Linear Factors That Repeat

$$\frac{5x^2+20x+6}{x^3+2x^2+x} = \frac{5x^2+20x+6}{x(x^2+2x+1)} = \frac{5x^2+20x+6}{(x)\underbrace{(x+1)(x+1)}_{\text{repeat}}}$$

$$\frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$ ← note! squared

LCD: $(x)(x+1)^2$

note ↑ special

$$A\cancel{(x+1)}(x+1)^2 + B(x)(x+1) + C(x)\cancel{(x+1)}$$

$$A(x^2+2x+1) + B(x^2+x) + Cx$$

$$Ax^2+2Ax+A+Bx^2+Bx+Cx$$

$$Ax^2+Bx^2+2Ax+Bx+Cx+A$$

$$x^2(A+B)+x(2A+B+C)+A$$

given $\begin{cases} A+B=5 \\ 2A+B+C=20 \end{cases}$

$A=6$ → $(6)+B=5$, $B=-1$

$2(6)+(-1)+C=20$

$11+C=20$

$C=9$

write $\frac{6}{x} - \frac{1}{x+1} + \frac{9}{(x+1)^2}$

$$\frac{3x^2+4x+5}{(x+2)(x-1)^2} \to \frac{A}{x+2} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

↑ LCD

$$A(x-1)^2 + B(x+2)(x-1) + C(x+2)$$

$$A(x^2+x-2) + B(x^2+x-2) + C(x+2)$$

$$Ax^2+Ax-2A+Bx^2+Bx-2B+Cx+2C$$

$$Ax^2+Bx^2-2Ax+Bx+Cx+A-2B+2C$$

$$x^2(A+B)+x(-2A+B+C)+A-2B+2C$$

$A + B = 3$

$-2A + B + C = 4$

$+\ A - 2B + 2C = 5$

rearrange

$C = 2A - B + 4$

$\cdot 2$

$\cancel{2A + 2B = 6}$

$\boxed{3B + C = 10}$

$\cdot 2$

$\cancel{2A - 4B + 4C = 10}$

$\cancel{\boxed{-3B + 5C = 14}}$

$\cancel{6B = -9}$

$\cancel{B = -1.5}$

$A - 2B + 2(2A - B + 4) = 5$

$A - 2B + 4A + 6A - 2B + 8 = 5$

$5A - 4B = -3$

$\cdot -5$

$-5A - 5B = -15$

$-9B = -18$

$B = 2$

$A + (2) = 3$

$A = 1$

$-2(1) + (2) + C = 4$

$-2 + 2 + C = 4$

$0 + C = 4$

$C = 4$

7.3 Multivariable Linear Systems

HW

11/14

p485 Get partial function

54 $\frac{1}{4x^2-9} \rightarrow \frac{1}{(2x-3)(2x+3)} \quad \frac{A}{2x-3} + \frac{B}{2x+3}$

LCD: $(2x+3)(2x-3)$

$$\frac{A(2x+3)\cancel{(2x-3)}}{\cancel{2x-3}} + \frac{B\cancel{(2x+3)}(2x-3)}{\cancel{2x+3}}$$

$$A(2x+3) + B(2x-3)$$
$$2Ax + 3A + 2Bx - 3B$$
$$2Ax + 2Bx + 3A - 3B$$
$$x(2A+2B) + 3A - 3B$$

numerator

$2A + 2B = 0 \xrightarrow{\text{elim}} 3A + 3B = 0$

$3A - 3B = 1$

numerator

$6A = 1$

$A = \frac{1}{6}$

$2\left(\frac{1}{6}\right) + 2B = 0$

$\frac{1}{3} + 2B = 0$

$2B = -\frac{1}{3}$

$B = -\frac{1}{6}$

---

57. $\frac{1}{2x^2+x} \rightarrow \frac{1}{(x)(2x+1)} \rightarrow \frac{A}{x} + \frac{B}{2x+1}$

$2x^2+x$

$x(2x+1)$

$$A(2x+1) + B(x)$$
$$2Ax + A + Bx$$
$$2Ax + Bx + A$$
$$x(2A+B) + A$$

why

= numerator

$2A + B = 0 \rightarrow 2(1) + B = 0 \rightarrow \frac{1}{x} + \frac{-2}{2x+1}$

$A = 1$

$B = -2$

why - from numerator

$2x-1=0$
$2x=1$
$x=\frac{1}{2}$

59. $\frac{5-x}{2x^2+x-1}$ $\quad \frac{5-x}{(x+1)(2x-1)}$ $\quad \frac{A}{(x+1)} + \frac{B}{(2x-1)}$

??

$A(2x-1)+B(x+1)$
$2Ax-A+Bx+B$
$2Ax+Bx-A+B$
$x(2A+B)-A+B$

??

(1) Fixed systems

multiplication of 2

system multiplication error

$2A+B=-1$
$-A+B=5$
$-2A+2B=-10$

$3B=4$
$B=3$

$-A+(3)=-1$
$-A=-4$
$A=4$

$\frac{4}{(x+1)}+\frac{3}{(2x-1)}$

61. $\frac{x^2+12x+12}{x^3-4x} \to \frac{x^2+12x+12}{(x)(x^2-4)} \to \frac{x^2+12x+12}{(x)(x+2)(x-2)}$

$\frac{A}{x}+\frac{B}{x+2}+\frac{C}{x-2}$ $\quad LCD=(x)(x+2)(x-2)$

$A(x+2)(x-2)+B(x)(x-2)+C(x)(x+2)$
$A(x^2-4)+B(x^2-2x)+C(x^2+2x)$

(✓) Fixed

carry error

$Ax^2-4A+Bx^2-2Bx+Cx^2+2Cx$
$Ax^2+Bx^2+Cx^2-2Bx+2Cx-4A$
$x^2(A+B+C)+x(-2B+2C)-4A$

can use matrix to ✓

carry

$A+B+C=1$
$-2B+2C=12$
$-4A=12$
$A=-3$

$-3+B+C=1$
$B+C=4$
$2B+2C=8$

$-2(4)+2C=12$
$-8+2C=12$
$2C=20$
$C=10$

$2B+2C=8$
$-2B+2C=12$
$4C=20$
$C=5$

$-3+B+(5)=1$
$2+B=1$
$B=-1$

$\frac{-3}{x}+\frac{-1}{(x+2)}+\frac{5}{(x-2)}$

63. $\frac{4x^2+2x-1}{x^2(x+1)} \rightarrow \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$

↑ LCD

$\frac{A(x^2)(x+1)}{x} + \frac{B(x^2)(x+1)}{x^2} + \frac{C(x^2)(x+1)}{x+1}$

$A(x^2+x) + B(x+1) + C(x^2)$

carry error ↓

$Ax^2 + Ax + Bx + B + Cx^2$

$Ax^2 + Cx^2 + Bx + A + B$

$x^2(A+C) + x(A+B) + A + B$

✓ now

carry error

$A+C=4$ $\quad$ $A+(-1)=2$ $\quad$ $3+C=4$

$A+B=2$ $\quad$ $A=3$ $\quad$ $C=1$

$B=-1$

$\frac{-3}{x} + \frac{2}{x^2} + \frac{7}{x+1}$

65. $\frac{27-7x}{x(x-3)^2} \rightarrow \frac{A}{x} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$

↑ LCD

$\frac{A(x)(x-3)^2}{x} + \frac{B(x)(x-3)^2}{x-3} + \frac{C(x)(x-3)^2}{(x-3)^2}$

$A(x^2-9) + B(x^2-3x) + Cx$

$Ax^2 - 9A + Bx^2 - 3Bx + Cx$

$Ax^2 + Bx^2 - 3Bx + Cx - 9A$

$x^2(A+B) + x(-3B+C) - 9A$

✓

$A+B=0$ $\rightarrow$ $-3+B=0$ $\quad$ $-3(3)+C=-7$

$-3B+C=-7$ $\quad$ $B=3$ $\quad$ $-9+C=-7$

$-9A=27$ $\quad$ $C=2$

$A=-3$

$\frac{-3}{x} + \frac{3}{x-3} + \frac{2}{(x-3)^2}$

66. $\dfrac{x^2-x+2}{x(x-1)^2}$ $\quad \dfrac{A}{x}+\dfrac{B}{x-1}+\dfrac{C}{(x-1)^2}$

$$\frac{A(x)(x-1)^2}{x}+\frac{B(x)(x-1)^2}{(x-1)}+\frac{C(x)(x-1)^2}{(x-1)^2}$$

multiplication error

$\cancel{(x-1)(x-1)}$
$\cancel{x^2-x-x+1}$ here

foil error
calc

$\cancel{(x-3)(x-3)}$
$\cancel{x^2-3x-3x+9}$

$\rightarrow A(x^2+1)+B(x^2-x)+Cx$

$A(x^2+1)+B(x^2-x)+Cx$

$Ax^2 \quad -A+Bx^2-Bx+Cx$

$Ax^2+Bx^2-Bx+Cx+A$

$x^2(A+B)+x(-B+C)+A$

$A+B=1$ $\quad (+2)+B=1$ $\quad -(-1)+C=-1$

$-B+C=-1$ $\quad B=-1$ $\quad C=-2$

$+A=2$

write error

$$\frac{2}{x}+\frac{-1}{x-1}+\frac{-2}{(x-1)^2}$$

Use Matrix on calc to ✓

7.3 Multivariable Linear Systems Day 2

11/15

ex1 $\dfrac{4x^2-9x-37}{x^3+2x^2-11x-12}$ ← do synthetic division to break into parts

$$\begin{array}{r|rrrr} -1 & 1 & 2 & -11 & -12 \\ & & -1 & -1 & 12 \\ \hline & 1 & 1 & -12 & 0 \end{array}$$

$(x+1)\ x^2+x-12$
$(x+4)(x-3)$ → $\dfrac{4x^2-9x-37}{(x+1)(x+4)(x-3)}$

$$\frac{A}{(x+1)} + \frac{B}{(x+4)} + \frac{C}{(x-3)}$$

$$A(x+4)(x-3) + B(x+1)(x-3) + C(x+1)(x+4)$$
$$A(x^2+x-12) + B(x^2+2x-3) + C(x^2+5x+4)$$
$$Ax^2+Ax-12A+Bx^2-2Bx-3B+Cx^2+5Cx+4C$$
$$Ax^2+Bx^2+Cx^2+Ax-2Bx+5Cx-12A-3B+4C$$
$$x^2(A+B+C)+x(A-2B+5C)-12A-3B+4C$$

$A+B+C=4$ $\xrightarrow{\cdot 2}$ $2A+2B+2C=8$
$A-2B+5C=-9$ $\quad 13A+7C=-1$
$-12A-3B+4C=-37$ $\xrightarrow{\cdot 4}$ $12A+28C=-4$
$-3B+32C=-41$

just do matrix

↑ or just do calc matrix

$A=2$
$B=3$
$C=-1$

$$\frac{2}{(x+1)} + \frac{3}{(x+4)} - \frac{1}{(x-3)}$$

Degree of numerator > degree (denominator)

$$\frac{x^3+2x^2-x+1}{x^2+3x-4}$$

$$\begin{array}{r} x-1 \\ x^2+3x-4 \overline{\big) x^3+2x^2-x+1} \\ -(x^3+3x^2-4x) \\ -x^2+3x+1 \\ -(-x^2-3x+4) \\ 6x-3 \end{array}$$

$$x-1+\frac{6x-3}{x^2+3x-4}, \quad \frac{6x-3}{(x+4)(x-1)}$$

$$\frac{A}{x+4}+\frac{B}{x-1} \qquad A(x-1)+B(x+4)$$

$$Ax-A+Bx+4B$$

$$Ax+Bx-A+4B$$

$$x(A+B)-A+4B$$

$$A+B=6, \quad -A+4B=-3$$

$$5B=3 \qquad B=\frac{3}{5}$$

$$A+\left(\frac{3}{5}\right)=6 \qquad A=5\frac{2}{5} \text{ or } \frac{27}{5}$$

$$x-1+\frac{\frac{27}{5}}{x+4}+\frac{3/5}{x-1}$$

$$x-1+\frac{27}{5}\cdot\frac{1}{x+4}+\frac{3}{5}\cdot\frac{1}{(x-1)}$$

remove complex fractions ↓

$$x-1+\frac{27}{5(x+4)}+\frac{3}{5(x-1)}$$

485

70. $\frac{4x^4}{(2x-1)^3} \to \frac{4x^4}{(2x-1)(2x-1)(2x-1)} \to \frac{4x^4}{(4x^2-4x+1)(2x-1)}$

$\frac{4x^4}{8x^3-12x^2+6x-1}$

$$\begin{array}{r} \frac{1}{2}x + \frac{3}{4} \\ 8x^3-12x^2+6x-1 \overline{\big) 4x^4+0x^3+0x^2+0x+0} \\ -(4x^4-6x^3+3x^2-\frac{1}{2}x) \\ 6x^3-3x^2+\frac{1}{2}x+0 \\ -(6x^3-9x^2+\frac{9}{2}x-\frac{3}{4}) \\ 6x^2-4x+\frac{3}{4} \end{array}$$

$\frac{6x^2-4x+3/4}{(2x-1)^3} \cdot 4 = \frac{24x^2-16x+3}{4(2x-1)^3}$ ← LCD

degree numerator = degree denominator −1

$24x^2-16x+3 = \frac{A}{(2x-1)} + \frac{B}{(2x-1)^2} + \frac{C}{(2x-1)^3} + \frac{0}{4}$ ← x^0, so 0

$\frac{A(4)(2x-1)^{\cancel{3}2}}{\cancel{2x-1}} + \frac{B(4)(2x-1)^{\cancel{3}}}{\cancel{(2x-1)^2}} + \frac{C(4)\cancel{(2x-1)^3}}{\cancel{(2x-1)^3}}$

$A(4)(2x-1)^2 + B(4)(2x-1) + 4C$

$4A(2x-1)(2x-1) + 4B(2x-1) + 4C$

$4A(2x^2-4x+1) + 8Bx - 4B + 4C$

$8Ax^2 - 16Ax + 4A + 8Bx - 4B + 4C$

$8Ax^2 - 16Ax + 8Bx + 4A - 4B + 4C$

$x^2(8A) + x(-16A + 8B) + 4A - 4B + 4C$

$8A = 24$

$-16A + 8B = -16$ → matrix on calc

$4A - 4B + 4C = 3$

7.3 Multivariable Linear Systems HW2

11/15

p 485

53. $\frac{1}{x^2-1} \rightarrow \frac{A}{(x+1)} + \frac{B}{(x-1)}$

$A(x+1) + B(x+1)$
$Ax - A + Bx + B$
$Ax + Bx - A + B$
$x(A+B) - A + B$

write at

$A + B = 0$ | $2B = 1$ $\quad A = -.5$
$-A + B = 1$ $\quad B = .5$

write as $\frac{1}{2}\left(\frac{1}{x+1} + \frac{1}{x-1}\right)$

55. $\frac{1}{x^2+x} \rightarrow \frac{1}{x(x+1)} \rightarrow \frac{A}{x} + \frac{B}{x+1}$

$A(x+1) + B(x)$
$Ax + A + Bx$
$x(A+B) + A$

First try

$\frac{1}{x} - \frac{1}{x+1}$

$A + B = 0$ $\quad (1) + B = 0$
$A = 1$ $\quad B = -1$

56. $\frac{3}{x^2-3x} \rightarrow \frac{3}{x(x-3)} \rightarrow \frac{A}{x} + \frac{B}{x-3}$

$A(x-3) + B(x)$
$Ax - 3A + Bx$
$x(A+B) - 3A$

$\frac{-1}{x} + \frac{1}{x-3}$

$A + B = 0$ $\quad (-1) + B = 0$
$-3A = 3$ $\quad B = 1$
$\frac{-3A}{-3} = \frac{3}{-3}$
$A = -1$

58. $\frac{5}{x^2+x-6} \rightarrow \frac{5}{(x-2)(x+3)} \rightarrow \frac{A}{(x-2)} + \frac{B}{(x+3)}$

$A(x+3) + B(x-2)$

$Ax + 3A + Bx - 2B$

$x(A+B) + 3A - 2B$

✓ 1st

$A + B = 0 \xrightarrow{\cdot -3} -3A - 3B = 0$

$3A - 2B = 5$

$-5B = 5$

$B = -1$

$A + (-1) = 0$

$A = 1$

$\frac{1}{(x-2)} + \frac{-1}{(x+3)}$

60. $\frac{x-2}{x^2+4x+3} \rightarrow \frac{x-2}{(x+1)(x+3)} \rightarrow \frac{A}{(x+1)} + \frac{B}{(x+3)}$

$A(x+3) + B(x+1)$

$Ax + 3A + Bx + B$

$x(A+B) + 3A + B$

✓ 1st

$A + B = 1$

$3A + B = -2 \xrightarrow{\cdot -1} -3A - B = 2$

$-2A = 3$

$A = -1.5$

$(-1.5) + B = 1$

$+1.5 \quad +1.5$

$B = 2.5$

$\frac{-1.5}{(x+1)} + \frac{2.5}{(x+3)}$

62. $\frac{x^2+12x-9}{x^3-9x} \rightarrow \frac{x^2+12x-9}{x(x^2-9)} \rightarrow \frac{x^2+12x-9}{(x)(x+3)(x-3)}$

$\frac{A}{x} + \frac{B}{x+3} + \frac{C}{x-3}$ LCD $= (x)(x+3)(x-3)$

✓

$A(x+3)(x-3) + B(x)(x-3) + C(x+3)(x\ \ )$

1st

$A(x^2-9) + B(x^2-3x) + C(x^2+3x)$

$Ax^2 - 9A + Bx^2 - 3Bx + Cx^2 + 3Cx$

$Ax^2 + Bx^2 + Cx^2 - 3Bx + 3Cx - 9A$

$x^2(A+B+C) + x(-3B+3C) - 9A$

$A+B+C = 1$ $\quad A = 1$

$-3B+3C = 12$ $\quad B = -2$

$-9A = -9$ $\quad C = 2$

calc matrix

$\frac{1}{x} - \frac{2}{x+3} + \frac{2}{x-3}$

64. $\frac{2x-3}{(x-1)^2} \rightarrow \frac{2x-3}{(x-1)(x-1)} \rightarrow \frac{A}{x-1} + \frac{B}{(x-1)^2}$

↑LCD

$\frac{A(x-1)^{\cancel{2}}}{\cancel{x-1}} + \frac{B\cancel{(x-1)}^{\cancel{2}}}{\cancel{(x-1)}^{\cancel{2}}}$

$Ax - A + B$

$A = 2$ $\quad -(2) + B = -3$

$-A + B = -3$ $\quad +2 \quad +2$

$B = -1$

$\frac{2}{x-1} - \frac{1}{(x-1)^2}$

67. $\dfrac{2x^3 - x^2 + x + 5}{x^2 + 3x + 2} \rightarrow \dfrac{2x^3 - x^2 + x + 5}{(x+1)(x+2)}$

$$
\begin{array}{r}
2x - 7 \\
x^2 + 3x + 2 \overline{\big) 2x^3 - x^2 + x + 5} \\
-(2x^3 + 6x^2 + 4x) \\
-7x^2 - 3x + 5 \\
-(-7x^2 - 21x - 14) \\
18x + 19
\end{array}
$$

$\dfrac{18x + 19}{(x+1)(x+2)} \rightarrow \dfrac{A}{x+1} + \dfrac{B}{x+2}$

$A(x+2) + B(x+1)$

$Ax + 2A + Bx + B$

$x(A+B) + 2A + B$

$A + B = 18 \Rightarrow -2A - 2B = -36, \quad -B = -17$

$2A + B = 19 \qquad B = 17$

$A + 17 = 18$

$A = 1$

$2x - 7 + \dfrac{1}{x+1} + \dfrac{17}{x+2}$

1

68. $\dfrac{x^3+2x^2-x+1}{x^2+3x-4}$

$$\begin{array}{r} x-1 \\ x^2+3x-4 \overline{\big) x^3+2x-x+1} \\ -(x^3+3x^2-4x) \\ -x^2+3x+1 \\ -(-x^2-3x+4) \\ 6x-3 \end{array}$$

small removing complex fraction error only

$$\frac{6x-3}{(x-1)(x+4)} \rightarrow \frac{A}{x-1}+\frac{B}{x+4}$$

$$A(x+4)+B(x-1)$$
$$Ax+4A+Bx-B$$
$$x(A+B)+4A-B$$

$A+B=6$
$4A-B=-3$) $5A=3$
$A=\frac{3}{5}$

$\left(\frac{3}{5}\right)+B=6$
$B=\frac{27}{5}$

write as

$$x-1+\frac{\frac{3}{5}}{x-1}+\frac{\frac{27}{5}}{x+4}$$

$$x-1+\frac{3}{5(x-1)}+\frac{27}{5(x+4)}$$

7.3 Multivariable Linear Systems Day 3

11/16

Partial Fractions with quadratic fractions in denomanator

ex1

$$\frac{3x^2+7x+4}{(x^2+2x+2)(x+2)} \rightarrow \frac{Ax+B}{x^2+2x+2} + \frac{C}{x+2}$$

quadratic factor, can't factor

linear factor

* degree numerator = degree denominator − 1

$LCD = (x^2+2x+2)(x+2)$

$$\frac{(Ax+B)\cancel{(x^2+2x+2)}}{\cancel{x^2+2x+2}} + x+2 + \frac{C(x^2+2x+2)\cancel{(x+2)}}{\cancel{x+2}}$$

$(Ax+B)(x+2) + C(x^2+2x+2)$

$Ax^2+2Ax+Bx+2B+Cx^2+2Cx+2C$

$x^2(A+C) + x(2A+B+2C) + 2B+2C$

$A+C=3$
$2A+B+2C=7$
$2B+2C=4$

calc → matrix

$A=2$
$B=1$
$C=1$

$$\frac{2x+1}{x^2+2x+2} + \frac{4}{x+2}$$

---

ex2

$$\frac{1}{(x^2+x+1)(x^2-x+1)} = \frac{Ax+B}{x^2+x+1} + \frac{Cx+D}{x^2-x+1}$$

LCD

other one

other one

Forgot other one

$(Ax+B)(x^2-x+1) + (Cx+D)(x^2+x+1)$

$Ax^3-Ax^2+Ax+Bx^2-Bx+B+Cx^3+Cx^2+Cx+Dx^2+Dx$

$x^3(A+C)+x^2(-A+B+C+D)+x(A-B+C+D)+B+D$ $\boxed{+D}$

$A+C=0$
$-A+B-C+D=0$
$A-B+C-D=0$
$B+D=1$

matrix → calc

$A=+\frac{1}{2}$
$B=\frac{1}{2}$
$C=-\frac{1}{2}$
$D=\frac{1}{2}$

$$\frac{+\frac{1}{2}x+\frac{1}{2}}{x^2+x+1} + \frac{-\frac{1}{2}x+\frac{1}{2}}{x^2-x+1}$$

or

$$\frac{x+1}{2x^2+2x+2} - \frac{x+1}{2x-2x+2}$$

3 cases of systems

1. Same line
   - ∞ # of solutions
   - consistant system
2. Parallel lines
   - no solutions
   - inconsistant system
3. One solution
   - consistant

485
84.

Standard equation for a circle

$$x^2+y^2+Dx+Ey+F=0$$

↑ Find D E F

Teacher copy error
- but I did it w/ #s provided

$(-6,-1)$
$(-4,-3)$
$(2,-5)$

$(-6)^2+(-1)^2+(-6)D+(-1)E+F$

$\boxed{-6D-E+F=-37}$

$(-4)^2+(-3)^2+(-4)D+(-3)E+F$

$\boxed{-4D-3E+F=-25}$

$(2)^2+(-5)^2(2)D+(-5)E+F$

$\boxed{2D-5E+F=-41}$

-1 $-2D+5E-F=41$

$\boxed{-6D+2E=16}$ $\cdot -2$

$\boxed{-8D+4E=4}$

$12D-4E=-32$

$4D=-28$

$D=-7$

$-6(-7)+2E=16$

$42+2E=16$

$E=-13$

$2(-7)-5(-13)+F=-41$

$-14+65+F=-41$

$51+F=-41$

$F=-92$

$-7=D$
$-13=E$
$-92=F$

$\hookrightarrow x^2+y^2+-7x+-13y-92$

486/86. Borrowed \$1 million

Some borrowed at 8%, 10%, 12%

Interest = \$97,200

$x = 2y$

$x + y + z = \$1{,}000{,}000$

$.08x + .1y + .12z = \$97{,}200$

$x = 2y$

$\boxed{x - 2y = 0}$

$\cdot 1$ $-x + 2y = 0$

$\boxed{3y + z = \$1{,}000{,}000}$

$\cdot -10$ $-.8x - y - 1.2z = -972{,}000$

$\boxed{.2x - .2z = 28{,}000}$

$\cdot 5$ $\boxed{x - z = 140{,}000}$

$\boxed{x + 3y = 1{,}140{,}000}$

$-x + 2y = 0$

$5y = 1{,}140{,}000$

$y = \$228{,}000$

$x + 3(228{,}000) = 1{,}140{,}000$

$x = \$456{,}000$

$3(228{,}000) + z = 1{,}000{,}000$

$z = \$316{,}000$

7.3 Multivariable Linear Systems HW Day 3

11/17

485

Find equation that passes through ($y = Ax^2 + Bx + C$)

77. $(0,0)$, $(2,-2)$, $(4,0)$

$A(0)^2 + B(0) + C = 0$

$C = 0$

$A(2)^2 + B(2) + C = -2$

$\boxed{4A + 2B + C = -2}$ → $4A + 2(-2) + (0) = -2$

$+4 \quad +4$

$4A = 2$

$A = \frac{1}{2}$

1

$\cdot -4$

$A(4)^2 + B(4) + C = 0$

$\boxed{16A + 4B + C = 0}$

$-16A - 8B + (0) = 8$

$-4B = 8$

$B = -2$

v. w/ calc matrix → $\frac{1}{2}x^2 - 2x + 0 = y$

79. $(2,0)$, $(3,-1)$, $(4,0)$

$A(2)^2 + B(2) + C = 0$

$\boxed{4A + 2B + C = 0}$

$A(3)^2 + B(3) + C = -1$

$\boxed{9A + 3B + C = -1}$

$\cdot 2$

$8A + 4B + 4C = 0$

$\cdot -4$

$A(4)^2 + B(4) + C = 0$

$\boxed{16A^2 + 4B + C = 0}$

$-16A^2 - 8B - 4C = 0$

→ $\boxed{8A + C = 0}$

$\boxed{-4B - 3C = 0}$

$-8A - C = 0$

$\cdot -1$

$\boxed{A + 3B = -1}$

or just calc

$A = 1$

$B = -6$

$C = 8$

$1x^2 + -6B + 8 = y$

$x^2 + y^2 + Dx + Ey + F = 0$

81. $(0,0)$
$(2,2)$
$(4,0)$

$(0)^2 + (0)^2 + D(0) + E(0) + F = 0$
$F = 0$

$(2)^2 + (2)^2 + D(2) + E(2) + F = 0$
$2D + 2E + F = -8$

$(4)^2 + (0)^2 + D(4) + E(0) + F = 0$
$4D + F = -16$

$2(-4) + 2E + (0) = -8$
$-8 + 2E = -8$
$2E = 0$
$E = 0$

$4D + 0 = -16$
$D = -4$

$x^2 + y^2 + (-4)x + (0)y + (0)$
$x^2 + y^2 - 4x$

---

83. $(-3,-1)$
$(2,4)$
$(-6,8)$

$(-3)^2 + (-1)^2 + D(-3) + E(-1) + F = 0$
$-3D - E + F = -10$

$(2)^2 + (4)^2 + D(2) + E(4) + F = 0$
$2D + 4E + F = -20$

$(-6)^2 + (8)^2 + D(-6) + E(8) + F = 0$
$-6D + 8E + F = -100$

Calc
$D = 6$
$E = -8$
$F = 0$

$x^2 + y^2 + 6x - 8y$

85. borrowed 775,000

A = ~~$~~ invested 8%

B = " " 9%

C = " " 10%

Interest = 67,000

$4C = A$

$A + B + C = 775{,}000$

$.08A + .09B + .1C = 67{,}000$

$4C = A$

$4C - A = 0$

$\cdot -10$

$B + 5C = 775{,}000$

$-.8A - .9B - C = -670{,}000$

$.2A + .1B = 105{,}000$

$\cdot -10$

$-2A - B = -105{,}000$

$B + 5C = 75{,}000$

$-2A + 5C = -30{,}000$

$-8C + 5C = -30{,}000$

$-3C = -30{,}000$

$C = 10{,}000$

$B + 5(10{,}000) = 775{,}000$

$B = 725{,}000$

$4(10{,}000) = A$

$A = 40{,}000$

wrong but oh well

486

92. Sports

34 points

11 plays

A = # 6 pt plays

B = # 1 pt plays

C = # 3 pt plays

A = C

$A + B + C = 11$

$6A + B + 3C = 39$

$A = C$

$A - C = 0$

$2A + B = 11$

$\cdot -3$ $-6A - 3B = -33$

$-2B + 3C = 6$

$3A - 2B = 6$

$\cdot 2$

$4A + 2B = 22$

$7A = 28$

$A = 4$ $C = 4$

$2(4) + B = 11$

$8 + B = 11$

$B = 3$

4 touchdowns

3 extra pts

4 field goals

7.3 Multivariable Linear Systems Day 5

11/19

Factoring in Getting Started

1. $\frac{1}{x^4-x^3} \to \frac{1}{x^3(x-1)} \quad \frac{A}{x-1}+\frac{B}{x}+\frac{C}{x^2}+\frac{D}{x^3}$

degree num = degree denom − 1

2. $\frac{1}{x^4-1} \to \frac{1}{(x^2+1)(x^2-1)} \to \frac{Ax+B}{(x^2+1)} + \cancel{\frac{B}{(x^2-1)}} + \frac{C}{x+1} + \frac{D}{x-1}$

1 more step

3. $\frac{x^3-4x^2+2}{(x^2+1)(x^2+2)} \to \frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{(x^2+2)}$

4. $\frac{x^4+x^2+1}{x^2(x^2+4)^2} \to \cancel{\frac{x^4+x^2+1}{x^2(x^2+4)(x^2+4)}}$

$$\frac{A}{x}+\frac{B}{x^2}+\frac{Cx+D}{x^2+4}+\frac{Ex+F}{(x^2+4)^2}$$

same as linear functions

5. $\frac{x^3-3x^2+12x-1}{x^3+(x^2+x+1)+(x^2+2)^3} = \frac{A}{x}+\frac{B}{x^2}+\frac{C}{x^3}+\frac{Dx+E}{x^2+x+1}$

can't factor any more

$$\frac{Fx+G}{x^2+2}+\frac{Hx+I}{(x^2+2)^2}+\frac{Jx+K}{(x^2+2)^3}$$

7.3 Multivariable Systems Review

11/19

1. $\dfrac{x}{8x^2-10x+3}$

24
ac
-6 -4 $\uparrow x$
-3 -2 $+$
-10

	$4x$	-3
$2x$	$8x^2$	$-6x$
-1	$-4x$	3

$$\frac{A}{4x-3} + \frac{B}{2x-1}$$

$$\frac{A\cancel{(4x-3)}(2x-1)}{\cancel{4x-3}} + \frac{B(4x-3)\cancel{(2x-1)}}{\cancel{(2x-1)}}$$

$$A(2x-1) + B(4x-3)$$

$$2Ax - A + 4Bx - 3B$$

$$x(2A+4B) - A - 3B$$

$$2A+4B = 1$$
$$-A-3B = 0 \quad \cdot 2$$
$$-2A-6B = 0$$

$$-2B = 1$$
$$B = -\tfrac{1}{2}$$

$$2A + 4\left(-\tfrac{1}{2}\right) = 1$$
$$+2 \qquad +2$$
$$2A = 3$$
$$A = \tfrac{3}{2}$$

2. $\dfrac{9x^2-9x+6}{2x^3-x^2-8x+4} = \dfrac{A}{x+2} + \dfrac{B}{x-2} + \dfrac{C}{2x-1}$

$$x^2(2x-1) - 4(2x-1)$$
$$(x^2-4)(2x-1)$$
$$(x+2)(x-2)(2x-1)$$

↓ factor

↑ factor wrong

← LCD

$$A(x-2)(2x-1) + B(x+2)(2x-1) + C(x+2)(x-2)$$
$$A(2x^2-x-4x+2) + B(2x^2-x+4x-2) + C(x^2-4)$$
$$2Ax^2 - 5Ax + 2A + 2Bx^2 + 3Bx - 2B + Cx^2 - 4C$$
$$2Ax^2 + 2Bx^2 + Cx^2 - 5Ax + 3Bx + 2A - 2B - 4C$$
$$x^2(2A+2B+C) + x(-5A+3B) + 2A - 2B - 4C$$

$$2A+2B+C = 9$$
$$-5A+3B = -9$$
$$2A-2B-4C = 6$$

$\rightarrow$ calc $\rightarrow$ $A = 3$, $B = 2$, $C = -1$

$$\frac{3}{x+2} + \frac{2}{x-2} - \frac{1}{2x-1}$$

3. $\frac{x-3}{x^3+3x} \to \frac{x-3}{x(x^2+3)} \to \frac{A}{x} + \frac{Bx+C}{x^2+3}$

$A(x^2+3) + (Bx+C)(x)$

$Ax^2 + 3A + Bx^2 + Cx$

$x^2(A+B) + x(C) + 3A$

$A+B=0$ $\quad$ $-1+B=0$

$C=1$ $\quad$ $B=1$

$3A=-3$

$A=-1$

$\frac{-1}{x} + \frac{1x+1}{x^2+3}$

4. $\frac{x^5-2x^4+x^3+x+5}{x^3-2x^2+x-2}$

$1x^2 + \cancel{x+2}$

$x^3-2x^2+x-2 \overline{)\, x^5-2x^4+x^3+0x^2+x+5}$

$-(x^5-2x^4+x^3-2x^2)$

bad subtraction

$\cancel{=x^4+0x^3+2x^2+x+5}$

$\cancel{-(-x^4+2x^3-x^2+2x)}$

$2x^2+x+5$

$\cancel{2x^3+3x^2-1x+5}$

remainder $\cancel{(-2x^2+4x^2-2x-4)}$

$\cancel{-x^2+3x+9}$

4

error!!

