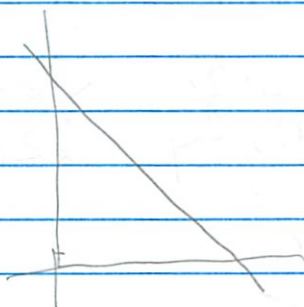
Match w/ Inverse



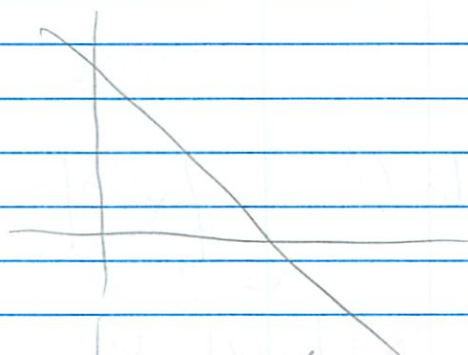
(c)



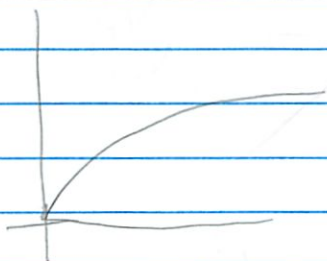
22,



(b)



23,



(a)



24,



(d)



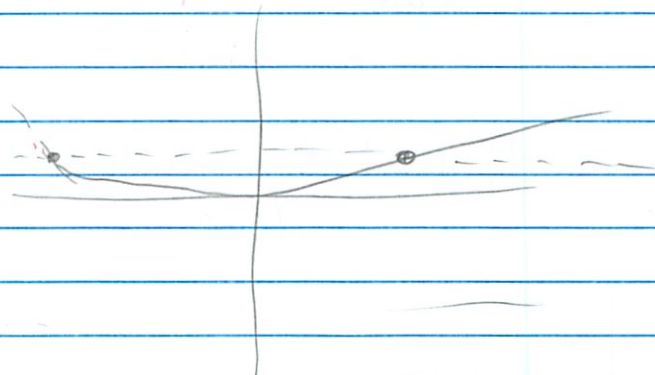
p 70

31,

Graph + — line test

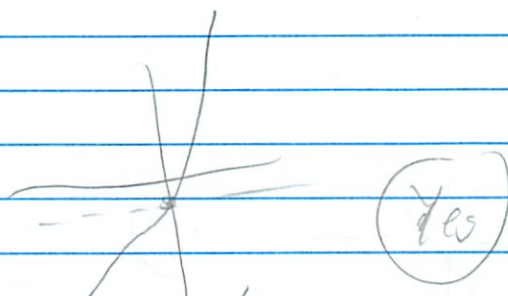
$$h(x) = \frac{x^2}{x^2 + 1}$$

No

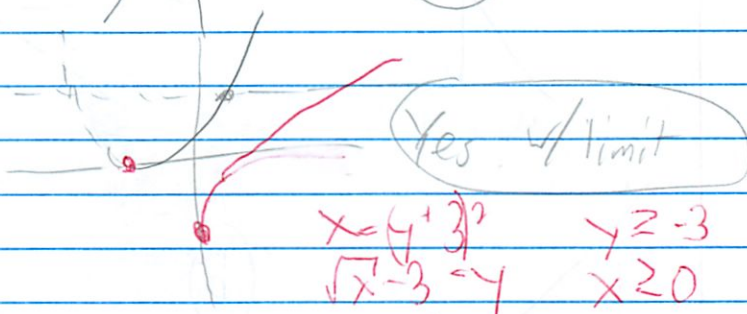


p70

38. $f(x) = x^5 - 7$

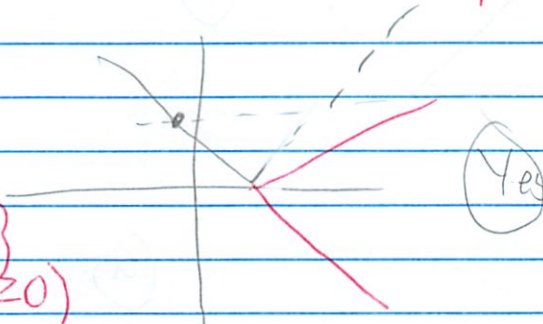


47. $f(x) = (x+3)^2$
 $x \geq -3$



$x = (y+3)^2$ $y \geq -3$
 $\sqrt{x-3} = y$ $x \geq 0$

51. $f(x) = |x-2|$
 $x \leq 2$



switch
domain range
 $x = |y-2|$
Range = $\{y: y \leq 2\}$
Domain = $\{x: x \geq 0\}$

84. $y = 8 + 1.75x$

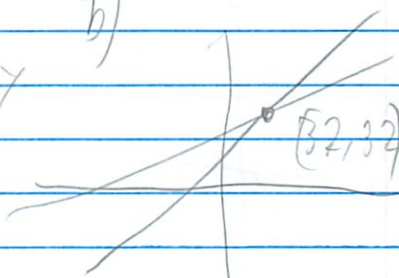
a) Inverse

$x = 8 + 1.75y$
 $-8 \quad -8$

$x - 8 = 1.75y$
 $\frac{x-8}{1.75} = y$

$y = \frac{x-8}{1.75}$

b)



output in terms
of wage

c) 10 units =
\$15.5

d) \$22.25 =
19 units

Pratice Problem

9/17/07

9/17

Q For all $\# x + y$ let $x \Delta y$ be defined as $x \Delta y = x^2 + xy + y^2$. What is the value $(3 \Delta 1) \Delta 1$?

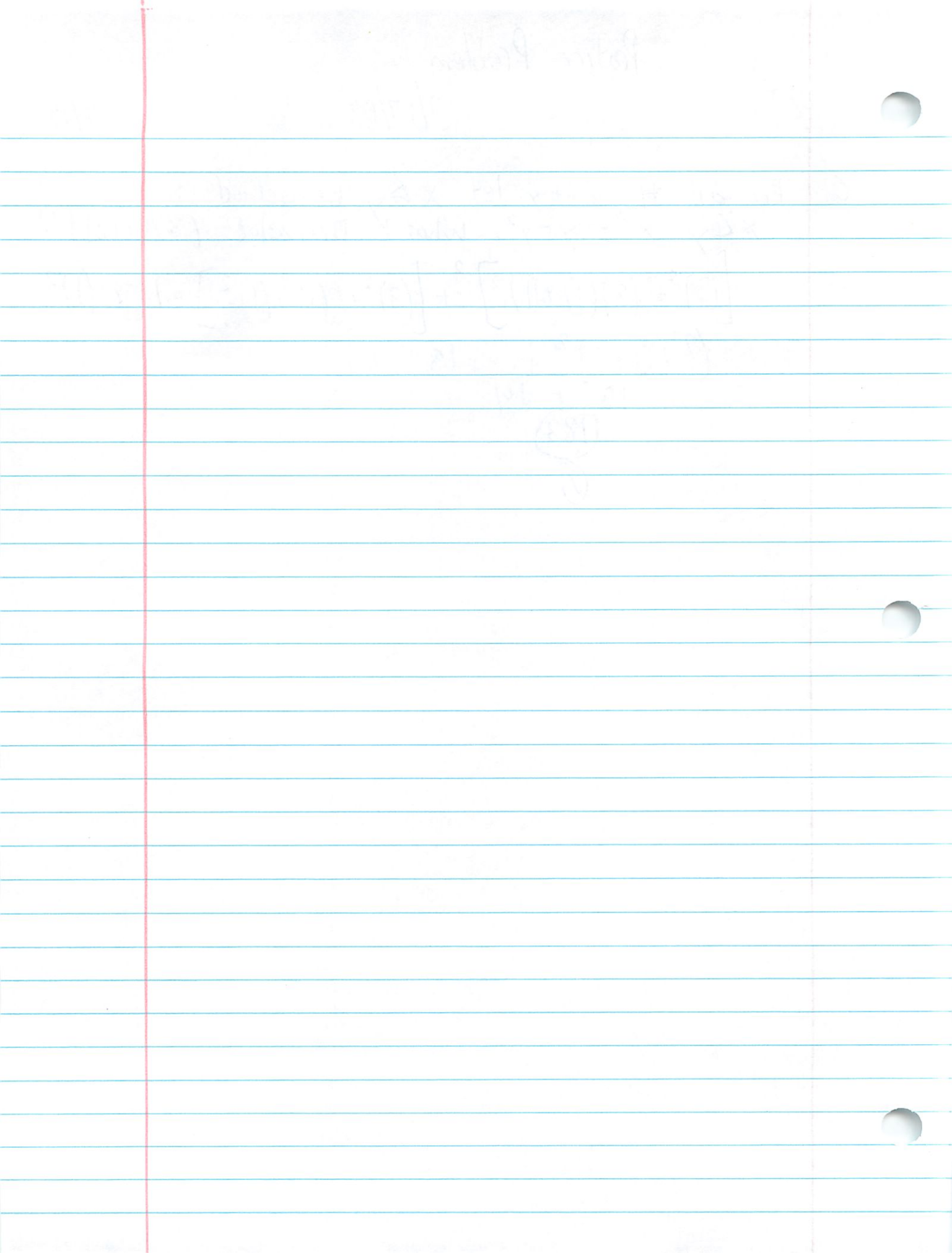
$$\left[(3)^2 + (3)(1) + (1)^2 \right]^2 + \left[(3)^2 + (3)(1) + (1)^2 \right] \cdot 1 + (1)^2$$

$$(9 + 3 + 1)^2 + 13 + 1$$

$$13^2 + 14$$

$$183$$

✓



1.7 Linear Models + Scatter Plots

9/14

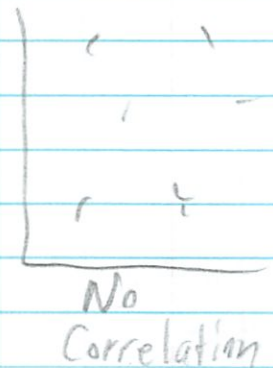
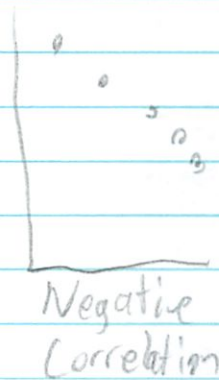
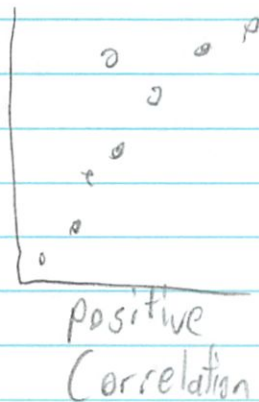
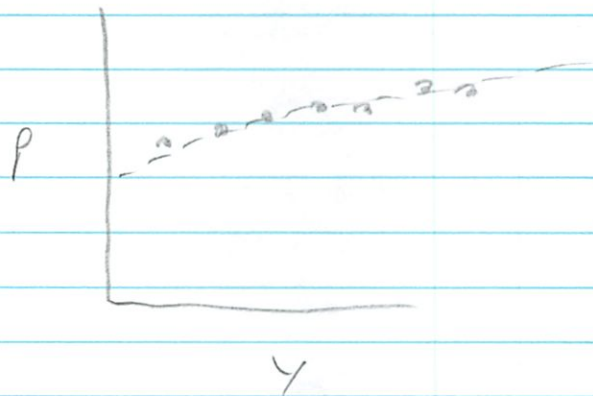
$t=0 = 1990$

year	P
1995	132
96	134
97	136
98	138
99	139
'00	141
01	142

$\rightarrow (5, 132)$

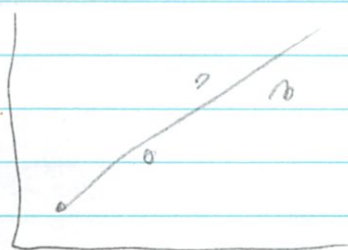
$$y = mx + b$$

$\uparrow$ linear regression



p73

ex2 Study Hours $\uparrow$ Positive Correlation
TV Hrs $\uparrow \rightarrow$ Scores $\downarrow$ Negative

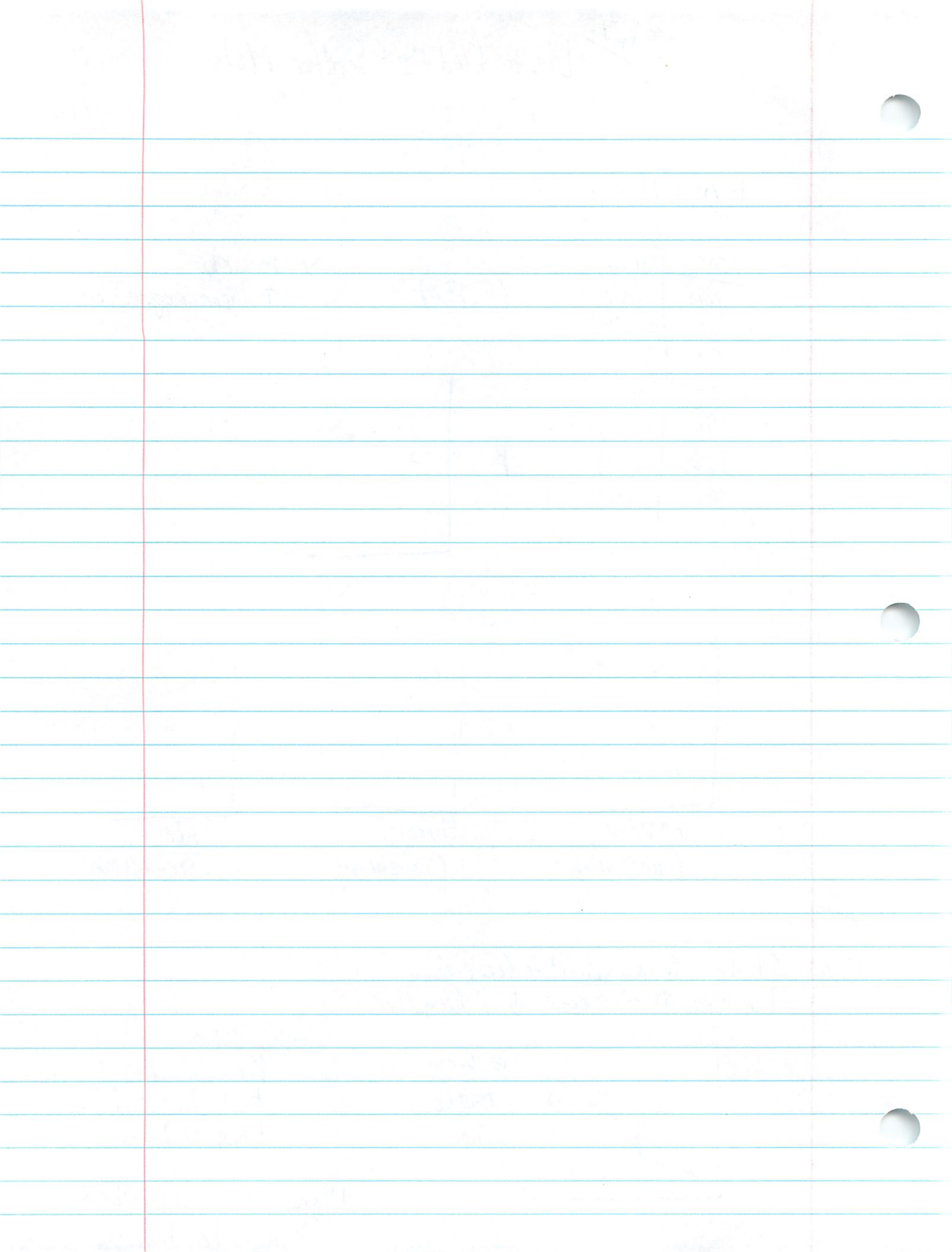


$\leftarrow$ Draw the middle of data

Find Slope

Pick 2 pts on line
Find slope + y-intercept
Plug $\rightarrow$ Formula

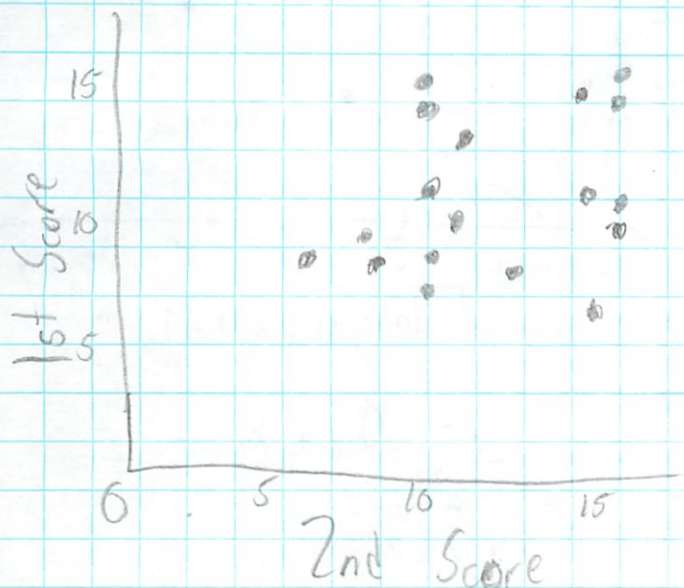
Higher r value = better
closer to 11



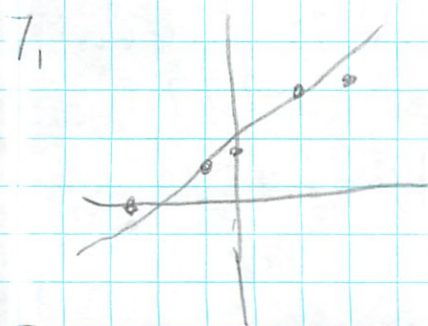
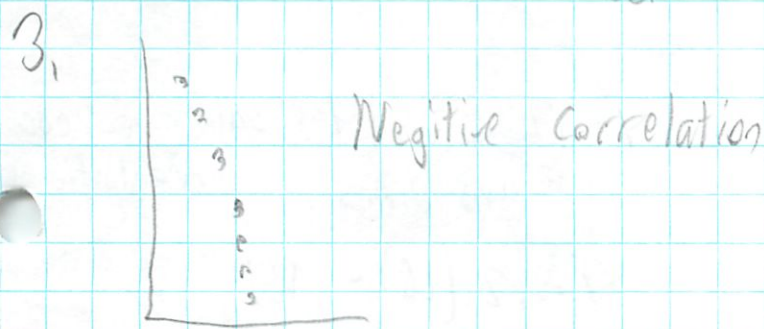
Q77

1.7 Linear Models + Scatter Plots

2. Create a Scatter Plot



b) Linear? No there does not really seem a correlation. Prior results do not guarantee future results



$$y = 1.46x + 1.61$$

The line fits the value pretty well

Hooke's Law

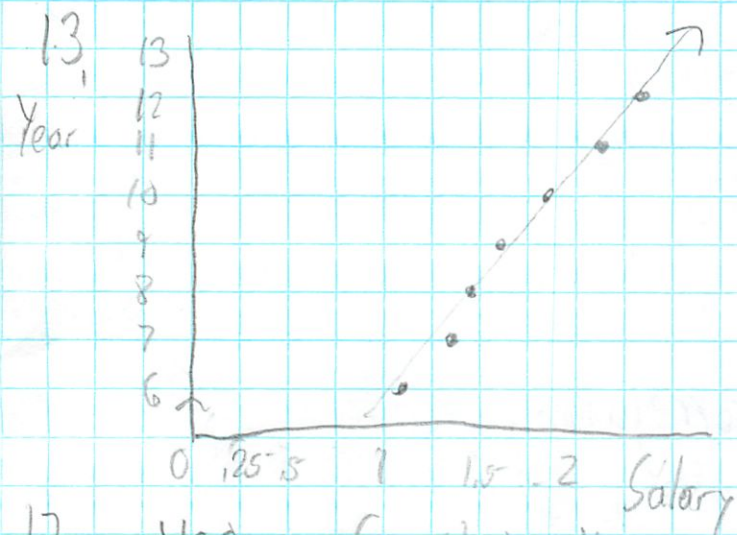
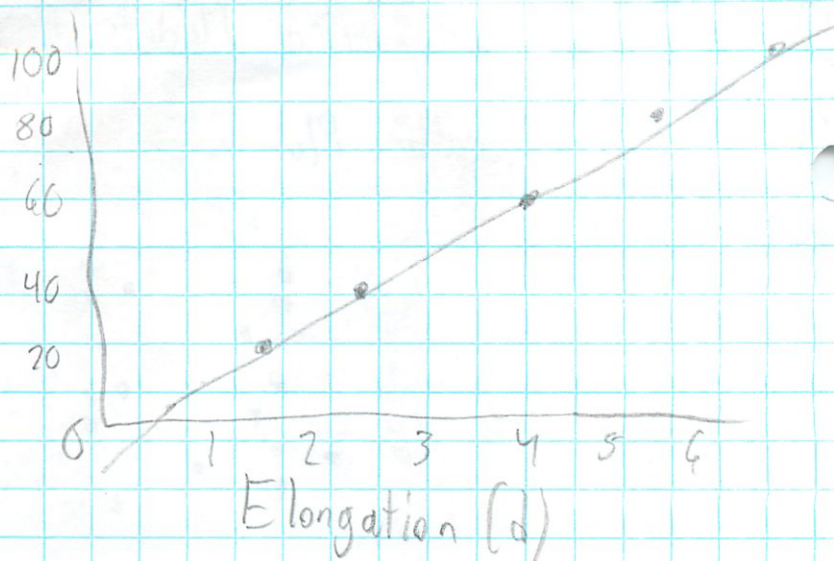
11, $F = kd$ ← distance
 Force ↑ stiffness

$$\frac{60 - 40}{4 - 2.5} = \frac{20}{1.5}$$

$$y = \frac{20}{1.5}x - 20$$

$$y = .066x$$

d) 55 kg? → $.066(55) = 3.63$



$$y = .12x - .14$$

↑ avg increase salary in 2000 unrealistic

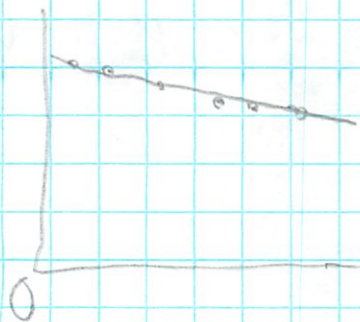
$$y = .2(16) - .14$$

$$y = 3.06$$

17, 400 m freestyle times
 $x = 0 = 1950$

$$y = -.025x + 5.05$$

The model fits Ok, the r value = .9277



It could but that assumes that training + tech can continue to improve

Chap 1 Review

9/18

Find y

1.

$$f(x) = x^2 - 4x - 12$$

~~$$x = y^2 - 4y - 12$$~~

~~$$x + 12 = y^2 - 4y$$~~

~~$$\frac{x+12}{4} = \frac{y^2}{4} - y$$~~

→ Complete the square →
→ take half + square

$$y + 12 = x^2 - 4$$

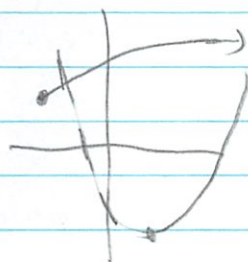
$$y + 12 + 4 = x^2 - 4x + 4$$

$$y + 16 = (x - 2)^2$$

$$y = (x - 2)^2 - 16$$

vertex (2, -16)

Restrict



$\frac{-4}{2} \rightarrow 4$
add to both sides

inverse

$$x = (y - 2)^2 - 16$$

$$x + 16 = (y - 2)^2$$

$$\sqrt{x + 16} = y - 2$$

$$\sqrt{x + 16} + 2 = y$$

Domain restriction
 $x \geq -16$

2.

$$y = x^2 + 2x + 4$$

$$y - 4 = x^2 + 2x \quad \text{complete the sq.} \quad \frac{2}{2} \rightarrow 1^2 \rightarrow 1$$

$$y - 3 = x^2 + 2x + 1$$

$$y - 3 = (x + 1)^2$$

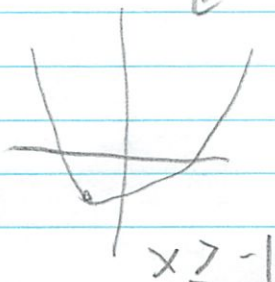
$$y = (x + 1)^2 + 3 \rightarrow -1 \rightarrow x = (y - 3)^2 + 3$$

$$x + 3 = (y - 3)^2$$

$$\sqrt{x + 3} = y - 3$$

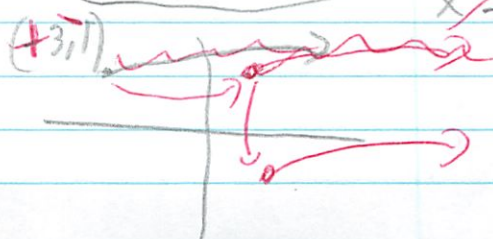
$$y = \sqrt{x + 3} + 3$$

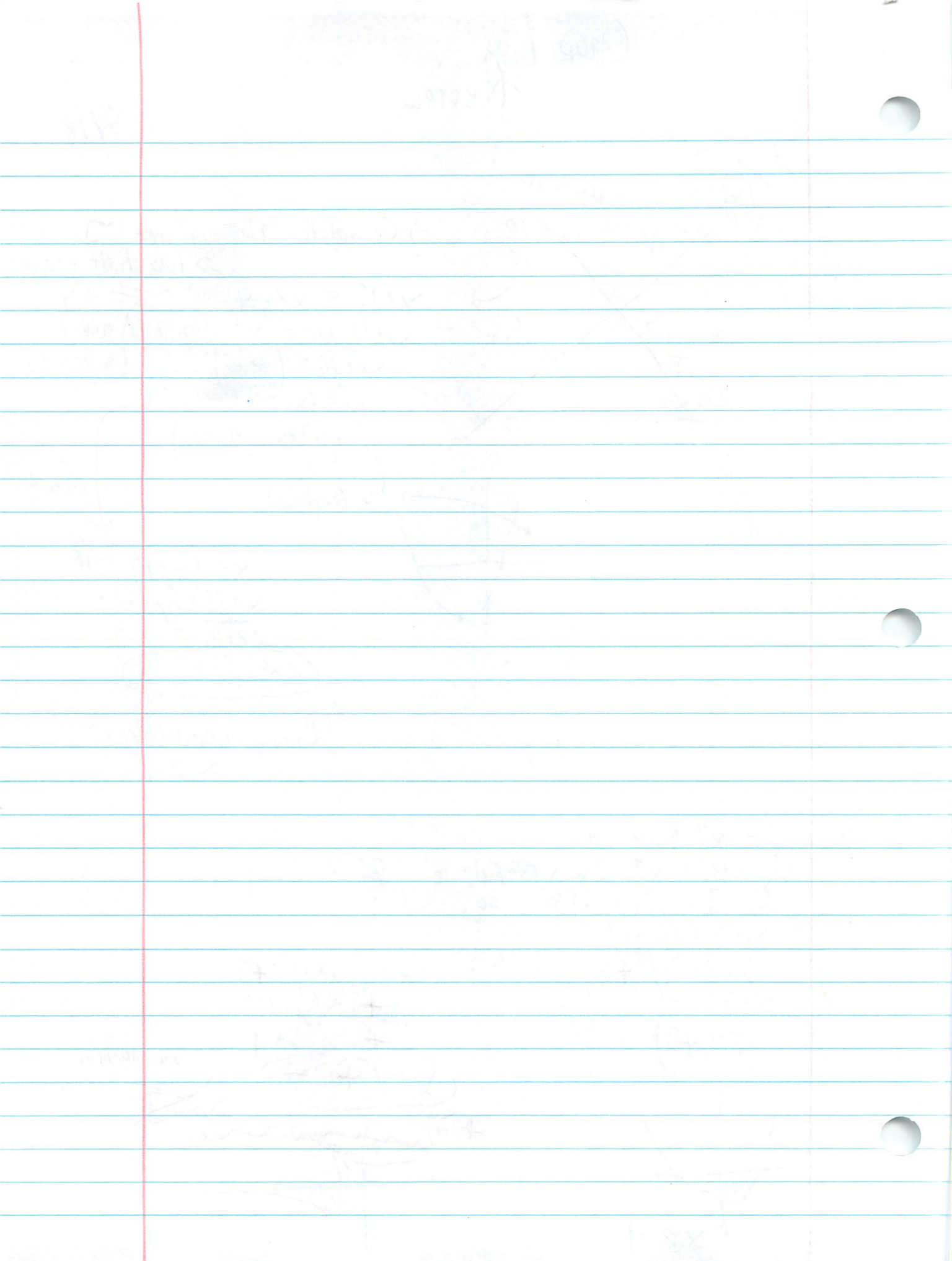
(-1, 3)



$x \geq -3$

no domain needed





More Chap 1

stuff

9/17

Distance from a point to a line

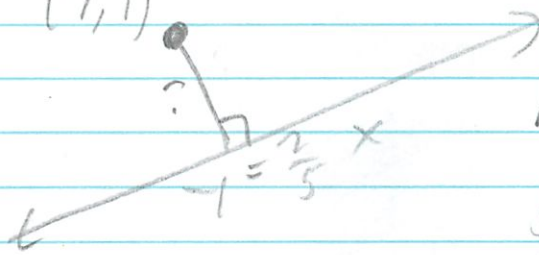


mathematically shortest distance which is perpendicular to the line

distance, d
given from a point (x, y)
to a non vertical line $= \boxed{Ax + By + C = 0}$

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

ex: $(7, 1)$



$$Ax + By + C = 0$$

$$y = \frac{2}{5}x$$

$$5y = 2x$$

$$0 = 2x - 5y$$

$$d = \frac{|2(7) - 5(1)|}{\sqrt{2^2 + (-5)^2}} \rightarrow \frac{|9|}{\sqrt{29}} \rightarrow \approx 1.67$$

p1

$$3x + 4y = 2$$

$$(5, -1)$$

$$3x + 4y = 2$$

$$-2$$

$$3x + 4y - 2 = 0$$

$$|3(5) + 4(-1) - 2|$$

$$\sqrt{3^2 + 4^2}$$

$$\frac{|9|}{5} \rightarrow \left(\frac{9}{5}\right)$$

P2

$$x=3$$
$$(5, 7)$$

$$1x - 3 = 0$$

$$\frac{|1(5) + 0(7) - 3|}{\sqrt{1^2 + 0^2}} \rightarrow \frac{|2|}{\sqrt{1}} \rightarrow (2)$$

Chap 1 Test

Topics

9/17

* Domain + Range

Increasing, Decreasing + Constant

Shifts of Functions (up, down, sideways)