$\frac{-x^2+3x+9}{x^3-2x^2+x-2}$ $\quad$ $x^2(x-2)+1(x-2)$

$(x^2+1)(x-2)$

copy error

$\frac{Ax+B}{x^2+1} + \frac{C}{x-2}$ $\quad$ $(Ax+B)(x-2) + C(x^2+1)$

$Ax^2-2Ax+Bx-2B+Cx^2+C$

$x^2(A+C)+x(-2A+B)+C-2B$

carry error

copy error

$A + C = 2$

$-2A + B = 1$

$+C = 5$

$\cancel{C = 15}$

$A + (+5) = -1$

$-5 \quad -5$

$A = -6$

-6

$-(6) + B = 3$

$-6 \quad -6$

$B = -3$

$$x^2 + \frac{-6x - 3}{x^2+1} + \frac{5}{x-2}$$

still wrong

8.1 Sequences + Series

Notes

11/27

A sequence has order
Has a first number (position)
A second position, etc

Is a function whose domain is the set of positive interger

a_1, a_2, a_3, a_4, a_n

ex: 1, 4, 7, 10 .... (infinate sequence)

+3 +3 +3

infinate

2, 4, 2, 4 ... (oscillating sequence)

$1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25} \ldots$

<u>finite sequence</u> - has a specific number of terms

consits of the first n positive intigers

ex The first 4 terms of the sequence given by

$a_n = 3n - 2$

$a_1 = n = 1 \rightarrow 1$

$a_2 = n = 2 \rightarrow 4$

$a_3 = n = 3 \rightarrow 7$

$a_4 = n = 4 \rightarrow 10$

P1 $a_n = 3 + (-1)^n$

$a_1 = 2$

$a_2 = 4$

$a_3 = 2$

$a_4 = 4$

563

18. $(-1)^n \left(\frac{n}{n+1}\right)$

$a_1 = -.5 \quad -\frac{1}{2}$

$a_2 = .667 \rightarrow \frac{2}{3}$

$a_3 = -.75 \quad -3/4$

$a_4 = .8 \quad \frac{4}{5}$

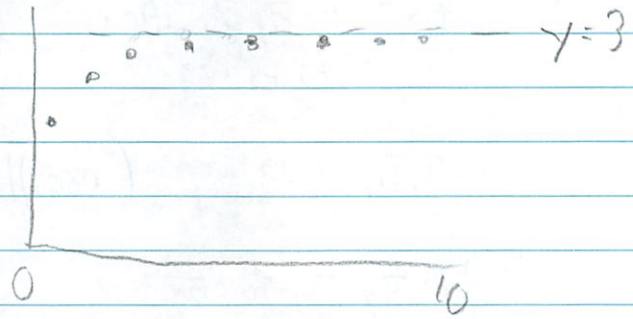
$a_5 = -.833 \quad -\frac{5}{6}$

30. $a_n = \frac{3n^2}{n^2+1}$ 3

$\rightarrow$

same degree

$a_{sy} = \frac{3}{1} \rightarrow 3$



To generate sequence - need more than 2 terms

ex $\quad$ ex: $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16} \cdots \frac{1}{2^n}$

$(n) = 1 \quad 2 \quad 3 \quad 4 \longrightarrow$

ex $\quad 1, 3, 5, 7 \cdots 2n-1 \leftarrow$ y-intercept

$\uparrow$ slope

ex $\quad 2, 5, 10, 17 \cdots n^2+1$

ex $\quad 1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25} \cdots \frac{1}{n^2}$

564

44. $\frac{2}{1}, \frac{3}{2}, \frac{4}{3}, \frac{5}{7}, \frac{6}{4} \cdots \frac{n+1}{2n-1}$ $\swarrow$ look at numerator + denomenater seperatly

46. $\frac{1}{3}, \frac{-2}{9}, \frac{4}{27}, \frac{-8}{81} \cdots \frac{-2^{(n-1)}}{3^n}$

48. $1+\frac{1}{2}, 1+\frac{3}{4}, 1+\frac{7}{8}, 1+\frac{15}{16}, 1+\frac{31}{32} \cdots 1+\frac{2^n-1}{2^n}$

564

52. $1, -1, 1, -1, 1 \quad \cdots \quad (-1)^{(n+1)}$

Some sequences are defined recusively
- uses a previous term to predict future term

ex Fibonacci Sequence

$1, 1, 2, 3, 5, 8, 13, 21, 34$

$a_0 \; a_1 \; a_2 \; a_3 \; a_4 \ldots$

add the previous two to get to the next term

$$a_k = a_{k-2} + a_{k-1}$$

$$a_4 = a_{4-2} + a_{4-1}$$
$$a_2 + a_3$$
$$2 + 3$$
$$a_4 = 5$$

54. $a_{k+1} = a_k + 3$ ← recursive sequence

$a_1 = 15$
$a_2 = 18$ +3
$a_3 = 21$
$a_4 = 24$
$a_5 = 27$

58. $a_{k+1} = a_k - 5$

$a_1 = 25$
$a_2 = 20$
$a_3 = 15$
$a_4 = 10$
$a_5 = 5$

Normal rule

$a_n = 30 - 5n$
$a_n = 5(6-n)$ or

60. $a_{k+1} = -2a_k$

Normal Rule(s)

$a_1 = 14$

$a_2 = -2(14) = -28$ $\quad -7(-2^n)$

$a_3 = -2(-28) = 56$ $\quad (-1)^n(n+1)(-7)$

$a_4 = -2(56) = -112$ $\quad 14(-2^{(n-1)})$

$a_5 = -2(-112) = 224$

8.1 Sequences + Series
HW

11/27

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5. $a_n = \left(-\frac{1}{2}\right)^n$

$a_1 = -\frac{1}{2}$
$a_2 = \frac{1}{4}$
$a_3 = -\frac{1}{8}$
$a_4 = \frac{1}{16}$
$a_5 = -\frac{1}{32}$

11. $a_n = \frac{1 + (-1)^n}{n}$

$a_1 = 0$
$a_2 = 1$
$a_3 = 0$
$a_4 = \frac{1}{2}$
$a_5 = 0$

15. $a_n = \frac{1}{n^{3/2}}$

$a_1 = 1$
$a_2 = .35$
$a_3 = .19$
$a_4 = \frac{1}{8}$
$a_5 = .09$

19. $a_n = (2n-1)(2n+1)$

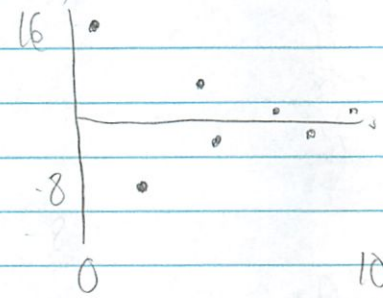
$a_1 = 3$
$a_2 = 15$
$a_3 = 35$
$a_4 = 63$
$a_5 = 99$

21. $a_n = (-1)^n(3n-2)$ $\quad a_{25} = -73$

23. $a_n = \frac{n^2}{n^2+1}$ $\quad a_{10} = .990$

Graph

27. $a_n = 16(-.5)^{n-1}$

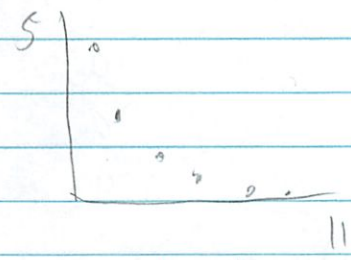


33. $a_n = 1 + \frac{n+1}{n}$

n	$a(n)$
1	3
2	2.5
3	$2\frac{1}{3}$
4	$2\frac{1}{4}$
5	$2\frac{1}{5}$
6	$2\frac{1}{6}$
7	$2\frac{1}{7}$
8	$2\frac{1}{8}$
9	$2\frac{1}{9}$
10	2.1

564

37. $a_n = 4(.5)^{n-1}$ (d)



Write sequence

(✓) 39. 1, 4, 7, 10, 13

$a_n = 3n - 2$

← try to make an $a_0 = -2$ ↗ y int piece

(✓) 41. 0, 3, 8, 15, 24

$a_n = n^2 - 1$

(✓) 43. $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}$

$a_n = \frac{n+2}{n+3}$

45. $\frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, -\frac{1}{16}$ $\quad a_n = \frac{(-1)^{n+1}}{2^n}$

47. $1+\frac{1}{1}, 1+\frac{1}{2}, 1+\frac{1}{3}, 1+\frac{1}{4}, 1+\frac{1}{5}$ $\quad a_n = 1+\frac{1}{n}$

49. $1, \frac{1}{2}, \frac{1}{6}, \frac{1}{24}, \frac{1}{120}$ $\quad a_n = \frac{1}{n!}$

51. $1, 3, 1, 3, 1$ $\quad a_n = (-1)^n + 2(1)^n$

$(-1)^n + 2$

57. $a_{k+1} = a_k + 2$

<u>Normal</u>

$a_1 = 6$

$a_2 = 8$ $\quad 4 + 2n$

$a_3 = 10$

$a_4 = 12$

$a_5 = 14$

8.1 Sequences + Series
Day 2

11/28

Factorials

ex I have 4 ice cream flavors. How many unique combos are there?

e Flavors = C, V, S, B

CVSB	VCSB			
CVBS	VCBS			
CBVS	→ VBCS	→ ...	→ ...	
CBSV	VBSC			
CSBV	VSCB			
CSVB	VSBC			
6	6	6	6	= 24

$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

ex2 $8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 40320$

↑ can type in calc [8] [MATH] [←] [4] [Enter]

ex3 $2n!$ n can start at 0

$0! = 1$

$a_0 = 2(0)! = 2$ ~~[illegible]~~ $2n! = 2(\underbrace{1 \cdot 2 \cdot 3 \cdot 4 \cdot n}_{n!})$

$a_1 = 2(1)! = 2$

$a_2 = 2(2 \cdot 1) = 4$

$a_3 = 2(3 \cdot 2 \cdot 1) = 12$

ex4 $a_n = \frac{2^n}{n!}$ $a_0 = 1$ ~~[illegible]~~

$a_1 = 2$

$a_2 = 2$

$a_3 = \frac{4}{3}$

$a_4 = \frac{2}{3}$

564

62. $a_n = \frac{1}{(n+1)!}$

$a_0 = 1$
$a_1 = \frac{1}{2}$
$a_2 = \frac{1}{6}$
$a_3 = \frac{1}{24}$
$a_4 = \frac{1}{120}$

64. $a_n = \frac{(-1)^{2n+1}}{(2n+1)!}$

$a_0 = -1$
$a_1 = -\frac{1}{6}$
$a_2 = -\frac{1}{120}$
$a_3 = -\frac{1}{5040}$
$a_4 = -\frac{1}{362{,}880}$

) same as $-\frac{1}{(2n+1)!}$

68. $\frac{5!}{7!} = \frac{\cancel{5\cdot4\cdot3\cdot2\cdot1}}{7\cdot6\cdot\cancel{5\cdot4\cdot3\cdot2\cdot1}} = \frac{1}{7\cdot6} = \frac{1}{42}$

70 $\frac{10!\cdot3!}{4!\cdot6!} = \frac{10\cdot9\cdot8\cdot7\cdot\cancel{6\cdot5\cdot4\cdot3\cdot2\cdot1}\cdot\cancel{3\cdot2\cdot1}}{\cancel{4\cdot3\cdot2\cdot1}\cdot\cancel{6\cdot5\cdot4\cdot3\cdot2\cdot1}}$

$\frac{10\cdot9\cdot8\cdot7}{4} = 1260$

72. $\frac{(n+2)!}{n!} = \frac{(n+2)(n+1)\cancel{n!}}{\cancel{n!}} = (n+2)(n+1)$

↑ solution

74. $\frac{(2n+2)!}{2n!}$ $\frac{(2n+2)(2n+1)\cancel{2n!}}{\cancel{2n!}}$ $(2n+2)(2n+1)$

Summation (Sigma)

Adds (sums) the terms of a sequence

$\sum$ ← sigma - greek letter

$$\sum_{i=1}^{n} a_i$$

n ← upper boundry (limit)

a_i ← index (what you are doing)

$i=1$ ← lower boundry (start)

← do index

$$= a_1 + a_2 + a_3 + a_4 + a_n \cdots$$

↑ start i here ↑ to upper limit

ex

$$\sum_{i=1}^{5} 3i = 3(1) + 3(2) + 3(3) + 3(4) + 3(5)$$

↑ stop

$$3 + 6 + 9 + 12 + 15$$

$$45$$

ex2

$$\sum_{k=3}^{6} (1+k^2) = (1+(3)^2) + (1+(4)^2) + (1+(5)^2) + (1+(6)^2)$$

$$10 + 17 + 26 + 37$$

$$90$$

564

76.

$$\sum_{i=1}^{6} (3i-1) = (3\cdot 1 - 1) + (3(2)-1) + (3(3)-1) + (3(4)-1) + (3(5)-1) + (3(6)-1)$$

$$2 + 5 + 8 + 11 + 14 + 17$$

$$57$$

78.

$$\sum_{k=1}^{5} 6 = 6 + 6 + 6 + 6 + 6$$

$$6 \cdot 5$$

$$30$$

ex $\sum_{i=1}^{10} \frac{3}{i+1}$ Calc: [2nd] [List] [←] [5] sum(
[''] [''] [→] [5] seq(
3/(n+1)
,
n
,
1
,
10
[)] [)]
[Enter]

6.06

94. $\left[1-\left(\frac{1}{6}\right)^2\right] + \left[1-\left(\frac{2}{6}\right)^2\right] \cdots \left[1-\left(\frac{6}{6}\right)^2\right]$ Write as Σ

$$\sum_{i=1}^{6} \left[1-\left(\frac{i}{6}\right)^2\right] = 3.4722$$

reale

98. $\frac{1}{1\cdot 3} + \frac{1}{2\cdot 4} + \frac{1}{3\cdot 5} + \cdots + \frac{1}{10\cdot 12}$ $\sum_{i=1}^{10} \frac{1}{i\cdot(i+2)}$

<u>Properties of Sums</u>

1. $\sum_{i=1}^{n} c = n\cdot c \rightarrow \sum_{i=1}^{3} 5 = 5+5+5$

$3\cdot 5$

15

2. $\sum_{i=1}^{n} c\cdot a_i = c\sum_{i=1}^{n} a_i \rightarrow \sum_{i=1}^{3} 3\cdot i^2 = 3(1^2+2^2+3^2)$

$3(14)$

42

3. $\sum_{i=1}^{n} (a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$

4. $\sum_{i=1}^{n} (a_i - b_i) = \sum_{i=1}^{n} a_i - \sum_{i=1}^{n} b_i$

Sum the terms of a finate or infinate sequence, it's called a series (not sequence)

Partial sums = $\sum_{i=1}^{n} a_i$

Adding all terms of an infinate sequence

$$\sum_{i=1}^{\infty} a_i$$

ex $\sum_{i=1}^{\infty} \frac{3}{10^i}$ ← infinate series

a) Find the 3rd partial sum

$$\sum_{i=1}^{3} = \frac{3}{10^i} = \frac{3}{10^1} + \frac{3}{10^2} + \frac{3}{10^3} = .333$$

b) Find the sum of the series

$.3 + .03 + .003 + .0003 \cdots$

$\frac{1}{3}$ ← limit $.33333333$

565/106.

$$\sum_{i=1}^{\infty} 4\left(\frac{1}{10}\right)^i = 4\left(\frac{1}{10}\right)^1 + 4\left(\frac{1}{10}\right)^2 + 4\left(\frac{1}{10}\right)^3 + 4\left(\frac{1}{10}\right)^4$$

$$.4 + .04 + .004 + .0004$$

$$.4444$$

$$\frac{4}{9}$$

8.1 Sequences + Series
Day 2 HW

11/28

564

57. $a_{k+1} = a_k + 2$

$a_1 = 6$
$a_2 = 8$
$a_3 = 10$
$a_4 = 12$
$a_5 = 14$

$4 + 2n$

59. $a_{k+1} = \frac{1}{3} a_k$

$a_1 = 81$
$a_2 = 27$
$a_3 = 9$
$a_4 = 3$
$a_5 = 1$

65. $a_n = \frac{(-1)^n}{(2n)!}$

$a_0 = 1$
$a_1 = \frac{1}{2}$
$a_2 = \frac{1}{24}$
$a_3 = \frac{1}{720}$
$a_4 = \frac{1}{40320}$

Simplify,

69. $\frac{12!}{4 \cdot 8!}$ $\quad \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot \cancel{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}{4 \cdot 3 \cdot 2 \cdot 1 \cdot \cancel{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}$ $\quad \frac{11880}{24} = 495$

71. $\frac{(n+1)!}{n!}$ $\quad \frac{(n+1)(\cancel{n!})}{\cancel{n!}}$ $\quad n+1$

77. $\sum_{k=1}^{4} 10 = 4 \cdot 10 = 40$

83. $\sum_{i=1}^{4} \left[(i-1)^2 + (i+1)^3\right] = \left[(1-1)^2 + (1+1)^3\right] + \left[(2-1)^2 + (2+1)^3\right]$

$+ \left[(3-1)^2 + (3+1)^3\right] + \left[(4-1)^2 + (4+1)^3\right] = 238$

✓ 89. $\sum_{k=0}^{4} \frac{(-1)^k}{k+1} = \frac{47}{60}$

93. $\left[2\left(\frac{1}{8}\right)+3\right] + \left[2\left(\frac{2}{8}\right)+3\right] \cdots \left[2\left(\frac{8}{8}\right)+3\right]$

$\sum_{i=1}^{8} \left[2\left(\frac{i}{8}\right)+3\right] = 33$

95. $3-9+27-81+243-729 \quad \sum_{i=0}^{3} (-1)^{i+1} 3^i = -546$

looked up

makes odd ⊖

97. $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots - \frac{1}{20^2} = \sum_{i=1}^{20} \frac{(-1)^{i+1}}{n^2} \approx .821$

99. $\frac{1}{4} + \frac{3}{8} + \frac{7}{16} + \frac{15}{32} + \frac{31}{64} = \sum_{i=1}^{5} \cancel{\frac{n^2-1}{2^2}} \; \frac{2^i-1}{2^{i+1}} = 2.0156$

4th partial sum

101. $\sum_{i=1}^{\infty} 5\left(\frac{1}{2}\right)^i = 5\left(\frac{1}{2}\right)^1 + 5\left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right)^3 + 5\left(\frac{1}{2}\right)^4$

$5/2 + 5/4 + 5/8 + 5/16$

$75/16$

3rd partial sum

103. $\sum_{n=1}^{\infty} 4\left(-\frac{1}{2}\right)^n \quad 4\left(-\frac{1}{2}\right)^1 + 4\left(-\frac{1}{2}\right)^2 + 4\left(-\frac{1}{2}\right)^3$

$-2 + 1 + -1/2$

$-3/2$

Find sum of infinite series

105. $\sum_{i=1}^{\infty} 6\left(\frac{1}{10}\right)^i$ $\quad 6\left(\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} \cdots\right) = 6 \cdot .111 = \frac{6}{9}$

107. $\sum_{k=1}^{\infty} \left(\frac{1}{10}\right)^k$ $\quad \frac{1}{10} + \frac{1}{100} + \frac{1}{1000} \cdots = .111 = \frac{1}{9}$

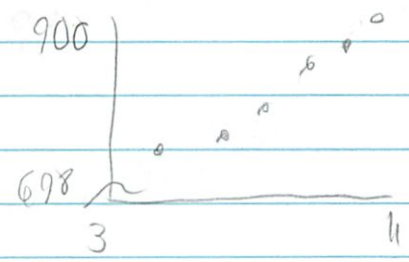
109. \$5,000 interest
3% intrest compounded quarterly

$A_n = 5000\left(1 + \frac{.03}{4}\right)^n$ $\quad$ n = number of quarters

balance

$a_1 = 5037.5$
$a_2 = 5075.28$
$a_3 = 5113.35$
$a_4 = 5151.69$
$a_5 = 5190.33$
$a_6 = 5229.26$
$a_7 = 5268.48$
$a_8 = 5308$
$a_{40} = 6741.74$

111. $a_n = 1.37n^2 + 3.1n + 698$ $\quad n = 3 = 1093$



Enrollment is rising

566/113. Net Profit

$a_n = 2.151n^2 + 235.9$ $n = 4 = 1994$

$3,491,000,000$ ← same as bar graph

8.2 Arithmetic Sequences + Partial Sums Notes

11/29

Arithmetic sequences - difference b/w consecutive terms which is the same (constant rate of change)

common difference = d

ex $7, 11, 15, 19$ so $d = 4$

$+4 \quad +4 \quad +4$

get $a_0 = 3$ ← y-intercept

so $a_n = mx + b \rightarrow 4n + 3$

ex2 $2, -3, -8, -13$ $d = -5$

$-5 \quad -5 \quad -5$

$a_0 = 7$ $a_n = -5n + 7$

P1 $1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}$ $d = \frac{1}{4}$

$\frac{4}{4} \quad \frac{6}{4}$

$y = \frac{1}{4}x + \frac{3}{4}$

P2 $1, 4, 9, 16$ not arithmetic

The nth term of an arithmetic sequence is the form of $mx + b$ or $dn + c = a_n$

d = common difference

c = y-intercept value $(c = a_1 - d)$.

ex3 $2, 6, 10, 14, 18$ $a_n = 4n - 2$

$a_0 = -2$

573

12. $a_n = 1 + (n-1)4$ ← recursive

$1 + 4(n-1)$

$1 + 4n - 4$

$4n - 3$ ← not recursive

22. $10, 5, 0, -5, -10$

$a_n = -5n + 15$

24. $a_1 = -4$

$a_5 = 16$ $\quad 20/\underset{5-1}{4} = 5$

$a_n = 5n - 9$

26. $a_5 = 190$

$a_{10} = 115$ $\quad \overset{115-190}{-75}/\underset{10-5}{5} = -15$

$a_n = -15n + 265$

30. $a_4 = 16$ $\quad 30/6 = 5 = d$

$a_{10} = 46$

$a_0 = a_4 - 4d$ $\quad a_n = 5n - 4$

$a_0 = 16 - 20$

$a_0 = -4$

$a_n = dn + c$ $\quad c = a_1 - d$

$a_n = dn + (a_1 - d)$

$a_n = dn - d + a_1$ $\quad$ Proof

$a_n = d(n-1) + a_1$

36. $a_1 = 200$

$a_{k+1} = a_k - 10$

$a_1 = 200$
$a_2 = 190$
$a_3 = 180$
$a_4 = 170$
$a_5 = 160$

$-10n + 210 = a_n$

40. $a_1 = 3$
$a_2 = 13$
$a_9 = \underline{83}$

$10n - 7 = a_n$

<u>Sum of a Finite Arithmetic Sequence</u>

$$S_n = \frac{n}{2}(a_1 + a_n)$$

↑ # of terms

ex $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19$

↑ $n = 10$ terms ↑ a_{10}

$$S_{10} = \frac{10}{2}(1 + 19)$$

100

<u>Sum of integers from 1 to 100</u>

$$S_n = \frac{100}{2} \cdot (1 + 100)$$

$$50 \cdot 101$$

$$5050$$

P3 $a_1 = 15$
$a_{100} = 307$ Probably infinite
Find the partial sum

$$S_{100} = \frac{100}{2}(15 + 307)$$
$$50 \cdot 322$$
$$16100$$

P4 Find the sum of the integers from -10 to 50

(a) don't forget 0

$$S_{60} = \frac{60}{2}(-10 + 50)$$
$$30 \cdot 40$$
$$\cancel{1200}\ 1220$$

P5 Find the 150th partial sum of this sequence

5, 16, 27, 38, 49 $d = 11$ $a_n = 11n - 6$
$5 - 11 = -6 = a_0$

$$11(150) - 6 = 1644$$

$$S_{150} = \frac{150}{2}(5 + 1644)$$
$$123675$$

p574
56. 4.2, 3.7, 3.2, 2.7 Find sum of first 12 terms
$-.5n + 4.7$

$$a_{12} = -.5(12) + 4.7$$
$$-6 + 4.7$$
$$-1.3$$

$$S_{12} = \frac{12}{2}(4.2 - 1.3)$$
$$17.4$$

Find sums

64. $\sum_{n=51}^{100} 7n$ $\quad S_{50} = \frac{50}{2}(\overset{7 \cdot 51}{357} + 700)$

52850

66. $\sum_{n=51}^{100} n - \sum_{n=1}^{50}$ $\rightarrow \frac{50}{2}(1+50)$

$\boxed{1275}$

$S_{50} = \frac{50}{2}(51+100)$

$\boxed{3775}$

$3775 - 1275$

2500

8.2 Arithmetic Sequences + Partial Sums HW

1/29

573/

7. Arithmetic?

3.2, 4.3, 4.9, 5.5, 6.1 — Arithmetic

.6 .6 .6 .6 — D = .6

11. $\frac{1}{n+1}$

$a_1 = \frac{1}{2}$
$a_2 = \frac{1}{3}$
$a_3 = \frac{1}{4}$
$a_4 = \frac{1}{5}$
$a_5 = \frac{1}{6}$

Not arithmetic

13. $a_n = 150 - 7n$

$a_1 = 143$
$a_2 = 136$
$a_3 = 129$
$a_4 = 122$
$a_5 = 115$

$d = 7$

19. $a_1 = 100$
$d = -8$

$108 - 8n$

21. $4, \frac{3}{2}, -1, -\frac{7}{2}$

-2.5

$6.5 - 2.5n$

25. $a_3 = 94$
$a_6 = 85$

$-9/3 = -3$

$a_0 = 94 + 3(-3)$
105

$105 - 3n$

35. $a_{k+1} = a_k + 4$

$a_1 = 15$
$a_2 = 19$
$a_3 = 23$
$a_4 = 27$
$a_5 = 31$

$11 + 4n$

37. $a_{k+1} = a_k - \frac{1}{4}$

$a_1 = \frac{7}{2}$

$a_2 = 3\frac{1}{4}$

$a_3 = 3$

$a_4 = 2\frac{3}{4}$

$a_5 = 2\frac{1}{2}$

$3.75 - .25n$

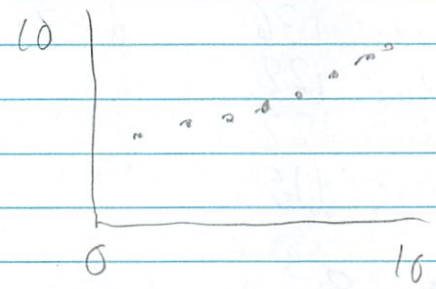
41. $a_1 = 4.2$

$a_2 = 6.6$

$a_7 = 18.6$

$1.8 + 2.4n$

45. $a_n = .5n + 4$



574

55. $.5, 1.3, 2.1, 2.9 \ldots n = 10$

$+.8$

Find Partial sum

$a_n = -.3 + .8n$

$a_{10} = -.3 + .8(10)$

7.7

✓

$S_{10} = \frac{10}{2}(.5 + 7.7)$

$S_{10} = 41$

57. $a_1 = 100$

$a_{25} = 220$

$n = 25$

$S_{25} = \frac{25}{2}(100 + 220)$

4000

59. First 100 odd positive integers

$1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21$

$a_1 = 1$

$a_{100} = 199$

$S_{100} = \frac{100}{2}(1 + 199)$

$10{,}000$

$a_n = a_1 + (n-1)d$

$1 + (n-1)2$

$1 + 2n - 2$

$2n - 1$

65. $\sum_{n=11}^{30} n - \sum_{n=1}^{10} n \rightarrow S_{10} = \frac{10}{2}(1+10)$

$\boxed{55}$

$\downarrow$

$S_{20} = \frac{20}{2}(11+30)$

$\boxed{410}$

$410 - 55$

355

67. $\sum_{n=1}^{500} (n+8) \quad S_{500} = \frac{500}{2}(9+508)$

129250

71. $\sum_{n=1}^{100} \frac{n+4}{2} \quad S_{100} = \frac{100}{2}(2.5+52)$

2725

75. Salary = \$32,500 $\quad a_n = 31000 + 1500n$

Annual raise = \$1,500 $\quad a_6 = 40,000$

$S_6 = \frac{6}{2}(32500 + 40000)$

217,500

79. Seating Capacity

15 rows

First row has 25 seats $\quad 23+2n$

Each row has 2 more seats

$S_{15} = \frac{15}{2}(25+53)$

585 seats

575/81. Sales $20,000 1st year 15 000 + 5000n

$5,000 more each year for 4 years

Profit for first 5 years

$S_5 = \frac{5}{2}(20,000 + 40,000)$

$150,000

8.3 Geometric Sequences + Series Notes

11/30

Consecutive terms of a geometric sequence have a common ratio
multiplier

r = common ratio, $r \neq 0$

ex, $2, 4, 8, 16$ $\quad r = 2 \quad = 2^n$

$\frac{4}{2} = 2 \quad \frac{16}{8} = 2$

ex2 $-\frac{1}{3}, \frac{1}{9}, -\frac{1}{27}, \frac{1}{81}$

$\frac{1/9}{-1/3} = -\frac{1}{3}$

582

8 $9, -6, 4, -\frac{8}{3}$ $\quad r = -\frac{2}{3}$

$\frac{-6}{9} = -\frac{2}{3}$

10. $\frac{1}{5}, \frac{2}{7}, \frac{3}{9}, \frac{4}{11}$

$\frac{2/7}{1/5} = \frac{2}{7} \cdot \frac{5}{1} = \frac{10}{7} = r$ ← for that – but not geometric

---

Nth term of a geometric sequence

$a_n = a_1 \cdot r^{n-1}$

$a_1 = a_1 \cdot r^{1-1} = a_1 \cdot r^0$

r^1

$a_2 = a_1 \cdot r^{2-1} = a_1 \cdot r^1$

r^1

$a_3 = a_1 \cdot r^2$

r^1

$a_4 =$

$a_5 =$

$a_n =$

14. $r = \frac{1}{3}$

$a_1 = 2$

$a_2 = 2 \cdot \left(\frac{1}{3}\right)^{2-1} = \frac{2}{3}$

$a_3 = 2 \cdot \left(\frac{1}{3}\right)^{2} = \frac{2}{9}$

$a_4 = 2 \cdot \left(\frac{1}{3}\right)^{3} = \frac{2}{27}$

$a_5 = 2 \cdot \left(\frac{1}{3}\right)^{4} = \frac{2}{81}$

18. $r = \sqrt{3}$

$a_1 = 4 = 4$

$a_2 = 4 \cdot \sqrt{3} = 4\sqrt{3}$

$a_3 = 4 \cdot (\sqrt{3})^{3-1} = 12$

$a_4 = 4 \cdot (\sqrt{3})^{3} = 4 \cdot 3 \cdot \sqrt{3} = 12\sqrt{3}$

$a_5 = 4 \cdot (\sqrt{3})^{4} = 36$

20. $a_{k+1} = \frac{1}{3} a_k$

$a_1 = 81$

$a_2 = 27$

$a_3 = 9$

$a_4 = 3$

$a_5 = 1$

26. $r = \frac{3}{2}$

$a_1 = 5$

$a_8 = 5 \cdot \left(\frac{3}{2}\right)^{8-1} = 5 \cdot \left(\frac{3}{2}\right)^{7} = 85,43$

$\frac{10935}{128}$ or

$a_5 = a_3 \cdot r^2$

$\frac{64}{27} = \frac{16}{3} \cdot r^2$

32. Geometric

$a_3 = \frac{16}{3}$

$a_5 = \frac{64}{27}$

$a_7 = ? = a_5 \cdot r^2$

$\frac{64}{27} \cdot \left(\frac{2}{3}\right)^2 \rightarrow \frac{256}{243}$

$\frac{16}{3} \cdot \frac{27}{64} = r^2 = \frac{4}{9} \rightarrow \sqrt[2]{4/9} = \pm\frac{2}{3}$

jumps

ex

$a_4 = 125$

$a_{10} = 125/64$

$a_{14} = ?$

$a_{10} = a_4 \cdot r^6$

$\frac{125}{64} = 125 \cdot r^6$

$\frac{1}{64} = r^6$

$\sqrt[6]{\ } \quad \sqrt[6]{\ }$

$\frac{1}{2} = r$

$a_{14} = a_{10} \cdot r^4$

$a_{14} = 125/64 \cdot \left(\frac{1}{2}\right)^4$

$a_{14} = \frac{125}{1024}$

34. $a_1 = 3$ $\quad \frac{36}{3} = 12$

$a_2 = 36$

$a_3 = 432$

$a_7 = \overset{a_1}{3} \cdot (\overset{r}{12})^6$

$8{,}957{,}952$

Sum of a $\boxed{\text{finite}}$ geometric sequence

$$S_n = \sum_{i=1}^{n} a_1 \cdot r^{n-1} = a_1\left(\frac{1-r^n}{1-r}\right)$$

ex

$$\sum_{n=1}^{12} 4(.3)^n = a_1 + a_2 + a_3 + \dots + a_{12}$$

$$4(.3)^1 + 4(.3)^2 + 4(.3)^3 + 4(.4)^4$$

$a_1 = 4(.3) = 1.2$

$r = .3$

$n = 12$

$$a_1\left(\frac{1-r^n}{1-r}\right)$$

$$1.2\left(\frac{1-.3^{12}}{1-.3}\right)$$

≈ 1.714

583

50. $$\sum_{n=0}^{15} 2\left(\frac{4}{3}\right)^n \qquad 2\left(\frac{1-\frac{4}{3}^{16}}{1-4/3}\right) = 592.65$$

1st term

$2\left(\frac{4}{3}\right)^0$

58. $15 - 3 + \frac{3}{5} - \dots - \frac{3}{625}$

$\frac{-3}{15} = -\frac{1}{5} = r$ → Write sequence in summation

$$\sum_{n=1} 15 \cdot \left(-\frac{1}{5}\right)^{n-1} = \frac{-3}{625} \cdot \frac{1}{5} = -\frac{1}{3125} \rightarrow 3125 = -5^{n-1}$$

see next problem for better steps

Note exponent is negitive and odd so through -1, n will be even

So Guess + ✓ from here

$$-5^{n-1} = -3125$$
$$n = 6$$
$$\sum_{n=1}^{6} 15\left(-\frac{1}{5}\right)^{n-1}$$

56. $7 + 14 + 28 + \ldots = 896$

$\frac{14}{7} = 2 = r$

$$a_n = a_1 \cdot r^{n-1}$$
$$896 = 7 \cdot 2^{n-1}$$
$$128 = 2^{n-1}$$
$$\log 128 = \log 2^{n-1}$$
$$\log 128 = n-1 \log 2$$
$$\frac{\log 128}{\log 2} = n-1$$
$$7 = n-1$$
$$8 = n$$

<u>Sum of Infinite Geometric Sequence</u>

r^{∞}

$$r^1 + r^2 + r^3 + r^4 \ldots r^{100} \ldots r^{\infty}$$
$$r = \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4 \ldots \left(\frac{1}{2}\right)^{100} \ldots \left(\frac{1}{2}\right)^{\infty}$$
$$r^n \to 0$$
$$r < 1$$

finite

$$a_1 \frac{(1-r^n)}{(1-r)} \to \frac{a_1(1-0)}{1-r} = \boxed{\frac{a_1}{1-r}}$$

infinite

If $|r| < 1$ and $n \to \infty$, then you have a definite limit - use $\frac{a_1}{1-r}$

ex) 1, .5, .25, .125, .0625, .03125

$r = \frac{1}{2}$ (<1)

$1 + .5 + .25 + .125 + .0625 + .03125$

approaches limit of $2 = \frac{a_1}{1-r}$

If $|r| > 1$

ex) $r = 2$ ← $1^2 + 2^2 + 3^2 + 4^2$

$1 + 4 + 16 + 64 + 256$

no limit

62. $\sum_{n=0}^{\infty} 2\left(-\frac{2}{3}\right)^n$ $\quad \left|-\frac{2}{3}\right| \overset{?}{<} 1$

$\frac{2}{3} < 1$

↑ limit approaches $\frac{a_1}{1-r} = \frac{2}{1-\left(-\frac{2}{3}\right)} = \frac{6}{5}$

64. $\sum_{n=1}^{\infty} \frac{1}{2}(4)^n$ $\quad 4 \overset{?}{>} 1$

$4 > 1$

↑ no limit

8.3 Geometric Sequences + Series
HW

12/1

582

Geometric?

3. 6, 18, 30, 42 ...

$\frac{18}{6}$ $\frac{30}{18}$ $\frac{42}{30}$

3 $\frac{5}{3}$ $\frac{7}{5}$

(X) Not Geometric

7. $\frac{1}{8}$ $\frac{1}{4}$ $\frac{1}{2}$ 1

$\frac{\frac{1}{4}}{\frac{1}{8}}$ $\frac{\frac{1}{2}}{\frac{1}{4}}$ $\frac{1}{\frac{1}{2}}$

2 2 2

(✓) Geometric

15. $r = -\frac{1}{10}$

$a_1 = 5$
$a_2 = -\frac{1}{2}$
$a_3 = \frac{1}{20}$
$a_4 = -\frac{1}{200}$
$a_5 = \frac{1}{2000}$

(✓)

21. $a_{k+1} = 2a_k$

$a_1 = 4$
$a_2 = 18$
$a_3 = 36$
$a_4 = 72$
$a_5 = 144$

23. $a_{k+1} = -\frac{3}{2}a_k$

$a_1 = 6$
$a_2 = -9$
$a_3 = 13.5$
$a_4 = -20.25$
$a_5 = 30.375$

27. $r = -\frac{1}{3}$ $\quad a_1 = 6$

$a_{12} = ? = 6 \cdot \left(-\frac{1}{3}\right)^{12-1} = -.00003387$

$-\frac{2}{3^{10}}$

31. ~~$r = \frac{1}{3}$~~ wrong variables

$a_2 = -18$

$a_2 = a_1 r^1$

$\cancel{18 = a_1\left(\frac{2}{3}\right)}$

$\cancel{27 = a_1}$

$\cancel{a_6 = -27\left(\frac{2}{3}\right)^{6-1}}$

$a_5 = \frac{2}{3}$

$a_5 = a_2 \cdot r^{5-2}$

$\frac{2}{3} = -18 \cdot r^3$

$\boxed{r = -\frac{1}{3}}$

$a_6 = a_5 \cdot r$

$a_6 = \frac{2}{3} \cdot \frac{1}{3}$

$a_6 = \frac{-2}{9}$

35. $a_1 = 5$

$a_2 = 30 \quad \frac{30}{5} = 6$

$a_3 = 180$

$a_{10} = ? = 5(6)^{10-1} = 50388480$

39. $a_n = 2(1.3)^{n-1}$

n	1	2	3	4	5	6	7	8	9
a(n)	2	2.6	3.38	4.394	5.712	7.43	9.6	12.55	16.3

10 - 21.21

41. $8, -4, 2, -1, \frac{1}{2}$

$s_1 = 8$

$s_2 = 8 + -4 = 4$

$s_3 = 8 - 4 + 2 = 6$

$s_4 = 8 - 4 + 2 - 1 = 5$

583

47. $\sum_{i=1}^{7} 64\left(-\frac{1}{2}\right)^{i-1} \quad 64\left(\frac{1-.5^7}{1--.5}\right) = 64\left(\frac{\frac{129}{128}}{3/2}\right) = 43$

49. $\sum_{n=0}^{20} 3\left(\frac{3}{2}\right)^n = 3\left(\frac{1-\frac{3}{2}^{20}}{1-3/2}\right) = 3(9973.77) = 29921.31$

55. $5 + 15 + 45 + \ldots + 3645$ Find sum

$3 = r$

$a_n = a_1 \cdot r^{n-1}$

$3645 = 5 \cdot 3^{n-1}$

$729 = 3^{n-1}$

$\log 729 = \log 3^{n-1}$

$\log 729 = n-1 \log 3$

$\frac{\log 729}{\log 3} = n-1$

$6 = n-1$

$7 = n$

$5\left(\frac{1-3^7}{1-3}\right) = 5\left(\frac{-2186}{-2}\right) = 5(1093) = 5465$

57. $2 - \frac{1}{2} + \frac{1}{8} - \ldots + \frac{1}{2048}$

$\frac{1}{2048} = 2 \cdot -\frac{1}{4}^{n-1}$

$\frac{1}{4096} = -\frac{1}{4}^{n-1}$

$\log(1/4096) = n-1 \log(-1/4)$

$\frac{\log(1/4096)}{\log(-1/4)} = n-1$

$6 = n-1$

$n = 7$

$2\left(\frac{1-(-\frac{1}{4})^7}{1--\frac{1}{4}}\right) = 1.6$

65. $\sum_{n=0}^{\infty} (.4)^n = \begin{matrix} .4 \ni 1 \\ .4 < 1 \end{matrix}$

bottom of sigma

$a_1 = (.04)^0 = 1$

$\frac{a_1}{1-r} = \frac{1}{1-.4} = \frac{1}{.6} = \frac{5}{3}$

67. $\sum_{n=0}^{\infty} -3(.9)^n = \frac{-3}{1-.9} = \frac{-3}{.1} = -30$

71. $3 \underset{-\frac{1}{3}}{-} 1 + \frac{1}{3} - \frac{1}{9} \ldots$

$\sum_{n=1}^{\infty} 3\left(-\frac{1}{3}\right)^n = \frac{3}{1--\frac{1}{3}} = \frac{3}{\frac{4}{3}} = 2.25 = \frac{9}{4}$

8.3 Geometric Sequences + Series Day 2

12/3

Annuities

A deposit of \$50 is made at the start of each month to a savings account which pays 6% interest compounded monthly.

What is the balance at the end of 2 years?

- 24 months = 24 deposits =

$$\frac{6 \div .01}{12} = \frac{.06}{12} = .005 = \boxed{A = P\left(1 + \frac{r}{n}\right)^{nt}}$$

$$A_1 + A_2 + A_3 + \dots + A_{24}$$

biggest deposit

$$A_{24} = 50\left(1 + \frac{.06}{12}\right)^{24} = \$56.35$$

$$A_{23} = 50\left(1 + \frac{.06}{12}\right)^{23} = \$56.077$$

finite, geometric series

multiplier $= 1 + \frac{.06}{12} = 1.005 = r$

$$S_{24} = a_1 \cdot \left(\frac{1 - r^n}{1 - r}\right)$$

$$50\left(1 + \frac{.06}{12}\right) \quad \frac{1 - 1.005^{24}}{1 - 1.005}$$

last deposit - since only earns interest for 1 month

\$1277.96

583

80. \$50 deposited in beginning of each month
8% compounded monthly
5 years
↑ 5·12 = 60 months

$$A_1 = 50\left(1+\frac{.08}{12}\right)^1 = 50.33$$

$$A_{60} = 50\left(1+\frac{.08}{12}\right)^{60} = 74.49$$

$r = 1 + \frac{.08}{12}$

don't forget

$$S_{60} = 50.33\left(\frac{1-\left(1+\frac{.08}{12}\right)^{60}}{1-1.08/12}\right)$$

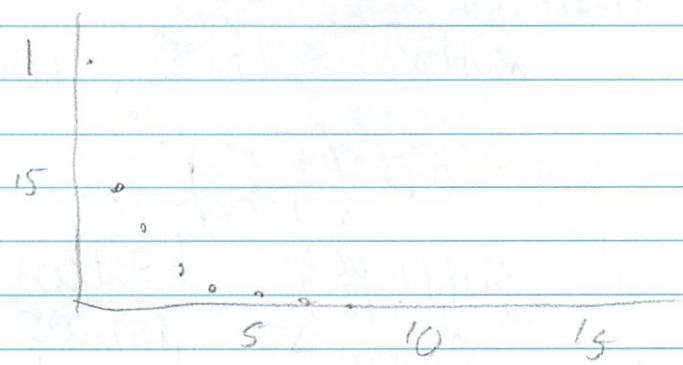
\$3698.34 ✓

Limits of Sequences

$a_n = \frac{1}{n}$

$\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5} \ldots$

Series - numbers you are adding



As n increases (w/o boundry) a_n gets closer, closer to 0

$\lim_{n\to\infty} \frac{1}{n} = 0$ converges to 0

ex2 $a_n = \frac{n+1}{n}$

n	1	2	3	4	...
a_n	2	3/2	4/3	5/4	
	2	1.5	1.33	1.25	

$\lim_{n \to \infty} \frac{n+1}{n} = 1$ ← note that when degree(num) = degree(denom) → horiz as, $= \frac{1}{1} = 1$

ex3 $a_n = b_n = \frac{3n+2}{n+1}$

degrees =, so limit $= \frac{3}{1} = 3$

ex4 $b_n = \frac{n^2+5}{3n^2}$

$\lim_{n \to \infty} = \frac{1}{3}$

ex5 $a_n = 2^n$

$2^1, 2^2, 2^3, 2^4$

$\lim_{n \to \infty} = \infty \approx 2, 4, 9, 16 \ldots$

diverges

ex6 $a_n = \sqrt{n}$

$\sqrt{1}, \sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}$

$1, 1.41, 1.73, 2, 2.2$

$\lim_{n \to \infty} = \infty$

divergent

ex7 $\frac{n^3+1}{2n^2-n}$

n	1	2	3	4	5	...
a_n	2	1.5	1.8	2.3	2.8	

$\lim_{n \to \infty} = \infty$ divergent

ex8 $b_n = (-1)^n$

n	1	1	2	3	4
b_n	-1	1	-1	1	-1

No limit = divergent (even though bandry)

ex9 $a_n = \left(-\frac{1}{2}\right)^n$

n	1	2	3	4	5	...
a_n	-.5	.25	-.125	$+\frac{1}{16}$	$-\frac{1}{32}$	

$\lim_{n \to \infty} = 0$ = convergent to 0

ex10 $\sqrt{9 - \frac{1}{n}}$

n	1	2	3	3	...
a_n	2.82	2.91	2.94	2.95	

$\lim_{n \to \infty} = 3$

ex11 $6 + \frac{4n-2}{n+8}$

n	1	2	3	4	5
a_n	6.22	6.6	6.9	7.1	7.3

$$\lim_{n \to \infty} \frac{4n-2}{n+8} = 4$$

$$6 + 4 = 10 = \lim_{n \to \infty} 6 + \frac{4n-2}{n+8}$$

ex12 $\frac{5+\sqrt{2}}{n+1}$

$$\lim_{n \to \infty} = 0$$

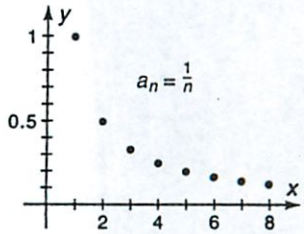
ex13 $b_n = 7$

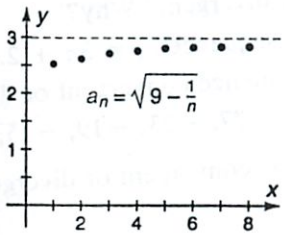
$$\lim_{n \to \infty} 7 = 7$$

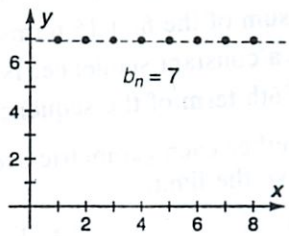
How are the notions of "getting larger and larger" and "getting closer and closer" measured? The notion of *arbitrarily close* provides a more precise definition of limit.

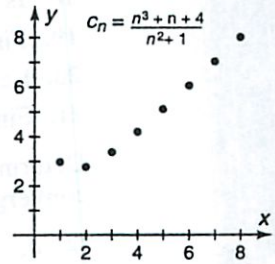
The limit of a sequence a_n is L if as n increases without bound, the terms of a_n get arbitrarily close to L. By "arbitrarily close" is meant simply "as close as you please." In other words, no matter how small a positive number ϵ (Greek epsilon) is chosen, for large enough N, all the terms after a_N are within ϵ of L. Stated even more precisely, a sequence a_n has a **limit** L if for any choice of $\epsilon > 0$, there exists a number N such that for all $n > N$, $|a_n - L| < \epsilon$.

For example, the limit of the harmonic sequence $\frac{1}{n}$ is 0. To illustrate this, let ϵ be a very small number, say $\epsilon = 0.000001$. For n large enough, in this case, if $n > N = 1{,}000{,}000$, then $|a_n - 0| = \left|\frac{1}{n} - 0\right| = \frac{1}{n} < 0.000001$. No matter how small a number ϵ is chosen, n can always be chosen large enough to guarantee that $\left|\frac{1}{n} - 0\right| < \epsilon$. As a further illustration of these ideas, the three sequences of Example 5 are graphed below.









Class Exercises

Find the limit of the indicated sequence as n increases without bound.

1. $a_n = \frac{1}{n^2}$ $\lim_{n\to\infty} = 0$ **2.** $b_n = \frac{3n + 1}{n^2}$ $\lim_{n\to\infty} = 0$ **3.** $c_n = \sqrt{2} + \left(\frac{1}{2}\right)^n$ $\sqrt{2}$

Determine whether each sequence converges. If it converges, give the limit.

4. $d_n = \left(\frac{1}{4}\right)^n$ **5.** $t_n = 4^n$ divergent **6.** $m_k = (-3)^k$

7. $c_n = \frac{2n^2 + 4}{n^2 + 2}$ 2 converges **8.** $\frac{2n^3 + 2n + 4}{n^2 + 1}$ 0 **9.** $\frac{2}{n^2 + 1}$ 0

10. $a_n = \cos n\pi$ **11.** $b_n = (-0.1)^n$ divergent **12.** $c_n = 10^n$

13. Thinking Critically A certain infinite sequence has the property that terms with odd subscripts are positive and those with even subscripts are negative. The sequence converges. What must its limit be? Why?

14. Does the sequence of prime numbers seem to be convergent? Why?

15. Find the first ten terms of the sequence of the reciprocals of the prime numbers. Does this sequence seem to be convergent or divergent? Why?

Practice Exercises

Use appropriate technology.

Find the first six terms of each sequence. Tell whether the sequence is convergent or divergent. Justify your response using a graph.

1. $a_n = \frac{3}{n^3}$ 0
2. $b_n = \sqrt{n}$ divergent
3. $c_n = -1$ −1

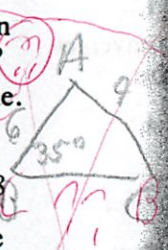
Determine whether each sequence converges. If it converges, give the limit.

4. $d_n = 5^n$ divergent
5. $t_n = \frac{n-3}{n+3}$
6. $r_n = \frac{n^2+n-2}{n+1}$
7. $a_n = (-1)^{n+1}\pi$ divergent
8. $b_k = \frac{2n+1}{3n+1}$
9. $c_t = \frac{2n^2}{n^2+1}$ 2
10. $r_n = \sin \frac{1}{2}n\pi$
11. $d_n = \cos \frac{1}{2}n\pi$ divergent
12. $m_k = \left(-\frac{1}{4}\right)^k$
13. $a_n = \frac{3n^5+n}{4n^5+8}$ divergent $\frac{3}{4}$
14. $b_n = \frac{3n^5+n}{4n^8+5}$
15. $c_n = \frac{3n^5+n}{4n^3+8}$ divergent
16. Is the sequence $-1, 1, -1, \frac{1}{2}, -1, \frac{1}{4}, -1, \frac{1}{8}, -1, \frac{1}{16}, \ldots$ convergent or divergent? Why?
17. Graph $f(x) = \sin 2x$, when $0 \le x < 2\pi$.
18. Is the Fibonacci sequence convergent or divergent? Why?
19. Find the sum of the first 15 terms of the sequence $c_n = 3n + 2$. $\frac{15}{2}(5+47) = 390$
20. Let a_n be a constant sequence. Is this sequence convergent or divergent? Why?
21. Find the 56th term of the sequence $-31, -27, -23, -19, -15, \ldots$ -4 $\frac{56}{2}(-31+189)$ 4648

Determine whether each geometric sequence is convergent or divergent. If it is convergent, give the limit.

22. $a_n = 4(3^n)$
23. $b_n = 4(0.2)^n$ 0
24. $c_n = 10\left(\frac{1}{2}\right)^n$ no sum
25. $r_k = \frac{1}{2}(10)^k$ divergent
26. $t_n = 6(-1)^n$
27. $d_n = 6\left(-\frac{1}{2}\right)^n$ 0
28. **Thinking Critically** Let ar^n be a geometric sequence. What conditions on the common ratio r will make the sequence convergent? Divergent? Why?
29. Given triangle ABC with $c = 6$ ft, $b = 9$ ft, and $\angle C = 35°$, solve the triangle.

Relier



$a = 13.44$
$A = 170°$
$B = 24°$

Determine whether each arithmetic sequence is convergent or divergent.

30. $a_n = 2n + 1$
31. $b_n = 4 - 2n$ divergent
32. $c_n = -2.8$
33. *Landscaping* Describe the rectangular plot of maximum area that can be enclosed using 180 ft of fencing given that one side of the plot is along a building and needs no fencing.
34. **Thinking Critically** Let $a_n = a + (n-1)d$ be an arithmetic sequence. What conditions on the common difference d will make the sequence convergent? Divergent? Why?
35. Find the first six terms of the sequence $a_n = e^n$. Is it convergent? Why?
36. Find the first six terms of the sequence $a_n = e^{-n}$. Is it convergent? Why?

divergent - keeps getting larger

Convergent to 0 $\left(\frac{1}{e^n}\right)$

8.3 Geometric Sequences + Series

12/3

583

77. \$1000 principal
3% interest
10 years
compounded:

annually (1)

$$1000\left(1+\frac{.03}{1}\right)^{1\cdot 10}$$

per year

years

\$1349.92

semi-annually (2)

$$1000\left(1+\frac{.03}{2}\right)^{2\cdot 10}$$

\$1346.86

Quarterly (4)

\$1348.35

Monthly (12)

\$1349.35

Daily (365)

\$1338.79

79. \$100/month
6% intrest
5 years

$$A = 100\left(1+\frac{.06}{12}\right)^{1} + \cdots 100\left(1+\frac{.06}{12}\right)^{60}$$

$$100\left(1+\frac{.06}{12}\right)\left(\frac{1-\left(1+\frac{.06}{12}\right)^{60}}{1-1+\frac{.06}{12}}\right)$$

\$7011.88

584/89.

$d = 1 + r$

$a_n = 3343(1.013)^n$

$n = 1995$

$n = 5, 6, \ldots, 11$

a) $r = .013$

1.3%

b) $a_{20} = a_1 \cdot r^d$

$a_{20} = \cancel{a_1 \cdot .013^{20}}$

$\cancel{3343}$

$3343(1.013)^{20}$

4328.379 thousand

4,328,379

c) $4100 = 3343(1.013)^n$

$1.126 = 1.013^n$

$\log(1.126) = n\log(1.013)$

$\frac{\log(1.126)}{\log(1.013)} = n$

$15.8 = n$

August

$\frac{8}{10}$ through 2005

91. Doubling Daily Wage $a_1 = .01$

$r = 2$ (doubles)

a) $S_{20} = .01\left(\frac{1-2^{29}}{1-2}\right)$

\$5,368,709.11

b) $S_{30} = .01\left(\frac{1-2^{30}}{1-2}\right)$

\$10,737,418.23

c) $S_{31} =$ \$21,474,836.47

8, extra

Sequence + Series

Limits Day 2

12/4

Limits of Infinite Series

Find the sum of an infinite series

sum of an infinite series of numbers = limit of its associate sequence of partial sums so the number of terms increases without bounds

ex Find the sum (if one exists) of the infinite series:

$$\sum_{k=1}^{\infty} \frac{1}{k(k+1)} \qquad \underbrace{\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \cdots}_{\frac{4}{5}}$$

Partial Sums

$S_1 = \frac{1}{2}$

$S_2 = \frac{1}{2} + \frac{1}{6} = \frac{2}{3}$

$S_3 = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} = \frac{3}{4}$

$S_4 = \text{11} + \frac{1}{20} \qquad \frac{4}{5}$

$\frac{n}{n+1}$

$$\lim_{n \to \infty} = \frac{n}{n+1} = \frac{1}{1} = 1$$

ex2

$$\sum_{k=1}^{\infty} \frac{1}{k(k+1)(k+2)}$$

$S_1 = \frac{1}{6}$

$S_2 = \frac{1}{6} + \frac{1}{24} = \frac{5}{24}$

$S_3 = \frac{5}{24} + \frac{1}{60} = \frac{9}{40}$

$S_4 = \frac{9}{40} + \frac{1}{120} = \frac{7}{30}$

$S_5 = \frac{7}{30} + \frac{1}{210} = \frac{5}{21}$

$\frac{n(n+3)}{4(n+1)(n+2)}$

$$\lim_{n \to \infty} = \frac{1}{4}$$

If the limit exist = Convergent series
If the limit doesn't exist = divergent series

ex3 $\sum_{k=1}^{\infty} 2k-1$ arithmetic

$S_n = 1 + 3 + 5 + 7 + \ldots$

$S_1 = 1$
$S_2 = 1+3 = 4$
$S_3 = 1+3+5 = 9$
$S_4 = 1+3+5+7 = 16$

n^2

$\lim_{n \to \infty} n^2$ = divergent (no limit)

P4 $\sum_{k=1}^{\infty} (-1)^k 10 = -10 + 20 - 40 + 80$

divergent = no limit
(oscilates the limit)

$r = -1$
$||r|| = 1$
$||r|| \geq 1$
✓ divergent

P5. $6 + 2 + \frac{2}{3} + \frac{2}{9} + \ldots$

$\cdot\frac{1}{3}$ $\cdot\frac{1}{3}$

$r = 1/3$

$r \geq 1$
$r < 1$

$\frac{a_1}{1-r} = \frac{6}{1-\frac{1}{3}} = 9$

P6 $.1 + .2 + .4 + .8 + \ldots$

$\cdot 2$ $\cdot 2$ $\cdot 2$

$r = 2$

$r \geq 1$
$r > 1$

no limit

P7 $5 + \underbrace{.2 + .04 + .008 + .0016}_{\text{look here}} + \ldots$

$\frac{.04}{.2} = \frac{1}{5} = r$ $\quad \frac{1}{5} \ ? \ 1$

$\frac{1}{5} < 1$

$\hookrightarrow \frac{.2}{1-\frac{1}{5}} + 5 = 5\frac{1}{4}$ Convergent by

P8 $10 + \underbrace{\frac{1}{9} + \frac{1}{9^2} + \frac{1}{9^3}}$ $\quad \frac{1}{9} \sim \frac{1}{9}$

do seperately $\rightarrow r = \frac{1}{9^1} = \frac{1}{9}$ $\quad \frac{1}{9} \ ? \ 1$

$\frac{1}{9} < 1$

has limit

$10 + \frac{1}{8} = 10\frac{1}{8}$ $\quad \frac{1}{8} \leftarrow \frac{\frac{1}{9}}{1-\frac{1}{9}}$

---

ex9 A ball is dropped from a height of 24 m
Rebounds to a height of 75 %
Find the total distance it travels

travels up+down

$d = 24 + 2(.75)24 + 2(.75^2)24 + 2(.75^3)24$

geometric series

factor out 2

$d = 24 + 2\left(\frac{a_1}{1-r}\right)$

$24 + 2\left(\frac{.75 \cdot 24}{1-.75}\right) = 168\text{m}$

P10

Dropped from 8ft
Bounces 70%
Find total difference it travels

$$d = 8 + 2(.7^1)8 + 2(.7^2)8 + 2(.7^3)8$$

$$8 + 2\left(\frac{.7 \cdot 8}{1-.7}\right) = 45\tfrac{1}{3}\text{ ft}$$

Determine whether each series converges or diverges. If it converges, find the sum. If it diverges, tell why it diverges.

5. $\frac{1}{4} + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 + \left(\frac{1}{4}\right)^4 + \cdots$ **6.** $1 + 5 + 5^2 + 5^3 + \cdots$

7. $\sum_{k=1}^{\infty} (0.15)^k$ **8.** $\sum_{t=1}^{\infty} 3\left(\frac{8}{9}\right)^t$ **9.** $\sum_{k=1}^{\infty} \left(-\frac{1}{4}\right)^k$ **10.** $\sum_{j=1}^{\infty} 3 \cdot \pi^j$

Express each repeating decimal as a ratio of two integers.

11. 0.2323232323 . . . **12.** 2.040404040 . . . **13.** 12.899999999 . . . **14.** −36.7777777 . . .

Practice Exercises

Use appropriate technology.

Find the first five terms of the sequence of partial sums for each series.

1. $3 + 6 + 9 + 12 + \cdots$ 3/9/18/30/45 **2.** $0.6 + (0.6)^2 + (0.6)^3 + \cdots$ $\lim \sum_{n=1}^{\infty} (.6)^n$ didn't do

3. $\sum_{k=1}^{\infty} \frac{1}{k^2}$ 1/1+1/4/13/16/61/144/1669/3600 **4.** $\sum_{t=1}^{\infty} 5(0.9)^t$ 4.5/8.55/12.195/15.4755/18.42795

5. Determine whether each series of Exercises 1 and 2 converges or diverges. If the series converges, find the sum. If it diverges, tell why it diverges.

1-diverge
2-converge 1.5
3-converge 0
4-diverge

Determine whether each series converges or diverges. If the series converges, find the sum. If it diverges, tell why it diverges.

6. $18 + 18(0.2) + 18(0.2)^2 + \cdots$ Converges 22.5 **7.** $1 + 0.1 + 0.001 + 0.0001 + \cdots$ Converges $\frac{1}{9}$

8. $\sum_{k=1}^{\infty} 8\left(\frac{2}{5}\right)^k$ Converges $5\frac{1}{3}$ **9.** $\sum_{j=1}^{\infty} \left(\frac{-3}{11}\right)^j$ Converges $-\frac{3}{14}$ **10.** $\sum_{k=1}^{\infty} -9(0.7)^k$ Converges −21

11. $\sum_{t=1}^{\infty} (1.1)^t$ divergent **12.** $\sum_{k=1}^{\infty} -3^k$ divergent **13.** $\sum_{n=1}^{\infty} (-3)^n$ divergent

14. Thinking Critically Suppose that the terms of a sequence $a_1, a_2, a_3, \ldots$ increase without bound. What conclusion can you draw about the related series $\sum_{n=1}^{\infty} a_n$? divergent 1+2+3+4+5+6+7

15. Solve the system of linear equations using an augmented matrix.

$$\begin{cases} x + 3y - z = 4 \\ 2x + 4y - 5z = 6 \\ 2y + 3z = 7 \end{cases}$$

Express each decimal as a ratio of two integers.

16. 0.45454545 . . . **17.** 12.999999 . . . **18.** 0.6758585858 . . . **19.** −8.9312424

20. Thinking Critically In the lesson, it is pointed out that $\lim_{n \to \infty} \frac{a(1 - r^n)}{1 - r} = \frac{a}{1 - r}$ because r^n gets closer and closer to 0 as n increases without bound, providing $|r| < 1$. Discuss why the restriction $|r| < 1$ is necessary.

21. In the class exercises of Lesson 13.4 it was proved that $1 + 2 + 3 + \cdots + n = \frac{1}{2}n(n + 1)$. Use this fact to show that the series $\sum_{k=1}^{\infty} k$ is divergent.

22. Find the first five partial sums S_1, S_2, S_3, S_4, and S_5 of the infinite series $\frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \frac{1}{15} + \frac{1}{21} + \frac{1}{28} + \cdots$. From the pattern, find a possible formula for S_n. Then use the definition of "infinite sum" to evaluate the sum of the infinite series. *Hint*: Express S_2 and S_4 as unreduced fractions.

Find the first five partial sums S_1, S_2, S_3, S_4, and S_5 of the following series.

$$\frac{3}{1\cdot 4} + \frac{5}{4\cdot 9} + \frac{7}{9\cdot 16} + \frac{9}{16\cdot 25} + \cdots + \frac{2n+1}{n^2(n+1)^2} + \cdots$$

23. From the pattern that you observe, what seems to be a formula for S_n?

24. Use your formula for S_n and the definition of "infinite sum" to find the sum of the infinite series.

25. Use the principle of mathematical induction to prove that

$$\frac{1}{2\cdot 5} + \frac{1}{5\cdot 8} + \frac{1}{8\cdot 11} + \cdots + \frac{1}{(3n-1)(3n+2)} = \frac{n}{2(3n+2)}$$

26. Use the result of Exercise 25 to find the sum of the series $\sum_{k=1}^{\infty} \frac{1}{(3k-1)(3k+2)}$.

27. Use the principle of mathematical induction to prove that

$$\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \cdots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$$

28. Use the result of Exercise 27 to find the sum of the series $\sum_{k=1}^{\infty} \frac{1}{(2k-1)(2k+1)}$.

29. *Physics* A ball is dropped from a height of 36 m. Each time it strikes the ground it rebounds to a height of 70% of the distance it fell. Find the total distance the ball travels after four bounces.

30. *Physics* A ball is dropped from a height of 40 m. Each time it strikes the ground it rebounds to a height of 45% of the distance it fell. Find the total distance the ball travels.

31. *Economics* A federal tax cut adds \$10 billion to the income of taxpayers. Assume that each taxpayer as a result of this tax cut spends 90% of the money and the recipients of this money in turn spend 90% of the money. Find the total amount of spending that results from the initial tax cut.

32. *Ecology* A radioactive substance is leaking from an underground storage tank. It spread 1000 m from the source during the first year, 800 m the second year, and 640 m the third year. If this pattern continues, how far will it spread in 6 years? Will the radioactive substance ever reach a river 6000 m from the source? Why or why not?

33. **Writing in Mathematics** Devise several situations that, like the wall-painting problem, can be described mathematically by the infinite series $\frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \cdots$. Show how one of them is described by the series.

34. Identify an infinite geometric series that has a sum of 6 and a first term of 2.

35. Identify an infinite geometric series that has a sum of $\frac{8}{3}$ and a common ratio of $\frac{1}{4}$.

36. **Thinking Critically** Find an example of a geometric series with a negative sum that has a positive common ratio. Can an infinite geometric series with a negative sum have a positive first term? Why or why not?

8.5 Expanding Binomials
Notes

12/5

Binomial - polynomial w/ 2 terms

$(x+y)^0 = 1$

$(x+y)^1 = x+y$

$(x+y)^2 = (x+y)(x+y) = x^2+2xy+y^2$

$(x+y)^3 = x^3+3x^2y+3xy^2+y^3$

$(x+y)^4 = x^4+4x^3y+6x^2y^2+4xy^3+y^4$

$(x+y)^5 = x^5+5x^4y+10x^3y^2+10x^2y^3+5xy^4+y^5$

$x^{5 \to 0}$ (5, 4, 3, 2, 1)

$y^{0 \to 5}$ (0, 1, 2, 3, 4)

↑ terms must add to the degree

$(x+y)^4 = \ldots 6x^2y^2 \ldots$

$2+2$

4 ✓

Binomial Theorem

$$(x+y)^n = x^n + nx^{n-1}y + \ldots {}_nC_r x^{n-r} y^r \ldots$$
$$\ldots + nxy^{n-1} + y^n$$

Combonations

$${}_nC_r = \frac{n!}{(n-r)!\,r!}$$

used to get binomial coefficents

ex ${}_8C_2$ = taking 8 objects - 2 at a time $= \binom{8}{2}$

A B C D E F G H

AB BC ...

AC BD ...

AD BE ...

A... ...

$$= \frac{8!}{(8-2)!\cdot 2!} = 28$$

599

6. $_{12}C_3 = \frac{12!}{(12-3)!(3)!} = \frac{12!}{9!\cdot 3!} = 220$

14. $_{52}C_{47}$ on Calc:

[5] [2] [Math] [E] [3] [4] [7] [Enter] $= 2598960$

Prb nCr

12. $_{17}C_4 = \frac{17!}{13!\cdot 4!} = 2380$

Pascal's Triangle

$(x+y)^0$	1	row 0
$(x+y)^1$	1 1	row 1
$(x+y)^2$	1 2 1	row 2
$(x+y)^3$	1 3 3 1	⋮
$(x+y)^4$	1 4 6 4 1	
$(x+y)^5$	1 5 10 10 5 1	
$(x+y)^6$	1 6 15 20 15 6 1	

Add up the 2 numbers above it

1 2 1

3 3

ex $(x+y)^7 = \overset{\downarrow\, {}_7C_1}{x^7} + \overset{\downarrow\, {}_7C_2}{7x^6y} + \overset{\downarrow\, {}_7C_3}{21x^5y^2} + \overset{\downarrow\, {}_7C_4}{35x^4y^3} + 35x^3y^4 + 21x^2y^5 + 7xy^6 + y^7$

26. $(y-2)^5$ $n=5 \rightarrow {}_5C_r$

$1y^5 + 5y^4(-2) + 10\cdot y^3(-2)^2 + 10\cdot y^2(-2)^3 + 5y(-2)^4 + (-2)^5$

$y^5 - 10y^4 + 40y^3 - 80y^2 + 80y - 32$

$(4x)^6 = 4^6 x^6$

$4096x^6$

30. $(4x+3y)^4 = {}_4C_0(4x)^4 + {}_4C_1(4x)^3(3y) + {}_4C_2(4x)^2 + (3y)^2 +$
${}_4C_3(4x)(3y)^3 + {}_4C_4(3y)^4$

$4^4 x^4 + 4 \cdot 4^3 \cdot x^3 \cdot 3 \cdot y + 6 \cdot 4^2 \cdot x^2 \cdot 3^2 \cdot y^2 +$
$4 \cdot 4 \cdot x \cdot 3^3 \cdot y^3 + 1 \cdot 3^4 \cdot y^4$

$256x^4 + 4 \cdot 64x^3 \cdot 3y + 6 \cdot 16x^2 \cdot 9y^2 +$
$4 \cdot 4x \cdot 9y^3 + 81y^4$

$256x^4 + 256x^3 \cdot 3y + 96x^2 \cdot 9y^2 +$
$16x \cdot 9y^3 + 81y^4$

$256x^4 + 768x^3y + 864x^2y^2 + \cancel{144}\,432xy^3 + 81y^4$

8.5 Binomial Theorem
Day 2 Notes

12/8

Review

$x^n + n x^{n-1} y \cdots {}_nC_r x^{n-r} y^r + n \cdot x y^{n-1} + y^n$

ex: $(x+y)^5 = x^5 + 5x^4 y + {}_5C_2 x^3 y^2 + {}_5C_3 x^2 y^3 + {}_5C_4 x y^4 + y^5$

599

32. $(2x-y)^5 = (2x)^5 + 5(2x)^4(-y) + 10(2x)^3(-y)^2 + 10(2x)^2(-y)^3 + 5(2x)(-y)^4 + (-y)^5$

$32x^5 + 5 \cdot 16x^4 - y + 10 \cdot 8x^3 y^2 + 10 \cdot 4x^2 (-y)^3 + 5 \cdot 2x y^4 + (-y)^5$

$32x^5 - 80x^4 y + 80x^3 y^2 - 40x^2 y^3 + 10xy^4 - y^5$

38. $\left(\frac{1}{x} + 2y\right)^6 = \left(\frac{1}{x}\right)^6 + 6\left(\frac{1}{x}\right)^5 (2y) + 15\left(\frac{1}{x}\right)^4 (2y)^2 + 20\left(\frac{1}{x}\right)^3 (2y)^3 + 15\left(\frac{1}{x}\right)^2 (2y)^4 + 6\left(\frac{1}{x}\right)(2y)^5 + (2y)^6$

$\frac{1}{x^6} + 6\frac{1}{x^5} 2y + 15\frac{1}{x^4} 4y^2 + 20\frac{1}{x^3} 8y^3 + 15\frac{1}{x^2} + 16y^4 + 6\frac{1}{x} \cdot 32y^5 + 64y^6$

$\frac{1}{x^6} + \frac{12y}{x^5} + \frac{60y^2}{x^4} + \frac{160y^3}{x^3} + \frac{240y^4}{x^2} + \frac{192y^5}{x} + 64y^6$

can put # in numerator too

$\frac{12y}{x^5}$

ex2 Find the 6th term of $(a+2b)^8$

$\leftarrow$ 1 minus what you are asked for

$_8C_5\ a^3(2b)^5$

$56a^3\ 32b^5$

$1792a^3b^5$

52. $(5a+6b)^5$

$n=5$

$\leftarrow n-1$

$_5C_4(5a)(6b)^4$

$5 \cdot 5a \cdot 1296b^4$

$32400ab^4$

ex3 Find the coefficent of the term a^6b^5 in the expression $(2a-5b)^{11}$

degree $=11$

$r=5$

$_{11}C_5(2a)^6(-5b)^5$

$462 \cdot 64a^6 - 3125b^5$

$-92{,}400{,}000\ a^6b^5$

56. Binomial Term $= \underbrace{ax^5}_{\text{1st term}}$ $(x+4)^{12}$ Find coefficent a

$\leftarrow 12-5$

$_{12}C_7\ x^5 \cdot 4^7$

$792 \cdot x^5 \cdot 16384$

$\underbrace{12{,}976{,}128}\ x^5$

$\hookrightarrow$ 12,976,128

58. $(4x-y)^{10}$

$\underbrace{a}_{\text{coefficent}}x^2y^8$

$_{10}C_2(4x)^2(-y)^8$

$45 \cdot 16 \cdot x^2y^8$

$720x^2y^8$

$\hookrightarrow$ 720

62. $(z^2-1)^{12}$

az^6

Find a

what shows up

12

${}_{12}C_9\left((z^2)^3(-1)^9\right)$

$220\ z^6 - 1$

$-220z^6$

8.5 Binomial Theorem

Day 2 HW

12/7

599 Find the binomial coefficent

3. $\binom{12}{0} = {}_{12}C_0 = 1$

9. $\binom{100}{98} = {}_{100}C_{98} = 4950$

13. ${}_{22}C_9 = 497420$

17. ${}_7C_4 = 35$

27. $(x+y)^5 = 1x^5 + {}_5C_1 x^4 y + {}_5C_2 x^3 y^2 + {}_5C_3 x^2 y^3 + 5xy^4 + y^5$

$x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^4$

34. $(5-2y)^3 = 5^3 + {}_3C_1 5^2(-2y) + {}_3C_2 5(-2y)^2 + (-2y)^3$

$125 + 3 \cdot 25 \cdot -2y + 3 \cdot 5 \cdot 4y^2 + -8y^3$

$125 - 150y + 60y^2 - 8y^3$

35. $(x^2+y^2)^4 = (x^2)^4 + 4(x^2)^3 y^2 + {}_4C_2 (x^2)^2 + (y^2)^2 + {}_4C_3 (x^2)(y^2)^3 + 1(y^2)^4$

$x^8 + 4x^6y^2 + 6x^4y^4 + 4x^2y^6 + y^8$

41. $-3(x-2)^3 - 4(x+1)^6$

$-3(x^3 + 3x^2(-2) + {}_3C_2 x + (-2)^2 + 1(-2)^3)$

$-3(x^3 - 6x^2 + 12x - 8)$

$-3x^3 + 18x^2 - 36x + 24$

$-4(x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1$

$+ \quad -4x^6 - 24x^5 - 60x^4 - 80x^3 - 60x^2 - 24x - 4$

$-4x^6 - 24x^5 - 60x^4 - 83x^3 - 42x^2 - 60 + 20$

43. $(3t-s)^3 = (3t)^3 + 3(3t)^2(-s) + 3(3t)(-s)^2 + (-s)^3$

$9t^3 - 9t^2 s + 9ts - s^3$

45. $(x+2y)^5 = x^5 + 5x^4(2y) + \overset{{}_5C_2}{10}x^3(2y)^2 + 10x^2(2y)^3 + 5x(2y)^4 + (2y)^5$

$x^5 + 10x^4y + 40x^3y^2 + 80x^2y^3 + 80xy^5 + 32y^5$

49. $(x-6y)^5$ $\quad {}_5C_2\, x^{\overset{n-1}{}}(x)^{5-\overset{r}{2}}(-6y)^{\overset{r}{2}}$

$n=3$ $\quad 10x^3\, 36y^2$

$360x^3y^2$

53. $(10x-3y)^{12}$ $\quad {}_{12}C_8\,(10x)^4(-3y)^8$

$n=4$ $\quad 495 \cdot 10000 \cdot x^4 \cdot 6561\, y^8$

$3.24 \cdot 10^{16} \cdot x^4 y^8$

59. $(3x-2y)^9$ $\quad {}_{10}C_6\,(3x)^6(-2y)^3$

$a\,x^6y^3$ $\quad 210 \cdot 729x^6 \cdot -8y^3$

$-1224720x^6y^3$

62. $(z^2-1)^{12}$ $\quad {}_{12}C_6\,(z^2)^6(-1)^6$ ← could be wrong

$a\,z^6$ $\quad 924\, z^{12}$

63. $(\sqrt{x}+5)^4 = (\sqrt{x})^4 + 4(\sqrt{x})^3 5 + \overset{{}_4C_2}{6}(\sqrt{x})^2 5^2 + 4(\sqrt{x})5^3 + 5^4$

$x^2 + 4x^{3/2}5 + 150x + 500x^{1/2} + 625$

600

66. $(u^{3/5}+v^{1/5})^5 = (u^{3/5})^5 + 5(u^{3/5})^4(v^{1/5}) + 10(u^{3/5})^3(v^{1/5})^2 +$

$10(u^{3/5})^2(v^{1/5})^3 + 5(u^{3/5})(v^{1/5})^4 + (v^{1/5})^5$

$v^{6/5}$

$u^3 + 5u^{12/5} + 10u^{9/5}v^{10/5} + 10u^{6/5}v^{6/5}$ multiply

$(x^{3/5})^2 = x^{(3/5)\cdot 2} = x^{6/5}$

67. $f(x) = x^3$ $\quad \dfrac{f(x+h) - f(x)}{h}$

$$\frac{(x+h)^3 - x^3}{h}$$

$$\frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}$$

$$\frac{3x^2h + 3xh^2 + h^3}{h}$$

$$3x^2 + 3h + h^2$$

71. $(1+i)^4 = 1^4 + 4 \cdot i + 6i^2 + 4i^3 + i^4$

$1 + 4i + 6(-1) + 4 - i + 1$

$1 + 4i - 4i - 6 + 1$

-4

85. Coin Toss

7 times

4 heads $= {}_7C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^3 \rightarrow \left(\frac{1}{2} + \frac{1}{2}\right)^7$

$35 \, (1/16) \, (1/8)$

$.2734375$

27.34%

8/ Sequences

Formula Sheet

12/10

Arithemetic Sequence

$a_n = \underset{\text{slope}}{d}n + c$ $\quad$ d = common diff slope

$c = a_1 - d$

$a_n = a_1(n-1)d$ ← recursive version

Sum of Finite Arithemetic Series

$S_n = \frac{n}{2}(a_1 + a_n)$ partial sum

next term =
$a_{n+1} = a_n + d$

Geometric Sequences

$\frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = r$ = common ratio

$a_n = a_1 r^{n-1}$ ← recursive
$a_n = (a_1 \cdot r)\left(\frac{1}{r}\right)^n$) same thing

Sum of a Finite Geometric Series

$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$

Sum of an Infinite Geometric Series

$S = \frac{a_1}{1-r}$ $\quad$ $|r| < 1$ → convergent
$|r| \geq 1$ → divergent

8/ Sequences + Series
Review HW

12/10

626/

3. Write the first 5 terms

$a_n = \frac{72}{n!}$

n	1	2	3	4	5
a_n	72	36	12	3	16

11. $\frac{2!\cdot 5!}{6!}$ $\quad \frac{2\cdot 1\cdot \cancel{5\cdot 4\cdot 3\cdot 2\cdot 1}}{6\cdot \cancel{5\cdot 4\cdot 3\cdot 2\cdot 1}}$ $\quad \frac{2\cdot 1}{6}$ $\quad \frac{2}{6}$ $\quad \frac{1}{3}$

15. $\sum_{j=1}^{4} 6/j^2 = 8.54$ ← calc sum(seq(thing, x, start, limit))

17. $\sum_{k=1}^{10} 2k^3 = 770$

23. $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\dots+\frac{9}{10}$ $\quad \sum_{x=1}^{9} \frac{x}{x+1} = 7.07$