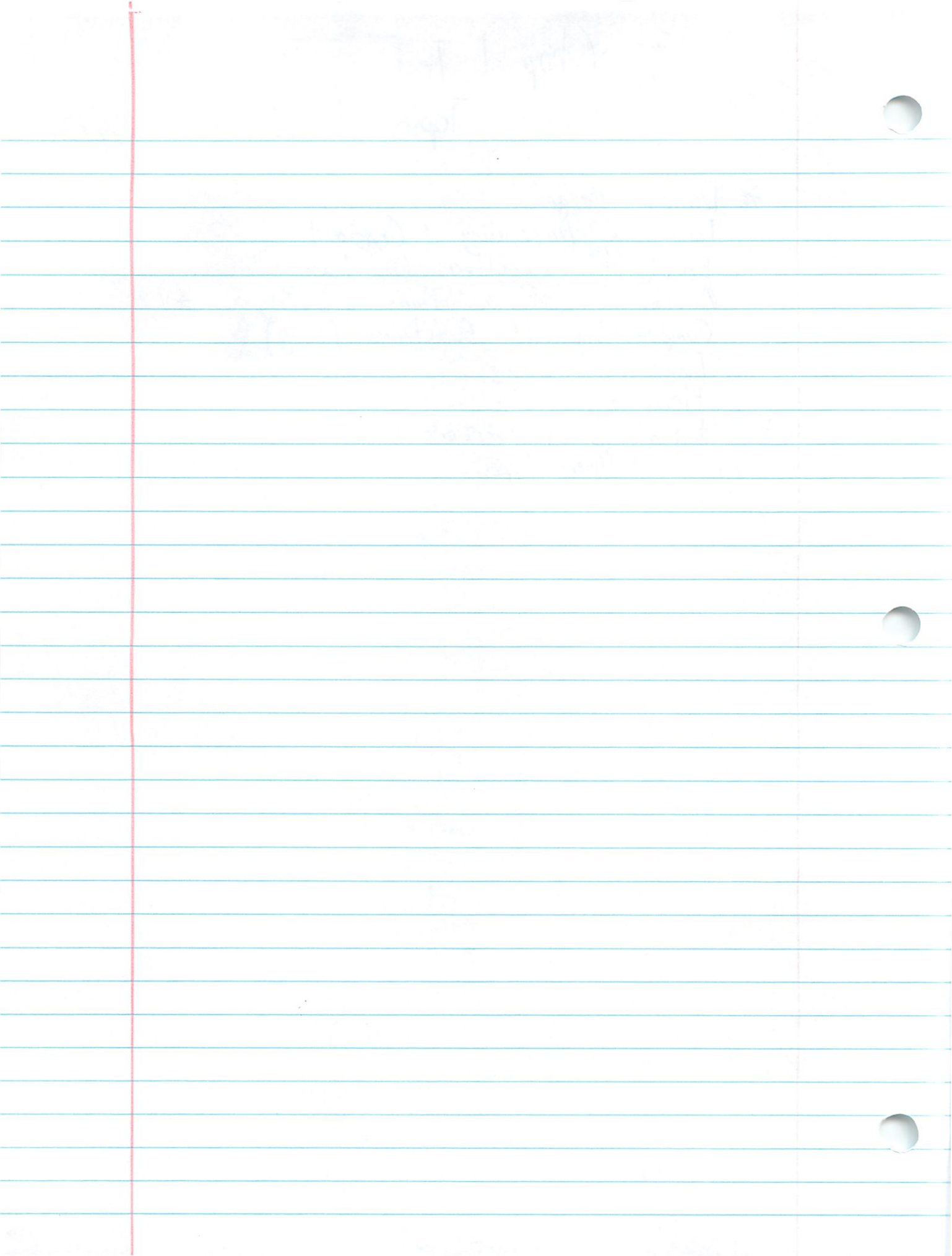
Arithmetic of functions ($+$, $-$, $\times$, $\div$)

Composition of functions ($f \circ g$)

Inverse functions

Piecewise functions

* Equations of functions
- Different types



Chap 1 Review H/W

9/17

p82

Find slope-intercept form

17. $(2, -1)$ $(4, -1)$ $\frac{-1 - (-1)}{4 - 2} = \frac{-1}{2} = -\frac{1}{2}$ $\cdot 5x$

One point

Point slope form $y - (-1) = -\frac{1}{2}(x - 2)$
 $y + 1 = -\frac{1}{2}x + 1$
 $y = -\frac{1}{2}x$

$y = -\frac{1}{2}x - 2$

19. $(-1, 0)$ $(6, 2)$ $\frac{2 - 0}{6 - (-1)} = \frac{2}{7}$ $\cdot x$

$y - (0) = \frac{2}{7}(x - (-1))$
 $y - 0 = \frac{2}{7}x + \frac{2}{7}$
 $y = \frac{2}{7}x + \frac{2}{7}$

$y = \frac{2}{7}x + \frac{38}{7}$

Parallel + Perpendicular

25. $(3, -2)$ $5x - 4y = 8$
 $-5x + 4y = -8$
 $-4y = -5x + 8$
 $y = \frac{5}{4}x - 2$

$y - (-2) = \frac{5}{4}(x - 3)$
 $y + 2 = \frac{5}{4}x - \frac{15}{4}$
 $y = \frac{5}{4}x - \frac{23}{4}$

$y + 2 = -\frac{4}{5}(x - 3)$
 $y + 2 = -\frac{4}{5}x + \frac{12}{5}$
 $y = -\frac{4}{5}x - \frac{2}{5}$

Eval + Simp.

35. $f(x) = x^2 + 1$ $f(-4) = (-4)^2 + 1 = 16 + 1 = 17$
 $f(2) = (2)^2 + 1 = 4 + 1 = 5$

$f(+2) = (+2)^2 + 1 = 4 + 1 = 5$

$-f(x) = -(x^2 + 1) = -x^2 - 1$

37. $\begin{cases} 2x+1, & x \leq -1 \\ x^2+2, & x > -1 \end{cases}$ $h(-2) = 2(-2) + 1$

$h(-1) = 2(-1) + 1$

(-1)

$h(0) = (0)^2 + 2$

(2)

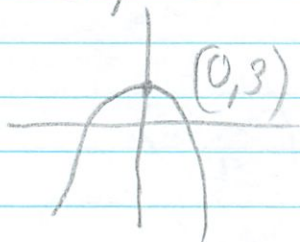
$h(2) = (2)^2 + 2$

(6)

Domain + Range

49.

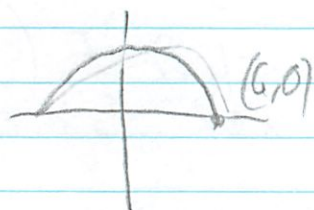
$f(x) = 3 - 2x^2$



D: $\{x \mid x \in \mathbb{R}\}$
R: $\{y \mid y \leq 3\}$

51.

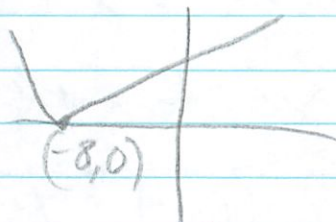
$h(x) = \sqrt{36 - x^2}$



D: $\{x \mid -6 \leq x \leq 6\}$
R: $\{y \mid 0 \leq y \leq 6\}$

60.

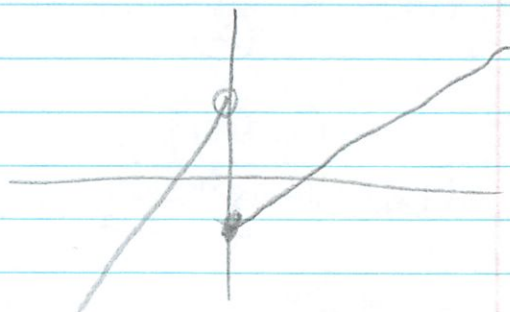
$f(x) = \frac{|x+8|}{2}$



Decreasing $(-\infty, -8]$
Increasing $[-8, \infty)$

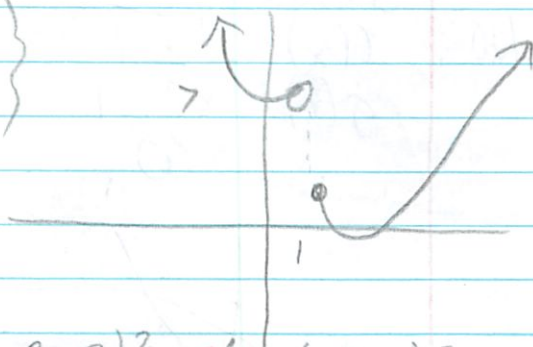
65.

$f(x) = \begin{cases} 3x+15, & x < 0 \\ x-4, & x \geq 0 \end{cases}$



83

$$66. f(x) = \begin{cases} x^2 + 7, & x < 1 \\ x^2 - 5x + 6, & x \geq 1 \end{cases}$$



p84

Review

67.

Even, Odd, Neither?

$$f(x) = (x^2 - 8)^2 \quad f(-x) = (-x^2 - 8)^2 \quad (x^2 - 8)^2$$

$$f(-x) = f(x) \text{ even}$$

$$f(-x) = -f(x) \text{ odd}$$

$$- [f(x)] = -(x^2 - 8)^2 = (-x^2 + 8)^2$$

even

neither

$$68. f(x) = 2x^3 - x^2 \quad f(-x) = 2(-x)^3 - (-x)^2 = -2x^3 - x^2$$

$$-f(x) = -(2x^3 - x^2) = -2x^3 + x^2$$

neither

Identify type

$$75. h(x) = \sqrt{x-5} \Rightarrow \text{sqrt function}$$



$$h = x - 5 \quad \text{hof}$$

$$f = \sqrt{x}$$

$$85. f(x) = 3 - 2x \quad f \circ g(4)$$

$$g(x) = \sqrt{x}$$

$$h(x) = 3x^2 + 2$$

$$3 - 2(4) - [\sqrt{4}]$$

$$93. f \circ h(-4) = 3 - 2(3x^2 + 2) \quad (-)$$

$$3 - 2(3[-4]^2 + 2)$$

$$3 - 2(50)$$

$$(-97)$$

100. Inverse?

$$f(x) = \sqrt{x+1}$$

$$g(x) = x^2 - 1$$

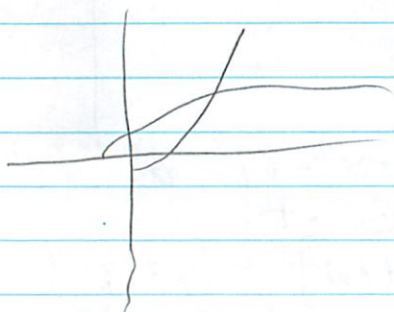
$$x \geq 0$$

$$f \circ g = \sqrt{(x^2 - 1) + 1} \quad \text{goF} = (\sqrt{x+1})^2 - 1$$

$$\text{X}$$

$$x+1-1$$

$$\text{X}$$



107. $f(x) = 4x^3 - 3$

$$x = \sqrt[3]{\frac{y+3}{4}}$$

$$\frac{x+3}{4} = \frac{y^3}{4}$$

$$\frac{x+3}{4} = y^3$$

$$\sqrt[3]{\frac{x+3}{4}} = y$$

108. $f(x) = \sqrt{x+10}$

$$x = \sqrt{y+10}$$

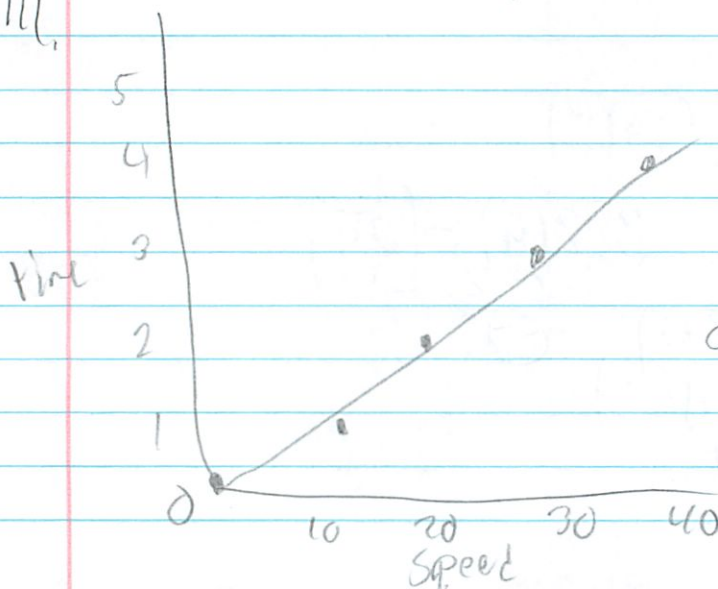
$$x^2 = y+10$$

$$-10 \quad -10$$

$$y = x^2 - 10$$

p85

111.



$$\frac{29,2 - 19,4}{3 - 2}$$

b) $\frac{9,8}{1} = 9,8x$

c) calc: $y = 9,7x + 4$

d) $? = 9,7(2,5) + 4$

24,65

2.1 Polynomials Quadratic Functions

9/20

ex: $f(x) = 5$ constant horizontal function

$$f(x) = 3x + 1 \quad \text{linear}$$

degree = 1

$$f(x) = 2x^2 + 3x - 1 \quad \text{quadratic}$$

degree = 2

$$f(x) = 3x^3 - 2x^2 + 3x - 1$$

degree = 3

$$f(x) = ax^2 + bx + c$$

degree = 2

$$h(t) = -16t^2 + 256t + 6 \quad \text{object falling in air (in ft)}$$

initial velocity, initial height

maximum: (8, 1030)

x-intercept: (16, 02, 0)

$$f(x) = x^2 \rightarrow g(x) = \frac{1}{3}x^2$$

going to get fatter

$$f(x) = x^2 \rightarrow g(x) = 2x^2$$

going to be skinnier

$$f(x) = x^2 \rightarrow g(x) = -x^2 + 1$$

vertex $\uparrow$

opens downward

$$f(x) = x^2 \rightarrow k(x) = (x+2)^2 - 3$$

vertex form (-2, -3)



Book's standard form = vertex form

$$(x-h)^2 + k$$

School's sifi. = $ax^2 + bx + c$

Convert to vertex form

$$2x^2 + 8x + 7$$

factor

$$2(x^2 + 4x) + 7$$

complete the sq.

$$2(x^2 + 4x + 4) + 7$$

$a = 8$

$-8 \in \text{balance}$

$$2(x^2 + 4x + 4) + 7 - 8$$

$$2(x+2)^2 - 1 \quad \text{vertex form} \rightarrow (-2, -1)$$

ex 2

$$h(x) = 4x^2 - 4x + 21$$

$$4(x^2 - x) + 21 \rightarrow \frac{1}{2} \quad \left(-\frac{1}{2}\right)^2 \rightarrow \frac{1}{4}$$

$$4\left(x^2 - x + \frac{1}{4}\right) + 21$$

$+1$

$$4\left(x^2 - x + \frac{1}{4}\right) + 22$$

$$4\left(x^2 - \frac{1}{2}\right)^2 + 22 \quad \in \text{vertex form}$$

x intercepts

factor using rooten $\rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$0 = -x^2 + 6x - 8$$

$$x^2 - 6x + 8$$

$$(x-4)(x-2) \quad \text{factor normally}$$

$$(4, 0) \quad (2, 0)$$

ex 3

$$f(x) = -x^2 + 2x + 5 \rightarrow \frac{-(-2) \pm \sqrt{(-2)^2 - 4(-1)(5)}}{2(-1)}$$

x-intercepts

$$(-1.4494, 0)$$

$$(3.4494, 0)$$

$$\frac{-2 \pm \sqrt{24}}{-2}$$

ex 3, cont. $f(x) = -x^2 + 2x + 5$

$$-\left(x^2 - 2x\right) + 5 \rightarrow \frac{-2}{2} = -1 \rightarrow (-1)^2$$

$$-\left(x^2 - 2x + 1\right) + 5 + 1$$

$$-\left(x^2 - 2x + 1\right) + 6$$

$$-(x-1)^2 + 6 \rightarrow (1, 6)$$

p1 $f(x) = -4x^2 + 24x - 41$ $\rightarrow \frac{-(24) \pm \sqrt{(24)^2 - 4(-4)(-41)}}{2(-4)}$

$$-4(x^2 - 6x) - 41$$

$$-4(x^2 - 6x + 9) - 41 + 36$$

$$-4(x-3)^2 - 5$$

$(3, -5)$ vertex

$\frac{-6}{2} = -3$
 $(-3)^2 = 9$

$$\frac{-24 \pm \sqrt{-80}}{-8}$$

$(1.88, 0)$
 $(4.11, 0)$ $\rightarrow$ not real
 γ -intercept

Write an equation in vertex form whose vertex is $(1, 2)$ and passes through $(3, 6)$

(h, k) (x, y)

$$f(x) = a(x-h)^2 + k$$

$$-6 = a(3-1)^2 + 2$$

$$-6 = 4a + 2$$

$$\frac{-8}{4} = \frac{4a}{4}$$

$$-2 = a$$

$$\rightarrow -2(x-1)^2 + 2$$



vertex point

p2 $\left(-\frac{5}{2}, 0\right) \quad \left(-\frac{7}{2}, -\frac{16}{3}\right)$

$$-\frac{16}{3} = a\left(-\frac{7}{2} - \left(-\frac{5}{2}\right)\right)^2 + 0$$

$$-\frac{16}{3} = a(-1)^2$$

$$\frac{-16}{3} = a$$

$$a = -\frac{16}{3}$$

$$y = -\frac{16}{3}\left(x - \frac{5}{2}\right)^2$$

p3 Find 2 pos # whose product is a maximum

$x = \text{first \#}$
 $y = \text{second \#}$ } maximum

The sum of the first + 3(second) = 42

$$x + 3y = 42$$

$$x = 42 - 3y$$

distribute $\hookrightarrow (42 - 3y)y = \text{vertex}$

$$f(x) = -3y^2 + 42y$$

$$-3(y^2 - 14y) \rightarrow -\frac{14}{2} = -7 \rightarrow (-7)^2$$

$$-3(y^2 - 14y + 49) + 147$$

$$-3(y - 7)^2 + 147 = f(x)$$

$$\boxed{\begin{matrix} 7 \\ y \end{matrix}} \quad \text{maximum}$$

$$x + 3(7) = 42$$

$$\boxed{x = 21}$$

$$21 \cdot 7 = 147 \quad (\checkmark)$$

2.1 Quadratic Functions

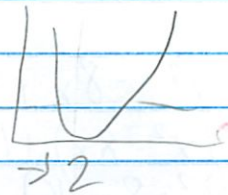
9/20

pg 95

Match

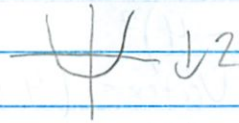
1.

$$f(x) = (x-2)^2 \quad g$$



3.

$$f(x) = x^2 - 2 \quad b$$



8.

$$f(x) = -(x-4)^2 \quad d$$



9.

Graph + Compare vs x^2

$$y = \frac{1}{2}x^2 \rightarrow \text{fatter}$$

$$y = \frac{1}{2}x^2 - 1 \rightarrow \text{fatter and } \downarrow 1$$

$$y = \frac{1}{2}(x^2 + 3)^2 \rightarrow \text{fatter + } \leftarrow 3$$

$$y = -\frac{1}{2}(x^2 + 3) - 1 \rightarrow \text{fatter, frown, } \leftarrow 3 + \downarrow 1$$

11.

$$y = -2x^2 \rightarrow \text{skinner, frown}$$

$$y = -2x^2 - 1 \rightarrow \text{skinner, frown, } \downarrow 1$$

$$y = -2(x-3)^2 \rightarrow \text{skinner, frown, } \leftarrow 3$$

$$y = 2(x-3)^2 - 1 \rightarrow \text{skinner, } \leftarrow 3 + \downarrow 1$$

Identify vertex + x-intercepts

17.

$$f(x) = (x+4)^2 - 3$$

$$y = (x+4)^2 - 3 \rightarrow \text{multiply out}$$

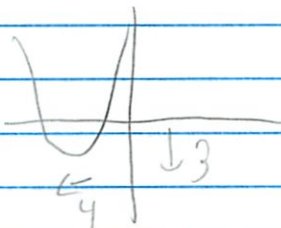
$$x^2 + 8x + 16 - 3$$

$$0 = x^2 + 8x + 13$$

$\downarrow$ rooten w/ quad. formula

$$(0, -2.26)$$

$$(0, -5.73)$$



$$\text{Vertex} = (-4, -3)$$

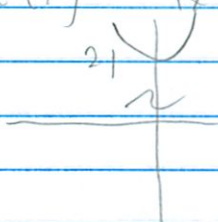
find
x-int
w/ this
form

19. $h(x) = x^2 - 8x + 16$

could do direct
 $(x^2 - 8x) + 16 \rightarrow -\frac{-8}{2} = -4 \rightarrow (-4)^2$
 $(x^2 - 8x + 16) + 16 - 16$
 $(x - 4)^2$
 Vertex = $(4, 0)$

$(x - 4)(x - 4)$
 $(4, 0) (4, 0)$
 x-intercept

25. $f(x) = 4x^2 - 4x + 21$



$4(x^2 - x) + 21 \rightarrow -\frac{-1}{2} \rightarrow (-1/2)^2 = (-4) \pm \sqrt{(-4)^2 - 4(4)(21)}$
 $4(x^2 - x + \frac{1}{4}) + 21 - 4$
 $4(x - \frac{1}{2})^2 + 17$
 $(-\frac{1}{2}, 17)$
 $2(4)$
 No real roots

Find vertex form

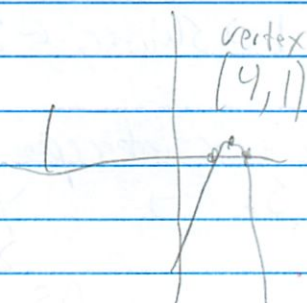
27. $-(x^2 + 2x - 3)$

Vertex
 $(-1, 4)$



$-(x^2 + 2x - 3)$
 $-x^2 - 2x + 3$
 $-\frac{2}{2} \rightarrow 1^2 \rightarrow 1$
 $-(x^2 + 2x) + 3$
 $-(x^2 + 2x + 1) + 3 + 1$
 $-(x + 1)^2 + 4$

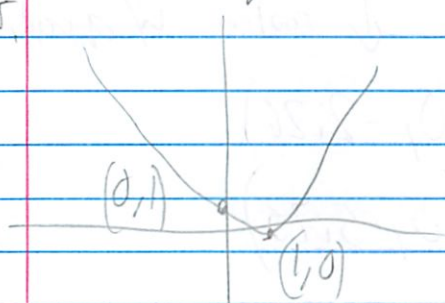
31. $-2x^2 + 16x - 31$



vertex
 $(4, 1)$
 $-2(x^2 - 8) - 31$
 $-2(x^2 - 8 + 16) - 31 + 32$
 $-2(x - 4)^2 + 1$

Write Equation

35.



$y = (x - 1)^2$

$\leftarrow +$ $\rightarrow -$

Write Vertex Form

39.

$(-2, 5)$ vertex
 $(0, 4)$ point

$$y = a(x-h)^2 + k$$

$$9 = a(0 - (-2))^2 + 5$$

$$9 = 4a + 5$$

$$\frac{4}{4} = \frac{4a}{4}$$

$$a = 1$$

$$(x+2)^2 + 5$$

41.

$(\frac{3}{2}, -\frac{3}{4})$ vertex
 $(-2, 4)$ pt

$$y = a(x-h)^2 + k$$

$$4 = a(-2 - \frac{3}{2})^2 - \frac{3}{4}$$

$$4 = 20.25a - \frac{3}{4}$$

$$4\frac{3}{4} = 20.25a$$

$$\frac{19}{20.25} = \frac{19}{20.25}$$

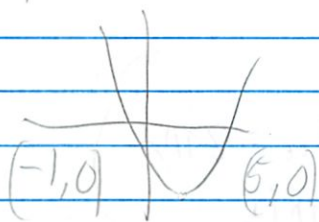
$$\frac{19}{81} = 0$$

$$\frac{19}{81} \left(x - \frac{3}{2} \right) = \frac{3}{4}$$

Find x-int. visually

43.

$$y = x^2 - 4x - 5$$



These are the solutions when
 $y = 0$

Graph + find x-int

47.

$$y = x^2 - 4x$$



$(0, 0)$
 $(4, 0)$

51.

$$y = -1.5(x^2 - 6x - 7)$$



55

Find 2 functions whose graphs have $(-\frac{1}{2}, 0)$ $(-3, 0)$
 $(2, 0)$ $(-5, 0)$

↓ create factors

$$f(x) = (x+3)(x+\frac{1}{2})$$

↑ up
a > 0

↓ down
a < 0

$$(x+3)(2x+1)$$

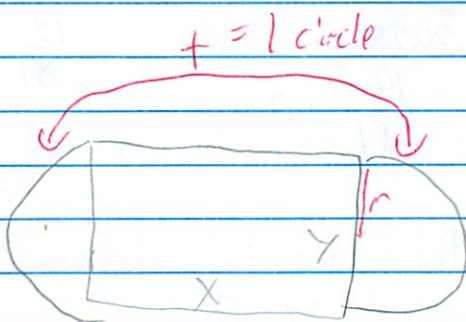
$$2x^2 + 7x + 3$$

$$-[(x+3)(2x+1)]$$

$$-[2x^2 + 7x + 3]$$

$$-2x^2 - 7x - 3$$

61



Outside = 200m

b) $r = \frac{1}{2}y$
 $d = \pi y$

Perce = $2\pi r$
 $2\pi(\frac{1}{2}y)$
 πy

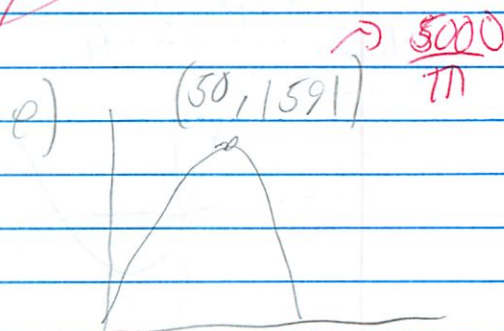
c) $2x + \pi y = 200$

$$\pi y = \frac{200 - 2x}{\pi}$$

$$y = \frac{200 - 2x}{\pi}$$

d) $A = xy$
 $A = x \left(\frac{200 - 2x}{\pi} \right)$

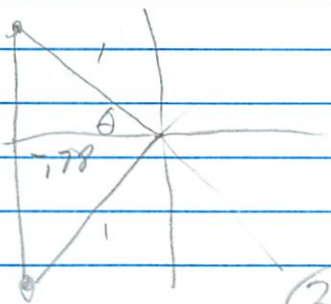
or $\hookrightarrow \frac{2}{\pi} [100 - x^2]$



At 50 ft length,
 the max area exists
 at 1591 ft²

$$\cos \theta = -.78$$

Determine θ where $0 \leq \theta \leq 2\pi$



$$\cos = \frac{x}{r} = \frac{-.78}{1}$$

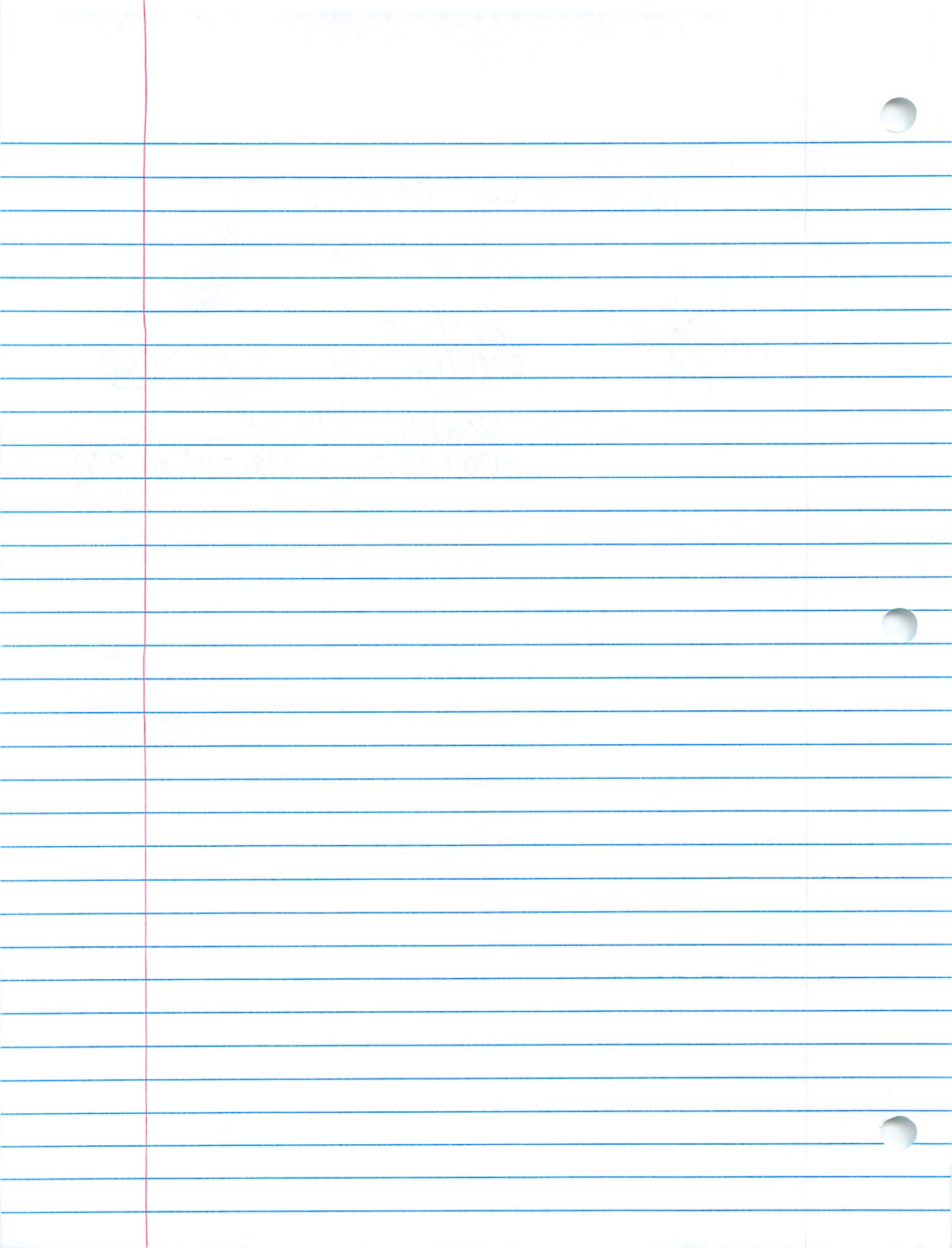
$$\cos^{-1}(-.78)$$

$$\theta = (141.26^\circ) \left(\frac{\pi}{180} \right) = (2.46\pi) \text{ (2nd)}$$

(3rd)

$$180 - 141.26 = 38.74^\circ$$

$$180 + 38.74 = (218.74^\circ \text{ or } 3.82\pi)$$



1. $2x^2 - 10x$

$2x(x-5)$

3. $x^2y - 3x^2$

$x^2(y-3)$

5. $x^2z + y^2z^2$

$z(x^2 + y^2z)$

7. $8x - 16y$

$8(x-2y)$

9. $4a + 20b$

$4(a+5b)$

11. $3x - 6y + 12$

$3(x-2y+4)$

13. $5 + 15n + 45m$

$5(1+3n+9m)$

15. $x^2 + 5x - 36$

$(x-4)(x+9)$

17. $x^2 + 7x - 98$

$(x-7)(x+14)$

19. $x^2 + 6x - 27$

$(x-3)(x+9)$

21. $x^2 - x - 30$

$(x-6)(x+5)$

23. $x^2 - x - 72$

$(x-9)(x+8)$

25. $6x^2 - 55x + 9$

$(x-9)(x-\frac{1}{6}) \Rightarrow (x-9)(6x-1)$

27. $3x^2 + 28x + 49$

$(x+7)(3x+7)$

29. $4x^2 - 9x - 28$

$(x-4)(4x+7)$

31. $2x^2 + 13x + 15$

$(x+\frac{3}{2})(x+5) \Rightarrow (2x+3)(x+5)$

33. $4x^2 - x - 3$

$(x-1)(4x+3)$

2. $4x^2 - 25$

$(2x+5)(2x-5)$

4. $x^2 - 64$

$(x+8)(x-8)$

6. $25x^2 - 9$

$(5x-3)(5x+3)$

8. $16x^2 - 1$

$(4x-1)(4x+1)$

10. $9 - 49x^2$

$(3+7x)(3-7x)$

12. $49 - x^2$

$(7-x)(7+x)$

14. $4x^2y^2 - 9$

$(2xy+3)(2xy-3)$

16. $x^2 + 6x - 16$

$(x+8)(x-2)$

18. $x^2 + x - 42$

$(x+7)(x-6)$

20. $x^2 + 3x - 4$

$(x+4)(x-1)$

22. $x^2 - 6x - 27$

$(x-9)(x+3)$

24. $x^2 - 7x - 8$

$(x-8)(x+1)$

26. $2x^2 - 7x - 72$

~~$(2x-1)(x+6)$~~ $(x+\frac{9}{2})(x-8)$

28. $5x^2 + 39x - 8$

$(5x+4)(x-8)$ $(2x+9)(x-8)$

30. $3x^2 - 23x + 30$

$(3+5)(x-6)$ *check do you know*

32. $5x^2 - 11x - 12$

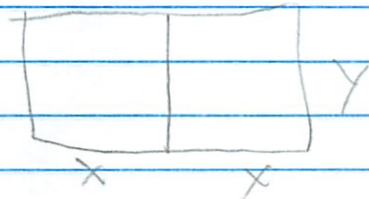
34. $3x^2 + 28x + 9$

2.1 Quadratic More Practice

9/21

p97
62.