27. $\sum_{k=1}^{\infty} 2\left(\frac{1}{100}\right)^k$

a) Find 4th term: $2\left(\frac{1}{100}\right)^4 = 2\cdot 10^{-8}$

b) Sum $= r = \frac{1}{100} < 1$ Convergent

$$\frac{\frac{2}{100}}{1-\left(\frac{1}{100}\right)} = .0202 = \frac{2}{99}$$

29. Compound Interest

n = quarters

$$a_n = 2500\left(1+\frac{.08}{4}\right)^n$$

quarter	1	2	3	4	5	6
total	2550	2601	2653	2706.1	2760.2	2815.4

7	8	40
2871.7	2929.1	5520.10

37. $a_4 = 10$ $\quad \frac{28-10}{10-4} = \frac{18}{6} = 3 \quad a_n = 3n + c$

$a_{10} = 28$

n	a_n
1	1
2	4
3	7
4	10
5	13

$d = 3$

$a_1 = a_4 - 3d$

$10 - 3 \cdot 3$

$a_1 = 1$

39. $a_{k+1} = a_k - 3$

$a_1 = 35$

$a_2 = 32$

$a_3 = 29$

$a_4 = 26$

$a_5 = 23$

627

43. Sum of first 20 terms

$a_1 = 100$

$d = -3$

$S_n = \frac{n}{2}(a_1 + a_{100}) \quad a_{100} = 100 + 99(-3)$

$S_{20} = \frac{20}{2}(100 + -197)$

$S_{20} = -970$

47. $\sum_{k=1}^{11} \left(\frac{2}{3}k + 4\right) =$

$a_1 = \frac{2}{3} + 4 = 4\frac{2}{3}$

$a_{11} = \frac{2}{3} \cdot 11 + 4 = 11\frac{1}{3}$

$S_{11} = \frac{11}{2}\left(4\frac{2}{3} + 11\frac{1}{3}\right)$

88

49. First 100 positive multiples of 5

$\sum_{k=1}^{100} 5k$ $\quad a_1 = 5$

$a_{100} = 500$ $\quad S_{100} = \frac{100}{2}(5 + 500)$

25250

50. $\sum_{k=20}^{80} k$ $\quad a_{20} = 20$, $a_{80} = 80$ $\quad S_{61} = \frac{61}{2}(20+80)$

~~3,000~~ 3,050

51. Salary = \$34,000 (year 1)
Raise = \$2250/year for 5 years

year 5 = 34,000 + 2250 · 4
\$43,000

5 year sum = $\frac{5}{2}$ = (34,000 + 43,000)
\$192,500

52. 1st trip = 123 bales
each trip = 11 fewer bales
total after ~~8~~ trips

trip + trip + 6 more trips
2 additional

Check

$a_1 = 123$
$a_8 = 123 - 11 \cdot \cancel{6}\ 7$
~~51~~ 46

$S_7 = \frac{8}{2}(123 + \cancel{51}\ 46)$

$S_7 = \cancel{810}\ 676$

Geometric

59. $a_1 = 9$
$a_3 = 4$

$a_3 = a_1 \cdot r^{3-1}$
$4 = 9 \cdot r^2$
$\frac{4}{9} = r^2$
$r = 2/3$

$a_1 = 9$
$a_2 = 6 \quad 2 \cdot 2/3$
$a_3 = 4$
$a_4 = 2\frac{2}{3}$
$a_5 = 16/9$

63. $a_1 = 25$
$a_{k+1} = -3/5\, a_k$

$a_1 = 25$
$a_2 = -15$
$a_3 = 9$
$a_4 = -5.4$
$a_5 = 3.24$

65. Find nth + sum of first 20 terms

$a_1 = 16$
$a_2 = -8$ $\quad r = \frac{-8}{16} = -\frac{1}{2}$

$a_n = 16 \cdot -\frac{1}{2}^{n-1}$ ← memorize $a_1 \cdot r^{n-1}$

~~$|r| < 1$ convergent~~ $\quad \cancel{\frac{16}{1 - -\frac{1}{2}}} = 10\,2/3$

not infinite

$16\left(\frac{1-\left(-\frac{1}{2}\right)^{20}}{1-\left(-\frac{1}{2}\right)}\right)$

75. $\sum_{i=1}^{10} 10\left(\frac{3}{5}\right)^{i-1}$ $\quad a_1 = 10$
$a_{10} = .10077696$

$S_9 = \frac{9}{2}\left(10 + .10077696\right)$
45.45

79. $\sum_{k=1}^{\infty} 4\left(\frac{2}{3}\right)^{k-1}$ $\quad a_1 = 4$
$2/3 < 1$ convergent

$\frac{4}{1 - 2/3} = 12$

81. Depreciation

\$120,000

30% year depreciation

a) $\$120{,}000(.7)^t$

b) $t=5 \rightarrow 120{,}000(.7)^5$

$\$20{,}168.4$

82. Annuity

\$75/month

4% intrest

finite, geometric sequence

$A = 75\left(1+\frac{.04}{12}\right)^1 + \dots 75\left(1+\frac{.04}{12}\right)^{48}$ $\leftarrow a_1$

$= \$87.99$

It was already revised

$a_{48} = 75.25$

$a_{47} = 75\left(1+\frac{.04}{12}\right)^2$

75.50

geometric

$r = \frac{75.50}{75.25} = 1.003333$

$\cancel{\$87.99}$ 75.25 $\left(\frac{1-1.00333^{48}}{1-1.00333}\right)$

a_1

$\cancel{\$4571.93}$ $\$3909.96$

628

93. $a_n = a_{n-1} - 1$

$a_1 = 16$

$a_1 = 16$

$a_2 = 15$

$a_3 = 14$

$a_4 = 13$

$a_5 = 12$

linear

+ study worksheets

97. $\binom{9}{4} = {}_9C_4 = 126$

101. $\binom{8}{4} = {}_8C_4 = 70$

105. $(a-4b)^5 = a^5 + 5a^4(-4b) + \overset{{}_5C_2}{10}a^3(-4b)^2 + 10a^2(-4b)^3 + 5a(-4b)^4 + (-4b)^5$

$a^5 + 5a^4 \cdot -4 \cdot b + 10a^3 \cdot 16 \cdot b^2 + 10a^2 \cdot -64b^3 + 5a \cdot 256b^4 - 1024b^5$

$a^5 - 20a^4b + 160a^3b^2 - 640a^2b^3 + 1280ab^4 - 1024b^5$

107. $(7+2i)^4 = 7^4 + 4 \cdot 7^3(2i) + 6 \cdot 7^2(2i)^2 + 4 \cdot 7 \cdot (2i)^3 + (2i)^4$

$2401 + 2744i + 1176i^2 + 224i^3 + 16i^4$

$2401 + 2744i - 1176 - 224i + 16$

$1241 + 2520i$

Ø/ Sequences + Series
Review 2

12/11

1. Find formula

arthmetic

$a_5 = 24$

$a_9 = 40$

$$\frac{40-24}{9-5} = \frac{16}{4} = 4$$

$$a_0 = 24 - 4 \cdot 5$$

$$a_0 = 4$$

$$a_n = 4n + 4$$

2. Find missing 3 terms

geometric

$a_1 = 4$

$a_5 = 324$

$$324 = 4 \cdot r^4$$

$$81 = r^4$$

$$\sqrt[4]{\ } \quad \sqrt[4]{\ }$$

$$r = 3$$

$a_1 = 4$

$a_2 = 12$

$a_3 = 36$

$a_4 = 108$

$a_5 = 324$

3 A firm offers \$26,000 salery

\$900 / year raise

What is salery in 9th year?

$$a_9 = 26{,}000 + 900(9-1)$$

$$\$33{,}200$$

4. Show w/ sigma : $3 - 6 + 9 - 12 + 15$

$$\sum_{x=1}^{5} 3n(-1)^{n+1}$$ ← memorize rotating negitive

5. Find sum of first 25 terms
3rd term = 24
diff = 3.5

differences

$a_1 = 24 - 2(3.5)$ $\quad$ $a_{25} = 17 + \cancel{25}\,24(3.5)$
$a_1 = 17$ $\quad$ ~~104.5~~ 101

$\frac{25}{2}(17 + \cancel{104.5}\,101)$
~~1518.75~~ 1475

6. Find a_5
$r = 1.75$
$S_5 = 20{,}775$

$S_5 = a_1\left(\frac{1-1.75^5}{1-1.75}\right)$
$20775 = a_1\, 20.55$
$a_1 = 1010.91$

$a_5 = a_1 r^4$
$a_5 = 1010.91 \cdot 1.75^4$
9481.23

7. Bouncing Ball
dropped 6 ft
rebounds $\frac{1}{3}$ of previous bounce
total distance?

$6 + 2(.33)6 + 2(.33^2)6 + 2(.33^3)6 + 2(.33^4)6 \ldots$

$6 + 2\left(\frac{a_1}{1-r}\right) \rightarrow 6 + 2\left(\frac{6 \cdot .33}{1-.33}\right) \rightarrow 11.94\text{ ft}$

8. A architect makes a theater with 15 seats in the first row

18 seats = 2nd

21 seats = 3rd

seating capacity = 870

How many rows 870 is the.

$\cancel{870 = 12 + 3d}$ → $870 = \frac{n}{2}(a_1 + a_n)$ ← SUM

$\cancel{858 = 3d}$

$\cancel{d = 286}$ rows

$870 = \frac{n}{2}(15 + a_n)$

$870 = \frac{n}{2}(15 + [12 + 3n])$

$870 = \frac{n}{2}(27 + 3n)$

$1740 = n(27 + 3n)$

$n^2 + 9n - 580$ → $1740 = 27n + 3n^2$

$3n^2 + 27n - 1740 = 0$

rooten

$n = 20$

$(n - 20)(n + 29)$

8/ Sequences + Series
Test Objectives

12/11

Write first 5 terms
Use $_nC_r$
Find sums of infinite series
Factorial notation
Finding nth term
Find formula for nth term
Use sigma to write the sum
Binomial theorem
- expanding
- getting specific term
- limit of sequences + series (worksheet)

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \cos^2 x$

Cofunction Identities

$\sin\left(\frac{\pi}{2} - x\right) = \cos x$ $\quad$ $\cos\left(\frac{\pi}{2} - x\right) = \sin x$

$\csc x\left(\frac{\pi}{2} - x\right) = \sec x$ $\quad$ $\tan\left(\frac{\pi}{2} - x\right) = \cot x$

$\sec\left(\frac{\pi}{2} - x\right) = \csc x$ $\quad$ $\cot\left(\frac{\pi}{2} - x\right) = \tan x$

Negative Angle Identities

$\sin(-x) = -\sin x$ $\quad$ $\cos(-x) = \cos x$

$\csc(-x) = -\csc x$ $\quad$ $\tan(-x) = -\tan x$

$\sec(-x) = \sec x$ $\quad$ $\cot(-x) = -\cot x$

Sum and Difference Formulas

$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$

$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

Double Angle Formulas

$\sin 2x = 2 \sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

Additional Trigonometric Formulas/Identities

Half - Angle Identities:

$$\sin^2 u = \frac{1-\cos 2u}{2} \qquad \cos^2 u = \frac{1+\cos 2u}{2} \qquad \tan^2 u = \frac{1-\cos 2u}{1+\cos 2u}$$

Half - Angle Formulas:

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}} \qquad \cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}} \qquad \tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

Alternative Half - Angle Formulas for Tangent:

$$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta} \qquad \tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta}$$

Product - to - Sum Formulas:

$$\sin u\cos v = \frac{1}{2}\left[\sin(u+v)+\sin(u-v)\right]$$

$$\cos u\sin v = \frac{1}{2}\left[\sin(u+v)-\sin(u-v)\right]$$

$$\cos u\cos v = \frac{1}{2}\left[\cos(u+v)+\cos(u-v)\right]$$

$$\sin u\sin v = \frac{1}{2}\left[\cos(u-v)-\cos(u+v)\right]$$

Sum - to - Product Formulas:

$$\sin u + \sin v = 2\sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\sin u - \sin v = 2\cos\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

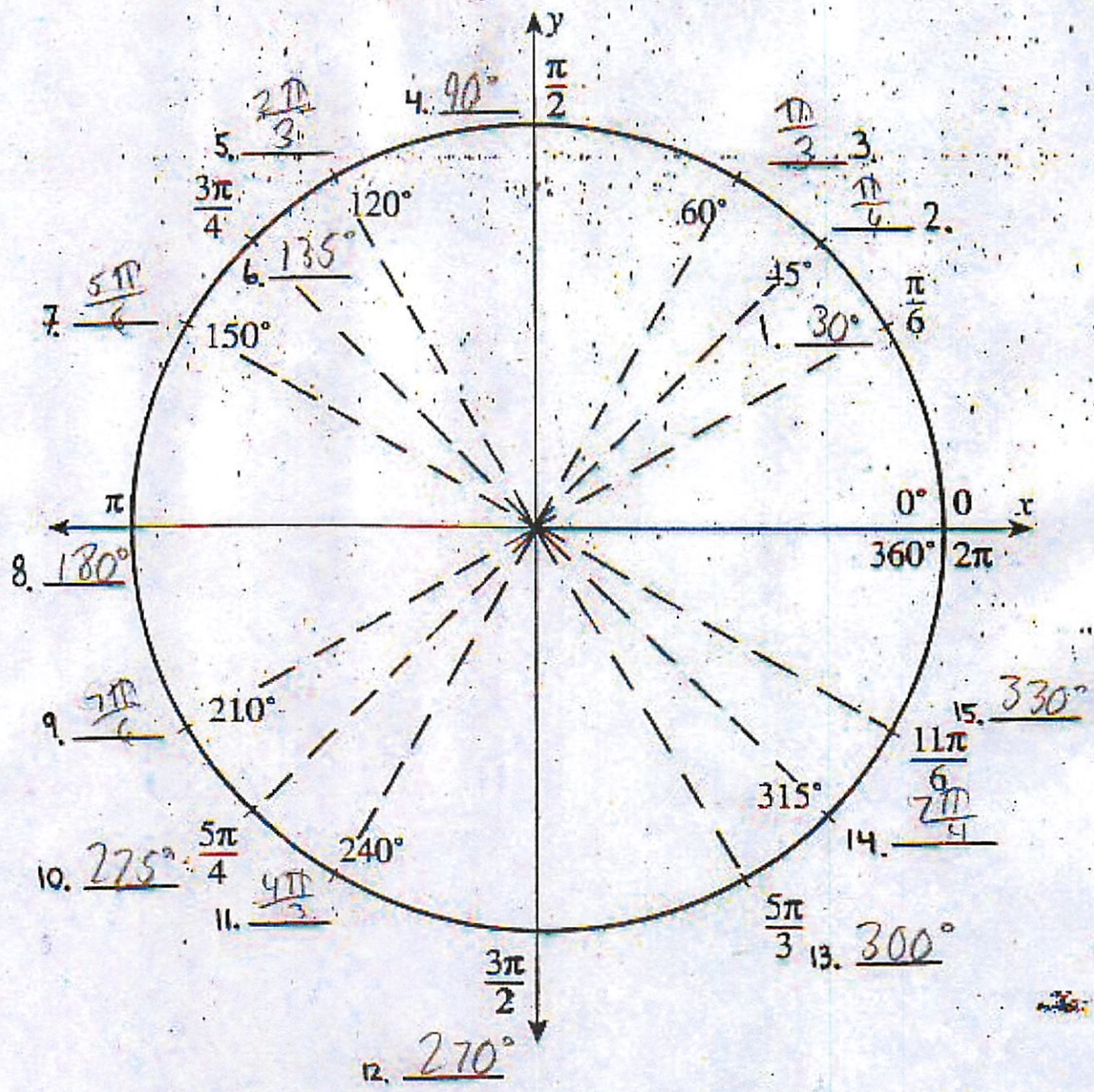
$$\cos u + \cos v = 2\cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\cos u - \cos v = -2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

Name Michael Plasmeier

Unit Circle in terms of Degrees & Radians
(in multiples of 30°, 45°, 60°)

Directions: Convert the radian measure to degrees and
Convert the given degree measure to radians.



5/ IS3

More Pratice + Review

12/13

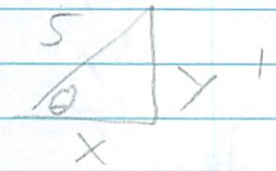
1. $\csc(-x) = -5$

$\downarrow$

$-\csc x = -5$

$\csc x = 5 = \frac{1}{\sin x} = 5$

$= \frac{r}{y} = 5$



$\rightarrow 5^2 = 1^2 + x^2$

$x = \sqrt{24}$

$\sin = \frac{y}{r} = \frac{1}{5}$ $\csc = 5$

$\cos = \frac{x}{r} \frac{\sqrt{24}}{5}$ $\sec = \frac{r}{x} \frac{5}{\sqrt{24}} \rightarrow \frac{5\sqrt{24}}{24} = \frac{5 \cdot 2\sqrt{6}}{24}$

$\tan = \frac{y}{x} \frac{1}{\sqrt{24}}$ $\cot = \frac{x}{y} = \frac{\sqrt{24}}{1} \rightarrow \sqrt{24}$ $\frac{5\sqrt{6}}{12}$

$\rightarrow \frac{\sqrt{24}}{24} \rightarrow \frac{\sqrt{4}\sqrt{6}}{24} = \frac{2\sqrt{6}}{24} = \frac{\sqrt{6}}{12}$

2. $\frac{\sin\theta + \cos\theta}{\sin\theta} - \frac{\cos\theta - \sin\theta}{\cos\theta} = \sec\theta \csc\theta$

$\frac{\cos\theta(\sin\theta + \cos\theta) - \sin\theta(\cos\theta - \sin\theta)}{\sin\theta \cos\theta}$

$\frac{\sin\theta\cos\theta + \cos^2\theta - \sin\theta\cos\theta + \sin^2\theta}{\sin\theta\cos\theta}$

$\frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta} \rightarrow \frac{1}{\sin\theta} \cdot \frac{1}{\cos\theta}$

$\csc\theta \sec\theta = \sec\theta\csc\theta$

3. $\frac{\tan x + \cot y}{\tan x \cot y} = \tan x + \cot y$

$\frac{\tan x}{\tan x \cot y} + \frac{\cot y}{\tan x \cot y}$

$\cot y + \tan x = \tan x + \cot y$

4. $\sec x \csc x = 2\csc x = \frac{2}{\sin x}$

$\frac{1}{\cos x} \quad \frac{1}{\sin x}$

$\sec x \csc x - 2\csc x = 0$

$\csc x(\sec x - 2)$

$\csc x = 0$

$\frac{1}{\sin x} = 0$

no solution

$\sec x - 2 = 0$

$\sec x = 2$

$\cos x = \frac{1}{2}$

1st: 60° $\frac{\pi}{3}$

4th: 300° $\frac{5\pi}{3}$

5. $\tan^2 3x = 3$ - Solve for x - no range

$\tan 3x = \sqrt{3}$

$\tan^{-1} \quad \tan^{-1}$ split

$\frac{3x}{3} = \pm \frac{60^\circ}{3}$, $\frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ then

$x = \pm 20^\circ$

1st:

2nd:

3rd:

4th:

$3x = \frac{\pi}{3} + n\pi$

$\frac{2\pi}{3} + n\pi$

3 then 3

$x = \frac{\pi}{9} + n\pi/3$

$x = \frac{2\pi}{9} + \frac{n\pi}{3}$

$\frac{5\pi}{9} + \frac{n\pi}{3}$

$\frac{7\pi}{9} + \frac{\pi n}{3}$

Equations Pratice

12/17

1. $5\sqrt{3}x - 3 = 8\sqrt{3}x$

$-5\sqrt{3}x \qquad -5\sqrt{3}x$

$\frac{-3}{3\sqrt{3}} = \frac{3\sqrt{3}x}{3\sqrt{3}}$

$-\frac{1}{\sqrt{3}} = x \rightarrow -\frac{\sqrt{3}}{3}$

2. $3x^2 + x - 1 = 0$

Rooten w/ quad formula → by hand

$x = .434$ or $\frac{1 \pm \sqrt{13}}{6}$ to get

$x = -.767$

3. $4x^2 - 8x = -3$

$4x^2 - 8x + 3 = 0$

Rooten

$x = 1.5$

$x = .5$

$(2x-3)(2x-1)$

12, -2, $-6x$, -8, $+x$

$2x - 1$

$2x$	$4x^2$	$-2x$
-3	$-6x$	3

4. $2x^2 - 3x - 2 = 0$

rooten

$x = 2$

$x = -.5$

$(2x+1)(x-2)$

5. $x^2 = \frac{3}{2}x$

$x^2 - \frac{3}{2}x = 0$

Rooten

$x = 0$

$x = 1.5$

$2x^2 - 3x = 0$

$x(2x-3)$

Test Corrections

12/17

9. $\cos x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2b}$

$\cos x = \dfrac{-1 \pm \sqrt{13}}{6}$

$\cos x = .434$ $\quad$ $\cos x = -.767$

$\cos^{-1}$ $\quad$ $\cos^{-1}$ $\quad$ $\cos^{-1}$ $\quad$ $\cos^{-1}$

1.12r $\quad$ 2.44r

⊕ 1st	1.12r	⊖ 2nd	2.44r
⊕ 4th	5.16r	⊖ 3rd	3.84r

10. $4\sin^2 x - 8\sin x = -3$

$4\sin^2 x - 8\sin x + 3 = 0$

$(2\sin x - 3)(2\sin x - 1)$

$\sin x = \frac{3}{2}$ $\quad$ $\sin x = \frac{1}{2}$

out of domain

⊕	30°	$\frac{\pi}{6}$
⊕	2nd	$\frac{5\pi}{6}$

11. $2\sec^2\frac{x}{2} - 3\sec\frac{x}{2} - 2 = 0$

$(2\sec\frac{x}{2} + 1)(\sec\frac{x}{2} - 2) = 0$

$\sec\frac{x}{2} = -\frac{1}{2}$ $\quad$ $\sec\frac{x}{2} = 2$

$\cos\frac{x}{2} = -2$ $\quad$ $\cos\frac{x}{2} = \frac{1}{2}$

Out of Range

$\frac{x}{2} = 60° = \frac{\pi}{3}$

$\frac{x}{2} = \frac{\pi}{3} \rightarrow \boxed{\frac{2\pi}{3}}$

12. $\tan^2\theta = \frac{3}{2}\sec\theta$

$\sec^2\theta - 1 = \frac{3}{2}\sec\theta$

$\sec^2\theta - 3/2\sec\theta - 1 = 0$

$2\sec^2\theta - 3\sec\theta - 2 = 0$

$(\sec\theta - 2)(2\sec\theta + 1)$

$\downarrow \qquad \downarrow$

$\sec\theta = 2 \qquad \sec\theta = -\frac{1}{2}$

$\downarrow \qquad \downarrow$

$\cos = \frac{1}{2} \qquad \cos\theta = -2$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

5/ Trig Review
Before Reviewz

12/17

346

Simplify

36. $\cot\left(\frac{\pi}{2} - x\right)\cos x$

$\tan x \cos x$

42. $\frac{1+\csc\theta}{\cot\theta + \cos\theta} = \sec\theta$

$\frac{1+\csc\theta}{\frac{\cos\theta}{\sin\theta} + \cos\theta}$

factor out cos

$\frac{1+\csc\theta}{\cos\theta\left(\frac{1}{\sin\theta} + 1\right)}$

$\frac{1}{\cos\theta} = \sec\theta$

48. $\frac{\tan\theta}{1+\sec\theta} + \frac{1+\sec\theta}{\sin\tan\theta} = 2\csc\theta$

$\frac{\tan\theta(\tan\theta)}{1+\sec\theta(\tan\theta)} + \frac{1+\sec\theta(1+\sec\theta)}{\tan\theta(1+\sec\theta)}$

$\frac{\tan^2\theta + 1 + 2\sec\theta + \sec^2\theta}{\tan\theta(1+\sec\theta)}$

$\tan^2\theta + 1 + \sec^2\theta + 2\sec\theta$

$\sec^2 x + \sec^2\theta + 2\sec\theta +$

$\frac{2\sec^2\theta + 2\sec\theta}{\tan\theta(1+\sec\theta)} = \frac{2\sec\theta(\sec\theta+1)}{\tan\theta(\sec\theta+1)}$

factor 2secθ

$\frac{2\sec\theta}{\tan\theta}$

$2\sec\theta \cdot \frac{\cos\theta}{\sin\theta}$

$2\frac{1}{\cos} \cdot \frac{\cos}{\sin}$

$2\frac{1}{\sin}$

$2\csc\theta$

66. $\dfrac{1}{\sec x+1} - \dfrac{1}{\sec x-1}$

$\dfrac{1(\sec x-1)}{\sec x+1(\sec x-1)} - \dfrac{1(\sec x+1)}{\sec x-1(\sec x+1)}$

$\dfrac{\sec x-1}{\sec^2 x+\sec x-\sec x-1} - \dfrac{\sec x+1}{\sec^2 x+\sec x-\sec x-1}$

$\dfrac{\sec x-1-(\sec x+1)}{\sec^2 x-1}$

$\dfrac{\sec x-1-\sec x-1}{\sec^2 x-1}$

$\dfrac{-2}{\sec^2 x-1}$

$\dfrac{-2}{\tan^2}$

$-2\cot x^2$

2 more steps

p 353/12. $\dfrac{\csc x-1}{1-\sin x} = \csc x$

$\dfrac{\csc x-1(1+\sin x)}{1-\sin x(1+\sin x)}$

$\dfrac{\frac{1}{\sin x}-1}{1-\sin x}$ convert

?? $\dfrac{\csc x+\csc x\cdot\sin x-1-\sin x}{1+\sin x-\sin x-\sin x^2}$

$\dfrac{\frac{1}{\sin}\cdot\sin x-1-\sin x}{-\sin x^2+1}$

$\dfrac{\frac{1-\sin x}{\sin x}}{1-\sin x}$

$\dfrac{1-1-\sin x+\csc x}{-\sin x^2+1}$

$\dfrac{\cancel{1-\sin x}}{\sin x}\cdot\dfrac{1}{\cancel{1-\sin x}}$

$\dfrac{-\sin x+\frac{1}{\sin x}}{-\sin x^2+1}$

$\dfrac{1}{\sin x}$

14. $\sec x - \cos x = \sin x \tan x$

$\frac{1}{\cos x} - \cos x = \frac{\sin x}{1} \cdot \frac{\sin}{\cos x}$

common denominator $\frac{\sin^2 x}{\cos x}$

$\frac{1}{\cos x} - \frac{\cos x \cdot \cos x}{\cos x}$

$\frac{1-\cos^2 x}{\cos x} \to \frac{\sin^2 x}{\cos x} \to \frac{\sin x \sin}{\cos}$

$\sin x \tan x$

354

22. $\sec^6 x(\sec x \tan x) - \sec^4 x(\sec x \tan x) = \sec^5 x \tan^3 x$

$\sec^5 x \tan x(\sec^2 x - 1)$

$\sec^5 x \tan x(\tan^2 x)$

$\sec x \tan^3 x$

30. $\frac{\tan x + \tan y}{1 - \tan x \tan y} = \frac{\cot x + \cot y}{\cot x \cot y - 1}$

expand

$\to \frac{\frac{1}{\cot x} + \frac{1}{\cot y}}{1 - \frac{1}{\cot x} \cdot \frac{1}{\cot y}}$

common denom

$\frac{\frac{1(\cot y)}{\cot x(\cot y)} + \frac{1(\cot x)}{\cot y(\cot x)}}{\frac{\cot x \cot y}{\cot x \cot y} - \frac{1}{\cot x \cot y}}$

$\cancel{\frac{\frac{\sin x}{\cos x} + \frac{\sin y}{\cos y}}{1 - \frac{\sin x}{\cos x} \frac{\sin y}{\cos y}} \cdot \cos x \cos y}$

$\cancel{\frac{\sin x \cos y + \sin y \cos x}{\cos x \cos y - \sin x \sin y}}$

xCecripical

$\frac{\cot y}{\cot x \cot y} + \frac{\cot x}{\cot x \cot y} \cdot \frac{\cot x \cot y}{\cot x \cot y - 1}$

$\frac{\cot y + \cot x}{\cot x \cot y - 1}$

52. $\frac{\cos x}{1-\tan x} + \frac{\sin x \cos x}{\sin x - \cos y} \rightarrow \frac{\cos x}{1-\frac{\sin x}{\cos x}} + \frac{\sin x \cos x}{\sin x - \cos y}$

common denominator

$\frac{\cos x(1+\tan x)}{(1-\tan x)(1+\tan x)}$

$\frac{\cos x}{\frac{\cos x - \sin x}{\cos x}} \rightarrow \frac{\cos x}{1} \cdot \frac{\cos x}{\cos x - \sin x}$

$\frac{\cos^2 x}{\cos x - \sin x}$

$\frac{\cos^2 x}{\cos x - \sin x} + \frac{\sin x \cos x}{\sin x - \cos y}$

take out $\ominus$

$\frac{\cos^2 x}{\cos x - \sin x} - \frac{\sin x \cos x}{-\sin x + \cos y}$

same

$\frac{\cos^2 x - \sin x \cos x}{\cos x - \sin x}$

$\frac{\cos x(\cancel{\cos x - \sin x})}{\cancel{\cos x - \sin x}} \rightarrow \cos x$

355

64. $\sec^4 x \tan^2 x = (\tan^2 x + \tan^4 x)\sec^2 x$

$\sec^2 x \sec^2 x \tan^2 x$

$\sec^2 x(1+\tan^2 x)\tan^2 x$

$\sec^2 x(\tan^2 x + \tan^4 x)$

364

12. $3\cot^2 x - 1 = 0$

$+1 \quad +1$

$3\cot^2 x = 1$

$\cot^2 x = \frac{1}{3}$

correct

$\cot x = \pm\sqrt{1/3}$

$\tan x = \pm\sqrt{3}$

$60° + 2n\pi$

$\frac{\pi}{3} + 2n\pi$

$\frac{2\pi}{3} + 2n\pi$

$\frac{4\pi}{3} + 2n\pi$

$\frac{5\pi}{3} + 2n\pi$

$\rightarrow \frac{\pi}{3} + n\pi$

$\frac{2\pi}{3} + n\pi$

16. $(3\tan^2 x - 1)(\tan^2 x - 3) = 0$ $\tan x = 3$

$3\tan^2 x = 1$

$\tan^2 x = \frac{1}{3}$

$\tan x = \pm\sqrt{1/3}$

$30°$

$\frac{\pi}{6} + 2n\pi$

$\frac{5\pi}{6} + 2n\pi$

$\frac{7\pi}{6} + 2n\pi$

$\frac{11\pi}{6} + 2n\pi$

$\frac{\pi}{6} + n\pi$

$\frac{5\pi}{6} + n\pi$

forgot

$\frac{\pi}{3} + 2n\pi$

$\frac{2\pi}{3}$ "

$\frac{4\pi}{3}$ "

$\frac{5\pi}{3}$ "

$\frac{\pi}{3} + n\pi$

$\frac{2\pi}{3} + n\pi$

22. $2\sin^2 x = 2 + \cos x$ → $2\sin^2 x - \cos x - 2 = 0$

$2(1-\cos^2 x) = 2 + \cos x$ or $2(1-\cos^2 x) - \cos x - 2 = 0$

$2 - 2\cos^2 x = 2 + \cos x$ $2 - 2\cos^2 x - \cos x - 2 = 0$

$-2 \quad +2\cos^2 x \quad -2 \quad +2\cos^2 x$ $-2\cos^2 x - \cos x = 0$

$\cos x + 2\cos^2 x$ same

~~factor~~ factor $\cos x$: $\cos x(2\cos x + 1) = 0$

$x = 0$

$x = -1/2$

$\cos^{-1}(0) = 90°$

$\cos^{-1}(-1/2) = 120°$

$240°$

unit circle error

$\cancel{\pi}$ $\frac{\pi}{2}, \frac{3\pi}{2}$

$\frac{2\pi}{3}$

$\frac{4\pi}{3}$

$2\cos x = -1$

$\cos x = -\frac{1}{2}$

24. $\sec x \csc x = 2\csc x$ $\boxed{\text{don't divide}}$

$\sec x \csc x - 2\csc x = 0$

$\csc x(\sec x - 2) = 0$

split out

$\csc x = 0$

$\frac{1}{\sin x} = 0$

no solution

$\sin x = \frac{1}{0}$

$\sec x = 2$

$\cos x = \frac{1}{2}$

$60°$

(+) 1st: $\frac{\pi}{3}$

(+) 4th: $\frac{5\pi}{3}$

34/ $\csc^2 x = 3\csc x + 4$

$\csc^2 x - 3\csc x - 4 = 0$

$(\csc x - 4)(\csc x + 1) = 0$

$\csc x = 4$ 1st .252r

$\sin x = \frac{1}{4}$ 2nd 2.88r

$\sin^{-1}$ $\sin^{-1}$

$\csc x = -1$

$\sin x = -1$

Θ 3rd 270° $\frac{3\pi}{2}$

Θ 4th 270°

365

40. $\sec 4x = 2$

$\cos 4x = \frac{1}{2}$

$\cos^{-1}$ $\cos^{-1}$

$\frac{60°}{4} + \frac{2\pi n}{4}$ $\qquad$ $\frac{300°}{4} + \frac{2\pi n}{4}$

$\frac{\pi}{12} + \frac{1}{2}\pi n$ write $\qquad$ $\frac{5\pi}{12} + \frac{1}{2}\pi n$

$\frac{\pi}{12} + \frac{n\pi}{2}$ $\qquad$ $\frac{5\pi}{12} + \frac{n\pi}{2}$

46. $\tan \frac{x}{3} = 1$

$\tan^{-1}$ $\tan^{-1}$ start w/ 1 for tan on diagonal

$45° + 1n\pi$ $\qquad$ $225° + 1\pi n$

$\times 3$ $\times 3$ $\qquad$ $\times 3$ $\times 3$

$\frac{3\pi}{4} + 3n\pi$ $\qquad$ $\frac{15\pi}{4} + 3n\pi$

50. $y = \sec^4\left(\frac{\pi x}{8}\right) - 4 \rightarrow 4 = \sec^4\left(\frac{\pi x}{8}\right)$

$(-2,0)$

$(2,0)$

$\frac{1}{4} = \cos^4\left(\frac{\pi x}{8}\right)$

$\sqrt[4]{\ }$ $\sqrt[4]{\ }$

$\pm\sqrt[4]{\frac{1}{4}} = \cos\left(\frac{\pi x}{8}\right)$

$\sqrt{2}\cdot\sqrt{2}\cdot\sqrt{2}\cdot\sqrt{2} = 4$ $\quad \pm\frac{\sqrt[4]{1}}{\sqrt[4]{4}} = \cos\frac{\pi x}{8}$

$\pm\frac{1}{\sqrt{2}} = \cos\frac{\pi x}{8}$

$\cos^{-1}$ $\cos^{-1}$

$\pm\frac{\pi}{4} = \frac{\pi x}{8}$

$\cdot 8$ $\cdot 8$

$\pm\frac{2\pi}{\pi} = \frac{\pi x}{\pi}$

$x = 2$

$x = -2$

60. $\cos^2 x - 2\cos x - 1 = 0 \quad [0, \pi]$

Rooten

$\frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$

$\frac{2 \pm \sqrt{8}}{2}$

correct

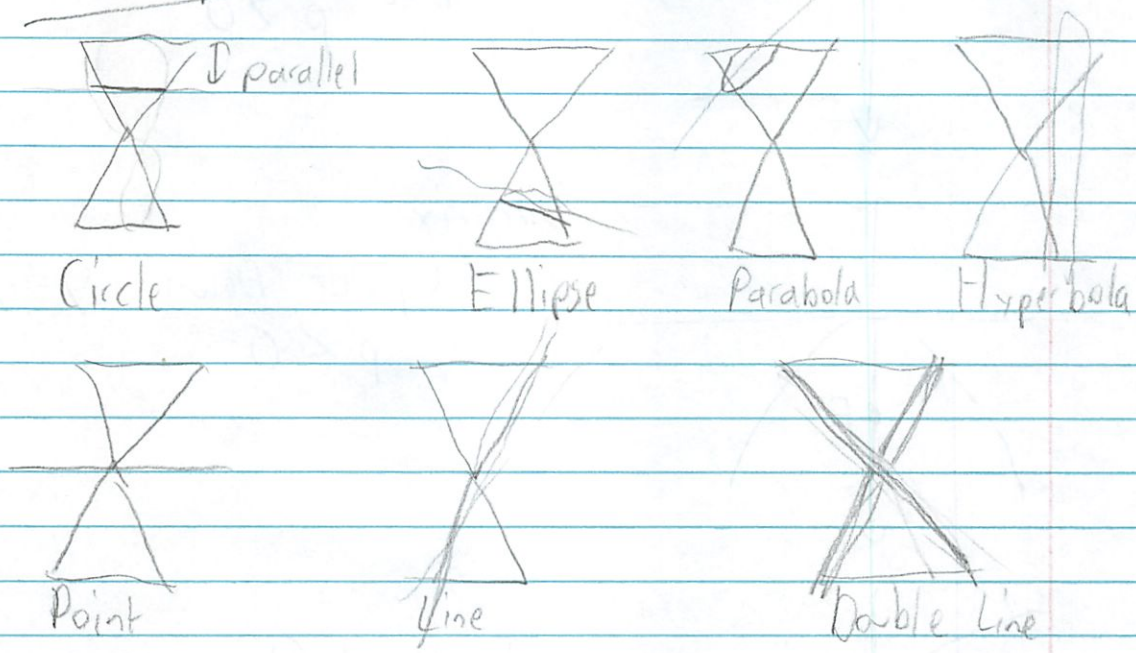
$2.414r \leftarrow 2.414r$

$-.414r$

9.1 Conics: Parabolas

1/2/08

Chart p632

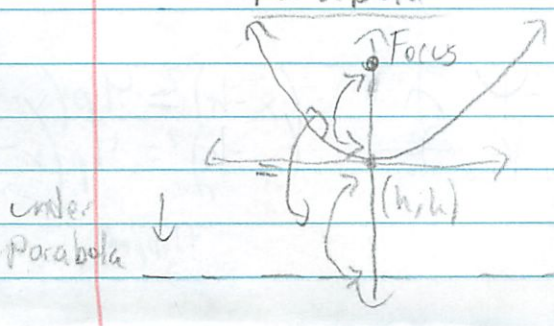


Can define conics:
- intersection of planes & cones
- algebraically in terms of 2nd degree equation
  $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$
- in terms of a locus (collection of points satisfying a certain geometric property)

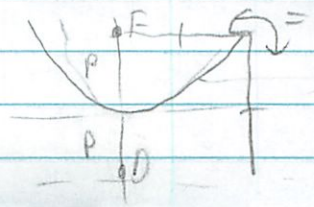
Circle - collection of all points that are equidistant from a fixed point (center $[h, k]$)

$$(x-h)^2 + (y-k)^2 = r^2$$
$$x^2 + y^2 = r^2$$

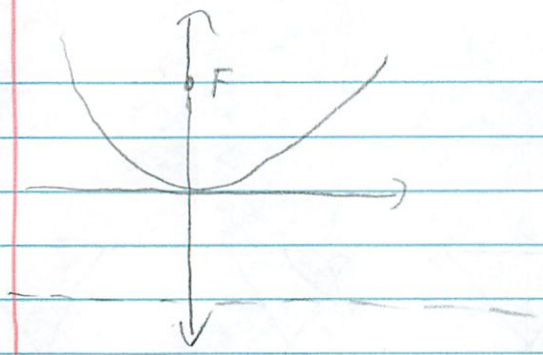
Parabola



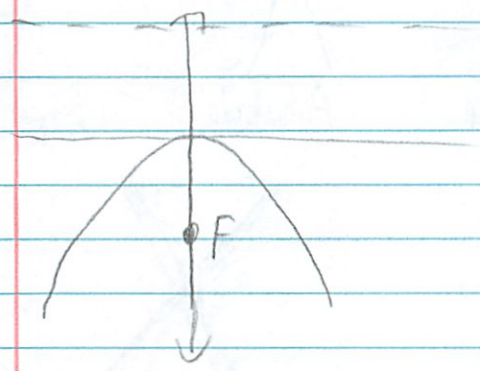
Distance from Vertex to Focus = Distance from Vertex to Directrix



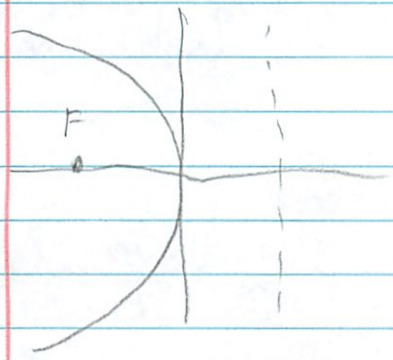
$|P|$ = Distance



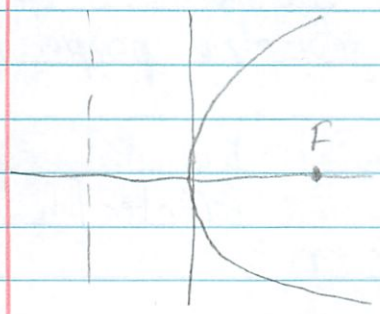
Upright $y = x^2$
$p > 0$



Upside Down $y = -x^2$
$p < 0$



To the left $x = -y^2$
$p < 0$



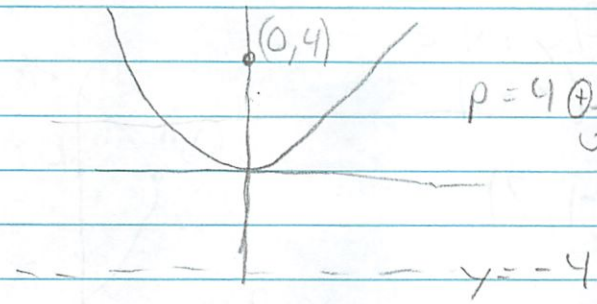
To the right $x = y^2$
$p > 0$

<u>Standard Equation</u>

$x^2 = 4py$ $\quad$ $(x-h)^2 = 4p(y-k)$
$y^2 = 4px$ $\quad$ $(y-k)^2 = 4p(x-h)$

flipped

ex 1 Find the standard form of the parabola with the vertex at the origin and focus is at $(0,4)$



$p = 4$ ⊕ ~~4~~

$x^2 = 4py$

$x^2 = 4(4)y$

$x^2 = 16y$

$\frac{1}{16}x^2 = y$

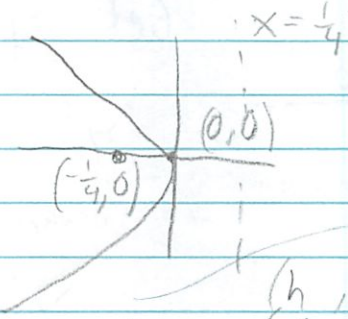
p 637

12. Find vertex, directrix, focus + sketch

$x + y^2 = 0$

$y^2 = -x$ → $p = \frac{-1}{4}$

Opens ⊖ ←



14. $(x-5) + (y+4)^2 = 0$

$(y+4)^2 = -(x-5)$

$(y-k)^2 = 4p(x-h)$

(h, k)

vertex = $(5, -4)$

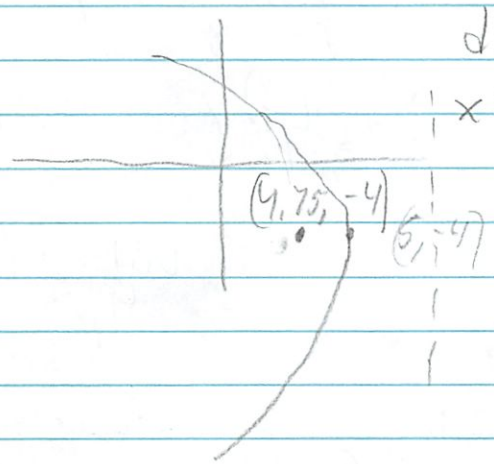
$p = -\frac{1}{4}$

focus $(-\frac{1}{4}, 0)$

← Opens

directrix → $x = \frac{1}{4} + 5$

original



16. $y^2 - 4y - 4x = 0$ ↓ Complete the square

$y^2 - 4y = 4x$

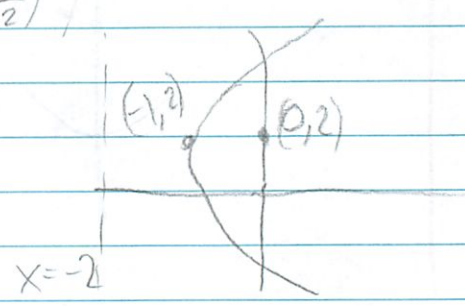
$y^2 - 4y + 4 = 4x + 4 \leftarrow \left(\frac{-4}{2}\right)^2$

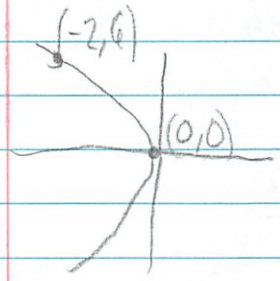
$(y-2)^2 = 4(x+1)$

vertex $= (-1, 2)$

$4p = 4$

$p = 1$



26. 

Get equation

$y^2 = 4px \rightarrow 6^2 = 4p(-2)$

↑ sideways

$36 = -8p$

$p = -4.5$

$y^2 = 4(-4.5)x$

$\boxed{y^2 = -18x}$

32. Directrix → $y = 3$

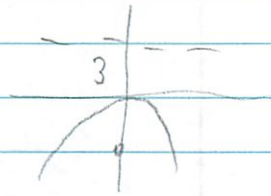
Vertex $(0,0)$

Focus $= (-3, 0)$

$p = -3$

$x^2 = 4(-3)y$

$x^2 = -12y$



ex2 Find focus

$y = -\frac{1}{2}x^2 - x + \frac{1}{2}$

$\quad +2 \quad +2 \quad +2 \quad +2$

$-2y = x^2 + 2x - 1$

$-2y + 1 = x^2 + 2x$ ↓ Complete the sq.

$x^2 + 2x + 1 = -2y + 1 + 1$

$(x+1)^2 = -2(y-1)$

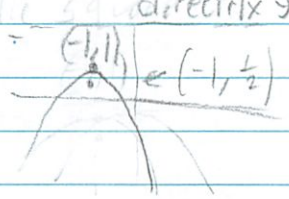
vertex $= (-1, 1)$

$4p = -2$

$p = -\frac{1}{2}$

focus $= (-1, \frac{1}{2})$

directrix → $y = \frac{3}{2}$



20. $x = \frac{1}{4}(y^2 + 2y + 33)$

$4x = y^2 + 2y + 33$

$4x - 33 = y^2 + 2y$

$y^2 + 2y + 1 = 4x - 33 + 1$

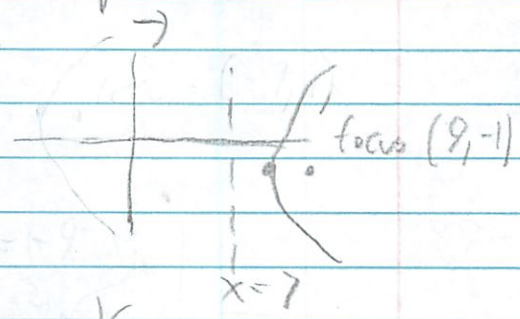
$(y+1)^2 = 4x - 32$

$(y+1)^2 = 4(x-8)$

vertex $= (8, -1)$

$4p = 4$

$p = 1$



Focus $= (h+p, k)$

Directrix $x = h - p$

sideways

22. $x^2 - 2x + 8y + 9 = 0$

$x^2 - 2x = -8y - 9$

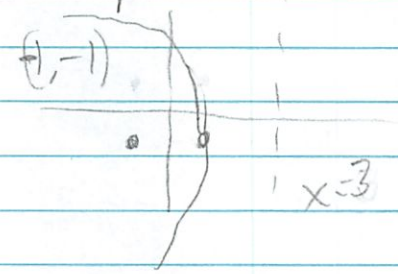
$x^2 - 2x + 1 = -8y - 9 + 1$

$(x-1)^2 = -8(y+1)$

vertex $= (1, -1)$

$4p = -8$

$p = -2$



ex3 Find standard form

vertex $(1,0)$

focus $(2,0)$

$p = 1$

$(y-0)^2 = 4(1)(y-1)$

$y^2 = 4(y-1)$

638

40. vertex $(3, -3)$

focus $(3, -\frac{9}{4})$

y changing

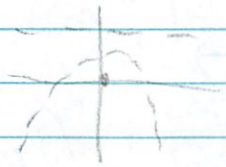
$(x-h)^2 = 4p(y-k)$

$(x-3)^2 = 4(\frac{3}{4})(y+3)$

$(x-3)^2 = 3(y+3)$

$p = -3 - \frac{9}{4} = +\frac{3}{4}$

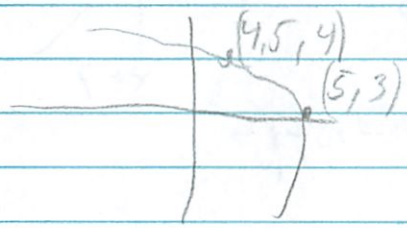
46. Focus $(0,0)$
directrix $\rightarrow y=4$



$p=-2$ (h,k)
vertex $=(2,0)$

$$(x-h)^2=4p(y-k)$$
$$(x-0)^2=4(-2)(y-2)$$
$$x^2=-8(y-2)$$

38.



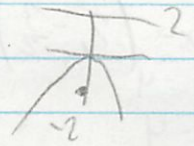
$$(y-k)^2=4p(x-h)$$
$$(4-3)^2=4p(4.5-5)$$
$$1^2=4p(-.5)$$
$$1=-2p$$
$$p=-\tfrac{1}{2}$$
$$(y-3)^2=4(-\tfrac{1}{2})(x-5)$$
$$(y-3)^2=-2(x-5)$$

9.1 Conics: Parabolas

HW

1/2/08

x^2 ~~#~~ 3. $x^2 = -8y$



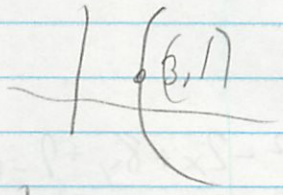
(d)

y^2 ~~7~~ ~~#~~ (d)

5. $(y-1)^2 = 4(x-3)$

(a)

vertex $= (3,1)$



7. $y = \frac{1}{2}x^2$

$\cdot 2 \quad \cdot 2$

$x^2 = 2y$

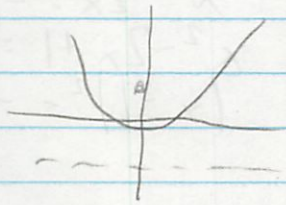
vertex $= (0,0)$

$4p = \cancel{1/2}\ 2$

$p = \cancel{1/8}\ \frac{1}{2}$

focus $(0, \frac{1}{2})$

directrix $x = -\frac{1}{2}$



10. $y^2 = 3x$

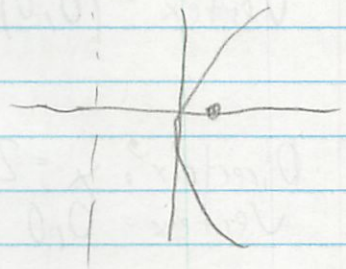
vertex $= 0,0$

$4p = 3$

$p = \frac{3}{4}$

focus $(\frac{3}{4}, 0)$

directrix $y = -3/4$



14. $(x-5) + (y+4)^2 = 0$

$(y+4)^2 + (x-5)$

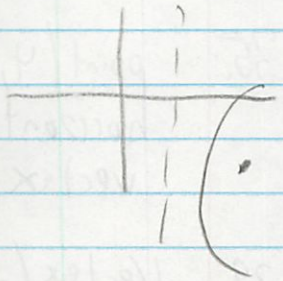
vertex $= (5,-4)$

$4p = 1$

$p = \frac{1}{4}$

directrix $= y = -4\frac{3}{4}$

focus $(5\frac{1}{4}, -4)$



15. $y^2 + 6y + 8x + 25 = 0$

$y^2 + 6y = -8x - 25$

$y^2 + 6y + 9 = -8x - 25 + 9$

$(y+3)^2 = -8x - 16$

$(y+3)^2 = -8(x-2)$

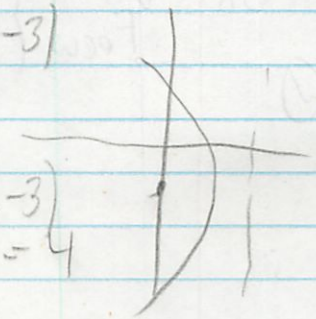
vertex $= (2,-3)$

$4p = -8$

$p = -2$

focus $= (0,-3)$

directrix $= y = 4$



17. $\left(x+\frac{3}{2}\right)^2 = 4(y-2)$

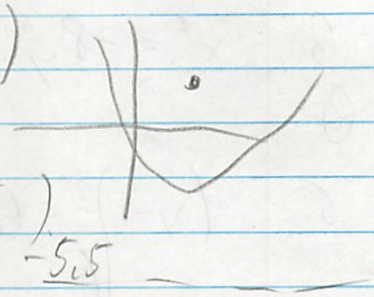
vertex $= \left(2, -\frac{3}{2}\right)$

$4p = 4$

$p = 4$

focus $= (2, 2.5)$

directrix: $y = -5.5$



22. $x^2 - 2x + 8y + 9 = 0$

$x^2 - 2x = -8y - 9$

$x^2 - 2x + 1 = -8y - 9 + 1$

$(x-1)^2 = -8(y+1)$

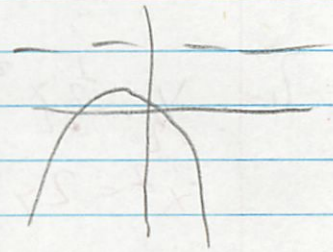
vertex $= (-1, 1)$

$4p = -8$

$p = -2$

focus $= (-1, -1)$

directrix: $y = 3$



29. Focus $= (-2, 0)$

Vertex $= (0, 0)$

$p = -2$

$y^2 = 4(-2)x$

$y^2 = -8x$

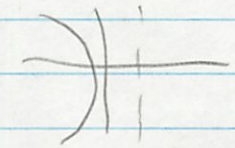
33. Directrix: $x = 2$

Vertex $= 0, 0$

$p = -2$

$y^2 = 4(-2)x$

$y^2 = -8x$



35. point $(4, 6)$

horizontal

vertex $(0, 0)$

mixed

$\cancel{y}^2 = 4p\cancel{x}$

$\cancel{6}^2 = 4p(\cancel{4})$

$36 \cancel{16} = \cancel{24}p\ 16$

$p = \cancel{\frac{3}{8}}\ 9/4$

$x^2 = 4\left(\cancel{\frac{3}{8}}\ 9/4\right)y$

$x^2 = \cancel{\frac{8}{3}}\ 9\ y$

638

39. Vertex $(-2, 0)$

Focus $(-3/2, 0)$

$(y-k)^2 = 4p(x-h)$

$(y-0)^2 = 4p(x+2)$

$y^2 = 4\left(\frac{1}{2}\right)(x+2$

$y^2 = 2(x+2)$

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41. Vertex $(5,2)$
Focus $(3,2)$

$(y-k)^2 = 4p(x-h)$
$(y-2)^2 = 4p(x-5)$
$(y-2)^2 = 4(-2)(x-5)$
$(y-2)^2 = -8(x-5)$

44. Vertex $(-2,1)$
directrix $x=1$

$(y-k)^2 = 4p(x-h)$
$(y-1)^2 = 4(-3)(x+2)$
$(y-1)^2 = -12(x+2)$

correct

= zero is in the middle

56. Road Design
32 ft wide
4 ft heigher in center
$(0,0)$ vertex

$(16,-.4)$

$x^2 = 4py$
$x^2 = 4(p)y$
$16^2 = 4(.4)(-2/5)$
$p = -160$
$-160 \quad -.1$

$\rightarrow -640y = x$

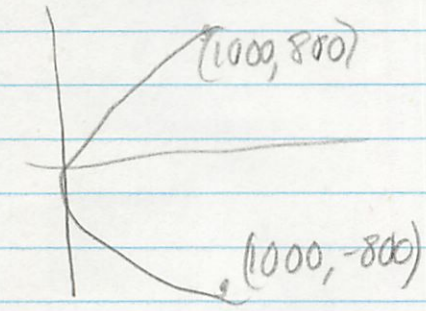
0,0 there

$(16,-.4)$

b) $(?,-.1)$ $\rightarrow$ $x^2 = 4(\cancel{.4})(.3)$
$x^2 = \cancel{.48ft}\ 64$
$x = \cancel{.69\ ft} = 8$ feet

639

57. Highway Design



$y^2 = 4px$
$800^2 = 4p(1000)$
$640{,}000 = 4000p$
$160 = p$

$y^2 = 4(160)x$
$y^2 = 640x$

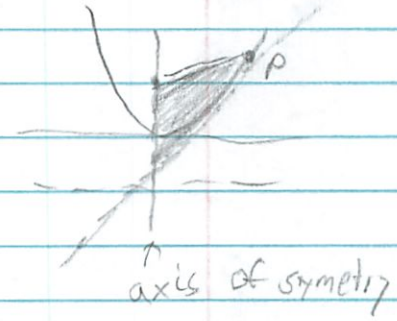
9.1 Conics: Parabolas

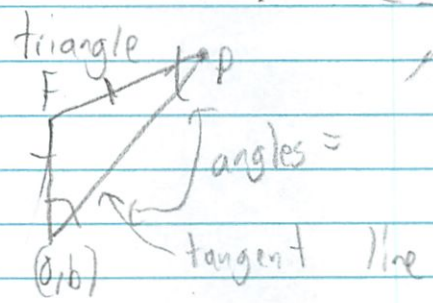
Day 2

1/3

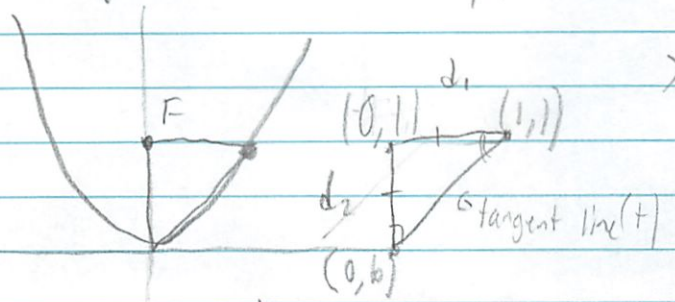
Tangent Lines to Parabolas
- have reflective properties related to the use of parabolas in constructing relitive surfaces

The tangent line to a parabola at point "p" makes equal angles with the following 2 lines:
- the line passing thru the point of tangency in the focus
- the axis of the parabola ←





ex1 Find equation of tangent line to parabola ($4y = x^2$) and point of tangency (1,1)



$x^2 = 4py$

$p = 1$

d_1/ $\sqrt{(0-1)^2 + (1-1)^2}$ d_2/ $d_2 = d_1 = 1$

$\sqrt{(-1)^2 (0)^2}$

$d_1 = 1$ b/ $1 - 1 = 0$

$b = 0$ (0,0)

t/

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{1-0}{1-0}$ slope = 1

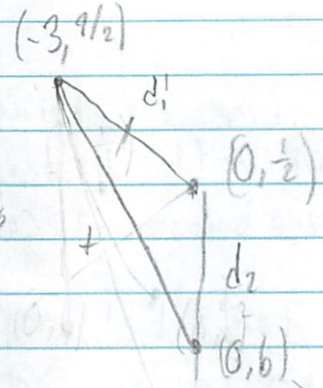
equation $= 1x + 0$

Q38

50. $x^2 = 2y \rightarrow 4p = 2$

$(-3, 9/2)$ tangent $\quad p = \frac{1}{2}$

$(0,0)$ vertex $\quad (0, \frac{1}{2})$ focus



d1/ $\sqrt{(-3-0)^2 + (9/2 - \frac{1}{2})^2}$

$d_1 = 5$

$d_2 = 5$

b/ $\frac{1}{2} - 5 = -4\frac{1}{2}$

$(0, -4\frac{1}{2})$

x/ $\frac{y_2 - y_1}{x_2 - x_1} \quad \frac{\frac{9}{2} - -9/2}{-3-0} = \frac{9}{-3} = -3$

$y = -3x - 4\frac{1}{2}$

9.1 Conics: Parabolas

HW Day 2

1/3

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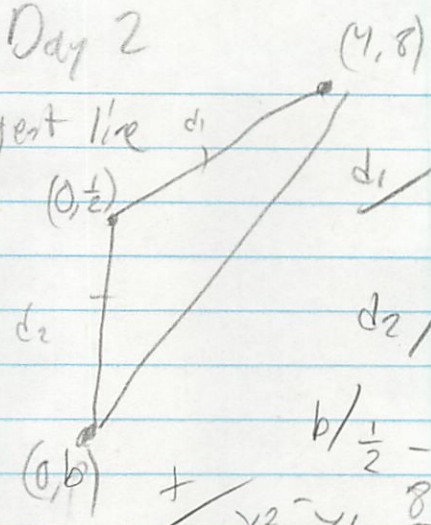
49. Find equation of tangent line

$x^2 = 2y \rightarrow 4p = 2$

$(4,8)$ tangent

$(0,0)$ vertex



d_1 / $\sqrt{(4-0)^2 + (8-\frac{1}{2})^2}$

$\sqrt{72.25}$

8.5

d_2 / $= 8.5$

b/ $\frac{1}{2} - 8.5 = -8$

+/ $\frac{y_2 - y_1}{x_2 - x_1}$ $\frac{8 - -8}{4-0} = \frac{16}{4} = 4$

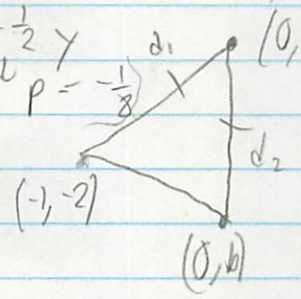
$y = 4x - 8$

51. $y = -2x^2 \rightarrow x^2 = -\frac{1}{2}y$

$p = -\frac{1}{8}$

$(-1,-2)$ tangent

$(0,0)$ vertex



d_1 / $\sqrt{(-1-0)^2 + (-2 - -\frac{1}{8})^2}$

$\sqrt{1 + 3.51}$

2.125

b/ $-\frac{1}{8} + 2.125 = \boxed{-4}$

add

+/ $\frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - -\frac{1}{8}}{-1-0} = \frac{-1.875}{-1}$

1.875

4

$y = 1.875 - 2.25$

$4x + 2$

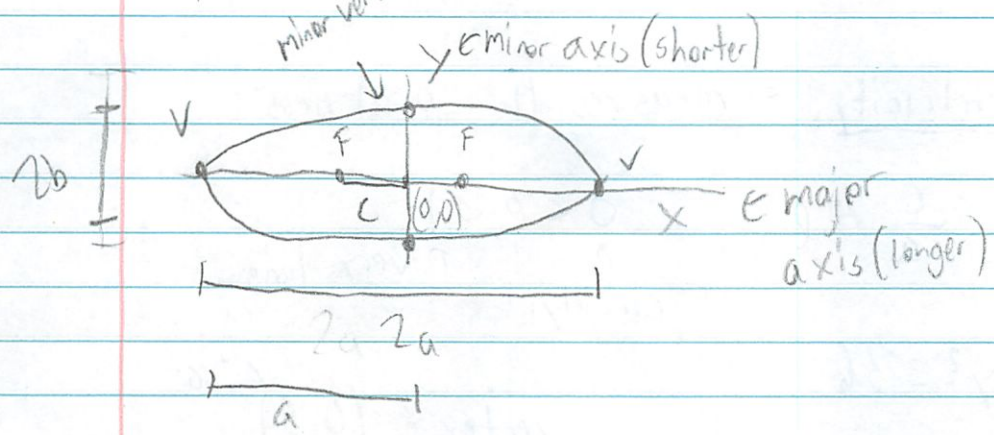
pressed up w/ b

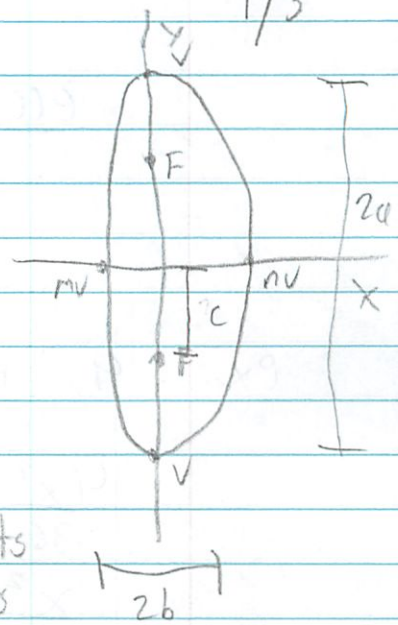
$(0,-2)$

9.2 Ellipses

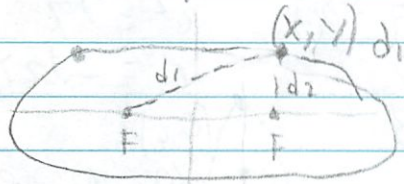
Notes

1/3

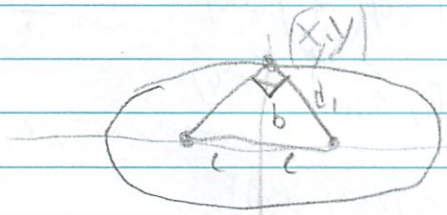




An ellipses is the set of all points in a plane, the sum of whoes distances from fixed points (foci) is constant



$d_1 + d_2 = \text{Constant} = 2a$



hypotenuse

$d_1 = \sqrt{b^2 + c^2} = 2a$

$a = \sqrt{b^2 + c^2}$

$a^2 = b^2 + c^2$

$\boxed{a^2 - b^2 = c^2}$

Standard Equations (0,0) center

biggest # y

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

x axis = major axis

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

y axis = major axis

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

eccentricity = measures the oval-ness

$\frac{c}{a} = e$ $\quad 0 < e < 1$

circle ↑ ↑ very long

ex $4x^2 + y^2 = 36$

$\frac{4x^2}{36} + \frac{y^2}{36} = 1$

$\frac{x^2}{9} + \frac{y^2}{36} = 1$

↑ bigger # $= a^2$

$a^2 = 36 \quad b^2 = 9$

$a = \pm 6 \quad b = \pm 3$

vertex $= (0,6)$ ← $\pm a$

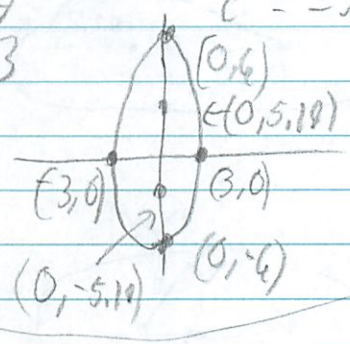
$(0,-6)$

$c^2 = a^2 - b^2$

$c^2 = 36 - 9$

$c^2 = 27$

$c = \pm\sqrt{27} = \pm 3\sqrt{3} = \pm 5.19$



eccentricity

$\frac{c}{a} = \frac{3\sqrt{3}}{6} = .577$

646

8. $\frac{x^2}{64} + \frac{y^2}{28} = 1$

↑a^2 ↑b^2

$a^2 = 64 \quad b^2 = 28$

$a = 8 \quad b = 5.29$

vertex $(8,0)$

$(-8,0)$

minor vertex $(0,5.29)$

$(0,-5.29)$

eccentricy $\frac{c}{a} = \frac{6}{8} = .75$

$c^2 = a^2 - b^2$

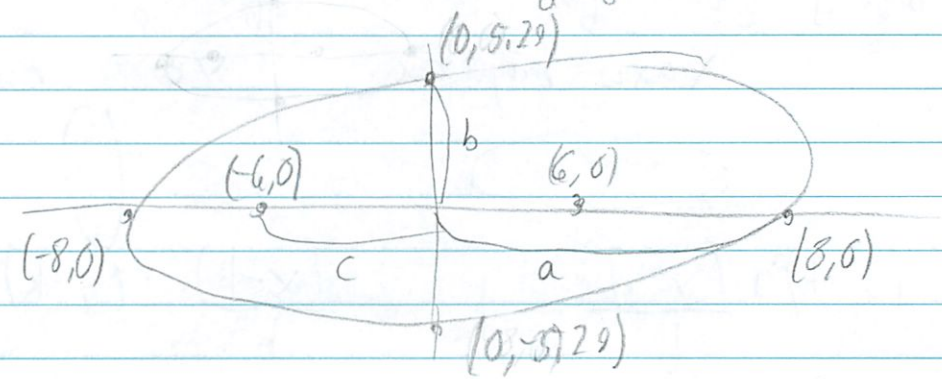
$64 - 28$

$c = \sqrt{36}$

$c = 6$

Focus $(6,0)$

$(-6,0)$



ex2 $4x^2 + y^2 - 8x + 4y - 8 = 0$

$4x^2 - 8x + y^2 + 4y = 8$ ← complete square twice

$4(x^2 - 2x + 1) + y^2 + 4y + 4 = 8 + 4 + 4$

$4(x-1)^2 + (y+2)^2 = 16$

$\frac{4(x-1)^2}{16} + \frac{(y+2)^2}{16} = 1$

$\frac{(x-1)^2}{4} + \frac{(y+2)^2}{16} = 1$

$\uparrow b^2 \quad \uparrow a^2$

major $(1, -2+4)$ $(1, 2)$
$(1, -2-4)$ $(1, -6)$

minor $(1+2, -2)$ $(3, -2)$
$(1-2, -2)$ $(-1, -2)$

$a^2 = 16 \quad b^2 = 4 \quad c^2 = a^2 - b^2$

$a = \pm 4 \quad b = \pm 2 \quad c^2 = 16 - 4$

$c = 2\sqrt{3} = \pm 3.46$

center at $(1, -2)$ Foci $(1, -2 + 2\sqrt{3})$
$(1, -2 - 2\sqrt{3})$

6.46

18. $x^2 + 4y^2 - 6x + 20y - 2 = 0$

$x^2 - 6x + 4y^2 + 20y = 2$

$(x^2 - 6x + 9) + (4y^2 + 20y + \underline{}) = 2 + 9$

$(x-3)^2 + 4(y^2 + 5y + 6.25) = 11 + 25$

$(x-3)^2 + 4(y + 2.5)^2 = 36$

$\frac{(x-3)^2}{36} + \frac{4(y+2.5)^2}{36} = 1$

$\frac{(x-3)^2}{36} + \frac{(y+2.5)^2}{9} = 1$

major $(3+6, -2.5)$ $(9, -2.5)$
$(3-6, -2.5)$ $(-3, -2.5)$

minor $(3, -2.5+3)$ $(3, .5)$
$(3, -2.5-3)$ $(3, -5.5)$

focus $(3 + 5.196, -2.5)$ $(8.19, -2.5)$
$(3 - 5.196, -2.5)$ $(2.19, -2.5)$

$a^2 = 36 \quad b^2 = 9$

$a = \pm 6 \quad b = \pm 3$

center $(3, -2.5)$

$c^2 = a^2 - b^2$

$c^2 = 36 - 9$

$c = \sqrt{27}, 5.196$

9.2 Ellipses HW

1/3

646

2. $\frac{x^2}{9} + \frac{y^2}{4} = 1$ $a^2 = 9$ major $(3,0)$ $(-3,0)$ (c)

$a = 3$

center $(0,0)$

5. $\frac{(x-2)^2}{16} + (y+1)^2 = 1$ (a)

center $(2,-1)$

6. $\frac{(x+2)^2}{9} + \frac{(y+2)^2}{4} = 1$ (e)

center $(-2,-2)$

8. $\frac{x^2}{64} + \frac{y^2}{28} = 1$ Did in school

9. $\frac{(x-4)^2}{16} + \frac{(y+1)^2}{25} = 1$

flipped a and b

$a^2 = 16$ $b^2 = 25$

$a = 4$ $b = 5$

$(4,-1)$ center

x^2

major $(4+4,-1)$ $(8,-1)$ $(4-4,-1)$ $(0,-1)$

minor $(4,-1+5)$ $(4,4)$ $(4,-1-5)$ $(4,-6)$

foci $(4+3.6,-1)$ $(7.6,-1)$ $(4-3.6,-1)$ $(.4,-1)$

$c^2 = a^2 - b^2$

$12 - 25$

$c = \sqrt{-13}$

± 3.60

eccentricity

$\frac{c}{a} = \frac{3.6}{4} = .9$

11. $\frac{(x+5)^2}{9/4} + \frac{(y-1)^2}{1} = 1$

$a^2 = 9/4$ $b = 1$

$a = 1.5$

center $(-5,1)$

(d)

major $(-5+1.5,1)$ $(-3.5,1)$ $(-5-1.5,1)$ $(-6.5,1)$

minor $(-5,1+1.5)$ $(-5,2.5)$ $(-5,1-1.5)$ $(-5,-.5)$

foci $(-5+1.12,1)$ $(-3.88,1)$ $(-5-1.12,1)$ $(-6.12,1)$

$c^2 = a^2 - b^2$

$9/4 - 1$

$\sqrt{1.25} = 1.12$

eccentricity

$\frac{c}{a} = \frac{1.12}{1.5} = .75$

15. $9x^2+4y^2+36x-24y+36=0$

$9x^2+36x+4y^2-24y=-36$

$9(x^2+4x+4)+4(x^2-6+9)=-36+36+36$

$9(x+2)^2+4(y-3)^2=36$

$\frac{9(x+2)^2}{36}+\frac{4(y-3)^2}{36}$

$\frac{(x+2)^2}{4}+\frac{(y-3)^2}{9}$

$b=2$ $a=3$

center $(-2,3)$

$c^2=a^2-b^2$

$9-4$

$c=\sqrt{5}$

2.23

major $(-2,3+3)$ $(-2,6)$

$(-2,3-3)$ $(-2,0)$

minor $(-2+2,3)$ $(0,3)$

$(-2-2,3)$ $(-4,3)$

foci $(-2,3+2.23)$ $(-2,5.23)$

$(-2,3-2.23)$ $(-2,.76)$

$\frac{c}{a}=\frac{2.23}{3}=.74$

19. $16x^2+25y^2-36x-50y+16=0$

$16x^2-36x+25y^2-50y=-16$

$16(x^2-2x+1)+25(y^2-2y+1)=-16+16+25$

$16(x-1)^2+25(y+1)^2=20$

$\frac{16(x-1)^2}{25}+\frac{25(y+1)^2}{25}$

$\frac{(x-1)^2}{(25/16)}+\frac{(y+1)^2}{1}=1$

$a=1.25$ $b=1$

center $(1,-1)$

$c^2=a^2-b^2$

$\frac{25}{16}-1$

$c=\sqrt{.5625}$

$c=.75$

major $(1+1.25,-1)$ $(2.25,-1)$

$(1-1.25,-1)$ $(-.25,-1)$

minor $(1,-1+1)$ $(1,0)$

$(1,-1-1)$ $(1,-2)$

foci $(1+.75,-1)$ $(1.75,-1)$

$(1-.75,-1)$ $(.25,-1)$

$\frac{c}{a}=\frac{.75}{1.25}=.6$

20. $9x^2+25y^2-36x-50y+61=0$

$9x^2-36x+25y^2-50y=-61$

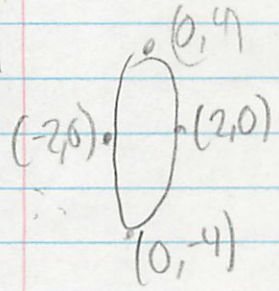
$9(x^2-4x+4)+25(y^2-2y+1)=-61+36+25$

$9(x-2)^2+25(y-1)^2=0$

degenerent conic

↑ that's all - just write

23.



$$\frac{x^2}{2^2}+\frac{y^2}{4^2}=\frac{x^2}{4}+\frac{y^2}{16}=1$$

9.2 Ellipses

Day 2

1/4

G47

Find the standard equation of an elipse

26. center $(0,0)$ major axis $\rightarrow \frac{x^2}{8^2} + \frac{x^2}{b^2} = 1$

vertex $(0, \pm 8)$ ← is y axis

foci $(0, \pm 4)$ ← c^2

a^2

$c^2 = a^2 - b^2$

$16 = 64 - b^2$

$-48 = -b^2$

$b^2 = 48$

$$\frac{y^2}{64} + \frac{x^2}{48} = 1$$

28. Foci $(\pm 2, 0)$ ← $c=2$ ← x axis

major axis lenght $= 6$ ← $2a$

$a = 3$

$$\frac{x^2}{3^2} + \frac{y^2}{b^2} = 1$$

$c^2 = a^2 - b^2$

$4 = 9 - b^2$

$b^2 = 5$

$b = \sqrt{5}$

$$\boxed{\frac{x^2}{9} + \frac{y^2}{5} = 1}$$

30. Center ≠ $(0,0)$ ← center $(2,1)$

Foci $(0,1)$ $(4,1)$ ← x axis, $c=2$

major axis lenght $= 6$ ← $2a$

$a = 3$

$c^2 = a^2 - b^2$

$4 = 9 - b^2$

$b^2 = 5$

$b = \sqrt{5}$

$$\frac{(x-2)^2}{9} + \frac{(y-1)^2}{5} = 1$$

34. Foci : $(0,0)$ $(4,0)$ ← center $(2,0)$

major axis lenght $= 8$ ← $2a$ x-axis

$a = 4$ $c = 2$

$c^2 = a^2 - b^2$

$4 = 16 - b^2$

$b^2 = 12$

$b = \sqrt{12}$

$$\frac{(x-2)^2}{16} + \frac{(y^2)}{12} = 1$$

36. center $(2,-1)$ ↓ y-axis

vertex $(2, -\frac{1}{2})$ $a = 1\frac{1}{2}$

minor axis $= 2$ ← $b = 1$

key

$$\frac{(y+1)^2}{9/4} + \frac{(x-2)^2}{1} = 1$$

← h

h always goes with x^2

proper way

$$\frac{(x-2)^2}{1} + \frac{4(y+1)^2}{9}$$

38. center $(3,2)$
$a = 3c$
foci $(1,2)(5,2)$ ← x axis
center $(3,2)$
$c = 2$

$a = 3(2)$
$a = 6$
$c^2 = a^2 - b^2$
$4 = 36 - b^2$
$b^2 = 32$
$b = \sqrt{32}$

$$\frac{(x-3)^2}{36} + \frac{(y-2)^2}{32} = 1$$

42. $e = \frac{1}{2}$ ← $\cancel{c = 1}$ $c = 4$
← $\cancel{a = 2}$ $a = 8$

vertex $(0, \pm 8)$ ← center $(0,0)$
& y axis
← $a = 8$

$c^2 = a^2 - b^2$
$16 = 64 - b^2$
$b^2 = 48$
$b = \sqrt{48}$

$$\frac{x^2}{48} + \frac{y^2}{256} = 1$$

9.2 Ellipses
HW Day 2

1/4

646

21. $12x^2+20y^2-12x+40y-37=0$

$12x^2-12x+20y^2+40y=37$

$12(x^2-x+\frac{1}{4})+20(y^2+2y+1)=37+3+20$

$12(x-\frac{1}{2})^2+20(y+1)^2=60$

$\frac{12(x-\frac{1}{2})^2}{60}+\frac{20(y+1)^2}{60}=1$

$\frac{(x-\frac{1}{2})^2}{5}+\frac{(y+1)^2}{3}=1$

$a^2=5$ $\quad b^2=3$

$a=\sqrt{5}$ $\quad b=\sqrt{3}$

center $(\frac{1}{2},-1)$

$c^2=a^2-b^2$

$c^2=5-3$

$c^2=2$

$c=\sqrt{2}$

major $(\frac{1}{2}+\sqrt{5},-1)$

$(\frac{1}{2}-\sqrt{5},-1)$

minor $(\frac{1}{2},-1+\sqrt{3})$

$(\frac{1}{2},-1-\sqrt{3})$

foci $(\frac{1}{2}+\sqrt{2},-1)$

$(\frac{1}{2}-\sqrt{2},-1)$

$e=\frac{c}{a}$

$\frac{\sqrt{2}}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}=\frac{\sqrt{10}}{5}$

647

25. Vertices $(\pm 3,0)$ ← $a=3$ ← x axis

Foci $(\pm 2,0)$ ← $c=2$

center $(0,0)$

$c^2=a^2-b^2$

$4=9-b^2$

$b=\sqrt{5}$

$\frac{x^2}{9}+\frac{y^2}{5}=1$

27. Foci $(\pm 4,0)$ ← $c=4$ ← x axis

major lenght $=10$ ← $a=5$

center $(0,0)$

$c^2=a^2-b^2$

$16=25-b^2$

$b^2=9$

$b=3$

$\frac{x^2}{25}+\frac{y^2}{9}=1$

29. Vertices $(0,\pm 5)$ ← $a=5$ ← y axis

passes through $(4,2)$

$\frac{x^2}{b^2}+\frac{y^2}{25}=1$

$\frac{4^2}{b^2}+\frac{2^2}{25}=1$

$-4/25 \quad -\frac{4}{25}$

$\frac{16}{b^2}=\frac{21}{25}$

$400=21b^2$

$400/21 \rightarrow 19=b^2$

4.36.