200 ft total
fence



x	Area
0	0
5	300
15	700
20	800
25	833
30	800
40	533

$$200 = 4x + 3y$$

$$-4x \quad -4x$$

$$\frac{-4x + 200}{3} = \frac{3y}{3}$$

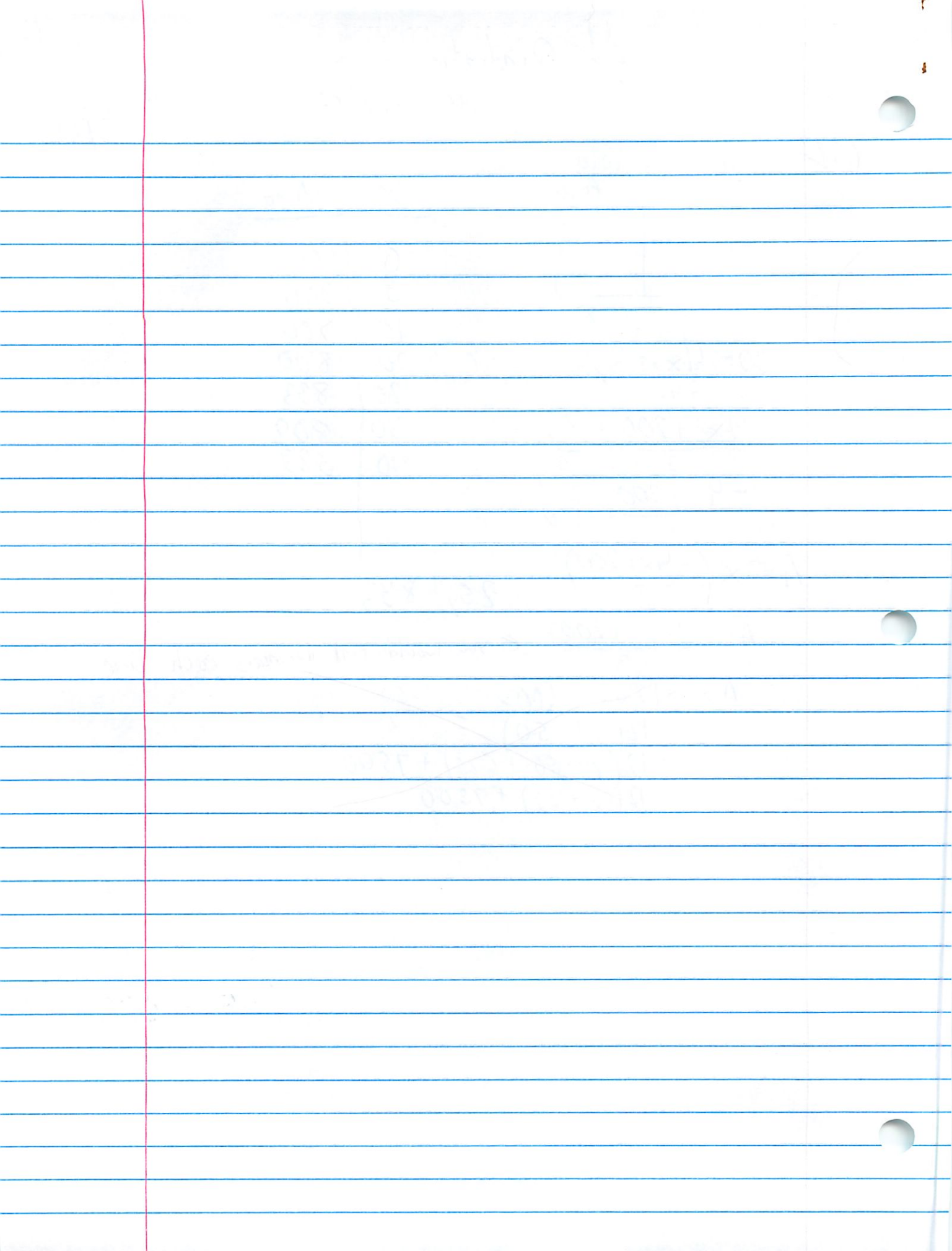
$$\frac{-4x + 200}{3} = y$$

$$A = x \left(\frac{-4x + 200}{3} \right) \quad 25, 833$$

$$A = \frac{-4x^2 + 200x}{3} \quad \leftarrow \text{you would first divide each time}$$

$$\begin{aligned} A &= -12x^2 + 600x \rightarrow -\frac{50}{2} \rightarrow 625 \\ &= -12(x^2 - 50) \\ &= -12(x^2 - 50 + 625) + 7500 \\ &= -12(x - 25) + 7500 \end{aligned}$$

On
Chap 2
Test



Chap 2 Supplement

9/22

Solving Quadratic Equations

- An Application -

Fund Raising

High School wants to buy a new printer for their computer lab.

It is selling hats with the school logo.

- 50 students will pay \$5 for the hat.

For each decrease of 25¢, 10 more students will purchase the hat.

the school must raise \$390.00 but would like to allow as many students as possible to purchase hats

Determine the price at which the school should sell the hats

x = price
y = profit

profit =

$$\text{profit} \geq \$390.00$$

Calc

$$y = -.025x^2 + 6.25$$

x = # 25¢ decrease

$$(5 - .25x)(50 + 10x) = 390$$

$$250 + 37.5 - 2.5x^2$$

25-

$$x^2 - 15x + 56 = 0$$

$$(x-7)(x-8)$$

7 or 8 decrease

$$7: 5 - .25(7) = 3.25$$

$$50 + 10(7) = 120$$

$$8: 5 - .25(8) = 3.00$$

$$50 + 10(8) = 130$$

more students buy

A company determines that it can sell

100 calculators per month if the selling price is \$25. For each \$2 decrease in the price, the

number of calculators sold per month increases by 20.

Find the revenue function and determine the

price the company should sell the calculators for

if they want a revenue of \$3000 per the month

price = people

$$(25 - 2(x)) \cdot (100 + 20x) = 3,000$$

$$2500 + 500x - 200x - 40x^2$$

$$-40x^2 + 300x + 2500 = 3,000$$

$$-40x^2 + 300x - 500 = 0$$

$$x = \frac{-300 \pm \sqrt{300^2 - 4(-40)(-500)}}{2(-40)}$$

$$x = \frac{-300 \pm \sqrt{90,000 - 80,000}}{-80}$$

$$x = \frac{-300 \pm \sqrt{10,000}}{-80}$$

$$x = \frac{-300 \pm 100}{-80}$$

$$x = \frac{-200}{-80} = 2.5 \text{ or } x = \frac{-400}{-80} = 5$$

$x = 2.5$ decrease

$$-40x^2 + 300x - 500 = 0$$

$$12x^2 - 15x + 25$$

$$-40(x^2 - 7.5x) - 500$$

$$-40(x^2 - 7.5x + 5.625) - 500 + 562.5$$

$$-40(x - 3.75)^2 + 6.25$$

$$-40(0)^2 + 6.25$$

$$6.25$$

find roots
not vertex

form

$$(2.5, 0) \rightarrow (25 - 2(2.5)) \cdot (100 + 20(2.5))$$

$$(5, 0) \rightarrow (25 - 2(5)) \cdot (100 + 20(5))$$

\$8000
\$10 decrease $\rightarrow$ \$15
\$5 decrease $\rightarrow$ \$20 + 150 units at 200 unit

A record store sells CDs at \$9.50 each.

For a CD on the top of the chart, the store expects to sell 150 CDs per week. For each

5¢ decrease in the price, the store will sell

approximately 20 more CDs per week. The

store wants to make a profit of \$1050. If each

CD costs the store \$3.00 determine the price

at which they should sell the CDs.

~~\$10.50 + 3 = \$13.50 target selling price~~

(price - people)

their profit $\rightarrow (9.50 - 3.00)(150 + 20(x)) = \1050

$1425 + 190x - 75x - 10x^2 - \$1050 = 0$

$-10x^2 + 115x + 375 = 0$

cooten

no vertices

$\begin{pmatrix} 2.5 & 0 \\ 3 & 0 \end{pmatrix} \rightarrow (9.50 - 3.00)(150 + 20(2.5)) = \1050

or $\begin{pmatrix} 2.5 & 0 \\ 3 & 0 \end{pmatrix}$ Sell for \$8.25 to 200
Sell for \$8 to 210

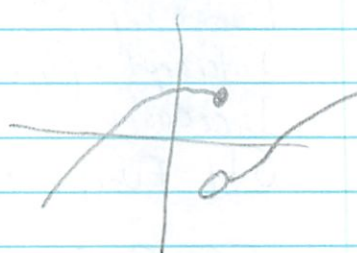
2.2 Polynomial Functions + Graphs

9/24

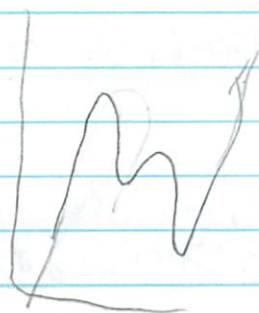
* are continuous



not



smooth, rounded curves



not



Degree of Polynomial

Degree = 1 $\rightarrow$ linear $f(x) = x + 1$

2 $\rightarrow$ quadratic $f(x) = x^2 + x + 1$ (parabolas)

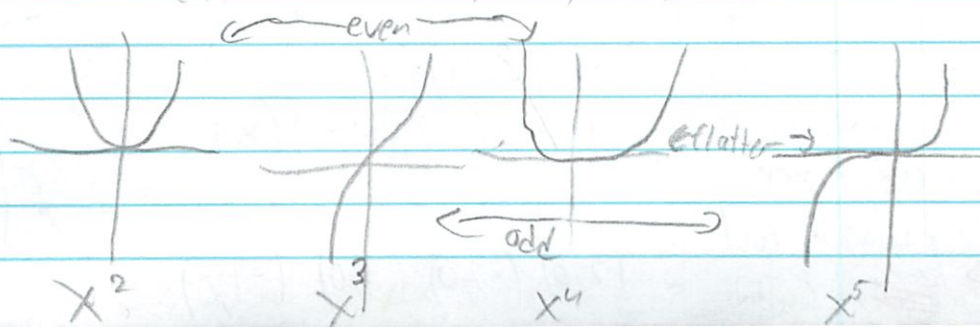
* A polynomial function of degree, n , has the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + b$$

$\uparrow$ linear $\uparrow$ constant

Simplist graphs are the monomials

$$f(x) = x^n \rightarrow x^1, x^2, x^3, x^4, \dots$$



Cubic Function

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

Intercept: $(0,0)$

Increasing

Odd Function (sym. to origin)

$n = \text{odd}$

$$f(x) = x^3 - 2x^2 - x + 1$$

Degree = 3

$n = \text{odd}$

leading coefficient $(a) = 1$

$> 0 \Rightarrow$ Falls to left + rises to right

$$f(x) = -2x^5 - x^2 + 5x + 3$$

rises to the left + falls on the right

leading co-eff $< 0 \Rightarrow n = \text{odd}$

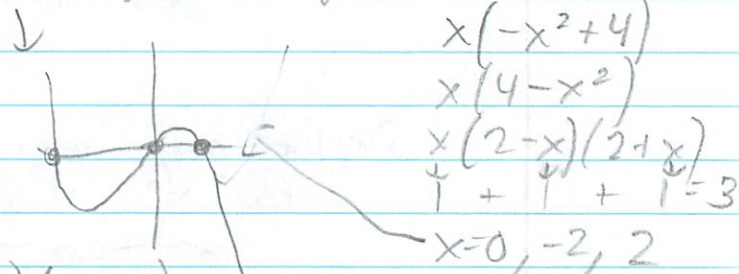
$n = \text{even}$

leading coefficient is $(+)$ $\Rightarrow$ rises to left + ~~falls~~ ^{rises} on the right
 $(-)$ $\Rightarrow$ graph falls to the left + right

p1

$$f(x) = -x^3 + 4x$$

rise to left + falls to right



p2

$$x^4 - 5x^2 + 4$$

$n = 4$

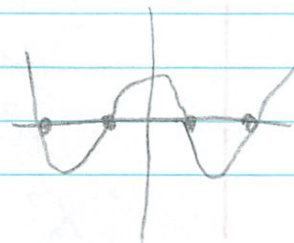
$a = 1 > 0$

rises to left

^{rises}

~~falls~~ right

$$\begin{aligned} &\Rightarrow (x^2 - 4)(x^2 - 1) \\ &(x-2)(x+2)(x-1)(x+1) \\ &\quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ &\quad 1 + 1 + 1 + 1 = 4 \\ &(2,0) (-2,0) (1,0) (-1,0) \end{aligned}$$



p3

$$f(x) = x^5$$

$$x(x^4)$$

$$x(x^2-1)(x^2+1)$$

$$x(x-1)(x+1)(x^2+1)$$

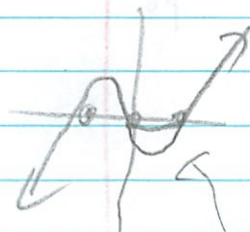
no real zero

falls to left
rises right

$$x=0$$

$$x=-1$$

$$x=1$$



Os of Polynomials

1. The function has at most n real Os
 $\uparrow$ degree

2. The graph has at most $(n-1)$ relative
 extrema

$\uparrow$ minimums + maximums

2
relative
extrema

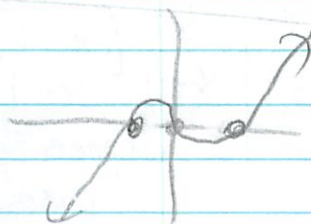
p4

$$f(x) = x^3 - x^2 - 2x \quad n=3$$

$$x(x^2 - x - 2) \quad \text{also}$$

$$x(x-2)(x+1)$$

at most $\left(\begin{array}{l} 3 \text{ real Os} \\ \uparrow x \text{ intercepts} \\ 2 \text{ relative extrema} \end{array} \right.$



p5

$$-2x^4 + 2x^2$$

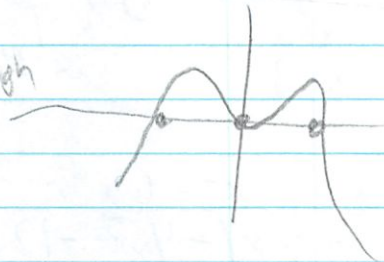
$$-2x^2(x^2 - 1) \leftarrow \text{odd goes through}$$

$$2x^2(x+1)(x-1)$$

$$x=0 \quad x=-1 \quad x=1$$

repeating

factor $\Rightarrow$ take characteristics
of quadratic



Repeated Os

For a polynomial function, a factor of $(x-a)^k$
 $k > 1$ yields a repeated zero of
 multiplicity, k

if $k = \text{odd}$, graph crosses x axis

if $k = \text{even}$, graph only "touches" axis

p6

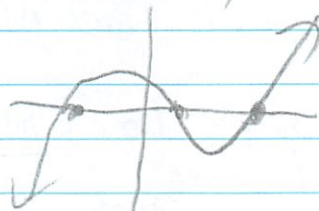
$$f(x) = \underset{\substack{\downarrow \\ 1}}{(x-3)} \underset{\substack{\downarrow \\ 1}}{(x+2)} \underset{\substack{\downarrow \\ 1}}{(x-1)}$$

= degree of k

$$k = 1 + 1 + 1 = k = 1 \text{ for each zero (odd)}$$

graph crosses at each 0

$$(3,0) \quad (-2,0) \quad (1,0)$$



p7

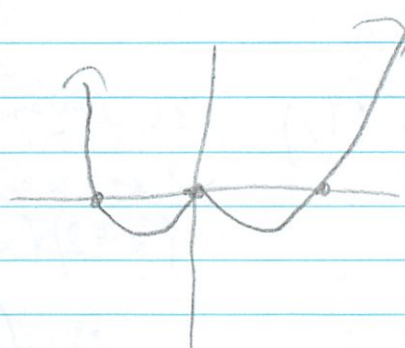
$$x^2 (x^2 - 1)$$

$$x^2 (x-1)(x+1)$$

$x=0 \quad x=1 \quad x=-1$

degree = 4 (↑)

↑ rises, both sides



p8

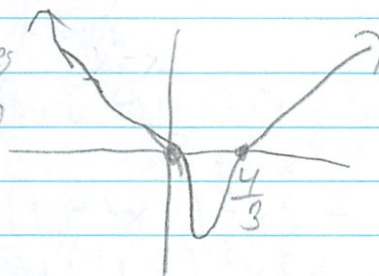
$$3x^4 - 4x^3$$

$$x^3 (3x - 4)$$

$x=0 \quad x=\frac{4}{3}$

$k=3$ $k=1$

④ even, rises both



p9

$$-2x^3 + 6x^2 - \frac{9}{2}x$$

$$-\frac{1}{2}x(-4x^2 + 12x - 9)$$

$$-\frac{1}{2}x(4x^2 - 12x + 9)$$

$x=0 \quad x=1 \quad x=\frac{3}{2}$

$k=1 \quad k=2$

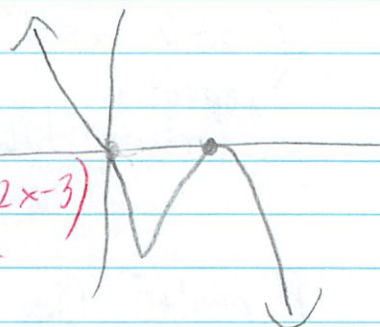
$$(2x-3)(2x-3)$$

$$(2x-3)^2$$

$x = \frac{3}{2}$

$k=2$

⊖ odd
↑ ↓

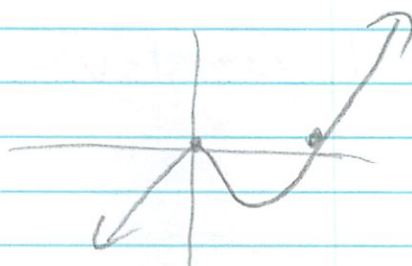


$$f(x) = x^3 - 5x^2$$

$$x^2(x-5)$$

$$x=0 \quad x=5$$

$$t=-2 \quad t=-1$$



⊖ odd
↓ π

↗ get other points to make it more specific

Find the polynomial
w/ given Os

ex

zeros: $-\frac{1}{2}, 3, 3 \rightarrow$

↗
set
up
factors

$$\begin{aligned} & \left(x + \frac{1}{2}\right)(x-3)^2 \\ & (2x+1)(x-3)^2 \\ & (2x+1)(x^2-6x+9) \\ & 2x^3-12x^2+18x+x^2-6x+9 \\ & \underline{(2x^3-11x^2+12x+9)} \end{aligned}$$

ex2

3, $2+\sqrt{11}$, $2-\sqrt{11} \rightarrow$

$$(x-3)(x-(2+\sqrt{11}))(x-(2-\sqrt{11}))$$

$$(x-3)[(x-2)-\sqrt{11}][(x-2)+\sqrt{11}]$$

$$(x-3)[(x-2)^2-11]$$

$$(x-3)[x^2-4x+4-11]$$

$$(x-3)(x^2-4x-7)$$

$$x^3-4x^2-7x-3x^2+12x+21$$

$$\underline{(x^3-7x^2+5x+21)}$$

→

p3

$$1+\sqrt{3}, 1-\sqrt{3} \rightarrow (x-[1+\sqrt{3}]) (x-[1-\sqrt{3}])$$

$$((x-1)+\sqrt{3}) ((x-1)-\sqrt{3})$$

$$(x-1)^2 - 3$$

$$x^2 - 2x + 1$$

$$\rightarrow \textcircled{x^2 - 2x - 2}$$

2.2 Polynomial + Rational Functions

H W

9/24

plus

3,

Match the polynomial function w/ graph

$$-2x^2 - 5x$$

$$x(-2x - 5)$$

$$x=0 \quad x=-\frac{5}{2}$$

$$t=1 \quad t=1$$

$$-2x-5=0$$

$$-\frac{2x-5}{-2} = \frac{5}{2}$$

Even

↑↓



7,

$$x^4 + 2x^3$$

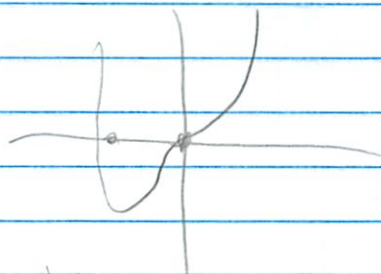
$$x^3(x+2)$$

$$x=0 \quad x=-2$$

$$t=3 \quad t=1$$

Even

↑↑



11,

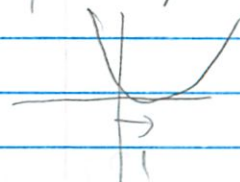
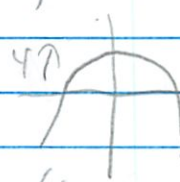
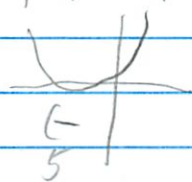
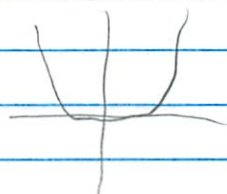
$$y = x^4$$

$$a) (x+5)^4$$

$$b) x^5 + 3$$

$$c) 4 - x^4$$

$$d) \frac{1}{2}(x-1)^4$$



13,

$$3x^3 + 9x + 1$$

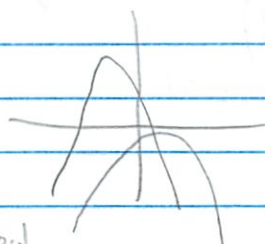
$$3x^3$$



15,

$$-(x^4 - 4x^3 + 6x^2)$$

$$-x^4$$



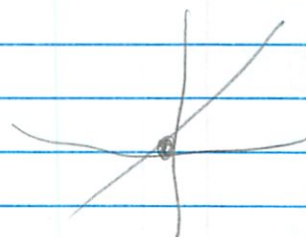
18,

Leading Co-efficient test

$$\frac{1}{2}x^3 + 5x$$

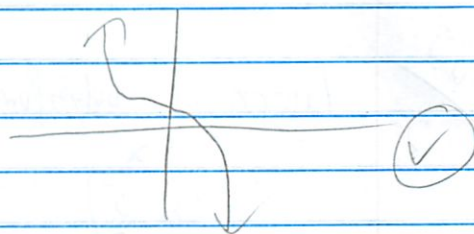
↑ pos, odd
↓

(V) →

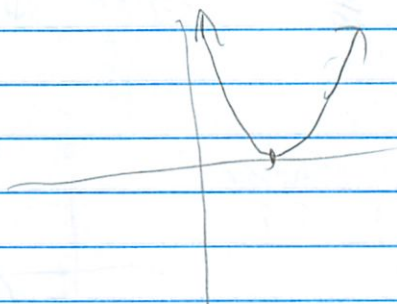


21, $\frac{6-2x+4x^2-5x^3}{3}$

0, odd
↑ ↓



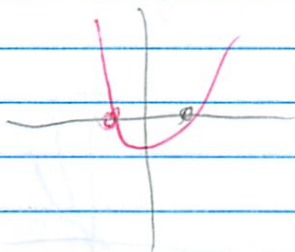
27, Find zeros
 $t^2 - 6t + 9$
 $(t-3)^2$
 $x=3$
 $t=2$



31, $t^3 - 4t^2 + 4t$
 $t(t^2 - 4t + 4)$
 $t(t-2)^2$
 $x=0$ $x=2$
 $t=1$ $t=2$

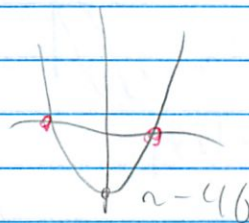


37, $\frac{1}{2}t^4 - \frac{1}{2}$
 $\frac{1}{2}(t^4 - 1)$
 $x=1$
 $t=4$



41, $2x^4 - 2x^2 - 40$
 $2(x^4 - x^2 - 20)$
 $2(x^2 - 5)(x^2 + 4)$
 $\pm\sqrt{5}$ $\uparrow$ non real

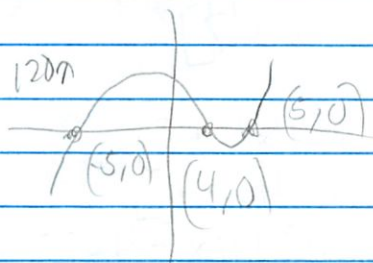
factor



(✓)

plot

43. $x^3 - 4x^2 - 25x + 100$

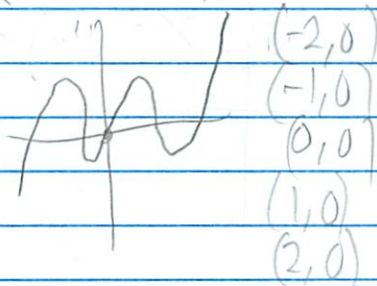


$$\begin{aligned} & x^3 - 4x^2 - 25x + 100 \\ & (x^3 - 4x^2) (-25x + 100) \\ & x^2(x-4) - 25(x-4) \\ & (x^2 - 25)(x-4) \\ & x = \pm 5 \quad x = 4 \end{aligned}$$

✓

46.

$$x^5 - 5x^3 + 4x$$

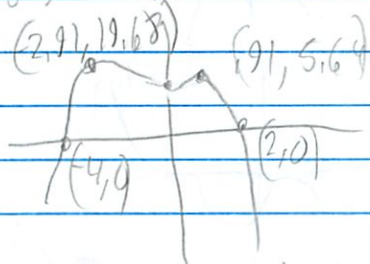


$$\begin{aligned} & x(x^4 - 5x^2 + 4) \\ & x(x^2 - 4)(x^2 - 1) \\ & x = 0 \quad x = \pm 2 \quad x = \pm 1 \end{aligned}$$

✓

48.

$$-\frac{3}{8}x^4 - x^3 + 2x^2 + 5$$



53.

$$0, -2, -3$$

$$\begin{aligned} & (x)(x+2)(x+3) \\ & (x^2 + 2x)(x+3) \\ & (x^3 + 3x^2 + 2x^2 + 6x) \\ & x^3 + 5x^2 + 6x \end{aligned}$$

57.

$$\frac{1+\sqrt{3}}{1-\sqrt{3}}$$

$$\begin{aligned} & (x - [1+\sqrt{3}]) (x - [1-\sqrt{3}]) \\ & [(x-1) + \sqrt{3}] [(x-1) - \sqrt{3}] \\ & (x-1)^2 - 3 \\ & x^2 - 2x + 1 - 3 \\ & x^2 - 2x - 2 \end{aligned}$$

60, $4, 2 + \sqrt{7}, 2 - \sqrt{7}$

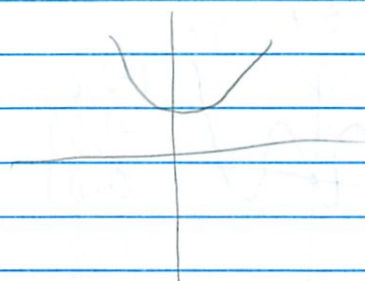
$$\begin{aligned} & (x-4)(x-[2+\sqrt{7}])(x-[2-\sqrt{7}]) \\ & (x-4)((x-2)+\sqrt{7})((x-2)-\sqrt{7}) \\ & (x-4)[(x-2)^2 - 7] \\ & (x-4)[x^2 - 4x + 4 - 7] \\ & (x-4)(x^2 - 4x - 3) \\ & x^3 - 4x^2 + 11x - 12 \\ & x^3 - 8x^2 - 5x - 44 \end{aligned}$$

63, $\frac{1}{4}(t^2 - 2t + 15)$

~~$x = \frac{1}{4}$ $x =$~~ even $\uparrow$

~~$t = 1$ $t =$~~ $\uparrow \uparrow$

no zeros

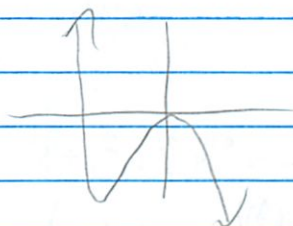


67, $-x^3 - 5x^2$ odd

$-x^2(x+5)$ $\uparrow \downarrow$

$x=0$ $x=-5$

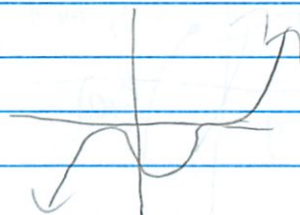
$t=2$ $t=1$



p110

80, $\frac{1}{8}(x+1)^2(x-3)^3$

no symmetry
 x -axis $2x$



85,

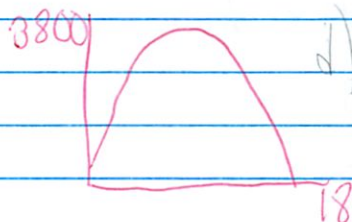


a) Volume = $x(36-2x)^2$

$\uparrow$ $\uparrow$
h base

b) Domain = $\{x: 0 < x < 18\}$

d) Max = $(6, 3456)$



c)

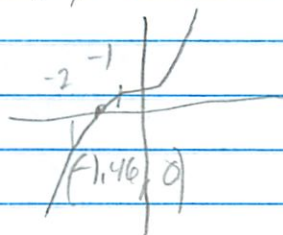
x	y
0	0
1	1156
2	2048
3	2700
4	3136
5	3380
6	3456
7	3388
8	3200