$\frac{x^2}{400/21}+\frac{y^2}{25}=1$

proper way

$\frac{21x^2}{400}+\frac{y^2}{25}=1$

G47

Find the equation

33. vertices $(0,2)(4,2)$ ← center $(2,2)$ ← x axis $a=2$
minor axis lenght $=2$ ← $b=1$

$$\frac{(x-2)^2}{4}+\frac{(y-2)^2}{2}=1$$

36. center $(2,-1)$
vertices $(2,\frac{1}{2})$
minor axis lenght $=2$ ← $b=1$

did in class

39. center $(0,4)$
$a=2c$ $\quad 4=2c \quad c=2$
vertices $(-4,4)(4,4)$
↑ x axis $a=4$

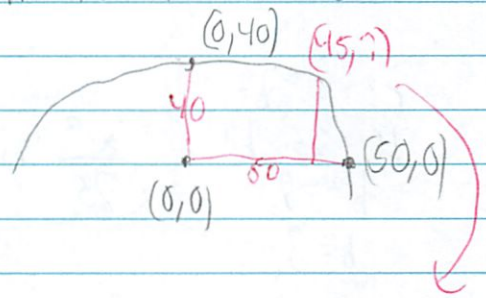
$c^2=a^2-b^2$
$4=16-b^2$
$b=\sqrt{12}$

$$\frac{x^2}{16}+\frac{(y-4)^2}{12}=1$$

43. Architecture ↓ x-axis
major axis $=100$ ft ← $a=50$
height $=40$ ft ← $b=40$

$$\frac{x^2}{50^2}+\frac{y^2}{40^2}=1$$

$$\frac{x^2}{2500}+\frac{y^2}{1600}=1$$

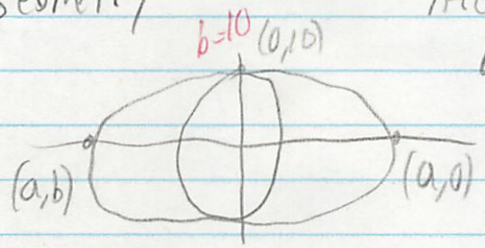


c) $(45,y) \rightarrow \frac{45^2}{2500}+\frac{y^2}{1600}=1$

$\sqrt{304}$

17,4 ft

647/45. Geometry



Area of elipse = 2 × area of circle

Area of elipse = πab

$\pi(10)^2$ ← wrong formula for area

area circle = ~~$2\pi(10)$~~

~~62.83~~ 100π

area elipse = ~~125.66~~ 200π

$200\pi = \pi a(10)$

$200 = 10a$

$20 = a$

lenght major axis = ~~8~~ 40

46. Astronomy - Halley's Comet

elliptical orbit

sun = focus

eccentricity = .97 ← $\frac{c}{a}$

lenght major axis = 35.88 AU = 2a

1 AU = 93 million miles

Equation?

x axis

(0,0) center

c/ $\frac{.97}{1} = \frac{c}{17.94}$

a = 17.94

c = 17.40

b/ $c^2 = a^2 - b^2$ 321.84 ← wrong #

$302.828 = \cancel{287.3744} - b^2$

$b = \sqrt{\cancel{284.55}\ 19.01}$

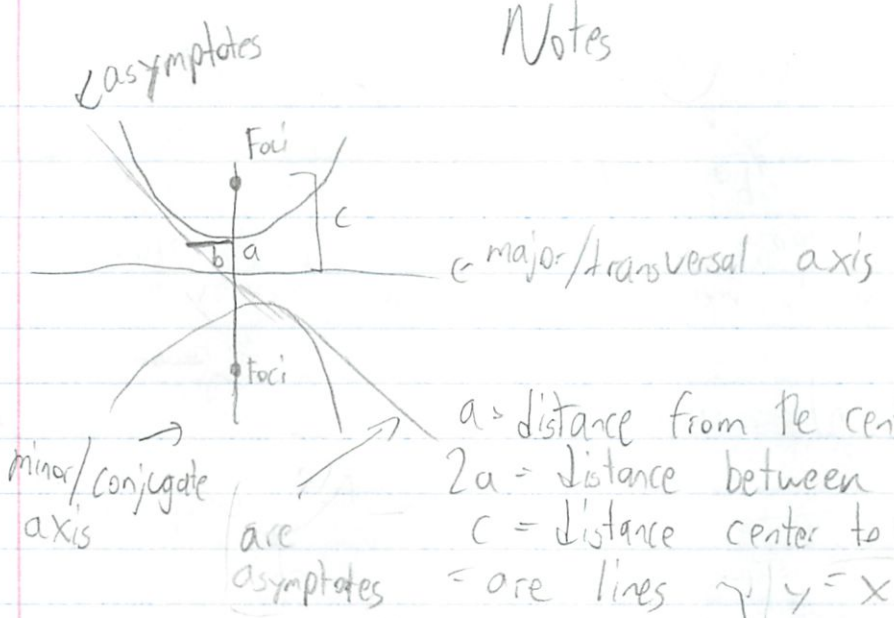
$b = \cancel{31.377}\ 4.36$

$$\frac{x^2}{\cancel{287.37}\ 321.84} + \frac{y^2}{\cancel{284.55}\ 19.01} = 1$$

9.3 Hyperbolas

Notes

1/7



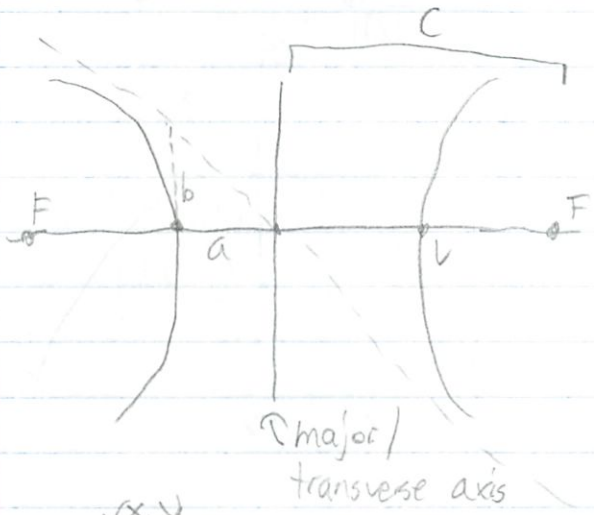
a = distance from the center to the vertex

2a = distance between vertices

c = distance center to focus

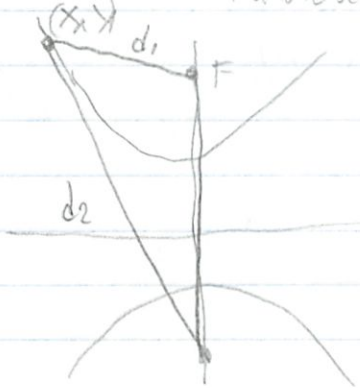
= are lines $\boxed{\begin{array}{l} y = x \\ y = -x \end{array}}$ $\frac{\text{rise}}{\text{run}} x$

$\pm \frac{a}{b} x$



asymptotes: $y = \pm \frac{b}{a} x$

$y - k = \pm \frac{b}{a}(x - h)$



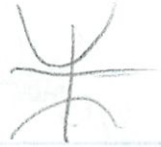
$\boxed{d_2 - d_1 = 2a}$

The set of all points (x,y) in a plane the differences of whose 2 distinct fixed points (foci) is constant = 2a

$c > a$ then $c^2 > a^2$ → $\boxed{c^2 = a^2 + b^2}$

ellipse: $c^2 < a^2 \rightarrow c^2 = a^2 - b^2$

$\frac{b}{a}$



$\frac{a}{b}$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{or} \quad \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

a^2 is always positive (+) so x is major axis

a^2 is always (+) so y is major axis

note

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \qquad \frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2}$$

656

8. $\frac{y^2}{9} - \frac{x^2}{1} = 1$

$a^2 = 9 \quad b^2 = 1$

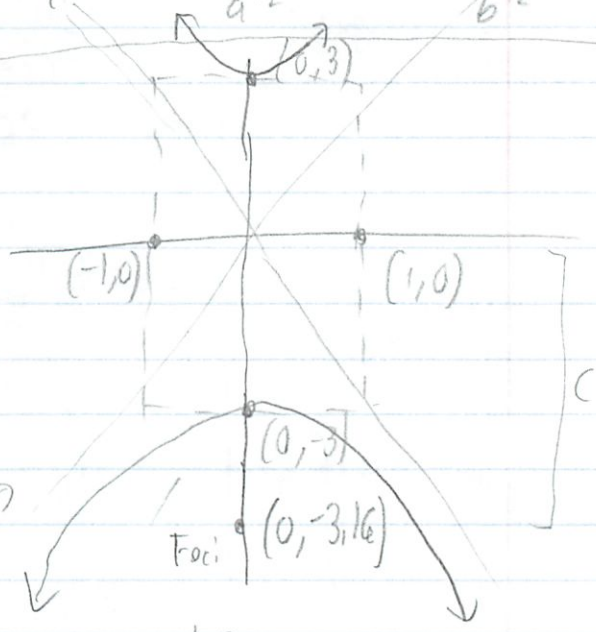
$a = \pm 3 \quad b = \pm 1$

$c^2 = a^2 + b^2$

$c = \pm\sqrt{10}$

$c = \pm 3.16$

$y = \pm \frac{a}{b}x = \pm\frac{3}{1}x = y = \pm 3x$



14. $\frac{(y-1)^2}{1/4} - \frac{(x+3)^2}{1/16} = 1$ ← center $(-3,1)$

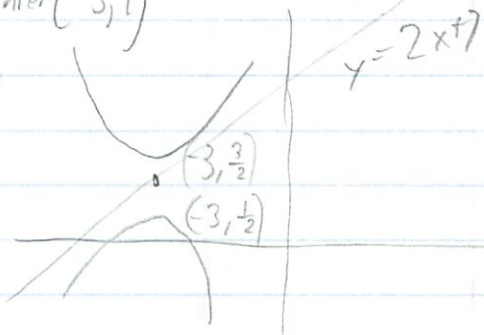
$a^2 = \frac{1}{4} \quad b^2 = \frac{1}{16}$

$a = \pm 1/2 \quad b = \pm \frac{1}{4}$

$c^2 = a^2 + b^2$

$c = \sqrt{5/16} \to \frac{\sqrt{5}}{\sqrt{16}} \to \frac{\sqrt{5}}{4}$

$c = \pm .559$



$y = \pm 2x \pm 6 + 1$

$\boxed{y = 2x+7 \quad y = -2x-5}$

vertices $(-3, 1 \pm \frac{1}{2})$

foci $(-3, 1 \pm \frac{\sqrt{5}}{4})$

asymptote $y - 1 = \pm \frac{a}{b}x + 3$

$y - 1 = \pm 2(x+3)$

18. $6y^2 - 3x^2 = 18$

$\frac{6y^2}{18} - \frac{3x^2}{18} = 1$

$\frac{y^2}{3} - \frac{x^2}{6} = 1$

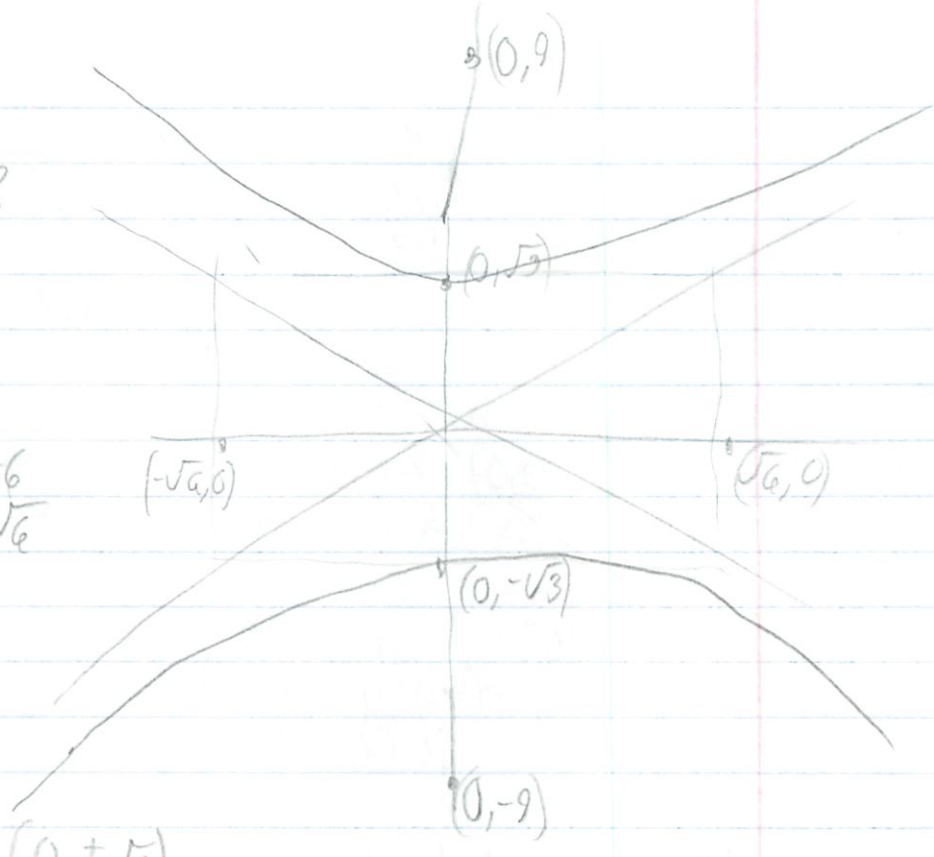
$a^2 = 3 \quad b^2 = 6$

$a = \sqrt{3} \quad b = \sqrt{6}$

$c^2 = a^2 + b^2$

$c = \sqrt{9}$

$c = 9$



vertecies $(0, \pm\sqrt{3})$

foci $(0, \pm 9)$

$y = \pm\frac{a}{b}x = \pm\frac{\sqrt{3}}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{18}}{6} = \frac{3\sqrt{2}}{6} =$ $\boxed{\frac{\sqrt{2}}{2}}$

20. $x^2 - 9y^2 + 36y - 72 = 0$

$x^2 - 9(y^2 - 4y + 4) = 72 - 36$

$x^2 - 9(y - 2)^2 = 36$

$\frac{x^2}{36} - \frac{(y-2)^2}{4} = 1$

$a^2 = 36 \quad b^2 = 4$

$a = \pm 6 \quad b = \pm 2$

$c^2 = a^2 + b^2$

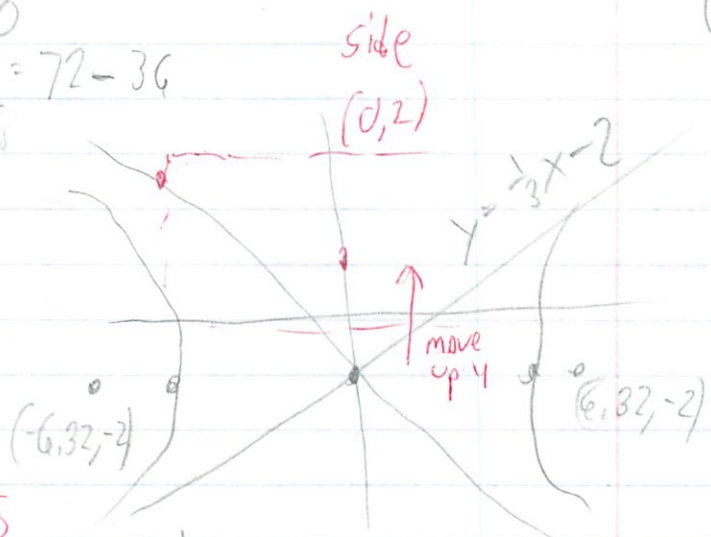
$c = \sqrt{40} \Rightarrow 2\sqrt{10}$

$c = \pm 6.32$

center $(0, 2)$

vertices $(\pm 6, 2)$

foci $(\pm 6.32, 2)$



$y - 2 = \pm\frac{b}{a}x$

$y - 2 = \pm\frac{2}{6}x$

$y = \pm 1/3x + 2$

$y = \frac{1}{3}x + 2$

$y = -\frac{1}{3}x + 2$

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28. verticies $(0, \pm 3)$ ← y axis $\frac{U}{n}$ ← $a=3$

asymptotes $y = \pm 3 \rightarrow \frac{a}{b} = \frac{3}{1}$

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$$\frac{y^2}{9} - \frac{x^2}{1} = 1$$

30. Foci $(\pm 10, 0)$ ← x axis)(← $c=10$

asymptotes $y = \pm \frac{3}{4}x$ ← $\pm \frac{b}{a}$ ← $b=3$ ← $a=4$

try

$10^2 = 3^2 + 4^2$ ⊗

$10^2 = (3 \cdot 2)^2 + (4 \cdot 2)^2$ ↓ ·2

$100 = 36 + 64$ ✓

$$\frac{x^2}{64} + \frac{y^2}{36} = 1$$

↑ a^2

657

36. Verticies $(2,1)$ $(-2,1)$ ← $a=2$ ← x axis)(← center $(0,1)$

Passes through $(5,4)$

$$\frac{x^2}{4} - \frac{(y-1)^2}{b^2} = 1$$

$$\frac{5^2}{4} - \frac{(4-1)^2}{b^2} = 1$$

$$\frac{25}{4} - \frac{9}{b^2} = 1$$

$$\frac{-21}{4} = \frac{-9}{b^2}$$

$$-36 = -21b^2$$

$$b^2 = \sqrt{36/21}$$

$$b = 1.30$$

$$\frac{x^2}{4} - \frac{(y-1)^2}{36/21} = 1$$

$$\frac{x^2}{4} - \frac{21(y-1)^2}{36} = 1$$

$$\frac{x^2}{4} - \frac{7(y-1)^2}{12} = 1$$

6.3 Hyperbole HW

1/7

657 9. Find stuff

$\frac{y^2}{25} - \frac{x^2}{81} = 1$

$a^2 = 25 \quad b^2 = 81$

$a = 5 \quad b = 9$

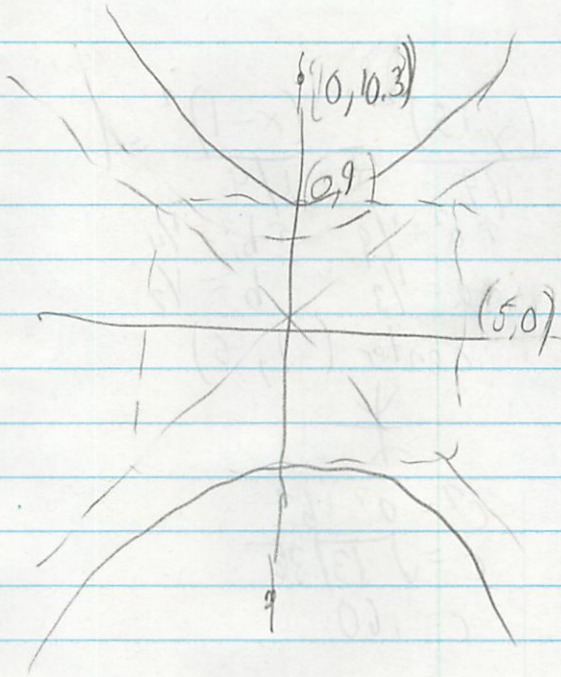
center $(0,0)$

$c^2 = a^2 + b^2$

$c^2 = 25 + 81$

$c = \sqrt{106}$

$c = 10.3$



$y = \pm \frac{a}{b} x = \pm \frac{5}{9} x \qquad y = 5/9\, x \qquad y = -5/9\, x$

vertices $(0, \pm 5)$

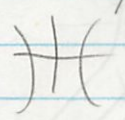
foci $(0, \pm \sqrt{106})$

11. $\frac{(x-1)^2}{4} - \frac{(y+2)^2}{1} = 1$

$a^2 = 4 \quad b^2 = 1$

$a = 2 \quad b = 1$

center $(1, -2)$

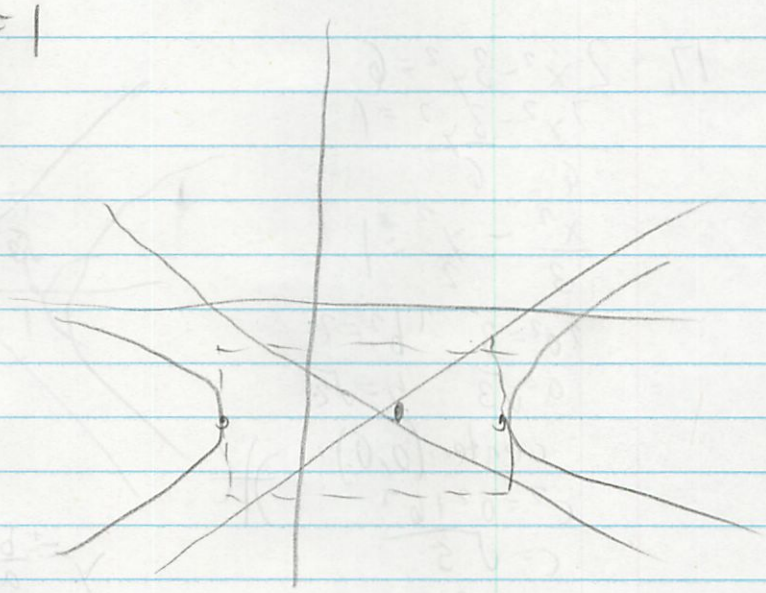


$c^2 = a^2 + b^2$

$c^2 = 4 + 1$

$c = \sqrt{5}$

$c = 2.23$



$y = \pm \frac{b}{a} x$

$y + 2 = \pm \frac{1}{2}(x - 1)$

$y = \pm 1/2 (x-1) - 2$

vertices $(1 \pm 2, -2)$

foci $(1 \pm \sqrt{5}, -2)$

13. $\frac{(y+5)^2}{1/9} - \frac{(x-1)^2}{1/4} = 1$

$a^2 = 1/9 \quad b^2 = 1/4$

$a = 1/3 \quad b = 1/2$

center $(1, -5)$

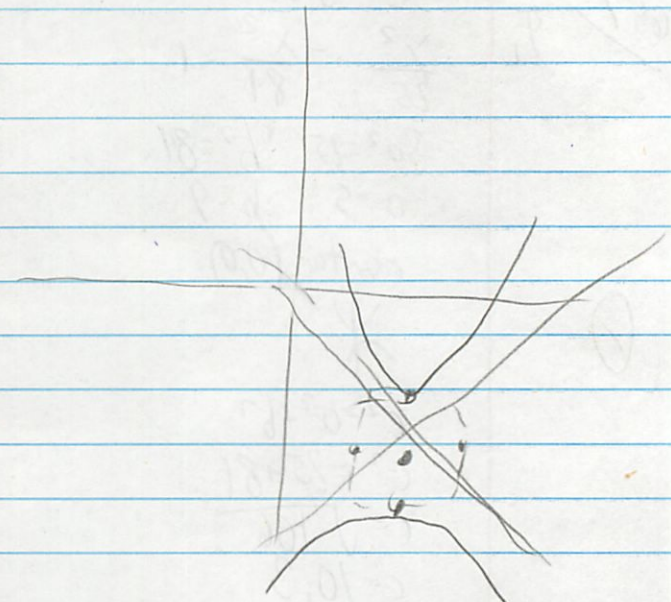
$c^2 = a^2 + b^2$

$c = \sqrt{13/36}$

$c = .60$

verticies $(1, -5 \pm 1/3)$

foci $(1, -5 \pm \sqrt{13/36})$



$y = \frac{a}{b}x \quad y + 5 = \pm\frac{1/3}{1/2}x - 1$

$y + 5 = \pm 2/3(x-1)$

$y = -5 \pm 2/3(x-1)$

17. $2x^2 - 3y^2 = 6$

$\frac{2x^2}{6} - \frac{3y^2}{6} = 1$

$\frac{x^2}{3} - \frac{y^2}{2} = 1$

$a^2 = 3 \quad b^2 = 2$

$a = \sqrt{3} \quad b = \sqrt{2}$

center $(0,0)$

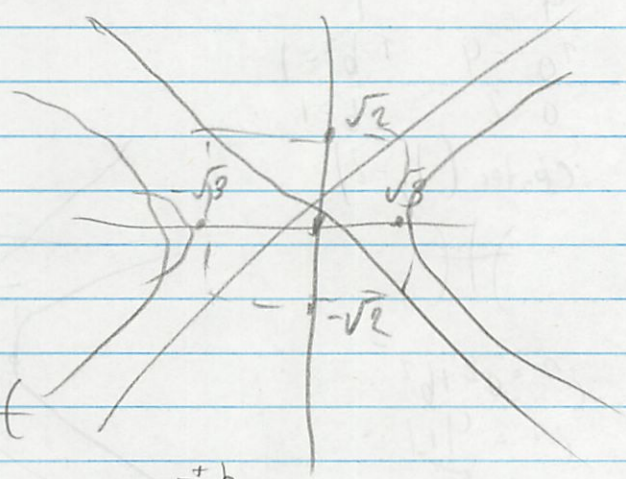
$c^2 = a^2 + b^2$

$c = \sqrt{5}$

$c = 2.24$

vertici $(\pm\sqrt{3}, 0)$

foci $(\pm\sqrt{5}, 0)$



$y = \pm\frac{b}{a}x$

$y = \pm\frac{\sqrt{2}}{\sqrt{3}}x$, $y = \frac{\pm\sqrt{6}}{3}$

19. $9x^2 - y^2 - 36x - 6y + 18$

$9x^2 - 36x - y^2 - 6y = -18$

$9(x^2 - 4x + 4) - (y^2 - 6y + 9) = -18 + 36 - 9$

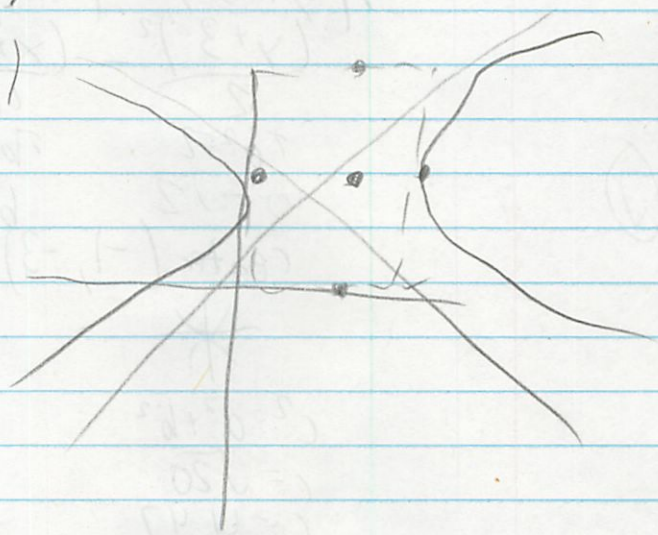
$9(x-2)^2 - (y-3)^2 = 9$

$\frac{(x-2)^2}{1} - \frac{(y-3)^2}{9} = 1$

$a^2 = 1$ $\quad$ $b^2 = 9$

$a = 1$ $\quad$ $b = 3$

center $(2, 3)$



$c^2 = a^2 + b^2$

$c = \sqrt{10}$

$c = 3.16$

vertici $(2 \pm 1, 3)$

foci $(2 \pm 3.16, 3)$

$y - 3 = \pm \frac{3}{1}(x+1)$

$y = 3 \pm 3(x+1)$

22. $16y^2 - x^2 + 2x + 64y + 63 = 0$

$16y^2 + 64y - x^2 + 2x = -63$

$16(y^2 + 4y + 4) - (x^2 - 2x + 1) = -63 + 64 - 1$

$16(y+2)^2 - (x-1)^2 = 0$

degenerent conic

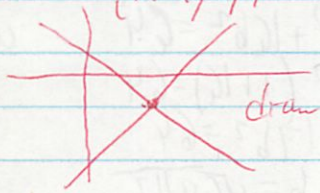
↓ can go furter → degenerent hyperbola

pair of intersecting lines

$(y+2)^2 = (x-1)^2 \cdot \frac{1}{16}$ ← set = to y

$y + 2 = \pm \sqrt{(x-1)^2 \cdot \frac{1}{16}}$

$y + 2 = \pm (x-1) \cdot 1/4$



draw like that

23. $9y^2 - x^2 + 2x + 54y + 62 = 0$

$9y^2 + 54y - x^2 + 2x = -62$

$9(y^2 + 6y + 9) - (x^2 + 2x + 1) = -62 + 81 - 1$

$9(y+3)^2 - (x+1)^2 = 18$

$\frac{(y+3)^2}{2} - \frac{(x+1)^2}{18} = 1$

$a^2 = 2 \qquad b^2 = 18$

$a = \sqrt{2} \qquad b = \sqrt{18}$

center $(-1, -3)$

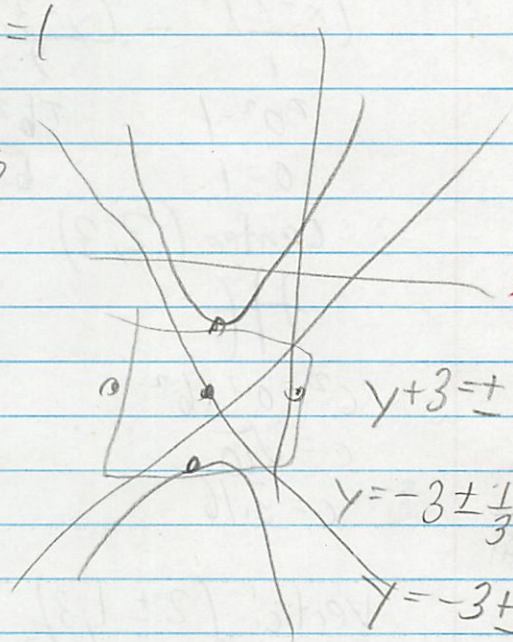
$c^2 = a^2 + b^2$

$c = \sqrt{20}$

$c \approx 4.47$

vertici $(-1, -3 \pm \sqrt{2})$

foci $(-1, -3 \pm \sqrt{20})$



$y + 3 = \pm \frac{\sqrt{2}}{\sqrt{18}} x + 1$

$y = -3 \pm \frac{1}{3}(x+1)$

$y = -3 \pm \frac{1}{3}(x+1)$

27. Vertic $(\pm 1, 0)$ ← $a = 1$ ← y axis

asymptote $y = \pm 5x$

$\frac{b}{a} \quad \frac{b}{1} = 5 \quad b = 5$

$\frac{y^2}{1^2} + \frac{x^2}{5^2} = 1$

$\frac{y^2}{1} + \frac{x^2}{25} = 1$

29. Foci $(0, \pm 8)$ ← $c = 8$, x axis

asymptotes → $y = \pm 4x$

center $(0,0)$ $\quad \frac{a}{b} = 4$

$a = 4b$

$a^2 = 16b^2$

$b^2 = c^2 - a^2$

$b^2 = 8^2 - 16b^2$

$b^2 + 16b^2 = 64$

$b^2(1 + 16) = 64$

$17b^2 = 64$

$b = \sqrt{64/17}$

$a^2 = 16(\sqrt{64/17})^2$

$16 \cdot (64/17)$

$a^2 = \frac{1024}{17}$

$\frac{x^2}{\frac{1024}{17}} + \frac{y^2}{\frac{64}{17}} = 1 \rightarrow \frac{17x^2}{1024} + \frac{17y^2}{64} = 1$

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31. Vertices $(2,0)(6,0)$ $\leftarrow a=2$ $\leftarrow$ center $(4,0)$

foci $(0,0)(8,0)$ $\leftarrow c=4$

x axis

$c^2=a^2+b^2$

$16=4+b^2$

$b=\sqrt{12}$

$$\frac{(x^2-4)}{4}+\frac{x^2}{12}=1$$

35. Verticies $(2,3)(2,-3)$ $\leftarrow (2,0)$ center

passes through pt $(0,5)$ $a=3$

y axis

$$\frac{y^2}{9}-\frac{(0-2)^2}{b^2}=1$$

$$\frac{25}{9}-\frac{(-2)^2}{b^2}=1$$

$$-\frac{16}{9}=-\frac{4}{b^2}$$

$$16\cdot 4=16b^2$$

$$b^2=9/4$$

$$b=3/2$$

$$\frac{y^2}{9}-\frac{(x-2)^2}{9/4}=1$$

$$\frac{y^2}{9}-\frac{4(x-2)^2}{9}=1$$

37. Verticies $(0,4)(0,0)$ $\leftarrow a=2$

point $(\sqrt{5},-1)$

y axis

center $(0,2)$

$$\frac{(y^2-2)}{4}-\frac{x^2}{b^2}=1$$

$$-\frac{1}{4}-\frac{5}{b^2}=1$$

$$\frac{5}{4}=-\frac{5}{b^2}$$

$$-20=5b^2$$

$$b=\sqrt{4}$$

$$b=2$$

$$\frac{(y^2-2)}{4}-\frac{x^2}{4}=1$$

39. Verticies $(1,2)(3,2)$ $\leftarrow$ center $(2,2)$

asymptote $y=x$ $a=1$

$y=4-x$ x axis

a point $(1,1)$

$$\frac{(y^2-2)}{1}-\frac{(x^2-2)}{b^2}=1$$

$$\frac{-1}{1}-\frac{-1}{b^2}=1$$

$+1$ $+1$

$$=\frac{1}{b^2}$$

$$2b^2=1$$

$$b=1$$

don't do

also now $\frac{(y^2-2)}{1}-\frac{(x^2-2)}{1}=1$

$\frac{b}{a}=\frac{b}{1}=1$

$b=1$

41. Vertices $(0,2)(6,2)$ ← Center $(3,2)$

$a = 3$

asy $= y = 2/3x$

x axis ⊥

$y = 4 - 2/3x$

point $(6,4)$

$$\frac{(x-3)^2}{9} - \frac{(y-2)^2}{b^2} = 1$$

$$\frac{(6-3)^2}{9} - \frac{(4-2)^2}{b^2} = 1$$

$$\frac{9}{9} - \frac{4}{b^2} = 1$$

$$-1 \qquad -1$$

$$-\frac{4}{b^2} = -\frac{1}{1}$$

$$-b^2 = -4$$

$$b = \sqrt{4}$$

$$b = 2$$

(1)

$$\frac{(x-3)^2}{9} - \frac{(y-2)^2}{4} = 1$$

9.3 Hyperbolas

More Notes

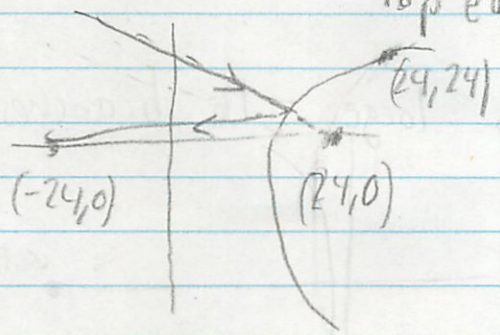
1/8

657/46. Hyperbolic mirror - reflects light coming to 1 focus to another - focus = (24,0)

top edge (24, 24) vertex = ?

↑ x axis)(

c = 24



$c^2 = a^2 + b^2$

$24^2 = a^2 + b^2$

$576 - a^2 = b^2$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

put in point

$$\frac{x^2}{a^2} - \frac{y^2}{(576-a^2)} = 1 \quad \text{if} \quad \frac{24^2}{a^2} - \frac{24^2}{(576-a^2)}$$

LCD

$$\frac{576}{a^2} - \frac{576}{576-a^2} = 1 \quad \frac{576(576-a^2)}{a^2(576-a^2)} - \frac{576(a^2)}{(576-a^2)a^2}$$

$$\frac{331776 - 576a^2}{576a^2 - a^4} - \frac{576a^2}{576a^2 - a^4} = \frac{297216 - 576a^2 - 576a^2}{576a^2 - a^4}$$

$$331776 - 576a^2 - 576a^2 = 576a^2 - a^4$$

$+a^4 \quad -576a^2 \quad -576a^2 + a^4$

$$a^4 - 1728a^2 + 331776 = 0$$

$$a^2 = \frac{-(-1728) \pm \sqrt{(1728)^2 - 4(1)(331776)}}{2(1)}$$

$a^2 = 1507,99$ $\quad$ $a^2 = 220,12$

$a = \pm 38,83$ $\quad$ $a = \pm 14,83$

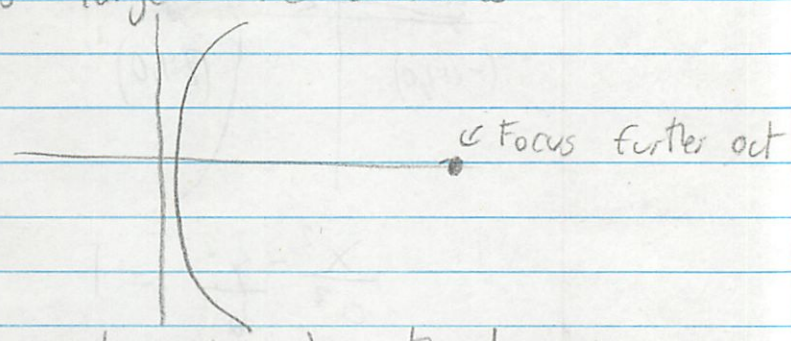
c > a) ⊗ ✓

(±14,83, 0)

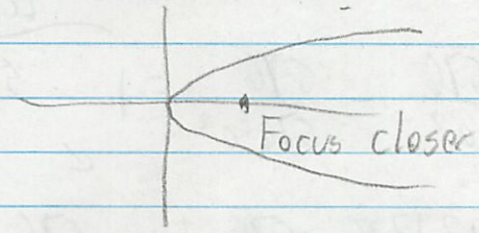
eccentricity $= \frac{c}{a}$ because $c > a$, $\underline{e > 1}$

ex ex $\frac{10}{4} \leftarrow \frac{c}{a}$ ← bigger c - the bigger the e

If eccentricity is large - the branches are flatter

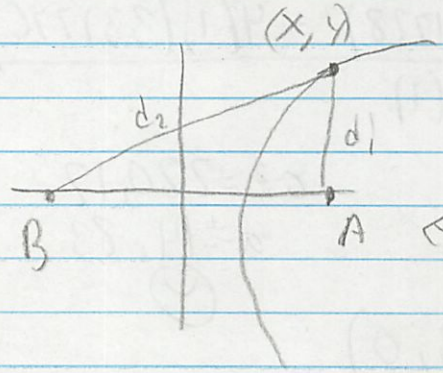


If eccentricity is close to 1 - the branches are more narrow



654

ex5 2 microphones are 1 mile (5280 ft) apart
Record explosion
Mic A recieves sound 2 sec before mic b
Where explosion?
Sound travels at 1100 ft/sec



$2c = 5280$
$c = 2640$

$\leftarrow d_2 - d_1 = 1100 \cdot 2 = 2a$
2200 ft $\quad \therefore a = 1100$ ft

$c^2 = a^2 + b^2$

$2640^2 = 1100^2 + b^2$

$b = \sqrt{5759600}$

$b = 2399.92$

path

$$\frac{x^2}{1100^2} + \frac{y^2}{5759600} = 1$$

$$\frac{x^2}{1210000} + \frac{y^2}{5759600} = 1$$

Type of Conic

all conics have been parallel to x or y axis

$$Ax^2 + Cy^2 + Dx + Ey + F = 0$$

Circle	$A = C$	$A \neq 0$
Parabola	$AC = 0$	$A = 0$ or $C = 0$ - not both
Elipse	$AC > 0$	$A + C$ like signs
Hyperbole	$AC < 0$	$A \cdot C$ have unlike signs

657

48 $x^2 + 4y^2 - 6x + 16y + 21 = 0$

$AC > 0$

$4 > 0$ elipse

658

50 $y^2 - 4y - 4x = 0$

$A = 0$ so $AC = 0$ parabola

54. $4x^2 + 4y^2 - 16y + 15 = 0$

$4 = 4$ circle

9.3 Hyperbole

th Day 2

1/8

657 43. Sound Location

You live 4 miles apart

East-West Street

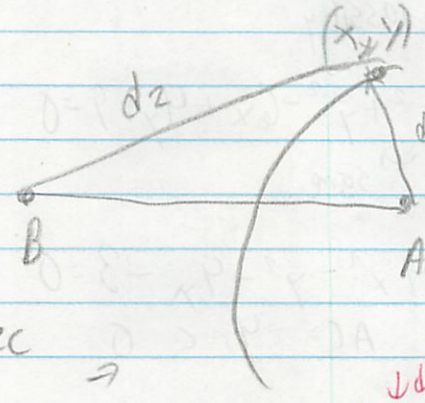
You hear thunder

Friend hears 18 sec later

Sound travels 1100 ft/sec

On test

Study



miles to feet

$2c = 21120$

$c = 10\,560$

differences in distances

$d_2 - d_1 = 1100 \cdot 18 = 2a$

$19800 = 2a$ $\quad a = 9900$

$c^2 = a^2 + b^2$

$10560^2 = 9900^2 + b^2$

$b^2 = 13503600$

$b = 3674.72$

path $\frac{x^2}{\cancel{11513600}\ 98010000} - \frac{y^2}{13503600} = 1$

a^2

hyperbolas are

44. Sound Location

3 listening stations Delay

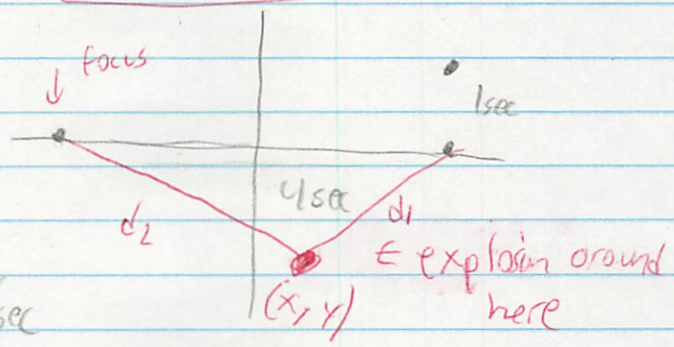
(3300, 0) 0 sec

(3300, 1100) 1 sec

(−3300, 0) 4 sec

Sound travels 1100 ft/sec

??



$c = 3300$

$d_2 - d_1 = 2a = 4 \cdot 1100$

$2a = 4400$

$a = 2200$

$(3300, -2750)$

$c^2 = a^2 + b^2$

$3300^2 = 2200^2 + b^2$

$b = \sqrt{6050000}$

$b = 2459$

assume $x = 3300$

find y

$\frac{3300^2}{2200^2} - \frac{y^2}{6050000} = 1$

$-1.25 = -\frac{y^2}{6050000}$

$y = \sqrt{7562500}$

$y = \pm 2750$ only negitive

The (3300, 1100) point only shows that point is in 4th quad

Classify

(1) 47. $x^2+y^2-6x+4y+9=0$ circle

same

658

(1) 49. $4x^2-y^2-4x-3=0$ hyperbole

$AC=-4<0$

(1) 51. $4x^2+3y^2+8x-24y+51=0$ elipse

$4\cdot 3=12>0$

(1) 53. $25x^2-10x-200y-119=0$ parapola

$\uparrow C=0$

9.4 Rotations + Systems Notes

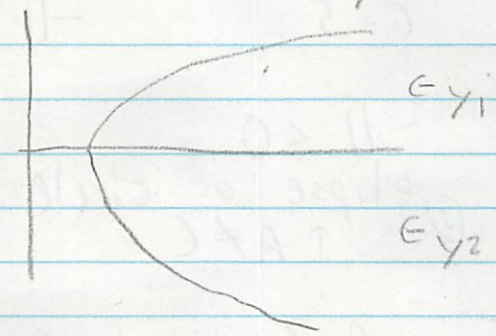
1/8

Are Parallel

Parabola Graphing

ex $y^2 = 4(x-1)$ $\quad y_1 = \sqrt{4(x-1)}$

$y_2 = -\sqrt{4(x-1)}$



Ellipse

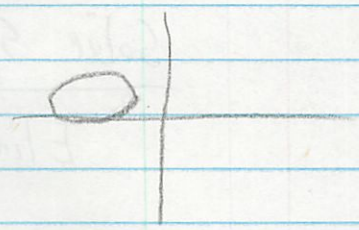
$$\frac{(x+3)^2}{4} + \frac{(y-1)^2}{1} = 1$$

Solve for y

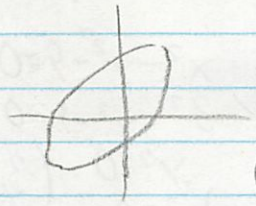
$$(y-1)^2 = 1 - \frac{(x+3)^2}{4}$$

$$y - 1 = \pm\sqrt{1 - \frac{(x+3)^2}{4}}$$

$$y = \pm\sqrt{1 - \frac{(x+3)^2}{4}} + 1$$



Not Parallel (slanted)



new B term

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

Use discriminant to classify conic

$(B^2 - 4ac)$

Elipse or Circle : $B^2 - 4AC < 0$

Parabola : $B^2 - 4AC = 0$

Hyperbole : $B^2 - 4AC > 0$

ex $3x^2+7xy+5y^2-6x-7y+15=0$

$A=3$ $\quad B^2-4AC$

$B=7$ $\quad 7^2-4(3)(5)$

$C=5$ $\quad -11$

$-11<0$

elipse or ~~circle~~

$\uparrow A \neq C$

P1 $x^2-6xy+9y^2-24+1=0$

$A=1$ $\quad (-6)^2-4\cdot1\cdot9$

$B=-6$ $\quad 0$

$C=9$

$0=0$ parabola

<u>Solve System of Conics</u>

Elimination or Substitution method

$x^2+y^2-16x+39=0$ → circle

$x^2-y^2-9=0$ → hyperbole

$+$ $2x^2-16x+30=0$

$2(x^2-8x+15)$ $\quad x^2-y^2-9=0$

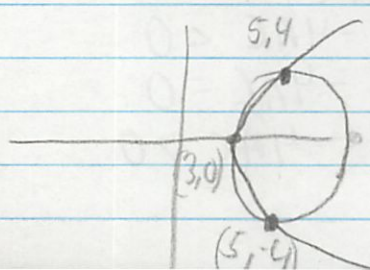
$2(x-5)(x-3)$ $\quad 3^2-y^2-9=0$

$\uparrow x=5$ $\quad \uparrow x=3$ $\quad y^2=0$ $\quad (3,0)$

$\uparrow y=5^2-16(5)+39$ $\quad (5,4)$

$y=\sqrt{16}$ $\quad (5,-4)$

$y=\pm4$



9.4 Rotation + Systems of Quadratics

HW

1/8

666

Classify

27. $16x^2-24xy+9x^2-30x-40y \rightarrow 9+(-24-40)y+(16x^2-30x)=0$

$A=16$ $\quad B^2-4AC$

$B=-24$ $\quad 24^2-4(16)(9)$

$C=9$ $\quad 0$

parabola

$$\frac{(24x+40)\pm\sqrt{(-24x-40)^2-4\cdot 9(16x^2-30)}}{2\cdot 9}$$

$$\frac{(24x+40)\pm\sqrt{576x^2+1600+1920x-576x^2+1080x}}{18}$$

$$\frac{(24x+40)\pm\sqrt{3000x+1600}}{18}$$

29. $15x^2-8xy+7y^2-45=0$

$A=15$

$B=-8$ $\quad 8^2-4(15)(7)$

$C=7$ $\quad -356 < 0$

ellipse / circle

$7y^2-8xy+(15x^2-45)=0$

factoring

$$y=8x\pm\sqrt{\frac{64x^2-4(7)(15x^2-45)}{2\cdot 7}}$$

$$\frac{8x\pm\sqrt{64x^2-420x^2+1260}}{14}$$

don't have to factor

$$y=\frac{8x\pm\sqrt{-356x^2+1260}}{14}$$

667

31. $x^2-6xy-5y^2+4x-22=0$

$A=1$

$B=-6$ $\quad 56$

$C=-5$

hyperbola

don't have to factor

factor

667

33. $x^2+4x+4y^2-5x-y-3=0$

$A=1$

$B=4$ 0 parabola

$C=4$

?? factoring don't have to factor

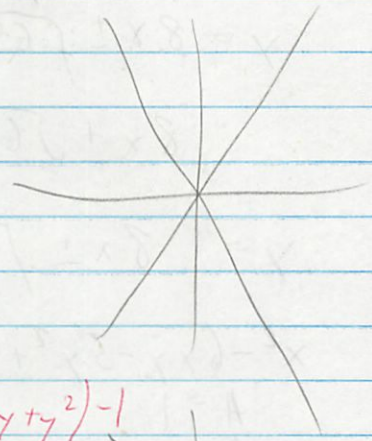
Sketch Degenerate Conic

35. $y^2-16x^2=0$

$y^2=16x^2$

$y=\pm\sqrt{16x^2}$

$y=\pm 4x$



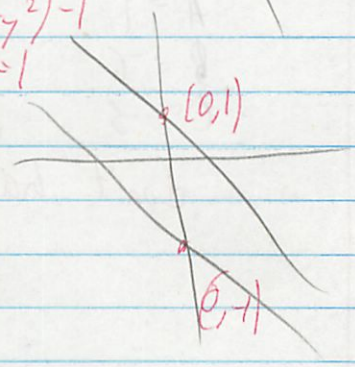
37. $x^2+2xy+y^2-1$ $\rightarrow (x^2+2xy+y^2)-1$

$y^2=x^2-2xy+1$ $(x+y)^2=1$

$y=\pm\sqrt{-x^2-2xy+1}$ $x+y=\pm 1$

$y=x+1$

$y=x-1$



9.4 Rotation + System of Quadratics

Day 2 Solving Systems

1/9

Elimination method

667/42.

$$\begin{cases} x^2-4y^2-20x-64y-172=0 & \leftarrow \text{hyperbola} \\ 16x^2+4y^2-320x+64y+1600=0 & \leftarrow \text{elipse} \end{cases}$$

$17x^2-340x+1428=0$

factor or quadratic formula

$(17x-238)(x-6)$

$x=14$ $\quad$ $x=6 \rightarrow (6)^2-4y^2-20(6)-64y-172=0$

$36-4y^2-120-64y-172=0$

$-4y^2-64y-256=0$

$-4(y^2+16y+64)$

$(y+8)^2$

$y=-8$

$(6,-8)$

↓ plug in $\quad$ plug in

$(14)^2-4y^2-20(14)-64y-172=0$

$196-4y^2-280-64y-172$

$-4y^2-64y-256=0$

$-4(y^2+16y+64)$

$(y+8)^2$

$y=-8$

$(14,-8)$

46. $16x^2-y^2+16y-128=0$

$y^2-48x-16y-32=0$

$16x^2-48x-160=0$

$16(x^2-3x-10)$

$x=5 \quad x=-2$

↓ pick term with most squared things

$y^2-48(-2)-16y-32=0$

$y^2-16y+64=0$

$(x-8)^2$

$y=8$

$(-2,8)$

$y^2-48(5)-16y-32=0$

$y^2-16y-272=0$

$y=26.33$

$x=-10.33$

$\frac{16\pm8\sqrt{21}}{2}$

$(5,26.33)$

$(5,-10.33)$

$(5,8\pm4\sqrt{21})$

44. $x^2+4y^2-2x-8y+1=0$
$-x^2+2x-4y-1=0$

$4y^2-12y=0$
$4y(y-3)=0$

$y=0$ | $y=3$

$x^2+4(0)^2-2x-8(0)+1$ | $x^2+4(3)^2-2x-8(3)+1=0$
$x^2-2x+1=0$ | $x^2+36-2x-24+1=0$
$(x-1)^2$ | $x^2-2x+13=0$
$(0,1)$ | No real solution

Substitution

50. $4x^2-y^2-8x+6y-9=0$ ① $\rightarrow 4y^2-8x-9=y^2-6y$
$2x^2-3y^2+4x+18y-43=0$ ② $\cdot 3$ $\cdot 3$
$12x^2-24x-27=-3y^2-18y$

$2x^2+4x-43-3y^2+18y$
$2x^2+4x-43-(3y^2-18y)$

$2x^2+4x-43-(12x^2-24x-27)$
$2x^2+4x-43-12x^2+24x+27$
$-10x^2+28x-16=0$
$10x^2-28x+16=0$
$2(5x^2-14x+8)$
$x=2$ $x=.8$ ← will be no solution

$x=3$

48. $6x^2+3y^2-12=0$
$x+y-2=0 \rightarrow y=2-x$

$6x^2+3(2-x)^2-12$
$6x^2+3(4-2x-2x+x^2)-12$
$6x^2+3(x^2-4x+4)-12$
$6x^2+3x^2-12x+12-12$
$9x^2-12x$
$3x(3x-4)$
$\rightarrow x=4/3$

$x=0$
$6(0)^2+3y^2-12=0$
$3y^2-12=0$
$3y^2=12$
$y^2=4$
$y=\pm 2$
$(0,\pm 2)$

$6(4/3)^2+3y^2-12=0$
$6(16/9)+3y^2=12$
$32/3+3y^2=12$
$\frac{4/3}{3}=\frac{3y^2}{3}$
$y^2=4/9$
$y=\pm 2/3$
$(4/3,\pm 2/3)$

9.4 Rotation / Systems of Quadratics

HW Day 2

1/9

Combine by elimination

~~667~~ 41. $\begin{cases} -4x^2 - y^2 - 16x + 24y - 16 = 0 \\ 4x^2 + y^2 + 40x - 24y + 208 = 0 \end{cases}$

$24x + 192 = 0$

$24x = -192$

$x = -8 \rightarrow 4(-8)^2 + y^2 + 40(-8) - 24y + 208$

$256 + y^2 - 320 - 24y + 208$

$y^2 - 24y + 144$

$y = 12$

$(-8, 12)$

43. $x^2 - y^2 - 12x + 16y - 64 = 0$

$x^2 + y^2 - 12x - 16y + 64 = 0$

$2x^2 - 24x = 0$

$2x(x - 12)$

$x = 0$

$(0)^2 + y^2 - 12(0) - 16y + 64$

$y^2 - 16y + 64$

$y = 8$

$(0, 8)$

$x = 12$

$(12)^2 + y^2 - 12(12) + 16y - 64$

$144 + y^2 - 144 + 16y - 64$

$x^2 + 16y - 64$

$x = 8$

$(12, 8)$

45. $-16x^2 - y^2 + 24y - 80 = 0$

$16x^2 + 25y^2 - 400 = 0$

$+24y^2 + 24y - 480 = 0$

~~cont~~ $24(y^2 + y - 20)$

$x = 4$

$x = -5$

cont

try $y = 0$

$\cancel{-16x^2 - (0)^2 + 24(0) - 80}$

$\cancel{-16x^2 - 80 = 0}$

$\cancel{-16(x^2 - 5)} \rightarrow$

Substitution

49. $10x^2 - 25y^2 - 100x + 160 = 0$

$y^2 - 2x + 16 = 0 \longrightarrow y^2 = 2x - 16$ ← just use $y = \sqrt{2x-16}$

$10x^2 - 25(2x - 16) - 100x + 160 = 0$

$10x^2 - 50x + 400 - 100x + 160 = 0$

$10x^2 - 150x + 560 = 0$

$10(x^2 - 15x + 56)$

$x = 8 \swarrow \longrightarrow x = 7$

$\cancel{10(8)^2 - 25y^2 - 100(8) + 160 = 0}$

$\cancel{640 - 25y^2 - 800 + 160 = 0}$

$\cancel{-25y^2 - 320 = 0}$

$\cancel{-5(5y^2 - 64)}$

$\cancel{\text{no answer}}$

$\cancel{0?}$

$\cancel{10(7)^2 - 25y^2 - 100(7) + 160}$

$\cancel{490 - 25y^2 - 700 + 160}$

$\cancel{-25y^2 - 50 = 0}$

$\cancel{-25(y^2 - 2)}$

ending

Use simpler equation

$(8,0)$ ✓

$x^2 = 2(8) - 16$

$x^2 = 0$

$x = 0$

$y^2 = 2(7) - 16$

$y^2 = -2$

$2i$

9.5 Parametrics

Notes

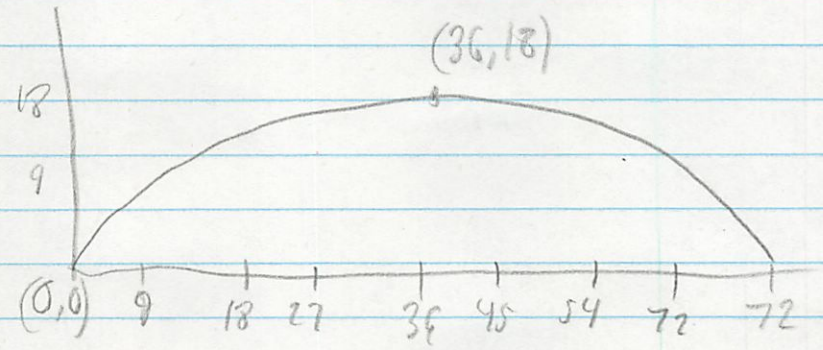
1/9

to this point graphs have been represented by a single equation in 2 variables (x, y)
↖ retangular equations

- can use a 3rd variable to represent a curve in a plane ↑ time

example; an object is propelled into the air at an angle of 45° with an initial velocity of 48 ft/sec
object follows a parabolic path

$$y = \frac{-x^2}{72} + x$$



↖ tells you where object is
- dosn't tell you when the object was at a given point (x, y)

SO introduce a 3rd variable
t = time ← parameter

Objective → write both x and y as a function of t

$x = f(t)$ $\quad$ $y = g(t)$

t	0	1	2	3	$\frac{3\sqrt{2}}{4}$
x	0	$24\sqrt{2}$			36
y	0	$-16+24\sqrt{2}$			18

$x = 24\sqrt{4}\,t$

$y = -16t^2 + 24\sqrt{2}\,t$

9.5 Parametrics

Notes Day 2

1/10

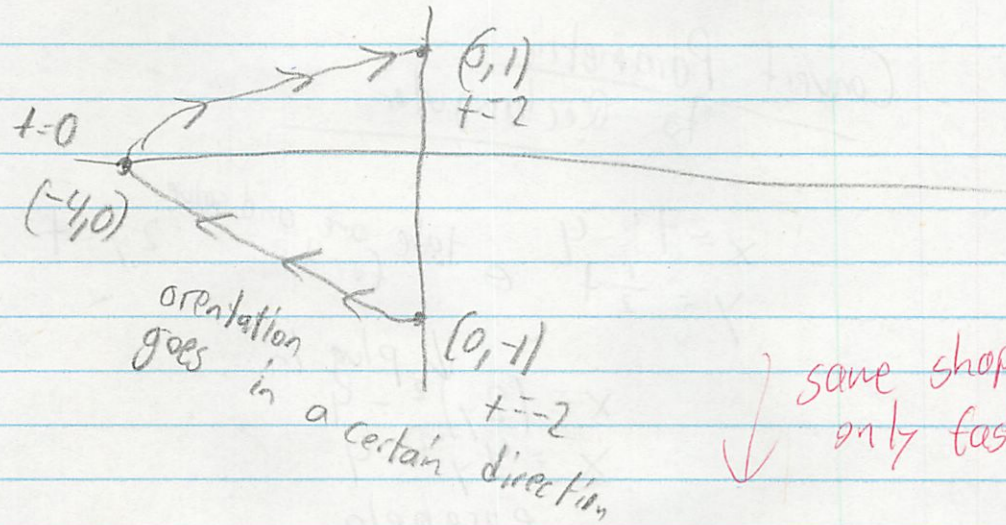
t is the parameter

ex1 Sketch the curve given by the parametric equation

$x = t^2 - 4 \qquad -2 \le t \le 3$

$y = \frac{t}{2}$

t	-2	-1	0	1	2	3
x	0	-3	-4	-3	0	5
y	-1	-.5	0	1/2	1	3/2



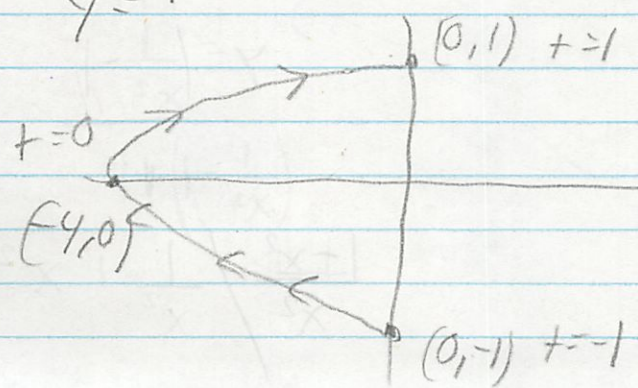
same shape only faster

ex2 2 different sets of parametrics

$x = 4t^2 - 4 \qquad -1 \le t \le 3/2$

$y = t$

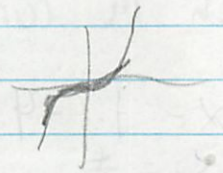
t	x	y
-1	0	-1
-1/2	3	-1/2
0	-4	0
1/2	-3	1/2
1	0	1
3/2	5	3/2



Calculator

Set to Radian mode
Set to Parametric mode (Par)

Put in $X_1 \to T$
$Y_1 \to T^3$



Graph (might have to adjust window)
$T_{min} = -4$
$T_{max} = 4$
$T_{step} = .1$

* Does matter which is X and Y

Convert Parametric to Rectangular

ex3

$x = t^2 - 4$
$y = \frac{1}{2}t$ ← take one and solve for t $\quad 2y = t$

↓ plug in

$x = (2y)^2 - 4$
$x = 4y^2 - 4$
parabola

ex4

$x = \frac{1}{\sqrt{t+1}} \quad t > -1 \qquad y = \frac{t}{t+1}$

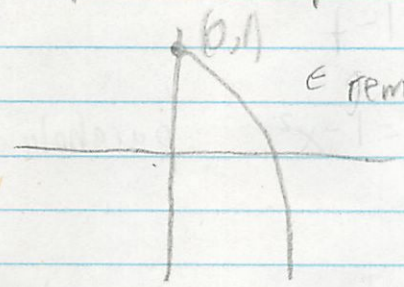
$x^2 = \frac{1}{t+1} \qquad y = \frac{\left(\frac{1}{x^2} - 1\right)}{\left(\frac{1}{x^2} - 1\right) + 1}$

$\frac{1}{x^2} = t + 1$

$\boxed{\left(\frac{1}{x^2}\right) - 1 = t}$

$\frac{1 - x^2}{x^2} \Big/ \frac{1}{x^2} + x^2 \to$

$y = 1 - x^2$ ← parabolic + vertex = $(0,1)$



← remember restriction

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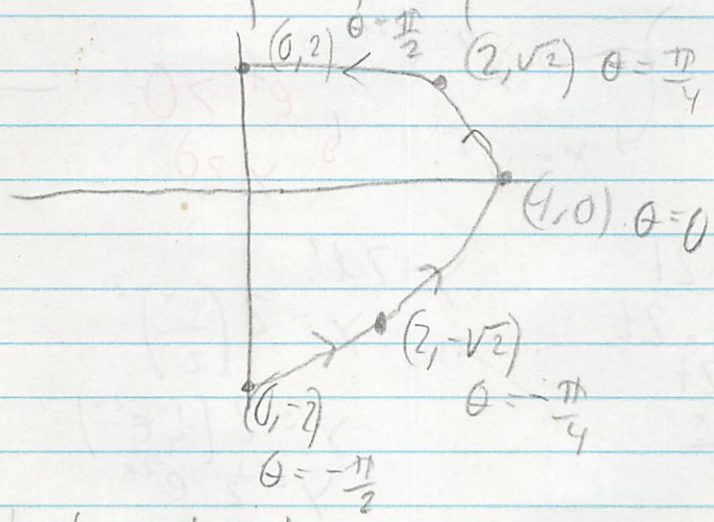
8. $x = 4\cos^2\theta \qquad y = 2\sin\theta$

θ	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\pi/4$	$\pi/2$
x	0	2	4	2	0
y	-2	$-\sqrt{2}$	0	$\sqrt{2}$	2



Convert to rectangular

$x = 4\cos^2\theta \qquad y = 2\sin\theta$

$\frac{x}{4} = \cos^2\theta \qquad \frac{y}{2} = \sin\theta$

$\frac{y^2}{4} = \sin^2\theta$

$\cos^2\theta + \sin^2\theta = 1$

$\frac{x}{4} + \frac{y^2}{4} = 1$ parabola

674

16. $x = \sqrt{t}$ $\qquad$ $y = 1 - t$

$x^2 = t$

$t \geq 0$ $\qquad$ $y = 1 - x^2$ $\qquad$ parabola

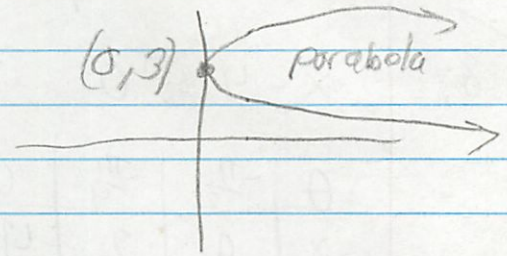
add restriction

18. $x = |t - 1|$ $\qquad$ $y = t + 2$

$y - 2 = t$

$x = |(y-2) - 1|$

$x = |y - 3|$

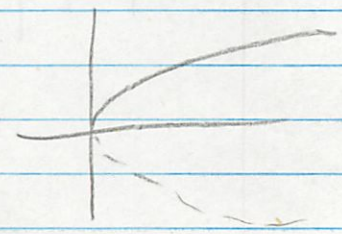


22. $x = e^{2t}$ $\qquad$ $y = e^t$

$y^2 = e^{2t}$

$x = y^2$ $\qquad$ $e^t > 0$

$y > 0$



24. $x = \ln 2t$ $\qquad$ $y = 2t^2$

$x = \log_e 2t$

$e^x = 2t$

$t = \frac{e^x}{2}$

$y = 2\left(\frac{e^x}{2}\right)^2$

$y = 2\left(\frac{1}{4}e^{2x}\right)$

$y = \frac{1}{2}e^{2x}$

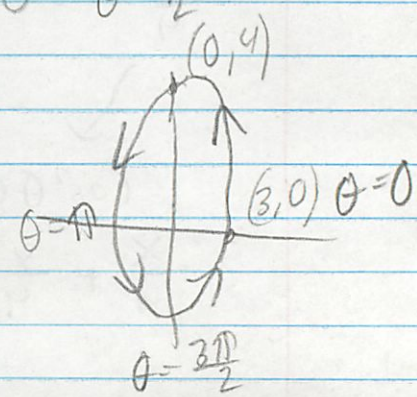
P1 $\quad y = 4\sin\theta$ $\qquad$ $\frac{x}{3} = \cos\theta$

$\frac{y}{4} = \sin\theta$ $\qquad$ $\frac{x^2}{3^2} = \cos^2\theta$ $\qquad$ $\theta = \frac{\pi}{2}$

$\frac{y^2}{4^2} = \sin^2\theta$ $\qquad$ $\frac{x^2}{9} = \cos^2\theta$

$\sin^2\theta + \cos^2\theta = 1$

$\frac{y^2}{16} + \frac{x^2}{9} = 1$ $\qquad$ ellipse



26. $x = 4 + 2\cos\theta$ $\qquad$ $y = -1 + 2\sin\theta$

$x - 4 = 2\cos\theta$ $\qquad$ $y + 1 = 2\sin\theta$

$\frac{x-4}{2} = \cos\theta$ $\qquad$ $\frac{y+1}{2} = \sin\theta$

$\left(\frac{x-4}{2}\right) = \cos\theta$ $\qquad$ $\left(\frac{y+1}{2}\right) = \sin\theta$

$\left(\frac{x-2}{2^2}\right)^2 + \left(\frac{y+1}{2^2}\right)^2 = 1$

$\frac{(x-2)^2}{4} + \frac{(y+1)^2}{4} = 1$ $\in$ circle

coefficents =

28. $x = \sec\theta$ $\qquad$ $y = \tan\theta$

$x^2 = \sec^2\theta$ $\qquad$ $y^2 = \tan^2\theta$

$\tan^2\theta + 1 = \sec^2\theta$

$y^2 + 1 = x^2$

$1 = x^2 - y^2$ $\in$ hyperbole

<u>Retangular to Parametric</u>

Find a set of parametric equations to represent the graph of $y = 1 - x^2$ with $t = x$

$y = 1 - t^2$

pair ✓

with $t = 1 - x$ instead

$t - 1 = -x$ & solve for x

$\boxed{1 - t = x}$ $\rightarrow$ $y = 1 - (1-t)^2$

$y = 1 - (1 - 2t + t^2)$

$y = 2t - t^2$

44, $y = 5 - 7x$ (no starting parametric)

so start with $t = x$

$y = 5 - 7t$

or $x = t^3$

so $y = 5 - 7t^3$

48 $y = 6x^2 - 5$

try $t = x$

$y = 6(t)^2 - 5$

9.5 Parametric Equations

Day 2 HW

1/10

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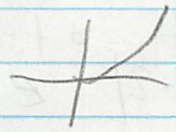
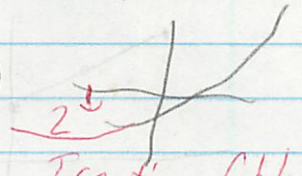
3. $x=\sqrt{t}$, $y=t$; $x=\sqrt{y}$, $y=x^2$; $t \geq 0$ (b)



5. $x=\ln t$; $e^x=t$ (f)

$y=\frac{1}{2}t-2$; $y=\frac{1}{2}(e^x)-2$

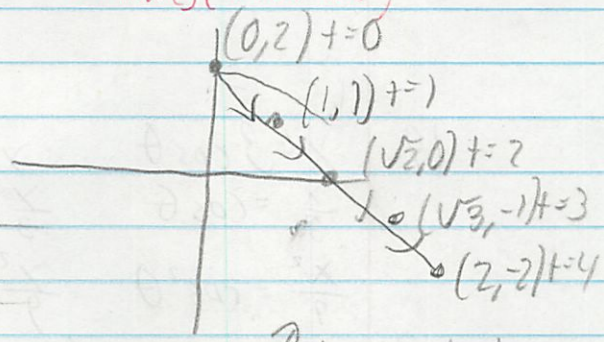


7. $x=\sqrt{t}$; $x^2=t$

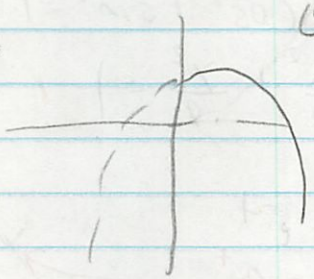
$y=2-t$; $\boxed{y=2-x^2}$

a)

t	0	1	2	3	4
x	0	1	$\sqrt{2}$	$\sqrt{3}$	2
y	2	1	0	-1	-2



$y=2-x^2$



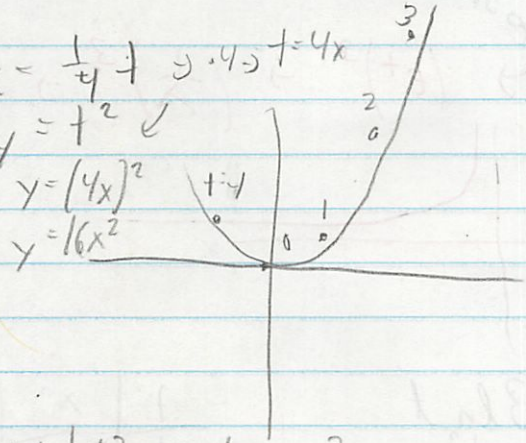
Suppose to be the same if this is $x \geq 0$

13. $x=\frac{1}{4}t$; $4x=t$

$y=t^2$

$y=(4x)^2$

$y=16x^2$



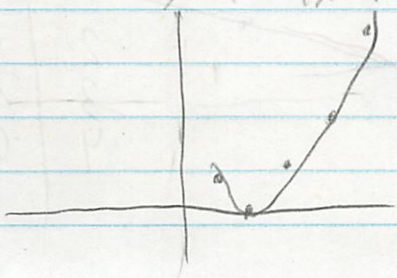
t	x	y
-1	$-1/4$	1
0	0	0
1	$1/4$	1
2	$1/2$	4
3	$3/4$	9

674

15. $x=t+2$; $t=x-2$

$y=t^2$; $y=(x-2)^2$

$y=x^2-4x+4$



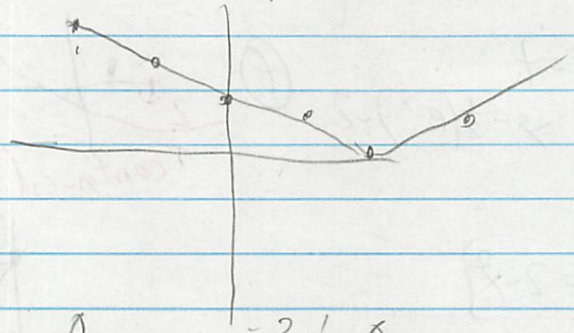
t	x	y
-1	1	1
0	2	0
1	3	1
2	4	4
3	5	9
4	6	16

17. $x = 2t \quad \to t = \frac{x}{2}$

$y = |t-2| \quad \Leftarrow \quad y = \left|\frac{x}{2} - 2\right| \to \frac{1}{2}|x-4|$

t	x	y
-2	-4	4
-1	-2	3
0	0	2
1	2	1
2	4	0
3	6	1

(V)



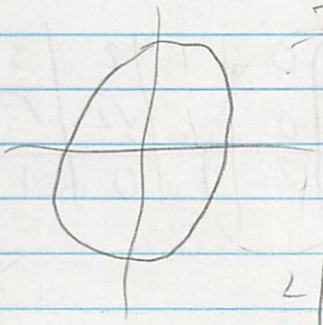
19. $x = 3\cos\theta \qquad y = 3\sin\theta$

$\frac{x}{3} = \cos\theta \qquad \frac{y}{3} = \sin\theta$

$\frac{x^2}{9} = \cos^2\theta \qquad \frac{y^2}{9} = \sin^2\theta$

$\cos^2 + \sin^2 = 1$

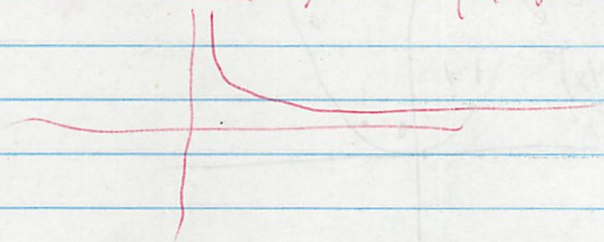
$\frac{x^2}{9} + \frac{y^2}{9} = 1$



t	x	y

21. $x = e^{-t}$

$\frac{1}{x} = e^{t} \longrightarrow y = e^{3t} \to (e^{t})^3 \to \left(\frac{1}{x}\right)^3 \to \frac{1}{x^3}$



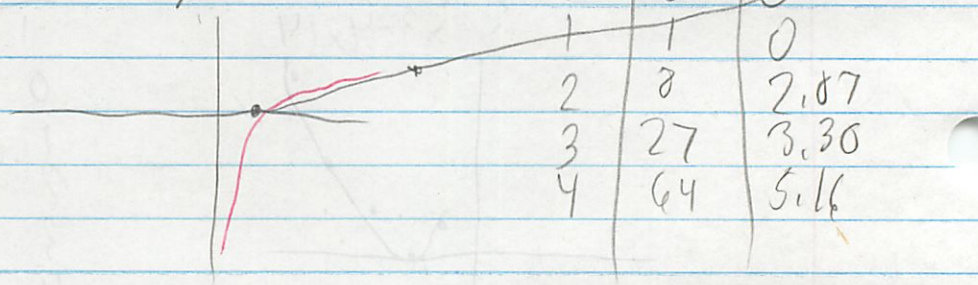
23. $x = t^3 \qquad y = 3\ln t$

$e^{y} = t^3$

$x = \sqrt[3]{e^{y}} \qquad \sqrt[3]{e^{y}}$

$\boxed{x = e^{y}}$

t	x	y
-1	-1	X
0	0	X
1	1	0
2	8	2.07
3	27	3.30
4	64	5.16



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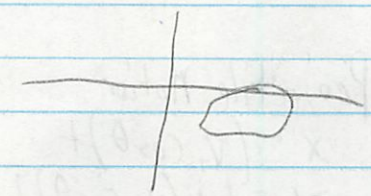
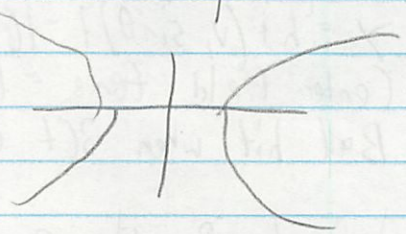
Graph with calculator

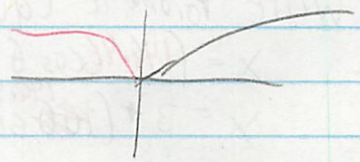
25. $x = -4 + 2\cos\theta$
$y = -1 + \sin\theta$



27. $x = 4\sec\theta$
$y = 3\tan\theta$



29. $x = t/2$
$y = \ln(t^2+1)$



43. Find 2 parametric equations
$y = 4x - 3$

$x = t$
$y = 4t - 3$

$x = \frac{1}{4}t$
$y = t - 3$

45. $y = \frac{1}{x}$

$x = t$
$y = \frac{1}{t}$

$x = 5t$
$y = \frac{5}{t}$

47. $y = x^2 + 4$

$x = t$
$y = t^2 + 4$

$x = \sqrt{t}$
$y = t + 4$

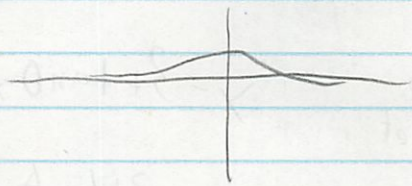
49. $y = x^3 + 2x$

$x = t$
$y = t^3 + 2t$

$x = 10t$ or $\frac{x}{10} = t$
$y = (10t)^3 + 20t$

Graph

53. Witch of Agnesi
$x = 2\cot\theta$
$y = 2\sin^2\theta$



675/59.