2.2 Intermediate Value Theorem

9/25

- helps you locate the real Os of a polynomial

ex $f(x) = x^3 + x^2 + 1$



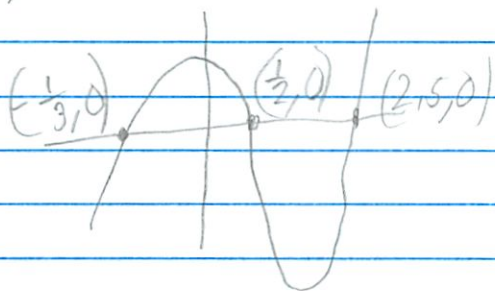
Evaluate at $(-1) = +1$

$(-2) = -3$

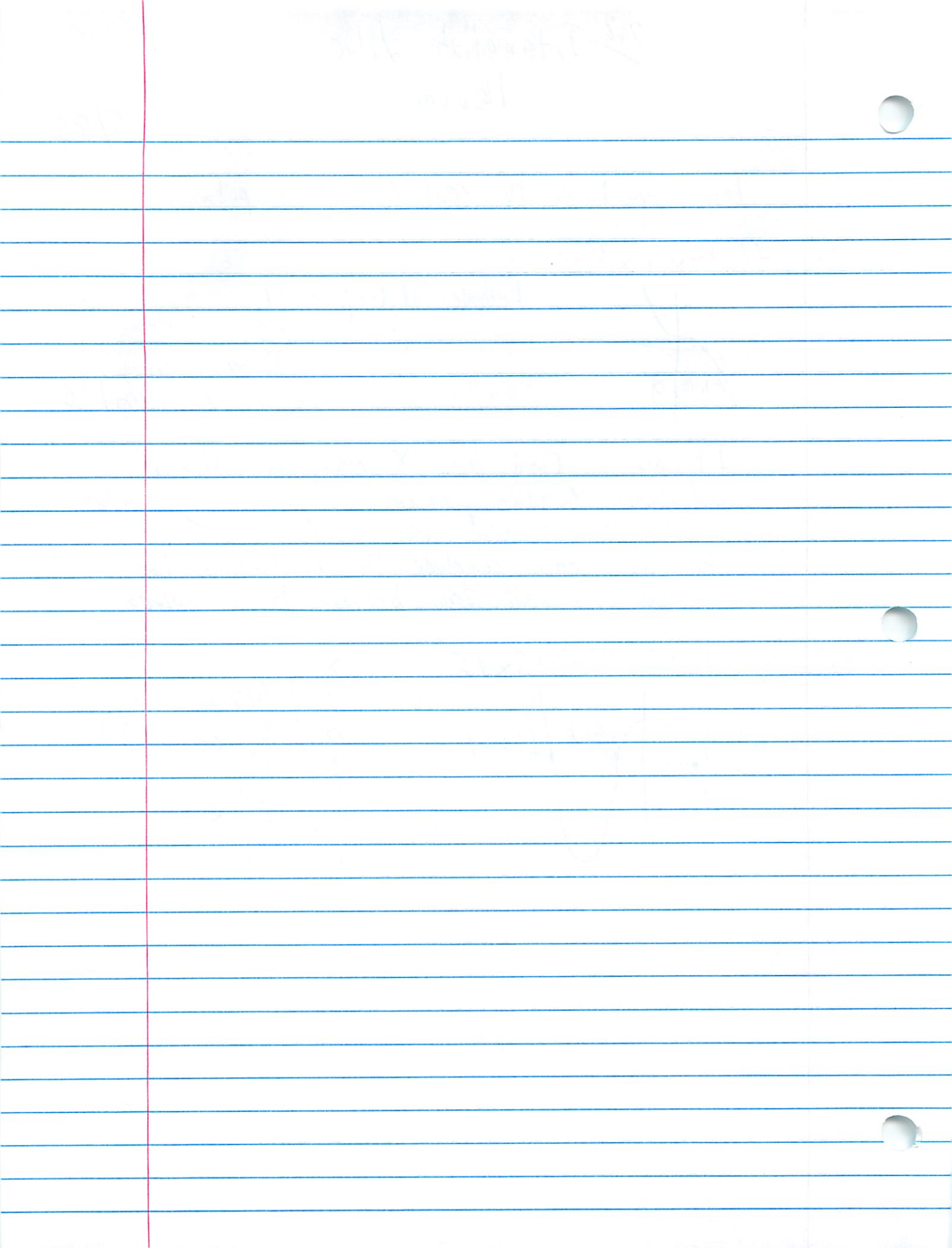
y-values change
side, so is real 0

* If you can find an X-value for which a polynomial function produces a positive y-value and an x-value for which $y = \ominus$ then you can conclude that the function has at least 1 real zero between the 2 values,

ex2 $y = 12x^3 - 32x^2 + 3x + 5$



x	y
-1	-42 $\in$
0	5 $\in$
1	-12
2	-21 $\in$
3	50 $\in$
4	273



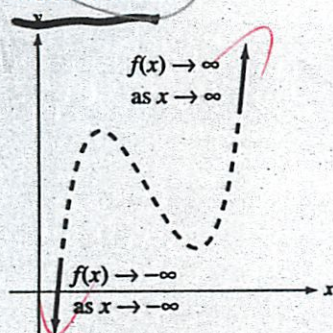
The Leading Coefficient Test

In Example 1, note that all three graphs eventually rise or fall without bound as x moves to the right. Whether the graph of a polynomial eventually rises or falls can be determined by the function's degree (even or odd) and by its leading coefficient, as indicated in the Leading Coefficient Test.

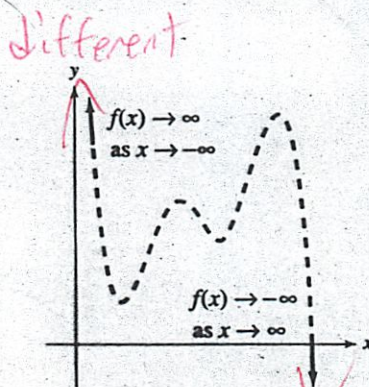
Leading Coefficient Test

As x moves without bound to the left or to the right, the graph of the polynomial function $f(x) = a_n x^n + \dots + a_1 x + a_0$, $a_n \neq 0$, eventually rises or falls in the following manner.

1. When n is odd:

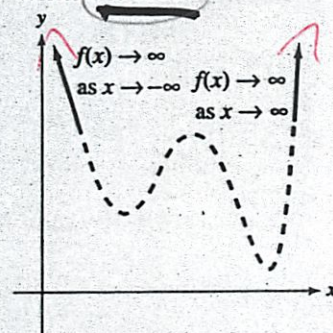


If the leading coefficient is positive ($a_n > 0$), the graph falls to the left and rises to the right.

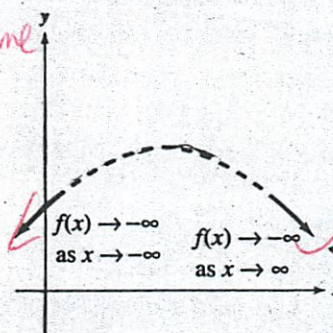


If the leading coefficient is negative ($a_n < 0$), the graph rises to the left and falls to the right.

2. When n is even:



If the leading coefficient is positive ($a_n > 0$), the graph rises to the left and right.



If the leading coefficient is negative ($a_n < 0$), the graph falls to the left and right.

that the dashed portions of the graphs indicate that the test determines the right-hand and left-hand behavior of the graph.

As you continue to study polynomial functions and their graphs, you will find that the degree of a polynomial plays an important role in determining characteristics of the polynomial and its graph.

Exploration

For each function, identify the degree of the function and whether the degree of the function is even or odd. Identify the leading coefficient and whether the leading coefficient is positive or negative. Use a graphing utility to graph each function. Describe the relationship between the degree and sign of the leading coefficient of the function and the right- and left-hand behavior of the graph of the function.

- $y = x^3 - 2x^2 - x + 1$
- $y = 2x^5 + 2x^2 - 5x + 1$
- $y = -2x^5 - x^2 + 5x + 3$
- $y = -x^3 + 5x - 2$
- $y = 2x^2 + 3x - 4$
- $y = x^4 - 3x^2 + 2x - 1$
- $y = -x^2 + 3x + 2$
- $y = -x^6 - x^2 - 5x + 4$

STUDY TIP

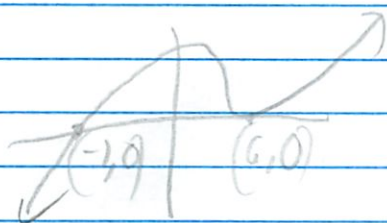
The notation " $f(x) \rightarrow -\infty$ as $x \rightarrow -\infty$ " indicates that the graph falls to the left. The notation " $f(x) \rightarrow \infty$ as $x \rightarrow \infty$ " indicates that the graph rises to the right.

A review of the shapes of the graphs of polynomial functions of degrees 0, 1, and 2 may be used to illustrate the Leading Coefficient Test.

2.2 Finding Equations Practice

9/25

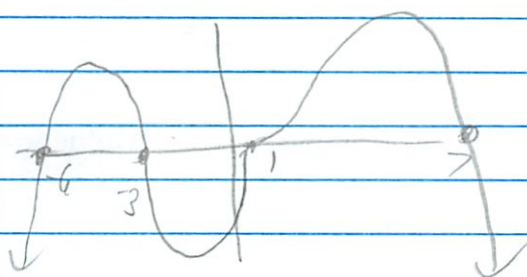
p1



$$(x+1)^1(x-6)^2$$

⊕ $a > 0$, odd ($n=3$)
 $\downarrow \uparrow$

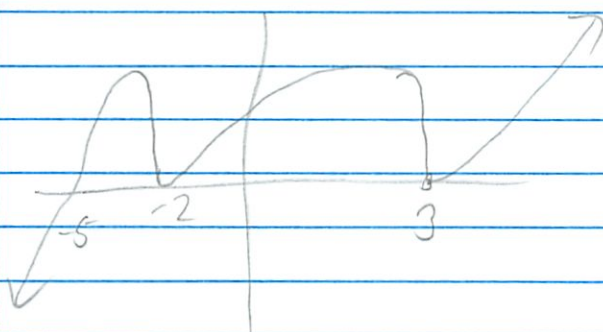
p2



$$(x+6)(x+3)(x-1)^3(x-7)$$

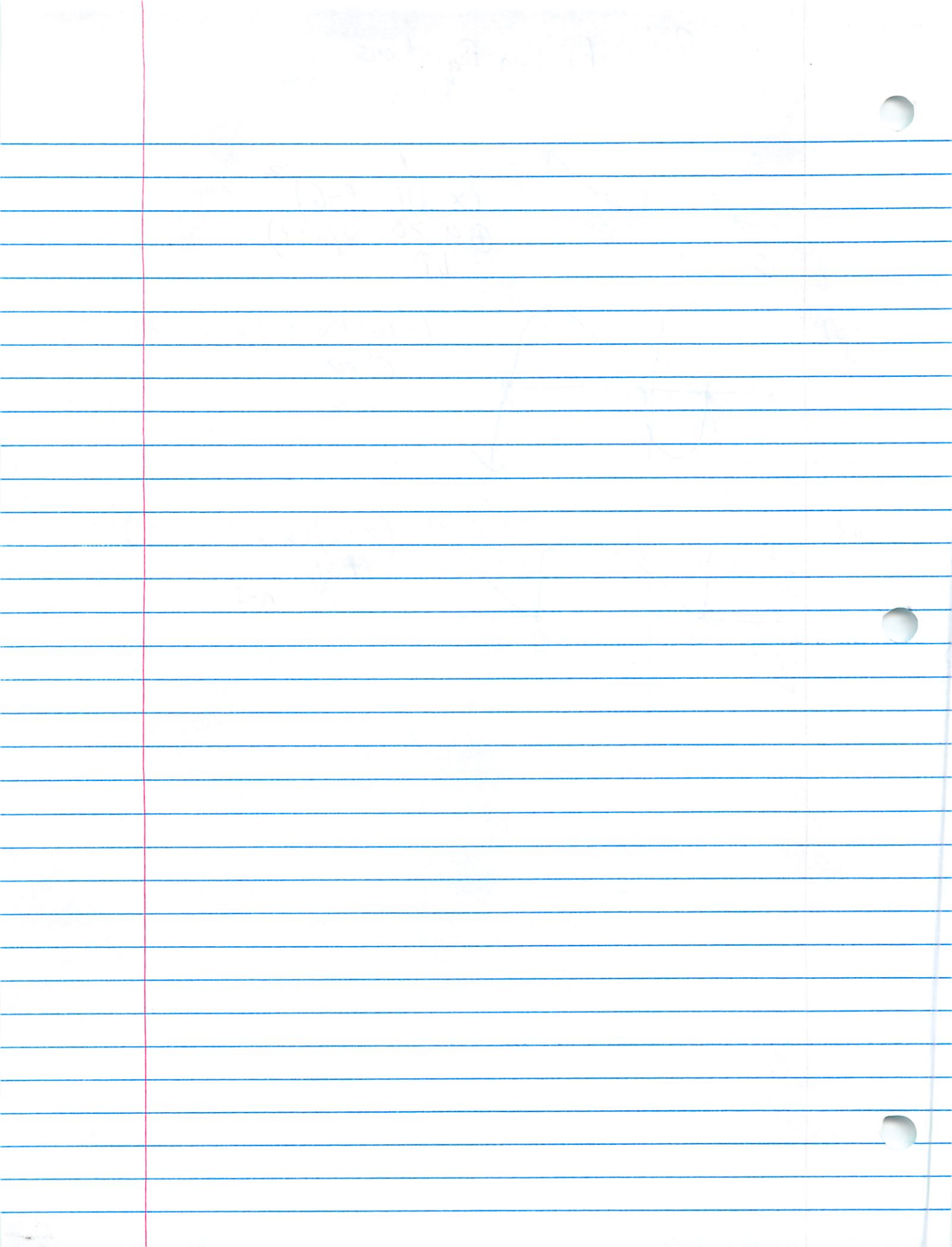
⊖ even
 $\downarrow \downarrow$

p3



$$(x+5)(x+2)^2(x-3)^2$$

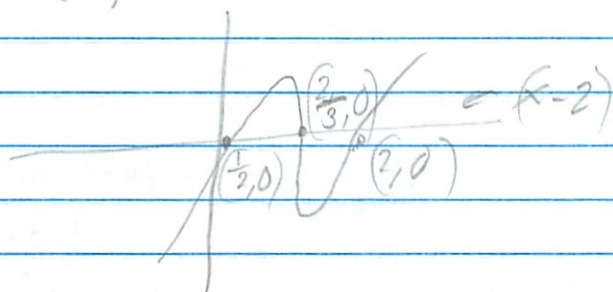
⊕ odd
 $\downarrow \uparrow$ $n=5$



2.3 Real Zeros of Polynomial Functions

9/25

$$f(x) = 6x^3 - 19x^2 + 16x - 4$$



Have a factor
Want other factor

2 methods to find zeros or points on polynomials

- long division + synthetic division

long division

ex1

$$\begin{array}{r} x-2 \overline{) 6x^3 - 19x^2 + 16x - 4} \end{array}$$

divisor dividend
answer = quotient

$$\begin{array}{r} \overline{) 6x^3 - 19x^2 + 16x - 4} \\ \underline{-(6x^3 - 12x^2)} \\ -7x^2 + 16x - 4 \end{array}$$

$$\begin{array}{r} -7x^2 + 16x - 4 \\ \underline{-(7x^2 + 14x)} \\ 2x - 4 \end{array}$$

$$\begin{array}{r} 2x - 4 \\ \underline{-(2x - 4)} \\ 0 \end{array}$$

0

No remainder

$$6x^3 - 19x^2 + 16x - 4 =$$

$$(x-2)(6x^2 - 7x + 2)$$

$$(x-2)(2x-1)(3x-2)$$

$$x=2 \quad x=\frac{1}{2} \quad x=\frac{2}{3}$$

ex2

$$\frac{x^2 + 3x + 5}{x + 1}$$

$$x + 2 \quad x + 1 \mid x^2 + 3x + 5$$

$$-(x^2 + x)$$

$$2x + 5$$

$$-(2x + 2)$$

$$3$$

$(x+2) \mid 3 \leftarrow$ point on graph

$$(-1, 3)$$

not an x-intercept

$$\frac{x^2 + 3x + 5}{x + 1} = x + 2 + \frac{3}{x + 1}$$

ex3

$$\frac{x^3 - 1}{x - 1}$$

$$x^2 + x + 1 \quad x - 1 \mid x^3 - 1$$

$$x^3 - 0x^2 - 0x - 1$$

$$-(x^3 - x^2)$$

$$+x^2 - 0x - 1$$

$$-(x^2 - x)$$

$$x - 1$$

$$-(x - 1)$$

$$0$$

$\rightarrow (1, 0)$ is a point

$$\frac{x^3 - 1}{x - 1} = x^2 + x + 1$$

$d(x) = \text{divisor}$

degree of $d(x)$ must be $\leq$ degree of function

ex4

is x

$$x^2 + 2x - 3$$

for

$$2x^2 + 1 \quad \sqrt{2x^4 + 4x^3 - 5x^2 + 3x - 2}$$

$$-(2x^4 + 4x^3 - 6x^2)$$

$$x^2 + 3x - 2$$

$$-(x^2 + 2x - 3)$$

$$x + 1$$

Synthetic Division

only works for divisors of $x - k$ (linear factor)

ex 4

$$x^4 - 10x^2 - 2x + 4 \div (x - 3)$$

$$\begin{array}{r|rrrrrr} -3 & 1 & 0 & -10 & -2 & 4 \\ & \downarrow & -3 & +9 & -29 & +87 \\ \hline & 1 & -3 & 9 & -31 & 91 \end{array}$$

↓ +
↓ =

↓ ← remainder

$$(x^3 - 3x^2 + 9x - 31 \text{ r } 91)$$

$x^3 - 3x^2 - x - 1 + \frac{1}{x-3}$

point $\odot (-3, 1)$

Synthetic Substitution

$$f(-3) = (-3)^4 - 10(-3)^2 - 2(-3) + 4$$

① remainder

corresponds

p1

$$5x^3 + 6x + 8 \div (x + 2)$$

$$\begin{array}{r|rrrr} -2 & 5 & 0 & 6 & 8 \\ & \downarrow & -10 & +20 & -32 \\ \hline & 5 & -10 & +26 & -24 \\ \hline & 5x^2 + 7x + 10 & -24 \end{array}$$

~~$x + 2$~~

$$5x^2 - 10x + 26 - \frac{24}{x+2}$$

test by subbing in

$$f(-2) = ?$$

$$5(-2)^3 + 6(-2) + 8$$
$$-44 \odot$$

$$p2 \quad 3x^3 - 4x^2 + 5 \div \begin{pmatrix} 2x-3 \\ x-\frac{3}{2} \end{pmatrix}$$

2.3 Real Zeros of

Polynomial Functions HW

9/25

p123

Divide using Long Division

1.

$$\begin{array}{r} 2x+4 \\ x+3 \overline{) 2x^2+10x+12} \\ \underline{-(2x^2+6x)} \\ 4x+12 \\ \underline{-(4x+12)} \\ 0 \end{array}$$

$$(2x+4)$$

$x \neq -3$

3.

$$\begin{array}{r} x^2-3x+1 \\ 4x+5 \overline{) 4x^3-7x^2-11x+5} \\ \underline{-(4x^3+5x^2)} \\ -12x^2-11x+5 \\ \underline{-(3x^2-15)} \\ 4x+5 \\ \underline{-(4x+5)} \\ 0 \end{array}$$

$$(x^2-3x+1)$$

5.

5.

$$\begin{array}{r} 7 \\ x+2 \overline{) 7x+3} \\ \underline{-(7x+14)} \\ -11 \end{array}$$

$$\frac{7 + \frac{-11}{x+2}}$$

7.

$$\begin{array}{r} 3x+5 \\ 2x^2+1 \overline{) 6x^3+10x^2+x+8} \\ \underline{-(6x^3+0x^2+3x)} \\ 10x^2-2x+8 \\ \underline{-(10x^2-0x+5)} \\ -2x+3 \end{array}$$

$$\frac{3x+5 - \frac{2x+3}{2x^2+1}}$$

9.

9.

$$\begin{array}{r} x \\ x^2+1 \overline{) x^3-0x^2-0x-9} \\ \underline{-(x^3-0x^2+x)} \\ -x-9 \\ \cdot 1 = x+9 \end{array}$$

$$\frac{x - \frac{x+9}{x^2+1}}$$

p12²

11. $\frac{2x}{(x-1)^2} \int \frac{2x^3 - 4x^2 - 15x + 5}{x^2 - 2x + 1} \int \frac{2x^3 - 4x^2 - 15x + 5}{(x-1)^2}$
 $-(2x^3 - 4x^2 + 2x)$
 $-17x + 5$

$\frac{2x - 17x + 5}{(x-1)^2}$

Synthetic Division

13. $(3x^3 - 17x^2 + 15x - 25) / (x - 5)$

$+5 \int$ 3 -17 15 -25
 $\downarrow +5 \nearrow +15 \downarrow +10 \nearrow +25$
 3 -2 5 0
 $(3x^2 - 2x + 5)$

15. $(6x^3 + 7x^2 - x + 26) / (x - 3)$

$+3 \int$ 6 7 -1 26
 $\downarrow +18 \nearrow +75 \downarrow +222$
 6 25 74 248
 $(6x^2 + 25x + 74 + \frac{248}{x-3})$

17. $(9x^3 - 18x^2 - 16x + 32) / (x - 2)$

$+2 \int$ 9 -18 -16 32
 $\downarrow +18 \nearrow 0 \downarrow -32 \nearrow +$
 9 0 -16 0
 $(9x^2 - 16)$

19. $(x^3 + 512) / (x + 8)$

$$\begin{array}{r|rrrr} 8 & 1 & 0 & 0 & 512 \\ & \downarrow & \nearrow 8 & \nearrow -64 & \nearrow -512 \\ & 1 & -8 & -64 & 0 \end{array}$$

$(x^2 - 8x - 64)$

21. $(4x^3 + 16x^2 - 23x - 15) / (x + \frac{1}{2})$

$$\begin{array}{r|rrrr} \frac{1}{2} & 4 & 16 & -23 & -15 \\ & \downarrow & \nearrow 2 & \nearrow 7 & \nearrow -15 \\ & 4 & 14 & -30 & 0 \end{array}$$

$(4x^2 + 14x - 30)$

23. $\frac{x^2}{x+2} = x - 2 + \frac{4}{x+2}$

Divide this to get



25. $x-4 \overline{) x^3 - x^2 - 14x + 11}$

$$\begin{array}{r} x^2 + 3x - 2 \\ -(x^3 - 4x^2) \\ \hline 3x^2 - 14x + 11 \\ -(3x^2 - 12x) \\ \hline -2x + 11 \\ -(2x - 8) \\ \hline 3 \end{array}$$

$x-k$ = linear factor

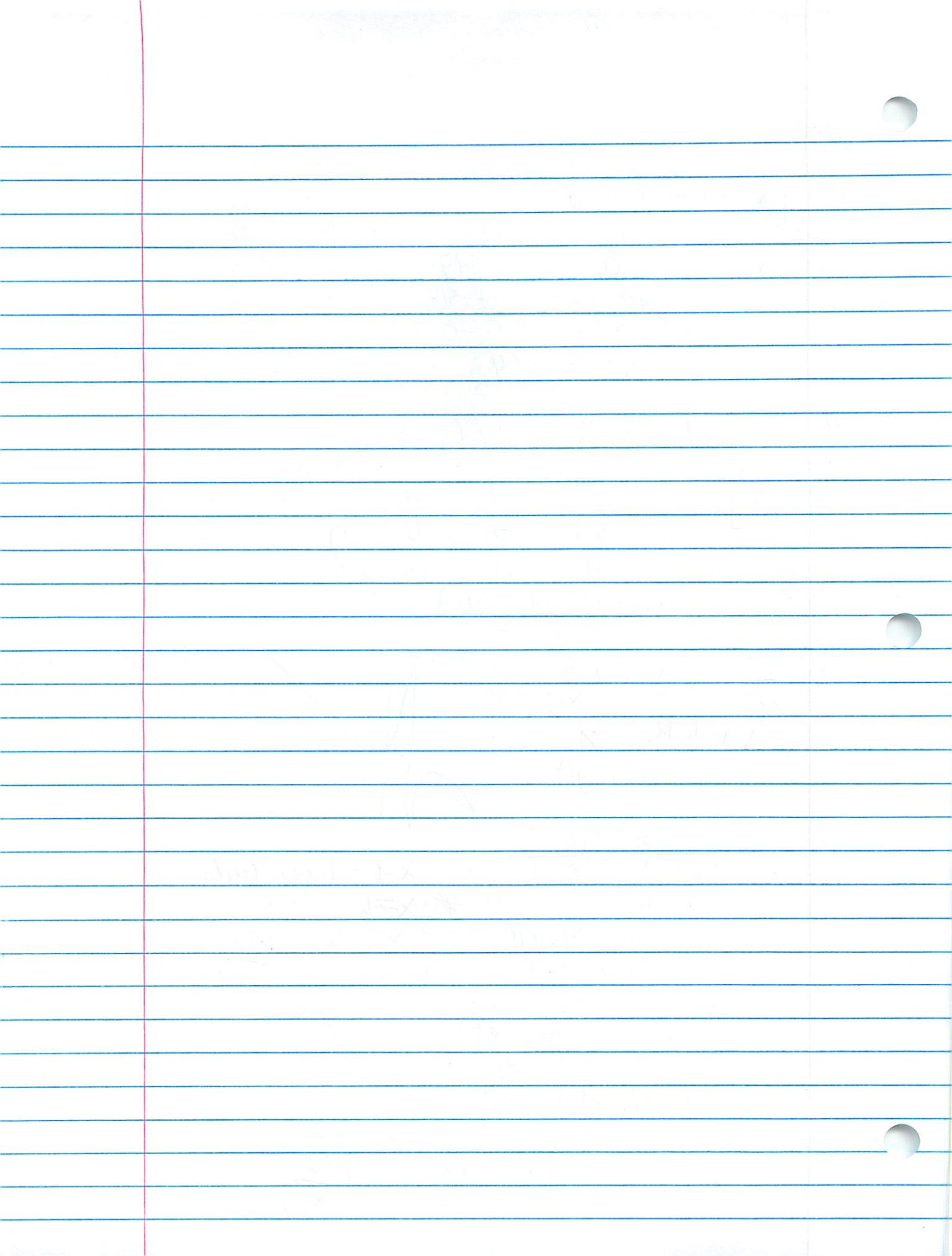
* $x=k$

$$\frac{x^2 + 3x - 2 + 3}{x-4}$$

what's missing

$f(x) = d(x)q(x) + r(x)$

$$(x-4) \overline{) x^2 + 3x - 2} + \frac{3}{x-4}$$



2.3 Part 2 Real Zeros of Polynomial Functions

9/26

Remainder Theorem

If a polynomial $f(x)$ is divided by $(x-k)$
the remainder, $r = f(k)$.

It tells you that synthetic division can be
used to evaluate a polynomial function

ex1 $f(x) = 3x^3 + 8x^2 + 5x - 7$ Use remainder theorem
to eval at $x = -2$

$$\begin{array}{r|rrrr} -2 & 3 & 8 & 5 & -7 \\ & \downarrow & \nearrow -6 & \nearrow -4 & \nearrow 2 \\ & 3 & 2 & 1 & (-9) \end{array} \quad \text{pt: } (-2, -9)$$

$$f(-2) = -3(-2)^3 + 8(-2)^2 + 5(-2) - 7$$

-9 (✓)

ex2 $f(x) = -3x^3 + 8x^2 + 10x - 8$ $k = 2 + \sqrt{2}$

$$\begin{array}{r|rrrr} 2+\sqrt{2} & -3 & 8 & 10 & -8 \\ & \downarrow & -6+3\sqrt{2} & 4+5\sqrt{2} & -2-4\sqrt{2} \\ & -3 & 2-3\sqrt{2} & 8-4\sqrt{2} & (0) \end{array}$$

foil

$$(2+\sqrt{2})(2-3\sqrt{2})$$

$$4 - 6\sqrt{2} + 2\sqrt{2} - 6$$

$$-2 = -4\sqrt{2}$$

$$(2+\sqrt{2})(8-4\sqrt{2})$$

$$16 - 8\sqrt{2} + 8\sqrt{2} - 8$$

$$8$$

$$-3x^2 + 2 - 3\sqrt{2}x + 8 - 4\sqrt{2}$$

$$(2+\sqrt{2}, 0)$$

Sub in
(2+√2)
for x
+ try $\Rightarrow 0 =$

The Factor Theorem

A polynomial has a factor $(x-k)$ if and only if $f(k)=0$

ex1

ex1 repeated division

show that $x-2$ and $x+3$ are factors of $f(x) = 2x^4 + 7x^3 - 4x^2 - 27x - 18$

$$\begin{array}{r} 2 \overline{) 2 \quad 7 \quad -4 \quad -27 \quad -18} \\ \underline{2 \quad 4 \quad 22 \quad 36} \\ 2 \quad 11 \quad 18 \quad 9 \end{array}$$

subset of

$$2x^3 + 11x^2 + 18x + 9$$

$$\begin{array}{r} -3 \overline{) 2 \quad 11 \quad 18 \quad 9} \\ \underline{-6 \quad -15 \quad -9} \\ 2 \quad 5 \quad 3 \end{array}$$

Can use here

~~a.c / r x~~
~~b / c +~~

$$2x^2 + 5x + 3$$

= find other zeros

$$\rightarrow (2x+3)(x+1) = 0$$

$$(-1.5, 0) \quad (-1, 0)$$

$$f(x) = (x-2)(x+3)(2x+3)(x+1)$$

ex2

$$48x^3 - 80x^2 + 41x - 6 = 0 \rightarrow x = \frac{2}{3}$$

$$\begin{array}{r} \frac{2}{3} \overline{) 48 \quad -80 \quad 41 \quad -6} \\ \underline{32 \quad -32 \quad 6} \\ 48 \quad -48 \quad 9 \end{array}$$

$$48x^2 - 48x + 9 = 0$$

GCF
Factor

$$3(16x^2 - 16x + 3)$$

$$3(4x-1)(4x-3)$$

$$f(x) = (3x-2)(4x-1)(4x-3) \cdot 3$$

Rational Zero Test

- gives all the possible rational zeros of a polynomial

factors of a constant term
factors of leading coefficient

ex Find any possible rational zeros of $f(x) = x^3 + x + 1$

$$\frac{\text{constant}}{\text{coefficient}} \quad \frac{1, -1}{1, -1} \rightarrow \frac{1}{1}, \frac{1}{-1}, \frac{-1}{1}, \frac{-1}{-1}$$

$(1, -1)$ are possible rational zeros

Need to check:

$$f(1) = (1)^3 + (1) + 1 = 3 \text{ (not actually a zero)}$$
$$f(-1) = (-1)^3 + (-1) + 1 = -1 \text{ (not a zero)}$$

Are no rational zeros

ex2 $f(x) = 4x^5 - 8x^4 - 5x^3 + 10x^2 - 2$

$$\begin{array}{l} \text{factor } (-2) \quad -\pm 1, \pm 2 \\ \text{factor } (4) \quad \pm 1, \pm 2, \pm 4 \end{array} \quad \begin{array}{l} 1, -1, 2, -2, 1, -1, 2, -2 \\ 1, 1, 1, 1, -1, -1, -1, -1 \end{array}$$
$$\frac{1}{2}, \frac{-1}{2}, \frac{2}{2}, \frac{-2}{2}, \frac{1}{4}, \frac{-1}{4}, \frac{2}{4}, \frac{-2}{4}, \frac{1}{4}, \frac{-1}{4}, \frac{2}{4}, \frac{-2}{4}, \frac{1}{4}, \frac{-1}{4}, \frac{2}{4}, \frac{-2}{4}$$

$$1, -1, 2, -2, \frac{1}{2}, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{4}$$

$$\pm \frac{1}{4}, \pm \frac{1}{2}, \pm 1, \pm 2$$

Put in calc + graph

$$(-1, -\frac{1}{2}, \frac{1}{2}, 1, 2)$$

find definite zeros

p1

$$2x^3 + 3x^2 - 8x + 3$$

$$\frac{\text{factors } (3)}{\text{factors } (2)} = \frac{\pm 1, \pm 3}{\pm 1, \pm 2} \rightarrow \pm 1, \pm 3, \pm \frac{3}{2}, \pm \frac{1}{2}$$

possible

pick one + try

$$\begin{array}{r} (x-1) \overline{) 2 \ 3 \ -8 \ 3} \\ \underline{2 \ 2 \ 5 \ -3} \\ 2 \ 5 \ -3 \ (0) \ (0) \end{array}$$

$$2x^2 + 5x - 3$$

$$(2x-1)(x+3) = 0$$

$$f(x) = (x-1)(2x-1)(x+3) \quad \text{actual}$$

p2

$$10x^3 - 15x^2 - 6x + 12$$

$$\left. \begin{array}{l} \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12 \\ \pm 1, \pm 2, \pm 5, \pm 10 \end{array} \right\} 32 \text{ possible rationals}$$

Input into calc + eyeball a possible root

$$\begin{array}{r} 2 \overline{) 10 \ -15 \ -6 \ 12} \\ \underline{20 \ 10 \ -12} \\ 10 \ 5 \ -6 \ (0) \ (0) \end{array}$$

factor $10x^2 + 5x - 6$
 not factorable
 use rooten

p124

52, $x^5 - x^4 - 3x^3 + 5x^2 - 2x = 0$ need to
 $x(x^4 - x^3 - 3x^2 + 5x - 2)$ get constant

$\pm 1, \pm 2$ $\pm 1, \pm 2$
 ± 1

1] $\begin{array}{cccccc} 1 & -1 & -3 & 5 & -2 & \\ \downarrow & \nearrow & \nearrow & \nearrow & \nearrow & \\ 1 & 0 & -3 & 2 & (0) & (0) \end{array}$

$x^3 - 3x + 2$

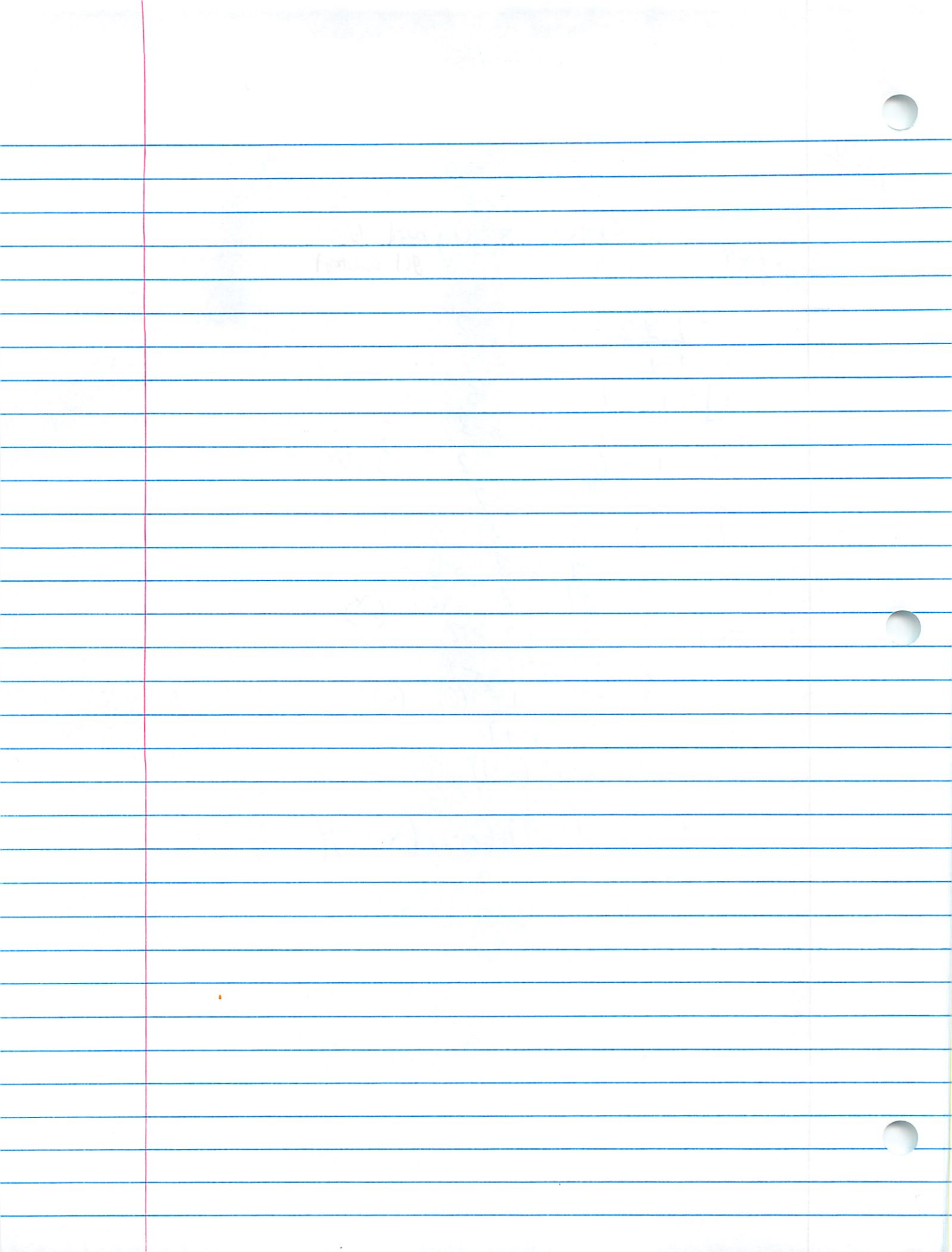
-1] $\begin{array}{cccc} 1 & 0 & -3 & 2 \\ \downarrow & \nearrow & \nearrow & \nearrow \\ 1 & -1 & 0 & (2) \end{array}$ (X)

-2] $\begin{array}{cccc} 1 & 0 & -3 & 2 \\ \downarrow & \nearrow & \nearrow & \nearrow \\ 1 & -2 & 1 & (0) \end{array}$ (0) (0)

$x^2 - 2x + 1$

$(x-1)(x-1)$

$f(x) = (x-1)(x-1)(x+2)(x-1)(x)$



Real Zeros Polynomial Functions

Part 2 HW

9/26

p124

27. $x^3 + 3x^2 - 2x - 14 \div \sqrt{2}$

$$\begin{array}{r} \sqrt{2} \overline{) \begin{array}{rrrr} 1 & 3 & -2 & -14 \\ \downarrow & \sqrt{2} & 3\sqrt{2}+2 & \rightarrow 6 \\ 1 & 3+\sqrt{2} & 3\sqrt{2} & (-8) \end{array}} \\ x^2 + (3+\sqrt{2})x + 3\sqrt{2} \end{array}$$

$f(x) = d(x)q(x) + r(x)$

$\sqrt{2} \mid (x^2 + (3+\sqrt{2})x + 3\sqrt{2}) + \frac{-8}{\sqrt{2}}$

29.

$$\begin{array}{l} \sqrt{x^3 - 6x^2 - 12x - 4} \div (1 - \sqrt{3}) \\ \left(\begin{array}{l} x = [1 - \sqrt{3}] \\ (x - 1 + \sqrt{3}) \end{array} \right) \mid \left(\begin{array}{l} 4x^2 + (-2 - 4\sqrt{3})x + (-2 - 2\sqrt{3}) \end{array} \right) \\ \begin{array}{r} 4x^3 - 6x^2 - 12x - 4 \\ - (4x^3 - (4 - 4\sqrt{3})x^2) \\ \hline (-2 - 4\sqrt{3})x^2 - 12x \\ - (-2 - 4\sqrt{3})x^2 - (10 - 2\sqrt{3})x \\ \hline (-2 - 2\sqrt{3})x - 4 \\ - (-2 - 2\sqrt{3})x - 4 \\ \hline 0 \end{array} \end{array}$$

foil

can also
do w/
synthetic
division

32. $g(x) = x^6 - 4x^4 + 3x^2 + 2$

a) $2 \overline{) \begin{array}{rrrrrrr} 1 & 0 & -4 & 0 & 3 & 0 & 2 \\ \downarrow & 2 & 8 & 0 & 0 & 6 & 12 \\ 1 & 2 & 0 & 0 & 3 & 3 & 8 \end{array}}$

b) $-4 \overline{) \begin{array}{rrrrrrr} 1 & 0 & -4 & 0 & 3 & 0 & 2 \\ \downarrow & -4 & 16 & -48 & 192 & -768 & 3072 \\ 1 & -4 & 12 & -48 & 195 & -766 & 3074 \end{array}}$

c) $3 \overline{) \begin{array}{rrrrrrr} 1 & 0 & -4 & 0 & 3 & 0 & 2 \\ \downarrow & 3 & -12 & 36 & -108 & 243 & -729 \\ 1 & 3 & 8 & 36 & -105 & 243 & -727 \end{array}}$

$$d) \begin{array}{r} -5 \end{array} \begin{array}{r} 1 \quad 0 \quad -4 \quad 0 \quad 3 \quad 0 \quad 2 \\ \downarrow \nearrow 5 \nearrow 25 \nearrow -105 \nearrow 525 \nearrow -2625 \\ 1 \nearrow -5 \nearrow 21 \nearrow -105 \nearrow 528 \nearrow -2623 \end{array}$$

$$33) 3x^3 + 5x^2 - 10x + 1$$

$$a) \begin{array}{r} 3 \end{array} \begin{array}{r} 3 \quad 5 \quad -10 \quad 1 \\ \downarrow \nearrow 9 \nearrow 42 \nearrow 96 \\ 3 \nearrow 14 \nearrow 32 \nearrow (97) \end{array}$$

$$b) \begin{array}{r} \frac{1}{3} \end{array} \begin{array}{r} 3 \quad 5 \quad -10 \quad 1 \\ \downarrow \nearrow 1 \nearrow 2 \nearrow \frac{8}{3} \\ 3 \nearrow 6 \nearrow -8 \nearrow (-5/3) \end{array}$$

$$c) \begin{array}{r} -2 \end{array} \begin{array}{r} 3 \quad 5 \quad -10 \quad 1 \\ \downarrow \nearrow -6 \nearrow 2 \nearrow 16 \\ 3 \nearrow -1 \nearrow -8 \nearrow (17) \end{array}$$

$$d) \begin{array}{r} -5 \end{array} \begin{array}{r} 3 \quad 5 \quad -10 \quad 1 \\ \downarrow \nearrow -15 \nearrow 50 \nearrow -200 \\ 3 \nearrow -10 \nearrow 40 \nearrow (-199) \end{array}$$

$$37) 2x^3 - 15x^2 + 27x - 10 = 0 \quad x = \frac{1}{2}$$

$$\begin{array}{r} \frac{1}{2} \end{array} \begin{array}{r} 2 \quad -15 \quad 27 \quad -10 \\ \downarrow \nearrow 1 \nearrow -7 \nearrow 10 \\ 2 \nearrow -14 \nearrow 20 \nearrow (0) \end{array}$$

$$2x^2 - 14x + 20$$

$$2(x^2 - 7x + 10)$$

$$(x-5)(x-2)$$

$$(x - \frac{1}{2})(x-5)(x-2)(2)$$

$$\hookrightarrow (2x-1)$$

41. $x^4 + 4x^3 - 15x^2 + 58x - 40$ $(x-5)(x+4)$

$$\begin{array}{r} -5 \end{array} \begin{array}{rrrrrr} 1 & -4 & -15 & 58 & -40 \\ & \downarrow & \nearrow +5 & \nearrow 5 & \nearrow -50 & \nearrow 40 \\ & 1 & & -10 & 8 & \textcircled{0} \end{array}$$

$$x^3 + x^2 - 10x + 8$$

$$\begin{array}{r} -4 \end{array} \begin{array}{rrrr} 1 & 1 & -10 & 8 \\ & \downarrow & \nearrow -4 & \nearrow 12 & \nearrow -8 \\ & 1 & & -3 & 2 & \textcircled{0} \end{array}$$

$$x^2 - 3x + 2$$

$$(x-2)(x-1)(x-5)(x+4)$$

43. $6x^3 + 41x^2 - 9x - 14$ $(2x+1)(3x-2)$

$$\begin{array}{r} -\frac{1}{2} \end{array} \begin{array}{rrrr} 6 & 41 & -9 & -14 \\ & \downarrow & \nearrow -3 & \nearrow -19 & \nearrow 14 \\ & 6 & & 38 & -28 & \textcircled{0} \end{array}$$

$$6x^2 + 38x - 28$$

$$\begin{array}{r} \frac{2}{3} \end{array} \begin{array}{rrr} 6 & 38 & -28 \\ & \downarrow & \nearrow 4 & \nearrow 28 \\ & 6 & & 42 & \textcircled{0} \end{array}$$

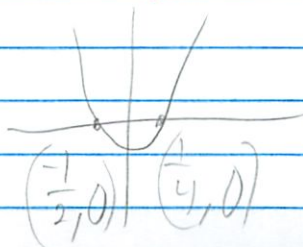
$$6x + 42$$

$$6(x+7)(2x+1)(3x-2)$$

Rational O Test

47. $2x^4 - 17x^3 + 35x^2 + 9x - 45$ E Cop - problem wrong

factors(-4) = $\pm 1, \pm 2, \pm 4$ different
factors(2) = $\pm 1, \pm 2$ $\pm 1, \pm 2, \pm 4, \pm \frac{1}{2}$



51. $2y^4 + 7y^3 - 26y^2 + 23y - 6 = 0$

calc) $\begin{pmatrix} 5, 0 \\ -6, 0 \end{pmatrix} \quad \frac{1}{2} \begin{array}{ccccc} 2 & 7 & -26 & 23 & -6 \\ \downarrow & \nearrow 1 & \nearrow 4 & \nearrow -11 & \nearrow 6 \\ 2 & 8 & -22 & 12 & \textcircled{0} \end{array}$

$2x^3 + 8x^2 - 22x + 12$

$-6 \begin{array}{ccccc} 2 & 8 & -22 & 12 \\ \downarrow & \nearrow -12 & \nearrow 24 & \nearrow -12 \\ 2 & -4 & 2 & \textcircled{0} \end{array}$

$2x^2 - 4x + 2$

$2(x^2 - 2x + 1)$

$(x-1)(x-1)$

$(2)(2x-1)(x+6)(x-1)(x-1)$
46

55. $x^5 - 7x^4 + 10x^3 + 14x^2 - 24x$

$x(x^4 - 7x^3 + 10x^2 + 14x - 24)$

calc) $\begin{pmatrix} -1, 4 \\ 0, 0 \end{pmatrix}$

$\begin{pmatrix} 1, 4 \\ 1, 4 \end{pmatrix}$

$\begin{pmatrix} 3, 0 \\ 4, 0 \end{pmatrix}$

$\begin{pmatrix} 3, 0 \\ 4, 0 \end{pmatrix}$

$\begin{pmatrix} 3, 0 \\ 4, 0 \end{pmatrix}$

$3 \begin{array}{ccccc} 1 & -7 & 10 & 14 & -24 \\ \downarrow & \nearrow 3 & \nearrow -12 & \nearrow -6 & \nearrow 24 \\ 1 & -4 & -2 & 8 & \textcircled{0} \end{array}$

$x^3 - 4x^2 - 2x + 8$

$4 \begin{array}{ccccc} 1 & -4 & -2 & 8 \\ \downarrow & \nearrow 4 & \nearrow 0 & \nearrow -8 \\ 1 & 0 & -2 & \textcircled{0} \end{array}$

$x^2 - 2$

$\begin{pmatrix} 1, 4 \\ -1, 4 \end{pmatrix} \begin{pmatrix} 1, 4 \\ -1, 4 \end{pmatrix}$

P125

71.

Rational 0s

$$x^4 - \frac{25}{4}x^2 + 9 \Rightarrow \frac{1}{4}(4x^4 - 25x^2 + 36)$$

$$= \frac{1}{4} \left(\cancel{x^4 - \frac{25}{4}x^2 + 9} \right) \text{ factor}$$

$$\frac{1}{4}(4x^2 - 9)(x^2 - 4)$$

$$\frac{1}{4}(2x-3)(2x+3)(x-2)(x+2)$$

$$x = \frac{3}{2} \quad x = -\frac{3}{2} \quad x = 2 \quad x = -2$$

now to do

73. $x^3 - \frac{1}{4}x^2 - x + \frac{1}{4} = \frac{1}{4}(4x^3 - x^2 - 4x + 1)$

60

$$\frac{1}{4}(4x^3 - x^2)(4x + 1)$$

factor

$$\frac{1}{4}(x^2(4x-1) - 1(4x-1))$$

$$\frac{1}{4}((x^2-1)(4x-1))$$

$$\frac{1}{4}(x-1)(x+1)(4x-1)$$

77

$$x^3 - x$$

↓ GCF

$$x(x^2 - 1)$$

$$x(x-1)(x+1)$$

$$0, 1, -1$$

3 rational zeros

0 irrational zeros

→ B

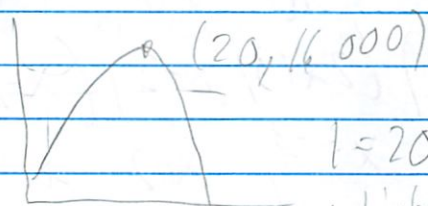
p126

81.

$$2(l+w) + g = 120$$

$$2(1+w) + g = 120$$

$$a) V = 4x^2(30-x)$$



which has a volume of 16,000 in³

y of 13,500 at $x = 24$

$$x = 15 \left\{ \begin{array}{l} 2(15+15) + g = 120 \\ 60 + g = 120 \end{array} \right.$$

$$60 + g = 120$$

$$\rightarrow 2(24+24) + g = 120$$

$$96 + g = 120$$

$$15, \frac{15 \pm 15\sqrt{5}}{2}$$

negative value

2.3 Real Zeros of Polynomials

Part 3

9/27

Other test for Zeros of Polynomials

Descartes Rule of Signs

- another way to get the possible # of real zeros

ex1

$$f(x) = x^2 + 1$$

$$\sqrt{-1} = x^2$$

$$x = \pm \sqrt{-1} \rightarrow \text{no real zeros}$$

$$x = \pm i$$

ex2

$$f(x) = x^3 + 1$$

$$-1 = x^3$$

$$\sqrt[3]{-1} = x$$

$$-1 = x \in 1 \text{ real zero}$$

ex3

Describe the possible real zeros of $f(x) = 3x^3 - 5x^2 + 6x - 4$ using Descartes..

positive real zeros

$\rightarrow x \rightarrow$ input is $\oplus$

look at original function

look for variations of signs

$$\overbrace{3x^3}^{\oplus} - \overbrace{5x^2}^{\ominus} + \overbrace{6x}^{\oplus} - \overbrace{4}^{\ominus} \Bigg) \text{ 3 variations in sign}$$

$\oplus$

$\Rightarrow$ either 3 $\oplus$ real zeros

scale down by 2

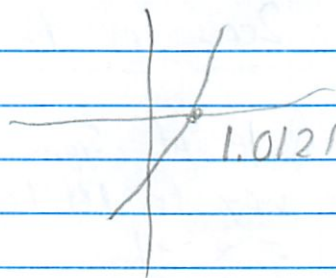
$\rightarrow$ or 1 $\oplus$ real zero

negative real zeros

$$f(-x) = 3(-x)^3 - 5(-x)^2 + 6(-x) - 4$$
$$= -3x^3 - 5x^2 - 6x - 4$$

0 sign variations $\rightarrow$ no $\ominus$ real zeros

→ Check on Calculator



∴ 1 real zero

p124
58.

$$3x^4 + 5x^3 - 6x^2 + 8x - 3$$

$\underbrace{\quad}_1 \quad \underbrace{\quad}_2 \quad \underbrace{\quad}_3$

3 possible ⊕ real zeros
or 1 ⊖ don't forget

$$f(-x) = 3(-x)^4 + 5(-x)^3 - 6(-x)^2 + 8(-x) - 3$$

$$3x^4 - 5x^3 - 6x^2 - 8x - 3$$

$\underbrace{\quad}_1 \quad \underbrace{\quad}_1$

1 possible ⊖ real zero

Remember even
powers don't
change sign

P2

$$4x^7 + 3x^6 + x^5 + 2x^4 - x^3 + 9x^2 + x + 1$$

$\underbrace{\quad}_1 \quad \underbrace{\quad}_2$

2 or 0

2 possible ⊕ real zeros

$$f(-x) = 4(-x)^7 + 3(-x)^6 + (-x)^5 + 2(-x)^4 - (-x)^3 + 9(-x)^2 + (-x) + 1$$

$$-4x^7 + 3x^6 - x^5 + 2x^4 + x^3 + 9x^2 - x + 1$$

$\underbrace{\quad}_1 \quad \underbrace{\quad}_2 \quad \underbrace{\quad}_3 \quad \underbrace{\quad}_4 \quad \underbrace{\quad}_5$

5, 3, 1 possible
⊖ real zeros

Least Test for Os → Upper + Lower Bounds for Os

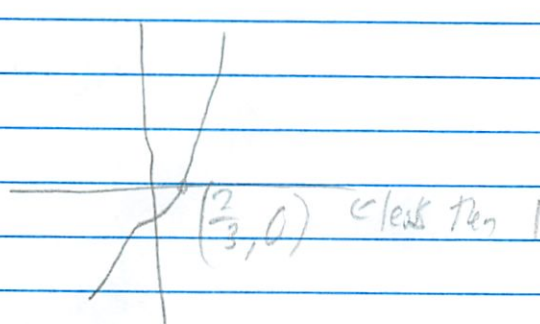
ex1

$$f(x) = 6x^3 - 4x^2 + 3x - 2$$

$$\frac{\pm 1, \pm 2}{\pm 1, \pm 2, \pm 3, \pm 6}$$

$$\rightarrow \pm 1, \pm 2, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{1}{6}, \pm \frac{1}{2}$$

$\begin{array}{r|rrrr} 6 & -4 & 3 & 2 \\ \downarrow 2 & 6 & 2 & 5 \\ \hline 6 & 2 & 5 & 3 \end{array}$
 If $k > 0$, and the last row of zeros (from synthetic division) is all ⊕, then that factor is an upper bound (can't have 2)



$$\begin{array}{l} (6x^3 - 4x^2)(3x-2) \\ 2x^2(3x-2) + 1(3x-2) \\ (2x^2+1)(3x-2) \\ \text{non real} \quad x = \frac{2}{3} \end{array}$$

p125

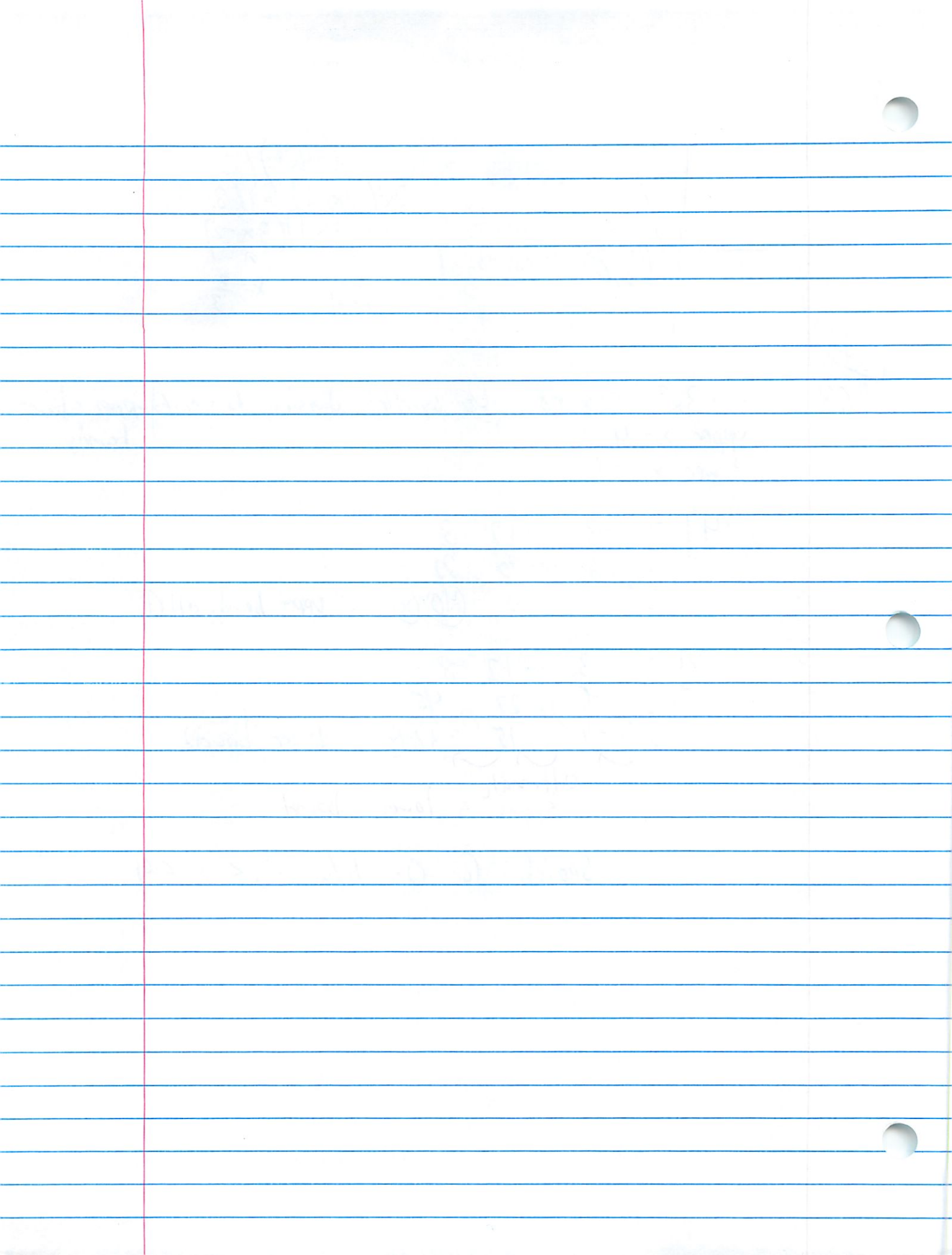
68. $2x^3 - 3x^2 - 12x + 8$ Use synthetic division to verify upper & lower bounds
 upper $x = 4$
 lower $x = -3$

$$\begin{array}{r} 4 \overline{) 2 \quad -3 \quad -12 \quad 8} \\ \underline{8 \quad 20 \quad 32} \\ 2 \quad 5 \quad 8 \quad (40) \otimes \end{array} \quad \leftarrow \text{upper bound, all } (+)$$

$$\begin{array}{r} -3 \overline{) 2 \quad -3 \quad -12 \quad 8} \\ \underline{-6 \quad 27 \quad -45} \\ 2 \quad -9 \quad 15 \quad (-37) \otimes \end{array} \quad \leftarrow \text{lower bound}$$

↗ ↘ ↗ ↘
 alternate signs = lower bound

Search for Os b/w $-3 < x < 4$



2.4 Complex Numbers "i"

9/27

$$x^2 + 1 = 0$$

$$x^2 = -1$$

$$\sqrt{\quad} \quad \sqrt{\quad}$$

$$x = \pm \sqrt{-1}$$

$$x = \pm i \text{ } i \text{ imaginary unit}$$

Real # + Imaginary #

Complex #s

*

$$\sqrt{-1} = i$$

$$i^2 = -1$$

$$\sqrt{-9} = \sqrt{9} \cdot \sqrt{-1}$$

$$3 \cdot i$$

$$3i$$

$$\sqrt{-16} = \sqrt{16} \cdot \sqrt{-1}$$

$$4i$$

Standard form of a complex #

$$a + bi$$

↑

real
part

↑

imaginary
part

$$9i = 0 + 9i$$

standard form

$$4 = 4 + 0i$$



Operations

ex1) $3 - (-2 + 3i) + (-5 + i)$ } add reg # 1st
 $3 + 2 - 3i - 5 + i$
 $0 - 2i$

P1 $(3 + 2i) + (4 - i) - (7 + i)$
 $3 + 2i + 4 - i - 7 + i$
 $0 + 0i$

p193

18. $(7 + \sqrt{-18}) - (3 + 3\sqrt{2}i)$
 $(7 + \sqrt{9}\sqrt{-2}) - 3 - 3\sqrt{2}i$
 $7 + 3\sqrt{2}i - 3 - 3\sqrt{2}i$
 $4 + 3\sqrt{2}i - 3\sqrt{2}i$
 4

Multiply

ex2 $\sqrt{-4} \cdot \sqrt{-16}$
 $\sqrt{4}i \cdot \sqrt{16}i$ } need to find i first, don't just x
 $2i \cdot 4i$
 $8i^2$

ex3 $(2 - i)(4 + 3i)$ } foil remember: $i^2 = -1$
 $8 + 6i - 4i - (3 \cdot -1)$
 $11 + 2i$

30. $(6 - 2i)(2 - 3i)$
 $12 - 18i - 4i + (6 \cdot -1)$
 $6 - 22i$

2.4 Complex Numbers

t/w

9/28

p133

3.

Find $A + B$

$$(a-1) + (b+3)i = 5 + 8i$$

$$a=6$$

$$b=5$$

↳ that was easy

Standard Form

9.

$$-5i + i^2$$

$$-5i + -1$$

$$-1 - 5i$$

10.

$$-3i^2 + i$$

$$-3 \cdot -1 + i$$

$$3 + i$$

Complete

17.

$$(-1 + \sqrt{-8}) + (8 - \sqrt{50})$$

$$-1 + \sqrt{8} \sqrt{-1} + 8 - \sqrt{50} \sqrt{-1}$$

$$-1 + 2\sqrt{2}i + 8 - 5\sqrt{2}i$$

$$7 - 3\sqrt{2}i$$

$$7 - 3\sqrt{2}i$$

with roots, you add like $\sqrt{s}$

↳ keep going

19.

$$13i - (14 - 7i)$$

$$13i - 14 + 7i$$

$$-14 + 20i$$

Complete

27.

$$(\sqrt{-10})^2$$

$$-10$$

$$\rightarrow -10 + 0i$$

33.

$$(\sqrt{14} + \sqrt{10}i)(\sqrt{14} - \sqrt{10}i)$$

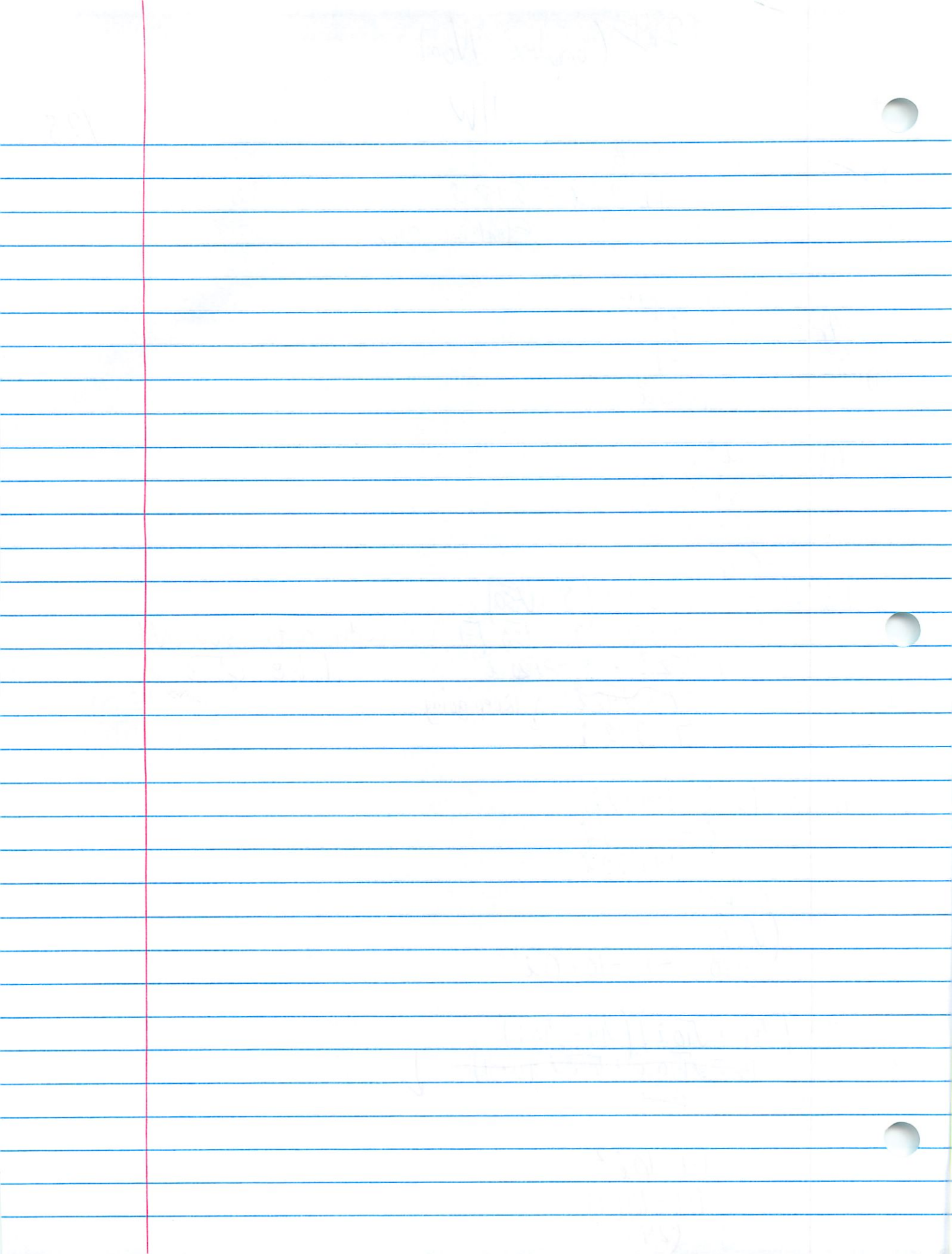
$$14 - \sqrt{140}i + \sqrt{140}i + 10(-1)$$

$$(4)$$

$$14 - 10i^2$$

$$14 - 10(-1)$$

$$(24)$$



2.4 Complex #s

Part 2

9/28

Additive Identity Property

$$(1 + \sqrt{3}i) + 0 = 1 + \sqrt{3}i$$

$$-7 + 0$$

$$0$$

Additive Inverse

$$-8 + 8$$

$$0$$

$$5 + -5$$

$$0$$

$$7 - 7$$

$$0$$

inverse of each other

$$a + bi = -a - bi$$

$$7 - 3i + -7 + 3i$$

$$0$$

Complex Conjugates

$$3 - 5i \rightarrow \text{change middle sign} \rightarrow 3 + 5i$$

$$\sqrt{-3} = \sqrt{3}i \rightarrow \begin{array}{l} 0 + \sqrt{3}i \\ 0 - \sqrt{3}i \\ -\sqrt{3}i \end{array} \text{ complex conjugate}$$

$$\sqrt{-20} \rightarrow \sqrt{4 \cdot 5}$$

$$2\sqrt{5}$$

$$2\sqrt{5}i$$

$$-2\sqrt{5}i$$

conjugate

p133

40. Multiply $-3 + \sqrt{2}i$ by its conjugate
 $(-3 + \sqrt{2}i) \cdot (-3 - \sqrt{2}i)$
 $9 + 3\sqrt{2}i - 3\sqrt{2}i - (\sqrt{2}i)^2$
 $9 - (2 \cdot -1)$
 (11)

ex1 Write a quotient $\rightarrow \frac{2+3i}{4-2i}$ in standard form

don't want $\rightarrow$ i on bottom
 $\frac{2+3i}{4-2i} \cdot \frac{(4+2i)}{(4+2i)} = \frac{8+4i+12i+6i^2}{16+8i-8i-4i^2} = \frac{8+16i-6}{16+4}$

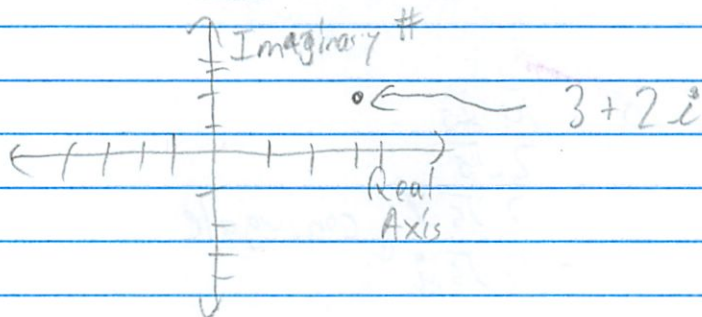
$\frac{2+16i}{20} \rightarrow \frac{2}{20} + \frac{16i}{20} \rightarrow \left(\frac{1}{10} + \frac{4}{5}i \right)$

p134
50.

$\frac{8-7i}{1-2i} \cdot \frac{(1+2i)}{(1+2i)} = \frac{8+16i-7i-14i^2}{1+2i-2i-4i^2} = \frac{8+9i+14}{1+4}$

$\frac{22+9i}{5} \rightarrow \left(\frac{22}{5} + \frac{9i}{5} \right)$

Complex # on Graphs



$$i^1 = \sqrt{-1} = i$$

$$i^2 = -1$$

$$i^3 = i^2 \cdot i = -1 \cdot i = -i$$

$$i^4 = i^2 \cdot i^2 = -1 \cdot -1 = 1$$

$$i^5 = i^4 \cdot i^1 = 1 \cdot i = i$$

$$i^6 = i^4 \cdot i^2 = 1 \cdot -1 = -1$$

$$i^7 = -i$$

$$i^8 = 1$$

p133

41.

Multiply by complex conjugate

$$\sqrt{-20} = (\sqrt{20}i) \cdot (-\sqrt{20}i)$$

$$-20 \cdot i^2$$

$$-20 \cdot -1$$

$$(20)$$

$$\sqrt{4} \sqrt{-5}$$

$$(2\sqrt{5}i) \cdot (-2\sqrt{5}i)$$

$$-4 \cdot 5 \cdot i^2$$

$$-4 \cdot 5 \cdot -1$$

$$(20)$$

same

Standard Form

47.

$$\frac{2}{4-5i} \cdot \frac{(4+5i)}{(4+5i)}$$

$$\frac{8+10i}{16-25i^2}$$

$$\frac{8+10i}{16-25 \cdot -1}$$

$$\frac{8+10i}{41}$$

$$\frac{8}{41} + \frac{10i}{41}$$

p134

55.

$$\frac{i}{3-2i} + \frac{2i}{3+8i}$$

$$\frac{i(3+8i)}{(3-2i)(3+8i)} + \frac{2i(3-2i)}{(3-2i)(3+8i)}$$

LCD

$$LCD = (3-2i)(3+8i)$$

$$\frac{i(3+8i)}{(3-2i)(3+8i)} + \frac{2i(3-2i)}{(3-2i)(3+8i)}$$

$$\frac{3i+8i^2}{9+24i-6i-16i^2} + \frac{6i-4i^2}{9+18i+16}$$

$$\frac{3i-8}{25+18i} + \frac{6i+4}{25+18i}$$

copy error

$$\frac{-4+9i}{25+18i} \cdot \frac{(25-18i)}{(25-18i)} \quad \frac{-100+72i+225i-162}{100-64-1} \quad -162$$

$$\frac{-100+297i-162}{625+324}$$

$$\frac{-100-162+297i}{164}$$

$$\frac{-124-43i}{164}$$

$$\frac{-124-43i}{164}$$

$$\frac{-100+297i+162}{949}$$

$$\frac{62+297i}{949}$$

$$\frac{-3+43i}{41}$$

2.1 - 4.1 Test

9/28

18
qu

1. Long Division of Polynomials

6 MC

2. Quadratics + Parabolas

Standard $\rightarrow$ Vertex forms

Find Vertex

Angle of Symmetry

x + y intercepts

3. Sketching Graphs of Polynomials

(2 qu)

w/ leading coefficients test
+ x-intercept

4. Determine a polynomial given the zeros

5. Word problems (x = \$25 & decrease)

6. Division of polynomials

- Synthetic division

- Synthetic substitution (check if correct)

- write in $(x-h) \cdot q(x) + r(x)$ form

- possible rational zeros test

- division

- Factor completely

- Ration

) determine all
real zeros

7. Intermediate value theorem (1 qu)

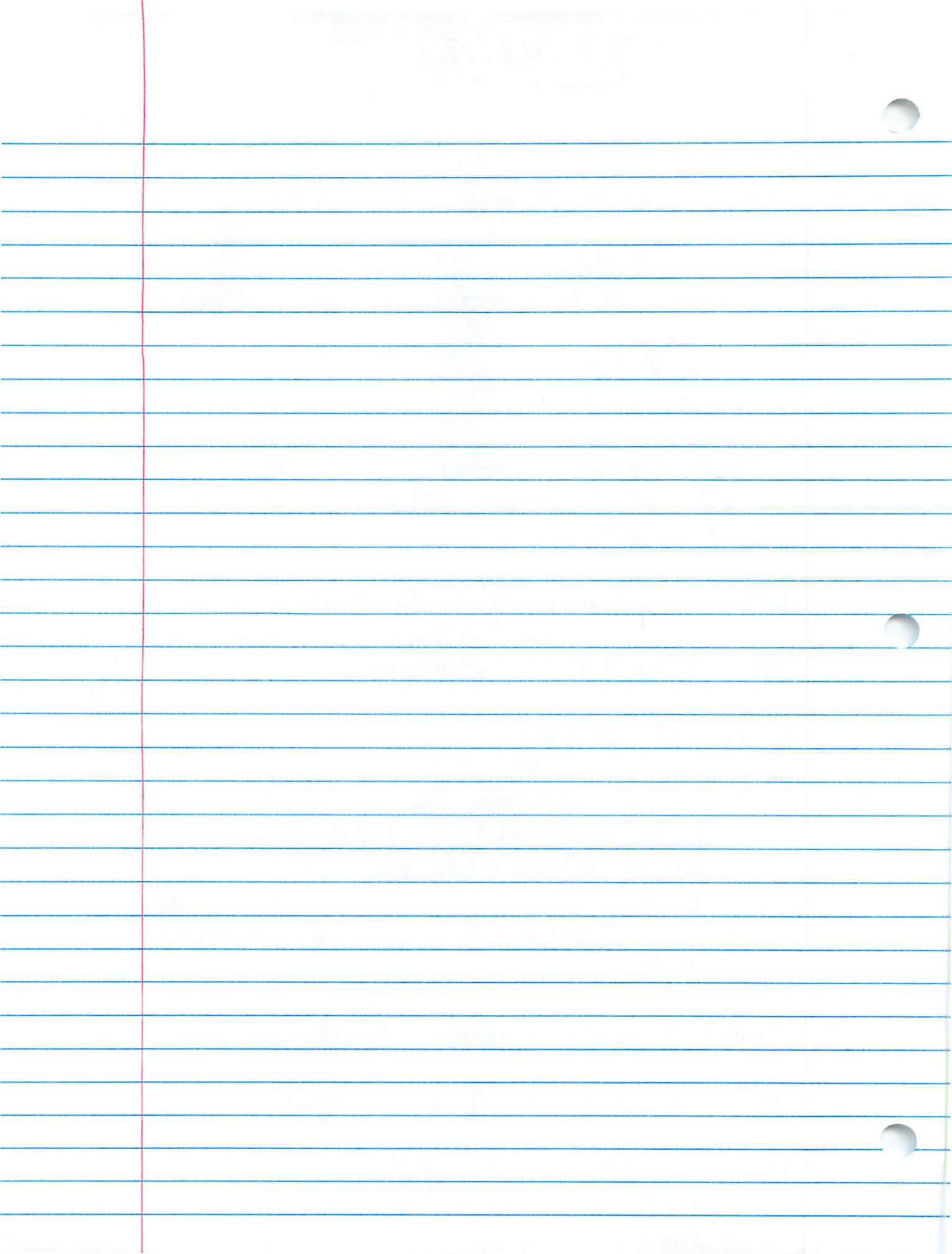
8. Descartes Rule of Thumb (1 qu)

9. Complex #s (2 qu)

- multiply, add, divide, subtract

- standard form

- use conjugate



2.1-2.4 Review

9/28

1. Find polynomial w/ 0, 2, $4+\sqrt{5}$, $4-\sqrt{5}$

$$(x-2)(x-(4+\sqrt{5}))(x-(4-\sqrt{5}))$$

$$(x-2)(x-4+\sqrt{5})(x-4-\sqrt{5})$$

$$(x-2)(x-4)^2 - 5$$

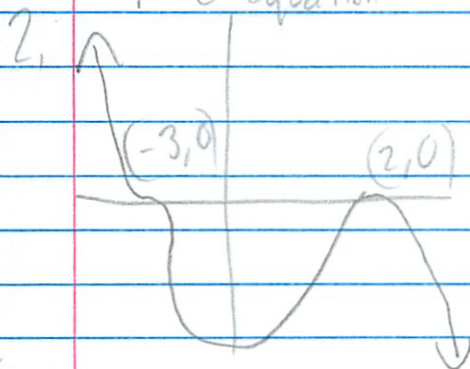
$$\cancel{x^2 - 4x + 4} \cancel{x^2 - 8x + 8} - 5$$

$$(x-2)(x^2 - 8x + 16 - 5)$$

$$x^3 - 8x^2 + 11x - 2x^2 + 16x - 22$$

$$x^3 - 10x^2 + 27x - 22$$

Find equation



$$(x+3)^3 (x-2)^2$$

$\uparrow$
a is positive

$a < 0$
 $\ominus \uparrow \downarrow$
 $n=5$ odd

p/69

Sketch

6.

$$3x^2 - 12x + 11$$

$$3(x^2 - 4x) + 11$$

$$3(x^2 - 4x + 4) + 11 - 12$$

$$3(x-2)^2 - 1$$

Vertex (2, -1)

$$\frac{11}{3} \left\{ \begin{array}{l} 3 \\ 1 \end{array} \right\} \begin{array}{l} -12 \\ 1 \end{array} \rightarrow \begin{array}{l} 11 \\ -1 \end{array} \rightarrow \begin{array}{l} 11 \\ 3 \end{array} \rightarrow \begin{array}{l} 22 \\ 3 \end{array} \otimes$$

Find possible zeros
w/ various whys

factors(11) $\pm 1, 11$
factors(3) $\pm 1, 3$

calc (1.422, 0)
(2.577, 0)

try others

Write Standard Form

7.

Vertex $(1, 4)$

Point $(2, -3)$

$$-3 = a(2-1)^2 - 4$$

$$-3 = a(1) - 4$$

$$\frac{-3}{-3} = \frac{a-4}{-3}$$

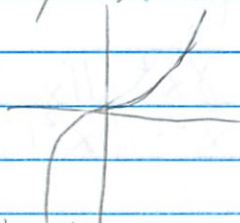
$$a = 1$$

$$y = (x-1)^2 - 4$$

Sketch + how does it Δ ?

16.

$$y = x^3$$



a) $-x^3 + 4$, flipped, $\pi/4$

b) $(x+2)^3 - 1$ $\in 2$ $\downarrow$

c) $-\frac{1}{3}x^3 + 1$ flipped, flatter $\pi/1$

d) $-(x+8)^3$ flipped $\in 8$

Leading co-efficient test

21.

$$\frac{3}{4}(x^4 + 3x^2 + 2)$$

$$\frac{3}{4}x^4 + \frac{9}{4}x^2 + \frac{3}{2}$$

easy

$\uparrow$ even

$\uparrow \uparrow$

Graph + Find zero

23.

$$x^4 - x^3 - 2x^2$$

$$x^2(x^2 - x - 2)$$

$$x^2(x-2)(x+1)$$

$$(0,0)$$

$$(0,2)$$

$$(0,-1)$$

24.

$$-2x^3 - x^2 + x$$

$$x(-2x^2 - x + 1)$$

-2	1	-1
-2	1	-1
-1	1	-1

$$(0,0)$$

$$(0,-1)$$

$$(0,\frac{1}{2})$$

$$x(x+1)(-2x+1)$$

170
29.

Intermediate Value Theorem

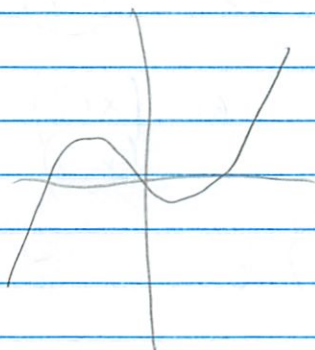
$$x^3 + 2x^2 - x - 1$$



x	y
-3	-7
-2	1
-1	1
0	-1
1	1
2	13

30.

$$.24x^3 - 2.6x - 1.4$$



x	y
-4	-6.86
-3	-1.08
-2	1.88
-1	.96
0	-1.4
1	-3.76
2	-4.68
3	-2.72
4	3.56

Long Divide

35.

$$\begin{array}{r} 8x+5 \\ 3x-2 \overline{) 24x^2 - x - 8} \\ \underline{-(24x^2 - 16x)} \\ 15x - 8 \\ \underline{-(15x - 10)} \\ 2 \end{array}$$

$$\frac{8x+5}{3x-2} + \frac{2}{3x-2}$$

Synthetic Divide

45.

$$(6x^4 - 4x^3 - 27x^2 + 18x) / (x - 2/3)$$

$$\begin{array}{r|rrrrrr} +\frac{2}{3} & 6 & -4 & -27 & 18 & 0 \\ \downarrow & & +4 & -\frac{10}{3} & -\frac{14}{3} & -\frac{2}{3} \\ \hline & 6 & 0 & -\frac{83}{3} & \frac{10}{3} & -\frac{2}{3} \end{array}$$

$$\underline{6x^3 - 8x^2 + 21x + 4} \quad \underline{-\frac{2}{3}}$$

$$6x^3 - 27x$$

49.

$$x^4 + 10x^3 - 24x^2 + 20x + 44$$

$$\begin{array}{r} -3 \overline{) 1 \quad 10 \quad -24 \quad 20 \quad 44} \\ \underline{\downarrow \nearrow -3 \quad -21 \quad 135 \quad -465} \\ 1 \quad 7 \quad -45 \quad 155 \quad -421 \end{array}$$

$$x^3 + 7x^2 - 45x + 155 + \boxed{-421} \leftarrow \text{function result}$$

$$\begin{array}{r} -1 \overline{) 1 \quad 10 \quad -24 \quad 20 \quad 44} \\ \underline{\downarrow \nearrow -1 \quad -9 \quad 33} \\ 1 \quad 9 \quad -33 \quad 53 \end{array} \quad \begin{array}{r} -3 \overline{) 44} \\ \underline{-53} \\ -9 \end{array}$$

Verify + Find Remaining Factors

53.

$$x^4 - 4x^3 - 7x^2 + 22x + 24 \quad \text{at } \begin{cases} x+2 \\ x-3 \end{cases}$$

$$\begin{array}{r} -2 \overline{) 1 \quad -4 \quad -7 \quad 22 \quad 24} \\ \underline{\downarrow \nearrow -2 \quad 12 \quad -10 \quad 24} \\ 1 \quad -6 \quad 5 \quad 12 \quad \textcircled{0} \quad \textcircled{0} \end{array}$$

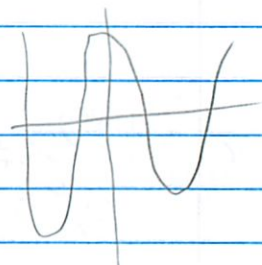
$$x^3 - 6x^2 + 5x + 12$$

$$\begin{array}{r} 3 \overline{) 1 \quad -6 \quad 5 \quad 12} \\ \underline{\downarrow \nearrow 3 \quad -9 \quad -12} \\ 1 \quad -3 \quad -4 \quad \textcircled{0} \quad \textcircled{0} \end{array}$$

$$x^2 - 3x - 4$$

$$f(x) = (x-4)(x+1)(x+2)(x-3)$$

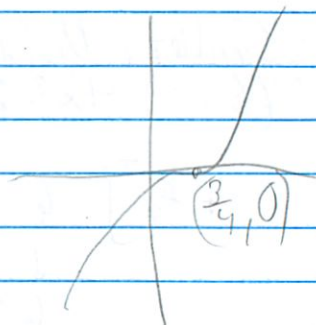
(+@even) $\nearrow \nearrow$



55 Rational Zero Test

$$4x^3 - 11x^2 + 10x - 3$$

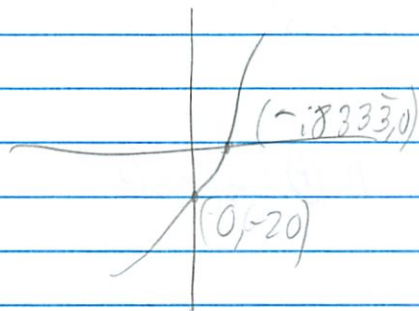
factors (3)	$\pm 1, \pm 3$
factors (4)	$\pm 1, \pm 2, \pm 4$



170
57.

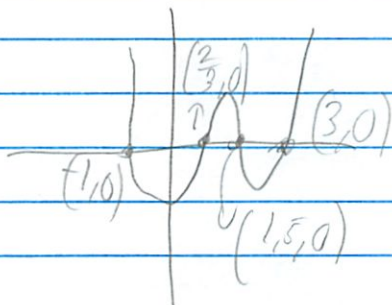
Find all zeros

$$6x^3 - 5x^2 + 24x - 20$$



171
58.

$$6x^4 - 25x^3 + 14x^2 + 27x - 18$$



Descartes' Rule of Signs

61.

$$5x^3 + 3x^2 - 6x + 9$$

↘₁

1 (+)

$$f(-x) = 5(-x)^3 + 3(-x)^2 - 6(-x) + 9$$

$$-5x^3 + 3x^2 + 6x + 9$$

↘₁

1 (-)

62.

$$-2x^5 + 0x^4 + 4x^3 - 2x^2 + 0x + 5$$

↘₁

↘₂

↘₃

3 or 1 (+)

$$f(-x) = -2(-x)^5 + 0(-x)^4 + 4(-x)^3 - 2(-x)^2 + 0(-x) + 5$$

$$2x^5 + 0x^4 - 4x^3 - 2x^2 - 0x + 5$$

↘₁

↘₂

2 or 0 (-)

171
63.

Upper + Lower Bounds

$$4x^3 - 3x^2 + 4x - 3$$

Upper $x=1$

lower $x = -\frac{1}{4}$

$$\begin{array}{cccc} 1 & 4 & -3 & 4 & -3 \\ & \downarrow & & & \\ & 4 & \nearrow & 1 & \nearrow & 5 \\ & 4 & & 1 & & 5 \end{array}$$

(2) $\in$ All (+) upper (1)

$$\begin{array}{cccc} -\frac{1}{4} & 4 & -3 & 4 & -3 \\ & \downarrow & & & \\ & -1 & \nearrow & 1 & \nearrow & -1.25 \\ & 4 & & -4 & & 5 \end{array}$$

(-4.25) $\in$ sign Δ lower (✓)

73. $\begin{array}{l} \text{Do} \\ (10-8i)(2-3i) \\ 20-30i-16i+24i^2 \\ 20-46i-24 \\ -4-46i \end{array}$

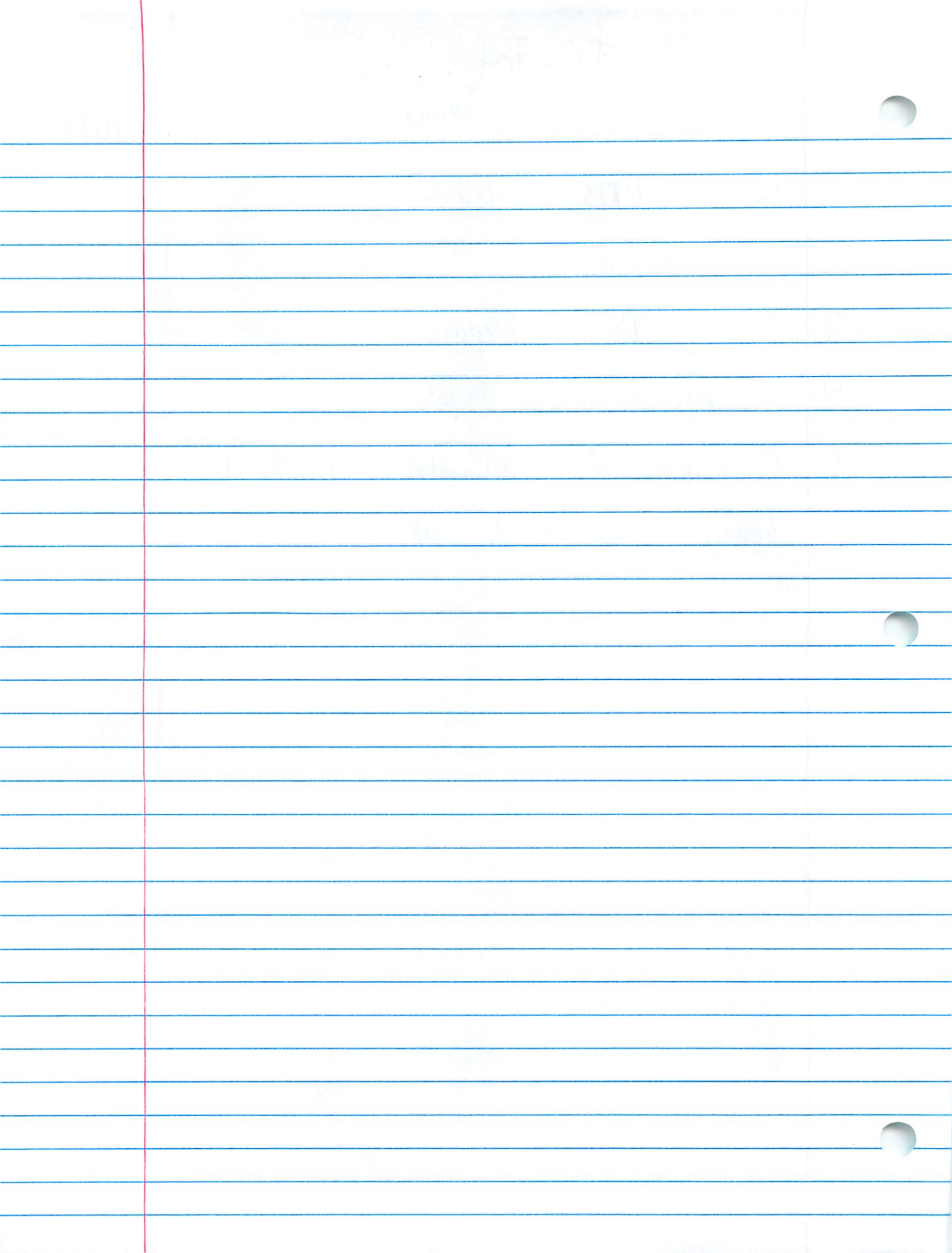
(✓)

79. $\frac{3+2i}{5+i} \cdot \frac{(5-i)}{(5-i)} = \frac{15-3i+10i-2i^2}{25-i^2} = \frac{17+7i}{26} = \frac{17}{26} + \frac{7}{26}i$

4/ IS Test Topics

10/2

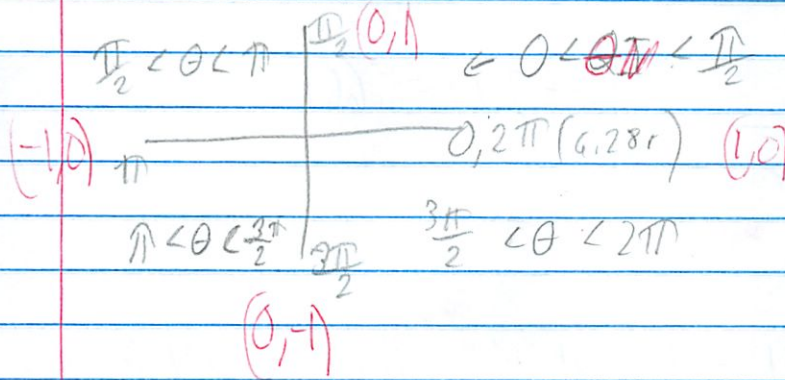
1. Converting DMS $\leftrightarrow$ degrees
2. Exact Value of Angle
3. Converting Degrees $\leftrightarrow$ radians
4. Word problem w/ arc length
5. Given point $\rightarrow$ find exact value of trig function
6. Knowing radians of a circle



4/15 Review

10/2

Radians of a Unit Circle

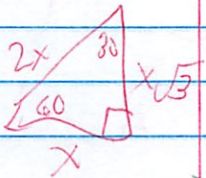
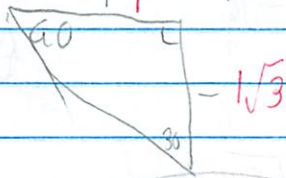


Sin	All	
+	+	Sin
=	+	cos
=	+	tan
=	+	
+	+	
tan	cos	

$$\sin \frac{3\pi}{2} = \frac{y}{r} = \frac{-1}{1} = -1$$

$$\cos \pi = \frac{x}{r} = \frac{-1}{1} = -1$$

$$\cot 300^\circ = \frac{x}{y} = \frac{1}{-\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

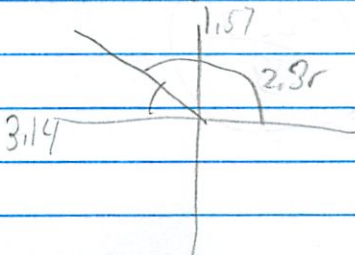


$$\left. \begin{array}{l} \tan^2 \theta - 1 \\ \cos^2 \theta - 1 \\ \sin^2 \theta - 1 \end{array} \right\} \text{diff of 2 squares} \rightarrow (\tan \theta - 1)(\tan \theta + 1)$$

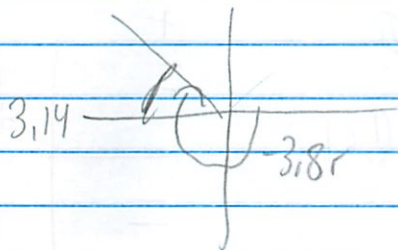
Find Reference Angles (θ')

$$A = 2.3r$$

$$3.14 - 2.3 = .84$$

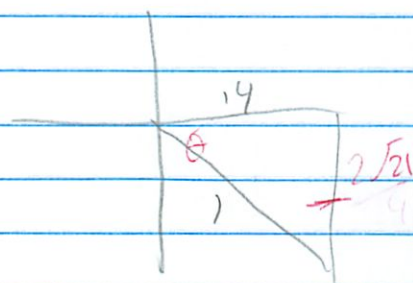


$$\theta = -3.8r$$



$$3.8 - 3.14 = .66r$$

If $\cos \theta = .4$ + terminal side lies in Q4
Find $\tan \theta$



$$\cos = \frac{x}{r}$$

$$\tan = \frac{y}{x} = \frac{14}{4} = \frac{7}{2}$$

$$10^2 = 4^2 + x^2$$

$$100 = 16 + x^2$$

$$84 = x^2$$

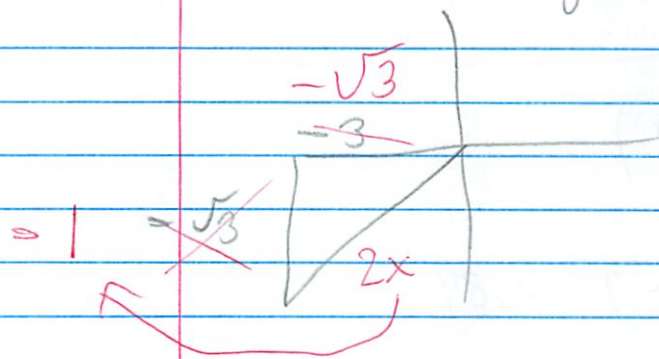
$$x = 9.16$$

$$\tan \theta = 1.75$$

$$\theta = 23.58^\circ$$

keep $\sqrt{84}$
 $-2\sqrt{21}$

Find $\sec \theta$ given that $\tan \theta = \frac{\sqrt{3}}{3}$ + Quad = 3



$$\tan = \frac{y}{x} \quad \sec = \frac{r}{x}$$

reduce $\frac{1}{\sqrt{3}}$ $r = -2$

$$\frac{-2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{-2\sqrt{3}}{3}$$

Review

10/4

p96

32.

Graph + Find Standard Form

$$f(x) = -4x^2 + 24x - 41$$

$$-4(x^2 - 6x) - 41$$

$$-4(x^2 - 6x + 9) - 41 + 36$$

$$-4(x-3)^2 - 5$$

$$\text{vertex } (3, -5)$$

x-intercepts:

on calc:

none

p109

59.

Find a function w/ these zeros

$$2, 4 + \sqrt{5}, 4 - \sqrt{5}$$

$$(x - [4 + \sqrt{5}]) (x - [4 - \sqrt{5}]) (x - 2)$$

$$((x - 4) + \sqrt{5}) ((x - 4) - \sqrt{5}) (x - 2)$$

$$((x - 4)^2 - 5) (x - 2)$$

$$\downarrow -4 - 4 = +16$$

- sign error

$$(x^2 - 8x + 16 - 5) (x - 2)$$

$$(x^2 - 8x + 11) (x - 2)$$

$$x^3 - 8x^2 + 11x - 2x^2 + 16x - 22$$

$$(x^3 - 10x^2 - 5x + 42)$$

$$x^3 - 10x^2 + 27x - 22$$

p124

38.

Synthetic Division + Real Zeros

$$48x^3 - 80x^2 + 41x - 6 = 0$$

$$\begin{array}{r|rrrr} \frac{3}{4} & 48 & -80 & 41 & -6 \\ & \downarrow & \nearrow 32 & \nearrow -32 & \nearrow 6 \\ & 48 & -48 & 9 & 0 \end{array}$$

$$48x^2 - 48x + 9$$

$$(x - \frac{3}{4}) (x - \frac{1}{4})$$

$$(4x - 3) (4x - 1)$$

$$\frac{3}{4} = x$$

$$3 = 4x$$

$$4x - 3$$

$$\begin{pmatrix} \frac{3}{4}, 0 \end{pmatrix}$$

$$\begin{pmatrix} \frac{3}{4}, 0 \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{4}, 0 \end{pmatrix}$$

Real Solutions

51. $2y^4 + 7y^3 - 26y^2 + 23y - 6 = 0$

Check calc for possibilities

$\pm 1, \pm 2, \pm 3, \pm 6$
 $\pm 1, \pm 2$

must have
 mistyped

~~$(-6, 462, 0)$~~
 ~~$(-2, 2722, 0)$~~
 ~~$(-4, 9807, 0)$~~ Rem

try -6

$\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}$

$(1, \frac{1}{2}, -6)$

58. Descartes Rule of Signs

$f(x) = 3x^4 + 5x^3 - 6x^2 + 8x - 3$

3 or 1 $\oplus$ ~~real~~ (s)
 zero

$f(-x) = 3(-x)^4 + 5(-x)^3 - 6(-x)^2 + 8(-x) - 3$

$3x^4 - 5x^3 - 6x^2 - 8x - 3$

1 $\ominus$ real ~~root~~
 zero
 not roots

2.5 Fundamental Theorem of Algebra

10/5

* The n th degree of a polynomial can have at most " n " real zeros

$$\begin{aligned}f(x) &= x^2 + 1 \\-1 &= x^2 \\x &= \pm i\end{aligned}$$

has no real zeros $\rightarrow$ but the total number of zeros = the degree of polynomial (2)

$$\begin{aligned}f(x) &= x^3 + 1 \\0 &= x^3 + 1 \\-1 &= x^3 \\\sqrt[3]{-1} &= x\end{aligned}$$

$-1 = x$ is a real zero which has multiplicity of 3
(# of zeros = degree) $(x+1)^3$

$$f(x) = -2x^5 + 3x^3 - 2x^2 + x - 1$$

at most - can have 5 real zeros

In the complex # system, every n th degree polynomial has exactly " n " zeros.

5 zeros $\rightarrow$ real
non-real

If $f(x)$ is a polynomial of degree " n ", where $n \geq 0$, then $f(x)$ has at least 1 zero in the complex system

$$f(x) = x^3 + 4x$$

$$\begin{aligned}&x(x^2 + 4) \\&\uparrow \\&x=0 \quad x^2 + 4 = 0 \\&\quad \quad x^2 = -4 \\&\quad \quad x = \pm \sqrt{-4}\end{aligned}$$

$$x = \pm 2i$$

$f(x)$ has 3 zeros $\leftarrow$ matches

real	non-real	degree of polynomial
$\downarrow$	$\downarrow$	
$x=0$	$x = \pm 2i$	

$$x = \pm 2i \rightarrow (x)(x+2i)(x-2i)$$

Linear Factorization Theorem

If $f(x)$ is a polynomial of degree " n " where $n > 0$, then $f(x)$ has exactly " n " linear factors,

ex) $f(x) = x^2 - 6x + 9$
 $(x-3)(x-3) \in 2 \text{ linear factors (real)}$
 $(x-3)^2$

P146

Factor $f(x) = x^3 - 4x^2 - 4x + 16$ multiplicity of 2

1) $\begin{array}{r|rrrr} 1 & 1 & -4 & -4 & 16 \\ & & 1 & -3 & -7 \\ \hline & 1 & -3 & -7 & 9 \end{array}$
 2) $\begin{array}{r|rrrr} 1 & 1 & -4 & -4 & 16 \\ & & 2 & & \\ \hline & 1 & & & \end{array}$

$\rightarrow \pm 1, \pm 2, \pm 4, \pm 16$

just factor by grouping

$x^2(x-4) - 4(x-4)$
 $(x^2-4)(x-4)$
 $(x+2)(x-2)(x-4)$

P2. $x^5 + x^3 + 2x^2 - 12x + 8 \hookrightarrow \pm 1, \pm 2, \pm 4, \pm 8$

1) $\begin{array}{r|rrrrrr} 1 & 1 & 0 & 1 & 2 & -12 & 8 \\ & & 1 & 1 & 2 & 4 & -8 \\ \hline & 1 & 1 & 2 & 4 & -8 & 0 \end{array}$
 $x^4 + x^3 + 2x^2 + 4x - 8$

1) $\begin{array}{r|rrrr} 1 & 1 & 1 & 2 & 4 & -8 \\ & & 1 & 2 & 4 & 8 \\ \hline & 1 & 2 & 4 & 8 & 0 \end{array}$
 $x^3 + 2x^2 + 4x + 8$

$(x-1)^2$
 multiplicity
 of 2
 (occurs 2x)

factor
by
grouping

$x^2(x+2) + 4(x+2)$
 $(x^2+4)(x+2)$
 $x^2+4=0 \quad (x+2)$
 $x^2=-4$

$(x+2)$
 $(x+2i)$
 $(x-2i)$

$(x \pm 2i)$

linear factorization

$$f(x) = (x-1)(x-1)(x-2i)(x+2i)(x+2)$$

5 zeros = degree of 5



pmo

20, $x^3 + 11x^2 + 39x + 29 \rightarrow \pm 1, \pm 29$

$$\begin{array}{r|rrrr} & 1 & 11 & 39 & 29 \\ \hline & & & & \end{array}$$

$$\downarrow \rightarrow -1 \rightarrow -10 \rightarrow -29$$

$$1 \rightarrow 10 \rightarrow 29 \quad \textcircled{0} \quad \textcircled{0}$$

$$x^2 + 10x + 29$$

don't use

ROOTEN

APRCA

Roots $\checkmark$ Quad Formula

$(x+3i)(x+7i)$ - nonreal

$$(-5+2i)(-5-2i)$$

$$(x+1)$$

$$(x+3i)$$

$$(x+7i)$$

$$(x-(-5+2i))$$

$$(x-(-5-2i))$$

degree of 3

$$\frac{-10 \pm \sqrt{116}}{2}$$

2

Complex zeros appear in conjugate pairs

$$\begin{array}{ll} -2 + \sqrt{5} & \text{and } -3 + 2i \\ -2 - \sqrt{5} & -3 - 2i \end{array}$$

Find a polynomial with given zeros $-1, -1$ and $3i$

* degree = 4

know also $-3i$

Long Way

need both conjugate

$$f(x) = (x+1)^2 (x-3i)(x-(-3i))$$

$$(x^2 + 2x + 1)(x^2 + 3ix - 3ix - 9i^2)$$

$$(x^2 + 2x + 1)(x^2 + 9)$$

$$x^4 + 2x^3 + x^2 + 9x^2 + 18x + 9$$

$$x^4 + 2x^3 + 10x^2 + 18x + 9$$

Short Method
(need 1 conjugate)

$x = -1$, multiply by 2

$$(x+1)(x+1) = 0$$

$$x^2 + 2x + 1 = 0$$

$$x = 3i$$

$$x^2 = (3i)^2$$

$$x^2 = 9i^2$$

$$x^2 = -9$$

$$x^2 + 9 = 0$$

$$(x^2 + 2x + 1)(x^2 + 9)$$

get same thing
as before

linear factors: $(x+1)(x+1)(x-3i)(x+3i)$

plu

38. Find polynomial w/ roots zeros: $-1, 6+5i, 6-5i$

$$(x+1) = 0$$

$$x+1 = 0$$

$$(6+5i) = x$$

$$(6+5i)^2 = x^2$$

$$36 + 25i^2 = x^2$$

$$36 - 25 = x^2$$

$$11 = x^2$$

$$x^2 - 11 = 0$$

know
also

* get i by itself

$$x - 6 = 5i$$

$$(x-6)^2 = (5i)^2$$

$$x^2 - 12x + 36 = -25$$

$$(x^2 - 12x + 61) = 0$$

$$(x+1)(x^2 - 11)$$

$$x^3 - 11x + x^2 - 11$$

$$x^3 + x^2 - 11x - 11$$

$$x^2 - 12x + 61$$

$$x^3 - 11x^2 + 61x + 61 = f(x)$$

cx

Finding Zeros of a Polynomial.

Find all the zeros of $f(x) = x^4 - 3x^3 + 6x^2 + 2x - 60$
 If $1 + 3i$ is a zero

$x = 1 + 3i \rightarrow$ create a factor

$x = 1 - 3i$

$$\begin{aligned} & (x - (1 + 3i))(x - (1 - 3i)) \\ & ((x-1) + 3i)((x-1) - 3i) \end{aligned}$$

$$\begin{aligned} & (x-1)^2 - 9i^2 \\ & x^2 - 2x + 1 - (9 \cdot -1) \end{aligned}$$

$$x^2 - 2x + 10$$

$\leftarrow$ foils cancel out

divide

$$\begin{array}{r} x^2 - x - 6 \\ x^2 - 2x + 10 \overline{) x^4 - 3x^3 + 6x^2 + 2x - 60} \\ \underline{-(x^4 - 2x^3 + 10x^2)} \\ -x^3 - 4x^2 + 2x - 60 \\ \underline{-(-x^3 - 2x^2 - 10x)} \\ -6x^2 + 12x - 60 \\ \underline{-(-6x^2 - 12x - 60)} \\ 0 \quad 0 \end{array}$$

$x^2 - x - 6 \in$ factor that

$$\begin{aligned} & (x+2)(x-3) \\ & (3,0) \quad (-2,0) \end{aligned}$$

$$\begin{aligned} & \text{foils} \\ & (1+3i)(1-3i)(3,0)(-2,0) \rightarrow f(x) \end{aligned}$$

4 = degree

Could do synthetic division in beginning

foils not shown

$$\begin{array}{r} 1+3i \overline{) 1 \quad -3 \quad 6 \quad \quad 2 \quad -60} \\ \downarrow \nearrow \end{array}$$

can do
 on calc in
 a + bi mode

$\rightarrow$

$$\begin{aligned} & \text{foils} \rightarrow (1+3i)(-2+3i) \rightarrow -2 + 3i - 6i + 9i^2 \rightarrow -11 - 3i \end{aligned}$$

On
final ✓

$$\begin{bmatrix} 1 & -3i \end{bmatrix}$$

$$\begin{array}{ccccccc} 1 & -2+3i & -5-3i & 6-18i & & & \\ \downarrow & \rightarrow & \rightarrow & \rightarrow & & & \\ 1 & -3i & -1+3i & -6+18i & & & \\ & -1 & -6 & 0 & \textcircled{\checkmark} & & \end{array}$$
$$\textcircled{x^2 - x - 6}$$

↓ i-shaft
drop

2.5 Fundamental Theorem of Algebra t/w

10/6

p/40

1.

Find zeros

$$f(x) = x^2(x+3)$$

$$x^2 = 0 \quad x = -3$$

$$x = 0$$

$$x = 0$$

$(0,0) \in \text{list } 2x$

$$(0,0)$$

$$(-3,0)$$

2

3.

$$(x+9)(x+2i)(x-2i)$$

$$x = -9 \quad x = -2i \quad x = 2i$$

leave

$$(-9,0)$$

$$(-2i,0)$$

$$(2i,0)$$

4

Find zeros

$$f(x) = x^3 - 4x^2 + x - 4$$

The # of real zeros is represented by the x-int. on the graph

$$x^3 - 4x^2 + x - 4$$

$$x^2 + 1$$

$$x^2 + 1 = 0$$

$$x^2 = -1$$

$$x^2 = -1$$

$$x = \pm \sqrt{-1}$$

$$x = \pm i$$

$$(4,0)$$

$$(i,0)$$

$$(-i,0)$$

5

9. Find linear factorization

$$h(x) = x^2 - 4x + 1$$

$$0 = x^2 - 4x + 1$$

quad formula

$$\frac{4 \pm \sqrt{12}}{2}$$

$$2$$

$$(2 \pm \sqrt{3})$$

$$(2 + \sqrt{3}, 0)$$

$$(2 - \sqrt{3}, 0)$$

13.

$$x^2 + 25$$

$$x^2 + 25 = 0$$

$$x^2 = -25$$

$$x = \pm 5i$$

$$(5i, 0)$$

$$(-5i, 0)$$

(✓)

19.

$$t^3 - 3t^2 - 15t + 125$$

↓ calc

$$\begin{array}{r|rrrr} -5 & 1 & -3 & -15 & 125 \end{array}$$

$$\downarrow \begin{array}{r} 1 & -5 & +10 & -125 \end{array}$$

$$\begin{array}{r} 1 & -8 & 75 & 0 \end{array} \quad (\checkmark)$$

$$x^2 - 8x + 25$$

$$(x+5)(x^2-8x+25)$$

Quad formula $\frac{8 \pm \sqrt{36}}{2}$
written $\rightarrow$

$$\frac{8 \pm 6i}{2}$$

$$4 \pm 3i$$

$$(5, 0)$$

$$(4+3i, 0)$$

$$(4-3i, 0)$$

23.

$$x^4 + 10x^2 + 9$$

$$(x^2)^2 + 10(x^2) + 9$$

$$(x^2+1)(x^2+9)$$

$$(x+i)(x-i)(x+3i)(x-3i)$$

↓ terms of squares

↓ factor

$$(i, 0)$$

$$(-i, 0)$$

$$(3i, 0)$$

$$(-3i, 0)$$

25.

$$x^4 - 4x^3 + 8x^2 - 16x + 16$$

$$\begin{array}{r|rrrrr} 2 & 1 & -4 & +8 & -16 & 16 \end{array}$$

$$\downarrow \begin{array}{r} 1 & 2 & -4 & 8 & -16 \end{array}$$

$$\begin{array}{r} 1 & -2 & 4 & -8 & 0 \end{array} \quad (\checkmark)$$

upper band $\rightarrow x^3 - 2x^2 + 4x - 8$

$$\begin{array}{r|rrrr} 2 & 1 & -2 & 4 & -8 \end{array}$$

$$\downarrow \begin{array}{r} 1 & 2 & 0 & 8 \end{array}$$

$$\begin{array}{r} 1 & 0 & 4 & 0 \end{array} \quad (\checkmark) \quad (\checkmark)$$

$$x^2 + 4 = 0$$

$$x^2 = -4$$

$$x = \pm 2i$$

$$(2, 0)$$

$$(2, 0)$$

$$(2i, 0)$$

$$(-2i, 0)$$

$$(x-2)^2 (x+2i)(x-2i)$$

$$27, \quad x^2 - 14x + 46$$

$$-b \pm \sqrt{b^2 - 4ac}$$

$$2a$$

$$14 \pm \sqrt{12}$$

$$2$$

$$14 \pm 2\sqrt{3}$$

$$2$$

$$7 \pm \sqrt{3}$$

$$(7 + \sqrt{3}, 0)$$

$$(7 - \sqrt{3}, 0)$$

$$(x - (7 + \sqrt{3}))(x - (7 - \sqrt{3}))$$

$$31, \quad x^3 - 11x + 150$$

factors (150)

1

+ test!

-6

$$1 \quad 0 \quad -11 \quad 150$$

$$\downarrow \quad 1 \quad -6 \quad 25 \quad 0 \quad 150$$

$$1 \quad -6 \quad 25 \quad 0 \quad (\checkmark)$$

$$x^2 - 6x + 25$$

$$6 \pm \sqrt{64}$$

$$2$$

$$3 \pm 4i$$

quad formula

$$33, \quad x^4 + 25x^2 + 144$$

$$(x^2)^2 + 25(x^2) + 144$$

$$(\cancel{x^2})(\cancel{x^2})$$

$$-25 \pm \sqrt{576}$$

$$2$$

$$-25 \pm 24$$

$$2$$

$$(x^2 + 9)(x^2 + 16)$$

$$x^2 = -9$$

$$x = \pm 3i$$

$$x^2 = -16$$

$$x = \pm 4i$$

$$(x - 3i)(x + 3i)(x - 4i)(x + 4i)$$

p141

35, Find function w/ Resp zeros: $3, i, -i$

$$\begin{aligned}
 & (x-3)(x-i)(x+i) \\
 & (x-3)(x^2 - i^2) \\
 & (x-3)(x^2 - (-1)) \\
 & (x-3)(x^2 + 1) \\
 & x^3 + x - 3x^2 - 3 \\
 & x^3 - 3x^2 + x - 3
 \end{aligned}$$

37, $2, -4+i, -4-i$

$$\begin{aligned}
 & (x-2)(x-[-4+i])(x-[-4-i]) \\
 & (x-2)((x+4)+i)((x+4)-i) \\
 & (x-2)((x+4)^2 - i^2) \\
 & (x-2)(x^2 + 8x + 16 - (-1)) \\
 & (x-2)(x^2 + 8x + 17) \\
 & x^3 + 8x^2 + 17x - 2x^2 - 16x - 34 \\
 & x^3 + 6x^2 + 1x - 34
 \end{aligned}$$

39, $-5, -5, 1+3i, 1-3i$

$$\begin{aligned}
 & (x+5)^2 (x-[1+3i])(x-[1-3i]) \\
 & (x^2 + 10x + 25)(x-1)^2 - 9i^2 \\
 & (x^2 + 10x + 25)(x^2 - 2x - 1 + 9) \\
 & x^4 - 2x^3 + 8x^2 + 10x^3 - 20x^2 + 80x + 25x^2 - 50x + 200 \\
 & x^4 + 8x^3 + 13x^2 + 30x + 200
 \end{aligned}$$

Find all zeros

47. $x^3 - 7x^2 - x + 87 \rightarrow 5+2i$ is a 0

Conjugate $\left\{ \begin{array}{r} 5+2i \quad | \quad -7 \quad -1 \quad 87 \\ \downarrow \rightarrow 5+2i \quad -14+6i \quad -87 \\ 1 \quad -2+2i \quad -15+6i \quad 0 \textcircled{0} \\ x^2 + (-2+2i)x - 15+6i \end{array} \right.$

Can't use calc when it has roots

$\uparrow$ $\begin{array}{r} 5-2i \quad | \quad -2+2i \quad -15+6i \\ \downarrow \rightarrow 5-2i \quad -15-6i \\ 1 \quad 3 \quad 0 \quad \textcircled{0} \end{array}$

Can also x^2+3x
those 2 together $(x)(x+3)$
+ long divide

$\begin{array}{l} (5+2i, 0) \\ (5-2i, 0) \\ (-3, 0) \end{array}$
 $\uparrow$
degree = 3

48. $3x^3 - 4x^2 + 14x + 20$ for $1-\sqrt{3}i$
 $1+\sqrt{3}i$ know

$\begin{array}{l} (x - [1-\sqrt{3}i]) \\ (x - [1+\sqrt{3}i]) \\ (x - [1+\sqrt{3}i]) (x - [1-\sqrt{3}i]) \\ (x-1)^2 - 3i^2 \\ x^2 - 2x + 1 + 3 \\ x^2 + 2x + 4 \end{array}$

$\begin{array}{r} 3x+2 \\ x^2+2x+4 \overline{) 3x^3-4x^2+14x+20} \\ \text{blah-blah} \end{array}$

long division

$(x - [1-\sqrt{3}i]) (x - [1+\sqrt{3}i]) (3x+2)$

53. $x^4 + 3x^3 - 5x^2 - 21x + 22$

find Os on calc or

factors (22)

factors (1)

$\pm 1, \pm 2, \pm 11, \pm 22$

$$\begin{array}{r} 1 \quad 3 \quad -5 \quad -21 \quad 22 \\ \downarrow \nearrow 1 \quad \nearrow 4 \quad \nearrow -1 \quad \nearrow -22 \\ 1 \quad 4 \quad -1 \quad -22 \quad 0 \end{array}$$

$$\begin{array}{r} 2 \quad 1 \quad 4 \quad -1 \quad -22 \\ \downarrow \nearrow 2 \quad \nearrow 12 \quad \nearrow 22 \\ 1 \quad 6 \quad 11 \quad 0 \end{array}$$

$$x^2 + 6x + 11$$

$$-6 \pm \sqrt{36 - 44}$$

$$2$$

$$-3 \pm \sqrt{2}i$$

$$(1, 0)$$

$$(2, 0)$$

$$(-3 + \sqrt{2}i, 0)$$

$$(-3 - \sqrt{2}i, 0)$$

56. $25x^3 - 55x^2 - 54x - 18$

$$\begin{array}{r} 3 \quad 25 \quad -55 \quad -54 \quad -18 \\ \downarrow \nearrow 75 \quad \nearrow 60 \quad \nearrow 18 \\ 25 \quad 20 \quad 6 \quad 0 \end{array}$$

$$25x^2 - 20x - 6$$

$$20 \pm \sqrt{400 + 600}$$

$$50$$

2.5 Fundamental Theorem of Algebra

More Practice

10/8

P140

16.

$$f(y) = y^4 - 625$$

$$(y^2)^2 - 625$$

$$(y^2 - 25)(y^2 + 25)$$

$$y^2 = 25$$

$$y = \pm 5$$

$$y^2 = -25$$

$$y = \pm 5i$$

$$(x-5)(x+5)(x-5i)(x+5i)$$

4.8.

$$h(x) = 4x^3 + 23x^2 + 34x - 10$$

$$\rightarrow -3 + i$$

$$-3 + i$$

$$\begin{array}{r} 4 \quad 23 \quad 34 \quad -10 \\ \downarrow \nearrow \end{array}$$

$$1 \quad -12 + 4i \quad -37 - i \quad 10$$

$$4 \quad 11 + 4i \quad -3 - i \quad 0 \quad (\checkmark)$$

conjugate

$$-3 - i$$

$$4x^2 + (11 + 4i)x - 3 - i$$

$$\begin{array}{r} 4 \quad 11 + 4i \quad -3 - i \\ \downarrow \nearrow \end{array}$$

$$1 \quad -12 - 4i \quad 3 + i$$

$$4 \quad -1 \quad 0 \quad (\checkmark)$$

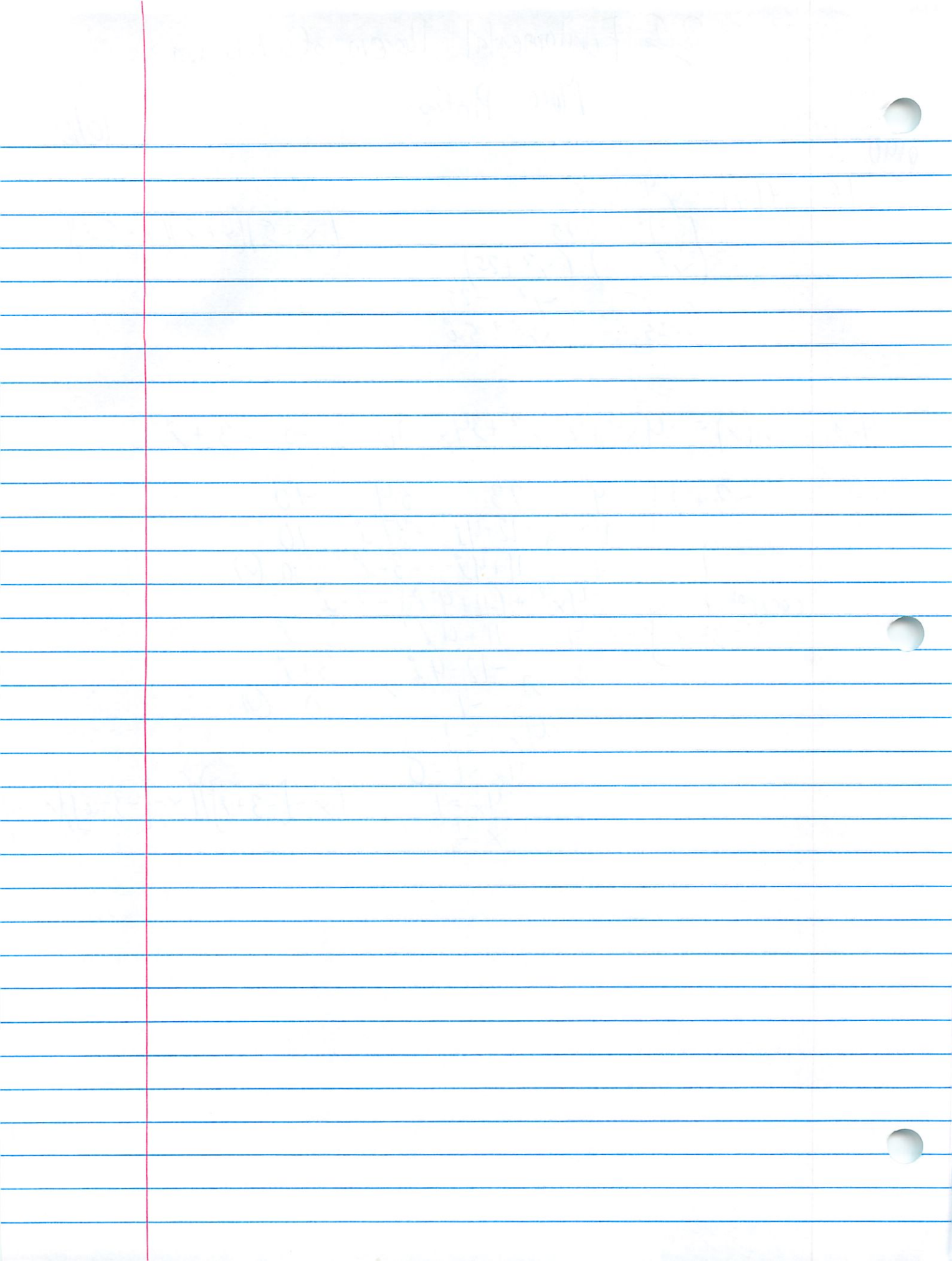
$$4x - 1$$

$$4x - 1 = 0$$

$$4x = 1$$

$$x = \frac{1}{4}$$

$$(x - [-3 + i])(x - [-3 - i])(x - \frac{1}{4})$$

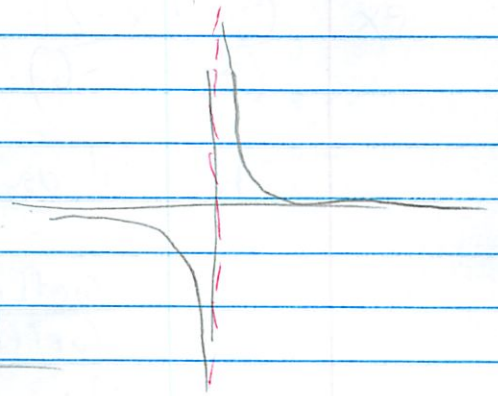


2.6 Rational Functions + Asymptotes

10/3

Asymptote = $f(x) = \frac{n(x)}{d(x)}$

$$f(x) = \frac{1}{x}$$



x	-1	-.5	-.1	-.01	-.001
f(x)	-1	-2	-10	-100	-1000

left as 0

approaching $-\infty$

domain = $\{x | x \neq 0\}$
 $[-\infty, 0) \cup (0, \infty)$
 unites w/

limit $(f(x))$
 $x \rightarrow 0^- = -\infty$

x	1	.5	.1	.01	.001
f(x)	1	2	10	100	1000

right of 0

approaching ∞

limit $(f(x))$
 $x \rightarrow 0^+ = \infty$

vertical asymptote $x=0$

horizontal asymptote $y=0$

No intercepts

decreasing

Symmetrical at origin = (odd)

range $(-\infty, 0) \cup (0, \infty)$

Definition of horizontal + vertical asymptotes

If the line $x=a$ is a vertical asymptote of $f(x)$, then $f(x)$ approaches $+\infty$, or $-\infty$ as x approaches a from both sides

The line $y=b$ is a horizontal asymptote if $f(x)$ approaches b , as x approaches ∞ , and $-\infty$ from both sides

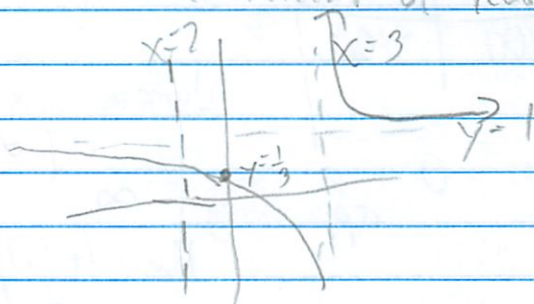
ex $\frac{(x^2 + x - 2)}{(x^2 - x - 6)} \rightarrow \frac{(x+2)(x-1)}{(x-3)(x+2)} = \frac{(x-1)}{(x-3)}$

$x \neq 3, -2$ $\in$ points of discontinuity

Vertical asymptote $x=3$

horizontal asymptote

$y = \frac{\text{Coefficient of leading term}}{\text{Coefficient of leading term}}$ if degrees =



y-intercept

$x=0 \rightarrow \frac{(0)^2 + (0) - 2}{(0)^2 + (0) - 6} = -\frac{1}{3}$

* Rules for H.A.

compare degrees of top to bottom

If degree (top) < degree (bottom)
Horiz asy = x axis ($y=0$)

$\frac{4}{x^2+1}$ horiz asy $y=0$
y-intercept = $(0, 4)$

$\frac{2x^2+1}{x^3+1}$

$x=0 \rightarrow y=1$

horiz asy $x=0$

y-intercept $(0, 1)$

vert asy: $0 = x^3 + 1$

$\sqrt[3]{-1} = -1$

$x = -1$

domain $\{x | x \in \mathbb{R}\}$



If degree(top) = degree(bottom),
use coefficient of leading terms

$$f(x) = \frac{2x+1}{x+1}$$

$$\text{horiz asy} = \frac{2}{1} = 2$$

$$y\text{-intercept} = +1 \rightarrow$$

$$\text{vert asy} = -1 \rightarrow x+1=0$$

$$x = -1$$

$$x\text{-intercept} \rightarrow \left(-\frac{1}{2}, 0\right) \rightarrow \text{want numerator} = 0$$

(zeros)

$$2x+1=0$$

$$2x = -1$$

$$x = -\frac{1}{2}$$

If degree(top) > degree(bottom)
no horizontal asymptote

$$f(x) = \frac{3x^3}{2x^2}$$

no horiz asy

$$\text{Vertical asymptote} = 0 \Rightarrow$$

$$\text{domain } \{x; x \neq 0\}$$

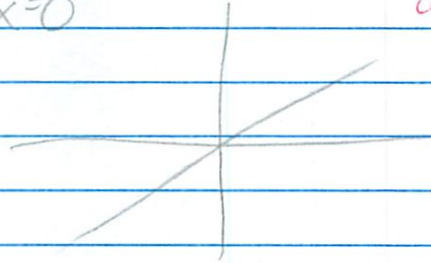
point of discontinuity
at $x=0$

↓ simplify

$$\frac{3x}{2}$$

↓

no vertical asymptote

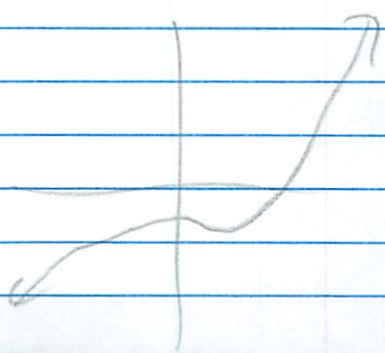


$$f(x) = \frac{3x^3 - 5x^2 + 4x - 5}{2x^2 - 6x + 7}$$

continuous everywhere

no horiz asy

no vertical asy



$$f(x) = \frac{2x}{3x^2 + 1}$$

horiz asy = 0

$$\text{vertical asy} \rightarrow 3x^2 + 1 = 0$$

$$3x^2 = -1$$

$$x^2 = -\frac{1}{3}$$

$$x = \pm \sqrt{-\frac{1}{3}}$$

$$x = \pm \sqrt{\frac{1}{3}} i$$

no real vertical

asymptote

Domain =

$$\{x \mid x \in \mathbb{R}\}$$

point at (0,0)

$$f(x) = \frac{2x^2}{x^2 - 1}$$

horizontal asy = $\frac{2}{1} \rightarrow 2$

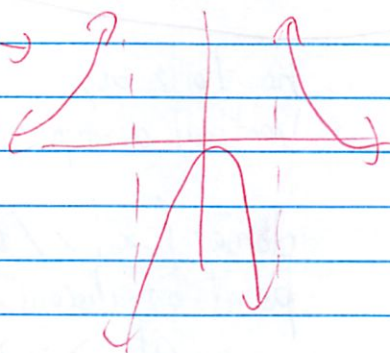
$$\text{vertical asy} \rightarrow x^2 - 1 = 0$$

$$x^2 = 1$$

$$x = \pm 1$$

Domain $\{x \mid x \neq \pm 1\}$

to find
direction $\rightarrow$
test -1, 0, 1
 $\rightarrow$ big (+)



2.6 Rational Functions + Asymptotes

HLW

10/8

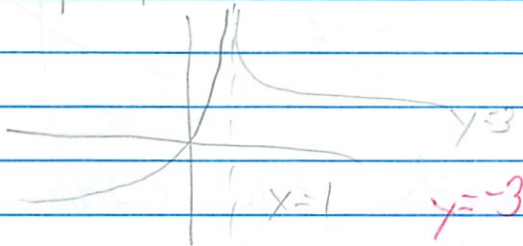
Q118

3.

Find Vertical + Horizontal Asymptotes

$$\frac{3x}{|x-1|}$$

r, reduce
asy

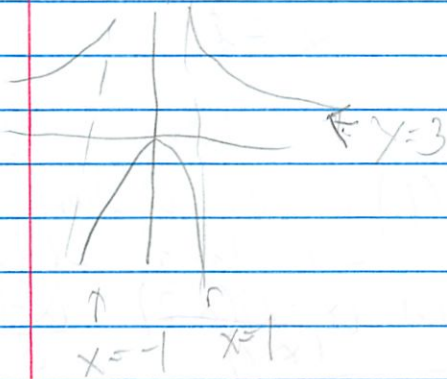


horizontal asy: degrees = $\frac{3}{1} = 3$
vertical asy: $x=1$

~~$x=1$~~ want to
 $x=1$ get 0
 $x+1=0$
 $x-1=0$, $x=1$
 $x=1$, $x=\sqrt{-1}$
 $x=\pm i$
non real

5.

$$\frac{3x^2}{x^2-1}$$



$$\text{horiz asy} = \frac{3}{1} = 3$$

$$\text{vertical asy: } x^2-1=0$$

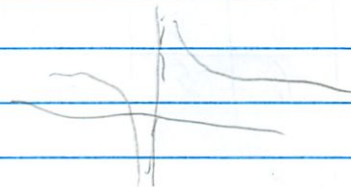
$$x^2=1$$

$$x=\pm 1$$

4. Match,

$$\frac{4x+1}{x}$$

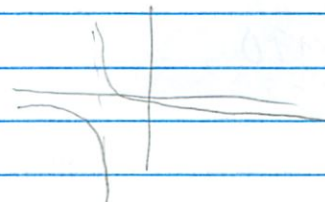
$\rightarrow$ ($\rightarrow$



12.

$$\frac{x+2}{x+4}$$

$\rightarrow$ ~~X~~
 $\downarrow$ f

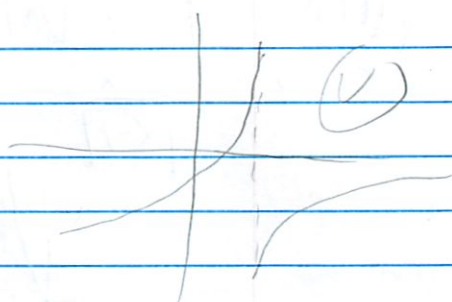


Very
similar - look
closely

150

15. $f(x) = \frac{2+x}{2-x}$

Domain: $\{x | x \neq 2\}$
 horiz asy = $\frac{1}{-1} = -1$
 vertical = $2-x=0$
 $-x = -2$
 $x = 2$



19. $f(x) = \frac{3x^2+x-5}{x^2+1}$

Domain: $\{x | x \in \mathbb{R}\} \in \mathbb{R} \neq \emptyset$
 horiz asy = $\frac{3}{1} = 3$
 vertical asy = $x^2+1=0$
 $x^2 = -1$
 $x = \pm \sqrt{-1}i$
 $x = \pm i$

Find domain + make table

23. $f(x) = \frac{x^2-4}{x+2}$ $g(x) = x-2$

$\{x | x \neq -2\}$ $\{x | x \neq 2\}$

$f(x)$ has a slight irregularity at $x = -2$

? why $\rightarrow$

vertical asy $\rightarrow$ none $\rightarrow$ reduce first $\frac{(x+2)(x-2)}{(x+2)}$ $\frac{(x-2)}{0}$ $\rightarrow$ none

x	-4	-3	-2.5	-2	-1.5	-1	0
f(x)	-6	-5	-4.5	X	-3.5	-3	-1
g(x)	-6	-5	-4.5	-4	-3.5	-3	-1

26. $f(x) = \frac{2x-8}{x^2-9x+20}$
 $\{x | x \neq 5, 4\}$
 vert asy:

$g(x) = \frac{2}{x-5}$
 $\{x | x \neq 5\}$
 vert asy: $x=5$

$x^2-9x+20 = (x-5)(x-4)$
 factor

differ at

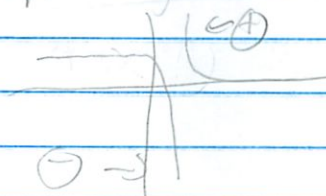
$x=4$

x	0	1	2	3	4	5	6
f(x)	-4	-1.5	$-\frac{2}{3}$	-1	X	X	2
g(x)	-4	-1.5	$-\frac{2}{3}$	-1	-2	X	2

$x=5 \in$ both
 undefined, so
 the same

28.

$$2 + \frac{1}{x-3}$$

It is approaching $x=3$ 

$$y=2$$

find zeros

31.

$$\frac{x^2-4}{x+3}$$

$$x^2-4=0$$

$$x^2=4$$

$$x=\pm 2$$

$$(2,0)$$

$$(-2,0)$$

want
numerator = 0

34.

$$5 + \frac{3}{x^2+1}$$

~~$$\frac{5(x^2+1)+3}{x^2+1}$$~~

~~$$\frac{5x^2+15}{x^2+1}$$~~

no real x -interceptshoriz-asy = 5 as the fraction
drops out

35.

$$C = \frac{255p}{100-p} \quad \text{where } 0 \leq p < 100$$

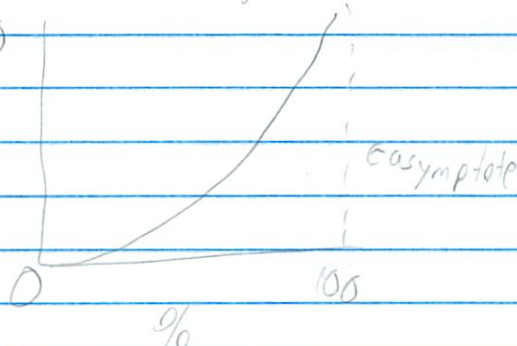
C Cost in millions
 p % pollutants

a) $10\% = \$28\frac{1}{3}$ million

b) $40\% = \$170$ million

c) $75\% = \$765$ million

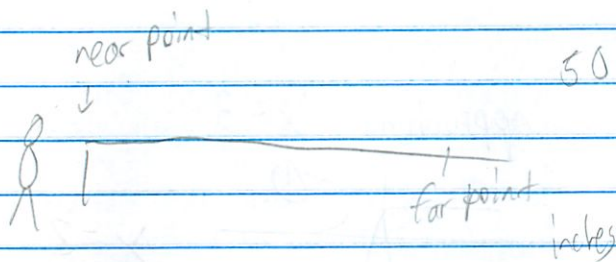
d) 3500



e) It's never possible to remove 100% - it's on asymptote

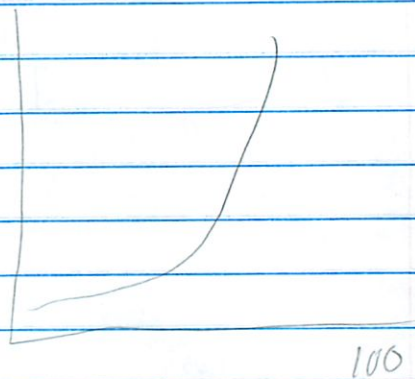
p156

37.



50

inches



100

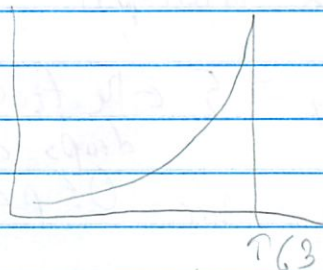
a) reciprocal

$$\frac{1}{y} = ax + b$$

$$\frac{1}{y} = 3,93x^2 + ,51$$

$$y = \frac{1}{0,445 - 1,007x}$$

b)



63

c) No. the model predicts we go blind at ~ age 63

2.6 Rational Functions + Asymptotes

Day 2

10/9

p149

18. $f(x) = \frac{x^2 - 25}{x^2 + 5x}$

Don't reduce for Domain + y intercept

copy the problem right

Domain = $\{x | x \neq 0, -5\}$ *no reducing*
 Horiz asy = $\frac{1}{1} = 1$
 vert asy = $x^2 + 5 = 0 \rightarrow$ factor

$x^2 + 5 = 0$
 $x^2 = -5$
 $x = \pm \sqrt{-5}$ *no real*
 $x = 0 \in \text{defined}$

x intercepts = $x^2 - 25 = 0$
 $x^2 = 25$
 $x = 5 \rightarrow (5, 0)$
 y intercept = $x = 0$
 $\frac{(0)^2 - 25}{(0)^2 + 5(0)} = \frac{-25}{0}$
undefined

20. $f(x) = \frac{3x^2 + 1}{x^2 + x + 9}$

$\frac{-1 \pm \sqrt{35}i}{2}$ *complex #*

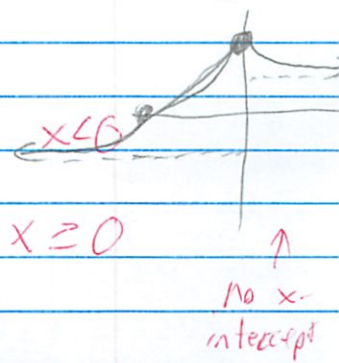
Domain = $\{x | x \in \mathbb{R}\}$
 horiz asy = $\frac{3}{1} = 3$
 vert asy = $x^2 + x + 9$
no real factors

x intercept = $3x^2 + 1 = 0$
 $x^2 = -\frac{1}{3}$
 $x = \pm \sqrt{-\frac{1}{3}}$ *no real x-int.*
 y intercept = $\frac{3(0)^2 + 1}{(0)^2 + (0) + 9} = \frac{1}{9}$

A graph w/ 2 horiz asy

$f(x) = \frac{x+10}{|x+2|}$ *test both cases*

$\begin{cases} \frac{x+10}{-x+2} \\ \frac{x+10}{x+2} \end{cases}$



becomes constant function

22.

$$\frac{x+1}{|x|+1}$$

$$\rightarrow \frac{x+1}{x+1}$$

$x < 0$

$$\frac{x+1}{1-x}$$

Domain doesn't matter

$$\{x: x \neq -1\}$$

$$\text{vertasy} = 0$$

$$y = (0, 1)$$

$$\text{horiz} = \cancel{x} = 1$$

$$x\text{-int} = \text{und}$$

is function

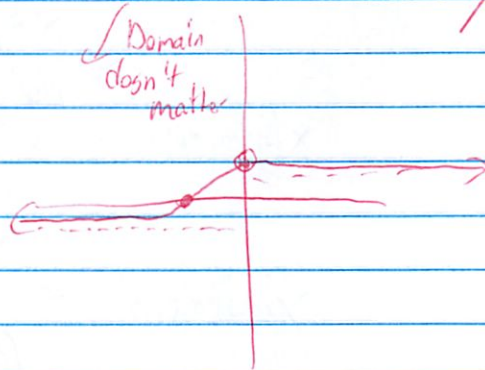
$$\{x: x \neq 1\}$$

$$\text{vertasy} = 1$$

$$y = (0, 1)$$

$$\text{horiz} = \cancel{x} = -1$$

$$x\text{-int} = (-1, 0)$$



2.7 Graphs of Rational Functions

10/9

ex1

$$g(x) = \frac{3}{x-2}$$

$$\text{Domain} = \{x : x \neq 2\}$$

$$\text{horiz asy} \rightarrow x = 0 \text{ (axis)}$$

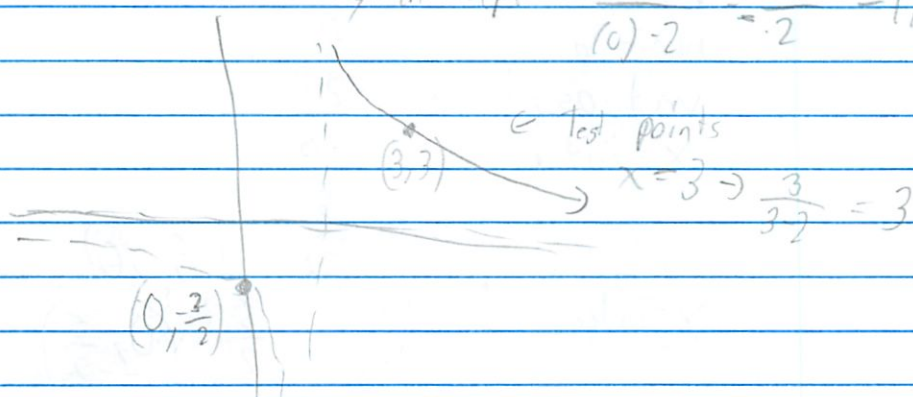
$$\text{vertical asy} \rightarrow x - 2 = 0$$

$$x = 2$$

$$x\text{-intercept} \rightarrow 3 = 0$$

none

$$y\text{-intercept} \rightarrow \frac{3}{(0)-2} = \frac{3}{-2} = -1.5$$



Notice that $\frac{3}{x-2}$ is a vertical stretch & a right shift of $\frac{1}{x-2}$, $f(x) = \frac{1}{x} \rightarrow 3[\frac{1}{x}]$ then shift 2 to the right

$$\frac{3}{x-2}$$

P1

$$\frac{2x-1}{x}$$

$$\text{Domain} = \{x : x \neq 0\}$$

$$\text{horiz} = \frac{2}{1} = 2 = \text{X}$$

$$\text{vertical} = x = 0$$

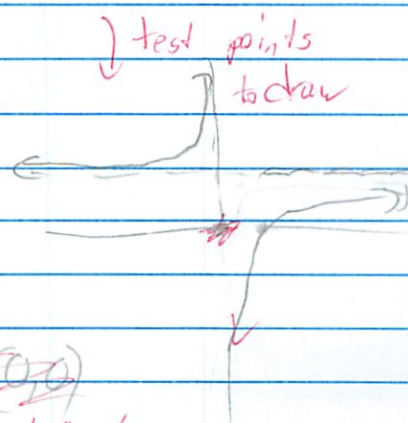
$$x\text{-int} \rightarrow 2x - 1 = 0$$

$$2x = 1$$

$$x = \frac{1}{2} \quad (\frac{1}{2}, 0)$$

$$y\text{-int.} \rightarrow \frac{2(0)-1}{(0)} = \frac{-1}{0}$$

Undefined



p157

lowest common denominator

Get LCD

14,

$$\frac{1}{x+2} + 2 \rightarrow \frac{1+2(x+2)}{x+2}, \quad \frac{1+2x+4}{x+2} \rightarrow \frac{5+2x}{x+2}$$

to $\odot \rightarrow$ look at calc graph
- should be =

Domain: $\{x; x \neq -2\}$ horiz asy: $y = \frac{2}{1} = 2$ vert asy: $x = -2$ x-int: $5+2x=0$

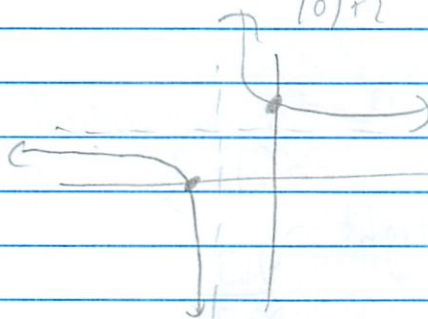
$$2x = -5$$

$$x = -\frac{5}{2}$$

$$\left(-\frac{5}{2}, 0\right)$$

y-intercept: $\frac{5+2(0)}{(0)+2}$

$$\frac{5}{2} \quad \left(0, \frac{5}{2}\right)$$



Test points

If it's on the line, need a hollow dot at this point

$$21. f(x) = 3x$$

$$x^2 - x - 2$$

$$(x-2)(x+1)$$

Domain: $\{x; x \neq 2, -1\}$ horiz asy: $y = 0$ vert asy: $x = 2$

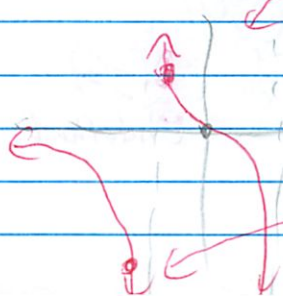
$$x = -1$$

x-int: $x = 0$

$$(0, 0)$$

y-int: $\frac{3(0)}{(0)^2 - (0) - 2}$

$$= \frac{0}{-2} \rightarrow 0 \quad (0, 0)$$



test points
to get
direction

$x = -99 \rightarrow$ high $\oplus$
 $x = -1.01 \rightarrow$ low $\ominus$

2.7 Graphs of Rational Functions

tlw

10/9

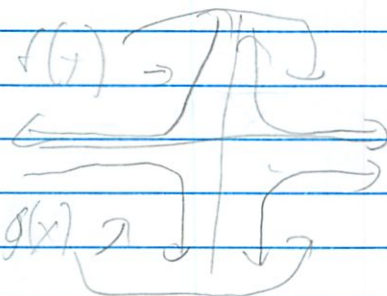
157/3

Graph

$$f(x) = \frac{2}{x}$$

$$g(x) = -f(x) = -\frac{2}{x}$$

$g(x)$ is $f(x)$ flipped along x-axis

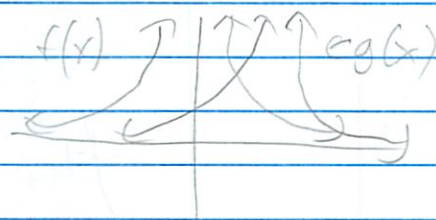


7.

$$f(x) = \frac{2}{x^2}$$

$$g(x) = \frac{2}{(x-2)^2}$$

$$g(x) \rightarrow 2$$



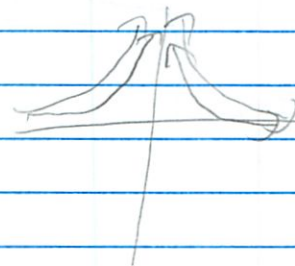
8.

$$f(x) = \frac{2}{x^2}$$

$$g(x) = \frac{1}{4} \cdot \frac{2}{x^2}$$

$$\frac{2}{4x^2} \rightarrow \frac{1}{2x^2}$$

$g(x) \neq f(x)$ but skinner



11.

$$\frac{5+2x}{1+x}$$

$$\text{Domain} = \{x, x \neq -1\}$$

$$\text{horiz asy} = \frac{2}{1} = 2 = y$$

$$\text{vert asy} = x = -1$$

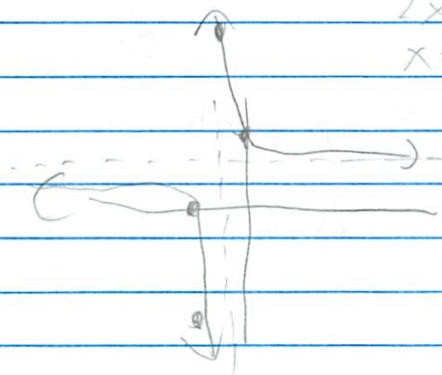
$$y\text{-intercept} = \frac{5+2(0)}{1+(0)} = \frac{5}{1} = 5 \quad (0, 5)$$

$$x\text{-intercept} = 5+2x=0$$

$$2x = -5$$

$$x = -\frac{5}{2}$$

$$\left(-\frac{5}{2}, 0\right)$$



Test

$$x = -0.99 \rightarrow 302(+)$$

$$x = -1.01 \rightarrow -298(-)$$

15.

$$\frac{x^2}{x^2-4}$$

$$\text{Domain} = \{x \mid x \neq 2, -2\}$$

$$\text{horiz asy} = 1$$

$$\text{vert asy} = x^2 - 4 = 0$$

$$x^2 = 4$$

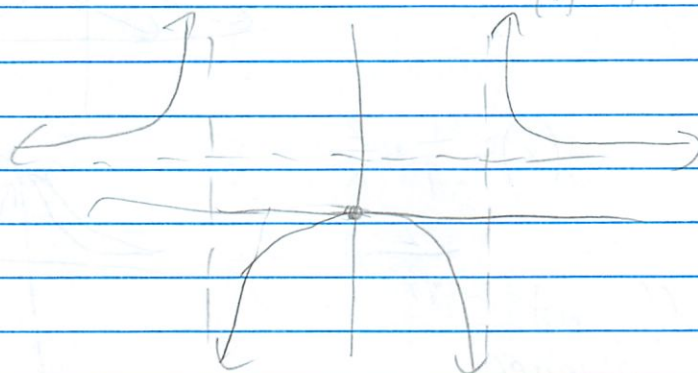
$$x = \pm 2$$

$$x\text{-int: } x^2 = 0$$

$$x = 0$$

$$(0,0)$$

$$y\text{-int: } \frac{(0)^2}{(0)^2-4} = \frac{0}{-4} = 0 \quad (0,0)$$



17.

$$\frac{x}{x^2-1}$$

$$\text{Domain} = \{x \mid x \neq \pm 1\}$$

$$\text{horiz asy} = y = 0$$

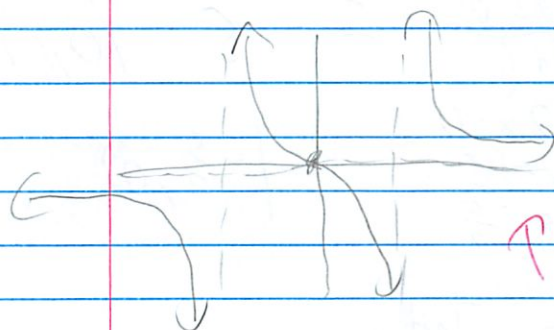
$$\text{vertical asy: } x^2 - 1 = 0$$

$$x^2 = 1$$

$$x = \pm 1$$

$$x\text{-int: } x = 0 \quad (0,0)$$

$$y\text{-int: } \frac{(0)}{(0)^2-1} = \frac{0}{-1} = 0 \quad (0,0)$$



↑ verify points

22.

$$\frac{2x}{x^2+x-2}$$

$$(x-1)(x+2)$$

$$\text{Domain: } \{x \mid x \neq 1, -2\}$$

$$\text{horiz asy} = y = 0$$

$$\text{vert asy} = x = 1, x = -2$$

$$x\text{-int: } 2x = 0$$

$$x = 0$$

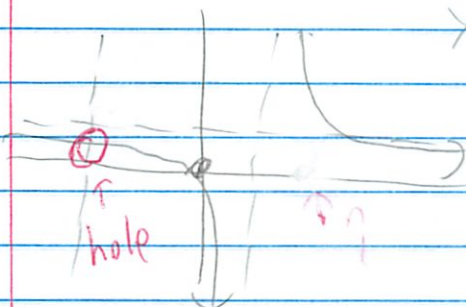
$$y\text{-int: } \frac{2(0)}{(0)^2+(0)-2} = \frac{0}{-2} = 0$$



23.

$$\frac{x^2 + 3x}{x^2 + x - 6}$$

$$\hookrightarrow \frac{(x-2)(x+3)}{(x-2)(x+3)}$$



$$\text{Domain } \{x; x \neq 2, -3\}$$

$$\text{horiz asy: } \frac{1}{1} = 1$$

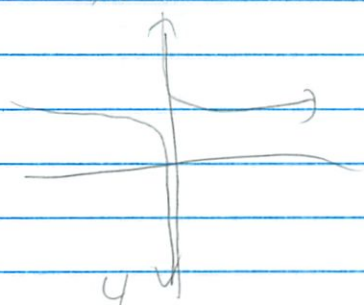
$$\text{vert asy: } x = 2, x = -3$$

$$x\text{-int: } \frac{x^2 + 3x}{(x)(x+3)} \quad (-3, 0)$$

$$y\text{-int: } \frac{(0)^2 + 3(0)}{(0)^2 + (0) - 6} = \frac{0}{-6} = 0$$

29.

$$\frac{3x+1}{x}$$



$$\text{Domain } \{x; x \neq 0\}$$

$$\text{horiz} = \frac{3}{1} = 3$$

$$\text{vert} = x = 0$$

31.

$$\frac{4}{t^2+1}$$



$$\text{Domain } \{x; x \in \mathbb{R}\}$$

$$\text{horiz asy: } y = 0$$

$$\text{vert asy: } t^2 + 1 = 0$$

$$t^2 = -1$$

$$t = \pm 1i$$

no real

35.

$$\frac{20x^2}{x^2+1} - \frac{1}{x}$$

$$\frac{20x^3 - x^2 + 1}{x^3 + x}$$

$$x^3 + x$$

$$\frac{19x^3 + 20x^2 - x + 1}{x(x^2+1)}$$

$$\text{Domain } \{x; x \in \mathbb{R}\}$$

$$\text{horiz asy} = \text{none}$$

$$\text{vert asy} = 0$$