Projectile Motion

$x = (v_1 \cos\theta)t$

$y = h + (v_1 \sin\theta)t - 16t^2$

Center field fence = 10 ft heigh + 400 ft from home plate

Ball hit when 3 ft above ground @ 160 mph @ θ angle

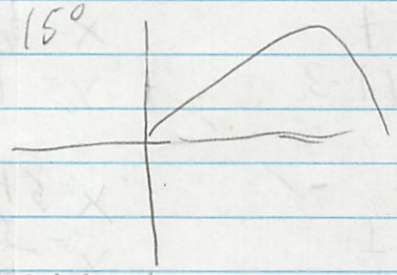
↳ 146.46 feet/sec

a) Write Parametic equation

$x = (146.46\cos\theta)t$

$y = 3 + (\cancel{160}\ 146.46 \sin\theta)t - 16t^2$

b) 15°



No

$[146.46\cos(15)]\,t$

$141.7t = x$

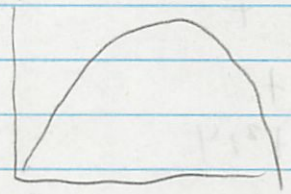
$3 + (146.46 \sin 15)t - 16t^2$

$3 + 38t - 16t^2 = y$

or set = to 400 to find t - whats y θ same

put in calc

c) 23°



Yes

when x = 400 height?

$400 = 146.46\cos 23\, t$

$t = 2.96$ sec

$y = 3 + (146.46 \sin 23)(2.96) - 16(2.96)^2$

$y = 21.39$

$y > 0$

d) Start by solving for t - $\frac{x}{146.46\cos\theta} = t$

Sub in $3 + 146.46\sin\theta\left(\frac{x}{146.46\cos\theta}\right) - 16\left(\frac{x}{146.46\cos\theta}\right)^2$

Rewrite in terms of tan

$y = 3 + \tan\theta\, x - \frac{16\sec^2\theta}{\boxed{146.67}^2} x^2$

$y = 3 + \tan\theta - \frac{16(1+\tan^2\theta)}{146.67^2} x^2$

Sub in problem

$10 = 3 + \tan\theta - \frac{16(1+\tan^2\theta)}{146.67} 400^2$

$10 = 3 + 400\tan\theta - 119.0\tan^2\theta - 119$

Quadratic formula

$119\tan^2\theta - 400\tan\theta + 126 = 0$ ← want min

$\tan\theta = .35 \to \boxed{19.4°}$ $\tan\theta = 3.01$ $\boxed{71.68°}$

9/ Conics + Parametrics
Test Topics

1/15

1. Determaine all parts of hyperbole, ellipse, and parabola then graph

2. Finding equation of a conic given foci, lenght of axis, etc.

3. Determin type of conic using $A \cdot C$ parallel
$B^2 - 4AC$ non parallel

4. Find equation of line tangent to parabola,

5. Systems of Conics

6. Given parametric equations, get retangular equations

7. Given rectangular equations come up with parametric equation.

8. Word problems
- going thru a tunnel
- hearing a sound
- hyperbolic mirror

9 / Analytic Geometry Review

1/15

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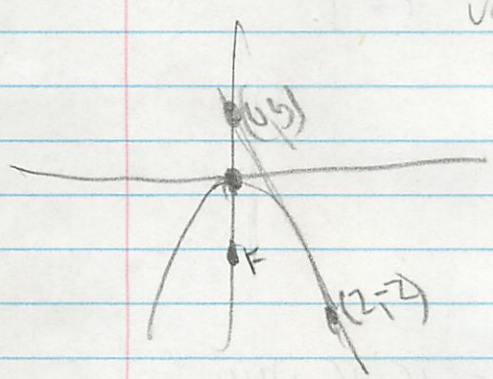
7. Find the equation of the tangent line

$x^2 = -2y$ ← $4p = -2$

$(2,-2)$ $\quad p = -\frac{1}{2}$

vertex $(0,0)$



d_1 / $\sqrt{(0-2)^2 + (-\frac{1}{2}-2)^2}$

$\sqrt{6.25}$

2.5

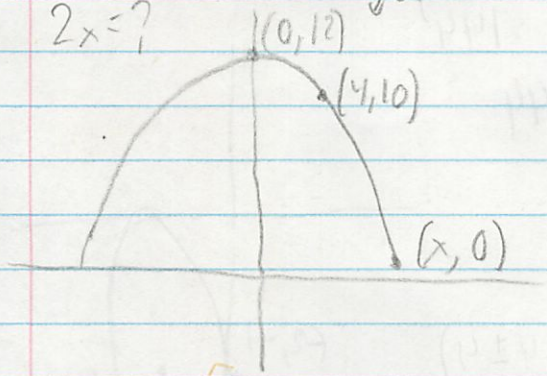
$d_2 = d_1 = 2.5$

b / $2.5 - \frac{1}{2} = 2$

t /

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - -2}{0-2} = \frac{4}{-2} = -2x + 2$

9. $2x = ?$



$x^2 = 4p(y-12)$

$16 = 4p(10-12)$

$16 = 4p \cdot -2$

$16 = -8p$

$p = -2$

$x^2 = 4(-2)(y-12)$

$x^2 = -8 \cdot -12$

$x^2 = 96$

$x = 9.79 \cdot 2 =$

19.59

13. Find standard form of ellipse

$(0,1)$ minor points $(2,0)$ ← $b=1$

$(4,1)$ ← $a=2$ $\quad (2,2)$ x axis major

vertex $(2,1)$

$\frac{(x-2)^2}{2^2} + \frac{(y-1)^2}{1^2} = 1$

$\frac{(x-2)^2}{4} + \frac{(y-1)^2}{1} = 1$

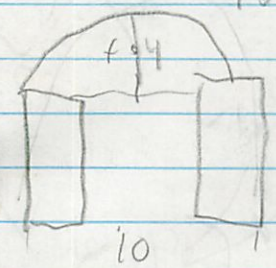
15. Architecture

switch

Foci = ?
c = ?

$\cancel{b}\,a = 4$
$c = ?$
$2b = 16$
$a\ \cancel{b} = 5$

$c^2 = a^2 - b^2$
$c^2 = 5^2 - 4^2$
$c^2 = 9$
$c = 3$



17. $16x^2 + 9y^2 - 32x + 72y + 16$

$16x^2 - 32x + 9y^2 + 72y = -16$

$16(x^2 - 2x + 1) + 9(y^2 + 8x + 16) = -16 + 16 + 144$

$16(x-1)^2 + 9(y+4)^2 = 144$

$\frac{(x-1)^2}{16} + \frac{(y+4)^2}{9} = 144$

$\frac{(x-1)^2}{9} + \frac{(y+4)^2}{16} = 1$

Center $(1, -4)$

$a = 4$ major $(1, -4 \pm 4)$

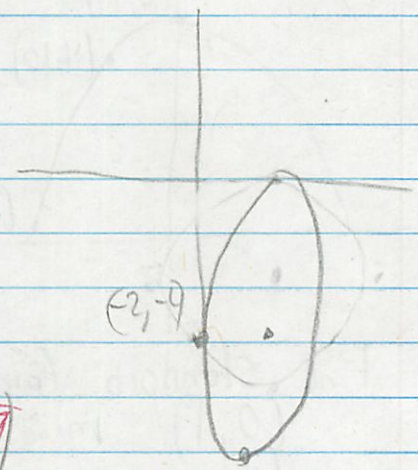
$b = 3$ minor $(1 \pm 3, -4)$

$c^2 = a^2 - b^2$ foci $(1, -4 \pm \sqrt{7})$

$c^2 = 4^2 - 3^2$ $e = \frac{c}{a}$ $\frac{\sqrt{7}}{4}$

$c = \sqrt{7}$

$c = 2.65$



19. $3x^2 + 8y^2 + 12x - 112y + 403 = 0$

$3x^2 + 12x + 8y^2 - 112y = -403$

$3(x^2 + 4x + 4) + 8(y^2 - 14y + 49) = -403 + 12 + 392$

$3(x+2)^2 + 8(y-7)^2 = 1$

$\frac{(x+2)^2}{1/3} + \frac{(y-7)^2}{1/8} = 1$ ex axis

Center $(-2, 7)$

$a^2 = 1/3$ y $\frac{1}{\sqrt{3}}$ $\frac{\sqrt{3}}{3}$ $b^2 = \frac{1}{8}$ $b = \frac{1}{\sqrt{8}}$ $\frac{\sqrt{8}}{8}$

$c^2 = a^2 - b^2$
$1/3 - 1/8$

$c = \sqrt{5/24}$ $\frac{\sqrt{5}}{2\sqrt{6}}$

major $(-2 \pm \frac{\sqrt{3}}{3}, 7)$

minor $(-2, 7 \pm \frac{\sqrt{2}}{8})$ $e = \frac{\sqrt{2}}{4}$

Focus $(-2 \pm \frac{\sqrt{5}}{2\sqrt{6}}, 7)$

$\sqrt{30}/12$

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Find standard form of hyperbole

23. Foci $(0,0)$ ← vertex $(4,0)$ ⟩|(

$(8,0)$ $\quad c=4$ $\quad \frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

asy: $y=\pm 2(x-4)$

slope $=\frac{b}{a}$

$b=2a$ $\quad a^2+b^2=c^2$ $\quad \frac{16}{5}+b^2=16$ $\quad$ remember vertex $(x-4)^2$

$b^2=4a^2$ $\quad a^2+4a^2=16$ $\quad b^2=12.8$ $\quad \frac{(x-4)^2}{\frac{16}{5}}-\frac{y^2}{\frac{64}{5}}=1$

$a^2(1+4)=16$ $\quad 64/5$

$5a^2=16$

$a^2=\frac{16}{5}$

700

25. $9x^2-16y^2-18x-32y-151=0$

$9x^2-18x-16y^2-32y=151$

$9(x^2-2x+1)-16(y^2+2y+1)=151+9-16$

$9(x-1)^2-16(y+1)^2=144$ ← remember!

$\frac{(x-1)^2}{9}-\frac{(y+1)^2}{16}=144$

$\frac{(x-1)^2}{16}-\frac{(y+1)^2}{9}=1$

$a^2=16$ $\quad b^2=9$ $\quad c^2=a^2+b^2$

$a=4$ $\quad b=3$ $\quad c^2=16+9$

$c=\sqrt{25}$

$c=5$

vertex $(1,-1)$

major $(1\pm 4,-1)$

minor $(1,-1\pm 3)$

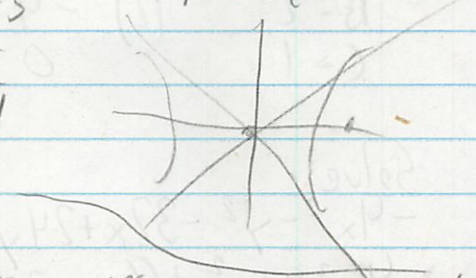
foci $(1\pm 5,-1)$

$e=\frac{c}{a}=\frac{5}{4}$

asymptotes

$y+1=\pm\frac{b}{a}(x-1)$

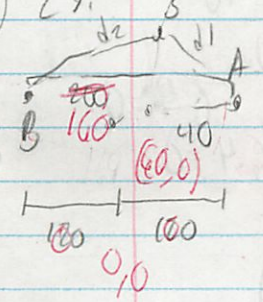
$y=\frac{\pm 3}{4}(x-1)-1$



29. travels 186,000 miles per sec

A gets .0005 sec sooner

How far north is ship?



$186{,}000 \cdot .0005 = 93$ miles $= 2a$

$2a=240$ $\quad a=93/2$

$a=120$

$2c=200$

$c=100$

$100^2=\left(\frac{93}{2}\right)^2+b^2$

$\frac{60^2}{120^2}+\frac{y^2}{b^2}$

$A=C$ Circle
$AC=0$ Parabola
$AC>0$ elipse
$AC<0$ hyperbole

Classify

33. $5x^2-2y^2+10x-4y+17=0$

$5\cdot -2$

$-20 < 0$ hyperbole

34. $16x^2-8xy+y^2-10x+5y$

$16=A$

$-8=B$ $\quad (-8)^2-4(16)(1)$

$1=C$ $\quad 0$ parabola

$y^2+(-8x+5)y+(16x^2-10x)=0$ solve for y

$$y=\frac{(8x-5)\pm\sqrt{(-8x+5)^2-4(1)(16x^2-10x)}}{2(1)}$$

$$y=\frac{(8x+5)\pm\sqrt{64x^2+25-80x-64x^2+40x}}{2}$$

41. $x^2+y^2+2xy+2\sqrt{2}x-2\sqrt{2}y+2=0$

$A=1$

$B=2$ $\quad (2)^2-4\cdot1\cdot1$

$C=1$ $\quad 0$ = parabola

43. Solve

$-4x^2-y^2-32x+24y-64=0$

$4x^2+y^2+64x-24y+304=0$

$24x+240=0$

$24x=-240$

$x=-10$

find y $\quad -4(-10)^2-y^2-32(-10)+24y-64=0$

$-400-y^2+320+24y-64=0$

$-y^2+24y-144=0$

$y=12$

$(-10,12)$

44. $x^2+y^2-25=0 \rightarrow y^2=-x^2+25$

$9x-4y^2=0 \quad \leftarrow \quad 9x-4(-x^2+25)=0$

$9x+4x^2-100=0$

$4x^2+9x-100=0$

$x=4 \qquad x=-6.25$

$9(4)-4y^2=0 \qquad 9(-6.25)-4y^2=0$

$-36 \quad -36 \qquad +56.25 \quad +56.25$

$-4y^2=-36 \qquad -4y^2=56.25$

$y^2=9 \qquad y^2=-14.0625$

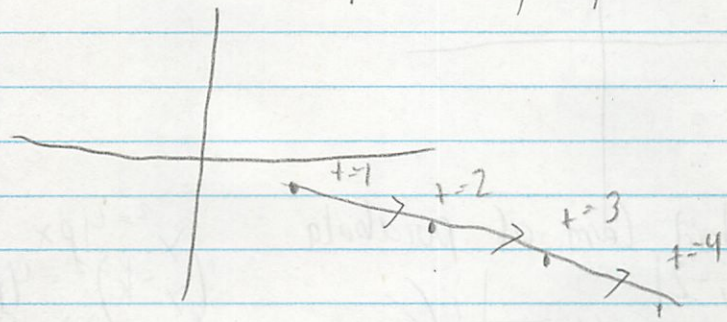
$y=3 \qquad$ non real

$(4,3)$

47. $x=1+4t$

$y=2-3t$

t	1	2	3	4	5
x	5	9	13	17	21
y	-1	-4	-7	-10	-13



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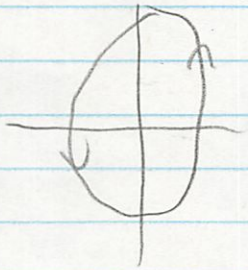
51. $x=6\cos\theta$

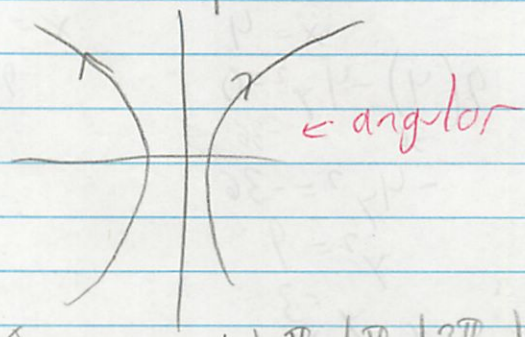
$y=6\sin\theta$

t	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
x	4.24	0	-4.24	-6
y	4.24	6	4.24	0



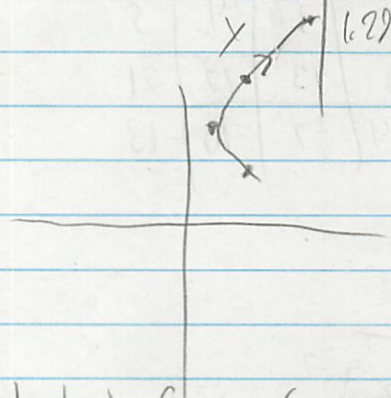
53. $x = \sec\theta$
$y = \tan\theta$

t	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
x	π	X	$-\pi$	-1
y	1	X	-1	0



54. $x = 2 - \sin\theta$
$y = 2 - \cos\theta$

t	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
x	1.29	1	1.29	2
y	1.29	2	2.70	3



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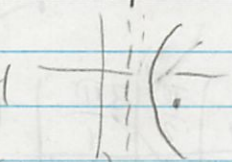
5. Find standard form of parabola
Focus $(8, -2)$
directrix $x = 4$



center $(6, -2)$
$p = 2$

$y^2 = 4px$
$(y-k)^2 = 4p(x-h)^2$
$(y+2)^2 = 4p(x-6)^2$
$(y+2)^2 = 4(2)(x-6)^2$
$(y+2)^2 = 8(x-6)^2$

Everything You Ever Wanted to Know about the Conic Sections

Use the following tables to guide you on your road to mathematical dominance of conic sections.

$x^2 =)($

$y^2 = \frac{\cup}{\cap}$

Circle Centered at (h, k)	
Standard Form Equation	$(x-h)^2+(y-k)^2=r^2$
Center	$(h,\ k)$
Radius	r

Ellipse Centered at the (*h*, *k*), with *a*>*b*		
Standard Form Equation	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$	$\frac{(y-k)^2}{a^2}+\frac{(x-h)^2}{b^2}=1$
Center	$(h,\ k)$	$(h,\ k)$
Major Axis Vertices	$(h\pm a,\ k)$	$(h,\ k\pm a)$
Minor Axis Vertices	$(h,\ k\pm b)$	$(h\pm b,\ k)$
Foci	$(h\pm c,\ k)$	$(h,\ k\pm c)$
Eccentricity	$e=\frac{c}{a}$	$e=\frac{c}{a}$
	$a^2=b^2+c^2$	

Parabola Centered at (*h*, *k*)		
Standard Form Equation	$(x-h)^2=4p(y-k)$	$(y-k)^2=4p(x-h)$
Vertex	$(h,\ k)$ $\cup$	$(h,\ k)$ $)($
Axis of Symmetry	$x=h$ $\cap$	$y=k$
Focus	$(h,\ k+p)$	$(h+p,\ k)$
Directrix	$y=k-p$	$x=h-p$
Eccentricity	$e=\frac{c}{a}$	$e=\frac{c}{a}$

Hyperbola Centered at (h, k)		
Standard Form Equation	$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$	$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$
Center	(h, k)	(h, k)
Transverse Axis Vertices	$(h \pm a, k)$	$(h, k \pm a)$
Conjugate Axis Vertices	$(h, k \pm b)$	$(h \pm b, k)$
Foci	$(h \pm c, k)$	$(h, k \pm c)$
Asymptotes	$(y-k)=\pm\frac{b}{a}(x-h)$	$(y-k)=\pm\frac{a}{b}(x-h)$
Eccentricity	$e=\frac{c}{a}$	$e=\frac{c}{a}$
	$a^2+b^2=c^2$	

Conic Section	Equation	Slope of Tangent at $P(x_0, y_0)$, $y_0 \neq 0$
Circle	$x^2+y^2=r^2$	$-\frac{x_0}{y_0}$
Ellipse	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ a>b$ $\frac{y^2}{a^2}+\frac{x^2}{b^2}=1,\ a>b$	$-\frac{b^2x_0}{a^2y_0}$ $-\frac{a^2x_0}{b^2y_0}$
Hyperbola	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$	$\frac{b^2x_0}{a^2y_0}$ $-\frac{a^2x_0}{b^2y_0}$
Parabola	$y^2=4cx$ $x^2=4cy$	$\frac{2c}{y_0}$ $\frac{x_0}{2c}$

IS 4

20/20

Reciprocal Identities

$\sin x = \dfrac{1}{\csc x}$ $\quad \sec x = \dfrac{1}{\cos x}$ $\quad \tan x = \dfrac{1}{\cot x}$

$\csc x = \dfrac{1}{\sin x}$ $\quad \cos x = \dfrac{1}{\sec x}$ $\quad \cot x = \dfrac{1}{\tan x}$

Tangent Identities

$\tan x = \dfrac{\sin x}{\cos x}$ $\quad \cot x = \dfrac{\cos x}{\sin x}$

Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \csc^2 x$

Cofunction Identities

$\sin\left(\dfrac{\pi}{2} - x\right) = \cos x$ $\quad \cos\left(\dfrac{\pi}{2} - x\right) = \sin x$

$\csc x\left(\dfrac{\pi}{2} - x\right) = \sec x$ $\quad \tan\left(\dfrac{\pi}{2} - x\right) = \cot x$

$\sec\left(\dfrac{\pi}{2} - x\right) = \csc x$ $\quad \cot\left(\dfrac{\pi}{2} - x\right) = \tan x$

Negative Angle Identities

$\sin(-x) = -\sin x$ $\quad \cos(-x) = \cos x$

$\csc(-x) = -\csc x$ $\quad \tan(-x) = -\tan x$

$\sec(-x) = \sec x$ $\quad \cot(-x) = -\cot x$

Sum and Difference Formulas

$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$

$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$

$\tan(u \pm v) = \dfrac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$

Double Angle Formulas

$\sin 2x = 2 \sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

$\tan 2x = \dfrac{2 \tan x}{1 - \tan^2 x}$

Power Reducing

$\sin^2 u = \dfrac{1 - \cos 2u}{2}$ $\quad \cos^2 u = \dfrac{1 + \cos 2u}{2}$ $\quad \tan^2 u = \dfrac{1 - \cos 2u}{1 + \cos 2u}$

6 Circular function definitions where θ is any angle

$\sin\theta = \dfrac{y}{r}$ $\quad \csc\theta = \dfrac{r}{y}$

$\cos\theta = \dfrac{x}{r}$ $\quad \sec\theta = \dfrac{r}{x}$

$\tan\theta = \dfrac{y}{x}$ $\quad \cot\theta = \dfrac{x}{y}$

$r = \sqrt{x^2 + y^2}$

CHART OF THE 3 TRIG FUNCTIONS OF SPECIAL ANGLES

Degrees	Radians	$\sin\theta$	$\cos\theta$	$\tan\theta$
0°	0	0	1	0
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

Additional Trigonometric Formulas/Identities

Half - Angle Identities:

$$\sin^2 u = \frac{1-\cos 2u}{2} \qquad \cos^2 u = \frac{1+\cos 2u}{2} \qquad \tan^2 u = \frac{1-\cos 2u}{1+\cos 2u}$$

Half - Angle Formulas:

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}} \qquad \cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}} \qquad \tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

Alternative Half - Angle Formulas for Tangent:

$$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta} \qquad \tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta}$$

Product - to - Sum Formulas:

$$\sin u\cos v = \frac{1}{2}\left[\sin(u+v)+\sin(u-v)\right]$$

$$\cos u\sin v = \frac{1}{2}\left[\sin(u+v)-\sin(u-v)\right]$$

$$\cos u\cos v = \frac{1}{2}\left[\cos(u+v)+\cos(u-v)\right]$$

$$\sin u\sin v = \frac{1}{2}\left[\cos(u-v)-\cos(u+v)\right]$$

Sum - to - Product Formulas:

$$\sin u + \sin v = 2\sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\sin u - \sin v = 2\cos\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

$$\cos u + \cos v = 2\cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\cos u - \cos v = -2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

IS Test

1/16

1. Sum or difference idenity to find exact value (2)

2. Determain exact value of trig expressions given sin v and cos v and a restriction on the quadrant

Don't have to do power reducing

3. Use double or half angle identity to find exact value of trig expression (2)

4. Express sum as a product and give exact value

Have formulas

5. Solving trig equations that have identies in them (sum identies, double angle) etc

6. Law of sines + cosines (4 word problems)

7. Herons area formula

B4 Pratice

1/16

1. $\dfrac{-1+\frac{\sqrt{3}}{3}}{1+\frac{\sqrt{3}}{3}}$ common denom $\rightarrow \dfrac{-\frac{3}{3}+\frac{\sqrt{3}}{3}}{\frac{3}{3}+\frac{\sqrt{3}}{3}} \rightarrow \dfrac{\frac{\sqrt{3}-3}{3}}{\frac{\sqrt{3}+3}{3}} \rightarrow \dfrac{\sqrt{3}-3}{3} \cdot \dfrac{3}{\sqrt{3}+3}$

$\dfrac{3\sqrt{3}-3}{3\sqrt{3}+3} \rightarrow \dfrac{\sqrt{3}-3}{\sqrt{3}+3} \cdot \dfrac{(\sqrt{3}-3)}{(\sqrt{3}-3)} = \dfrac{3-3\sqrt{3}-3\sqrt{3}+9}{3+3\sqrt{3}-3\sqrt{3}-9}$

$\dfrac{3-6\sqrt{3}+9}{-6} \rightarrow \dfrac{12-6\sqrt{3}}{-6} \rightarrow \boxed{-2+\sqrt{3}}$ ← proper answer

2. Express the sum $\sin\frac{7\pi}{12} + \sin\frac{\pi}{12}$ as a product

$\sin u + \sin v = \sin(u+v)$

~~$\sin u \cos v + \cos u \sin v$~~

not sum + diff formula ~~$\sin\left(\frac{7\pi}{12}\right)\cos\left(\frac{\pi}{12}\right) + \cos\left(\frac{7\pi}{12}\right)\sin\left(\frac{\pi}{4}\right)$~~

$\sin\frac{7\pi}{12} = 105^\circ$ $60+45^\circ$ $\quad \sin\frac{\pi}{12} = 15^\circ$ $60-45^\circ$

$u \quad v \qquad u \quad v$

$\sin(u+v) + \sin(u-v) = 2\sin u \sin v$

$\sin\left(\frac{\pi}{3}+\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right)$

$\sin\frac{\pi}{3}\sin\frac{\pi}{4} = \sqrt{6}$

$\boxed{2 \cdot \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6}}{2}}$

3. Find the exact value of $\sin\left(\frac{\pi}{12}\right)$ ← 15°

$60-45$

$\frac{\pi}{3} - \frac{\pi}{2}$

$\sin u-v = \sin u \cos v - \cos u \sin v$

$\sin 60^\circ \cos 45^\circ - \cos 60^\circ \sin 45^\circ$

$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

4. Find exact value of $\sin(u+v)$

$\sin u = \frac{4}{5}$ $\quad 0 < u < \frac{\pi}{2} \rightarrow$ ~~53.13~~ ✓limit

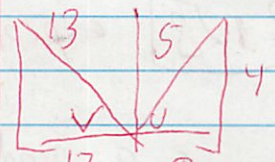
$\cos v = -\frac{12}{13}$ $\quad \frac{\pi}{2} < v < \pi \rightarrow$ ~~157.38~~ ✓limit

$\sin(u+v) = \sin u \cos v + \cos u \sin v$

$\frac{4}{5} \cdot -\frac{12}{13} + \cos(\cancel{53.13})\cancel{\sin(157.38)}$

$(3/5)(5/13)$

Do this



Find $\rightarrow$ 3

Find $\rightarrow$ 5/13

$\leftarrow$ use graph

$-\frac{48}{65} + \frac{15}{65} = -\frac{33}{65}$

5. Evaluate using double angle + half angle

$\cos\theta = \frac{5}{13}$ $\quad \frac{3\pi}{2} < \theta < 2\pi \rightarrow \boxed{4th}$

exact value

$\cos 2\theta = 2\cos^2\theta - 1$

$2(5/13)^2 - 1$

$-.704$

$\frac{2}{1} \cdot \frac{25}{169} - \frac{1}{1} \rightarrow \frac{50}{169} - \frac{169}{169} = \boxed{\frac{-119}{169}}$

$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}} \rightarrow \pm\sqrt{\frac{1+5/13}{2}} \rightarrow \pm\sqrt{\frac{18/13}{2}} \rightarrow \pm\sqrt{\frac{9}{13}}$

$\pm\frac{\sqrt{9}}{\sqrt{13}} \rightarrow \frac{\cdot\sqrt{13}}{\cdot\sqrt{13}} = \pm\frac{3\sqrt{13}}{13}$ $\leftarrow$ 4th quad (+)

6. Find all solutions of for $[0, 2\pi)$

$\sin + \cos = 1$
$-\cos$

$2 - \sin^2 x = 2\cos^2\left(\frac{x}{2}\right)$

$2 - (1 - \cos^2 x) = 2\left(\pm\sqrt{\frac{1+\cos x}{2}}\right)^2$

$2 - 1 + \cos^2 x = 2\left(\frac{1+\cos x}{2}\right)$

$\cos^2 x + 1 = 1 + \cos x$

$\cos^2 x - \cos x + 0 = 0$

factor $x = 0$

$x = 1$

$\cos x = 0$

$\cos^{-1}$ $\quad \cos^{-1}$

$\frac{\pi}{2}, \frac{3\pi}{2}$

$\cos x - 1 = 0$

$\cos x = 1$

$\cos$ $\quad \cos$

$x = 0$

~~2π~~ $\leftarrow$ not included

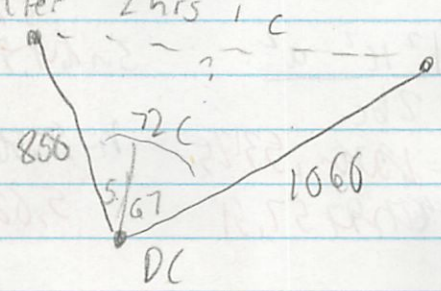
7. Two planes leave DC, at same time

Plane 1 ⇒ 425 mph 355° bearing → ·2 = 850 miles

Plane 2 = 530 mph 67° bearing → ·2 = 1060 miles

Distance after 2 hrs ?

(✓)



$c^2 = a^2 + b^2 - 2ab\cos C$

$850^2 + 1060^2 - 2(850)(1060)\cos 22$

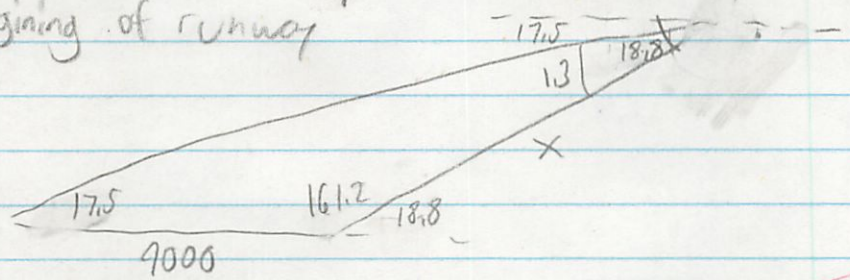
$c^2 = 1289251.376$

1135.45 miles

8. Pilot landing on a 9000 ft runway

Angle of depression at ends of runway = 17.5° + 18.8°

Find air distance plane must travel before touching begining of runway



$\frac{\sin 1.3}{9000} = \frac{\sin 17.5}{x}$ = 119289 feet

Or get height

$\sin 20.5 = \frac{h}{31} = 10.85$

if $h < B < A \Rightarrow$ 2 cases

9. 2 solutions?

a = 12

b = 31

A = 20.5°

B_1/ $\frac{\sin 20.5}{12} = \frac{\sin B}{31}$ $B_1 = 64.78°$

C_1/ ⇒ 180 − 20.5 − 64.78 = 94.72°

Test B_2/ 180 − 64.78 = 115.22

← If another B

$A + (180 - B_1) < 180$

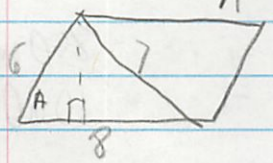
C_2/ 180 − 20.5 − 115.22 = 44.28°

C_1/ $\frac{\sin 20.5}{12} = \frac{\sin 94.72}{c}$ c = 34.15

C_2/ $\frac{\sin(20.5)}{12} = \frac{\sin 44.28}{c}$

c = 23.922

10.



$A = ? = b \cdot h$

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$\cos A = \cancel{1.376}\ .5375$

$A = \cancel{67.97}\ 57.91$

$\sin \cancel{67.97}\ 57.91 = \frac{h}{6}$

$h = \cancel{5.56} = h$

5.08

$A = \cancel{5.56} \cdot 8$

$\cancel{44.50}$

40.66

Formulas

Sum and Difference Identities

(a) $\cos(\alpha \pm \beta) = \cos\alpha \cdot \cos\beta \mp \sin\alpha \cdot \sin\beta$

(b) $\sin(\alpha \pm \beta) = \sin\alpha \cdot \cos\beta \pm \cos\alpha \cdot \sin\beta$

(c) $\tan(\alpha \pm \beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha \cdot \tan\beta}$

Double-Angle Identities

(d) $\sin(2\alpha) = 2 \cdot \sin\alpha \cdot \cos\alpha$

(e) $\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha = 2 \cdot \cos^2\alpha - 1 = 1 - 2 \cdot \sin^2\alpha$

(f) $\tan(2\alpha) = \dfrac{2 \cdot \tan\alpha}{1 - \tan^2\alpha}, \quad \tan\alpha \neq 1$

Half-Angle Identities

(a) $\sin\left(\dfrac{\alpha}{2}\right) = \pm\sqrt{\dfrac{1 - \cos\alpha}{2}}$

(b) $\cos\left(\dfrac{\alpha}{2}\right) = \pm\sqrt{\dfrac{1 + \cos\alpha}{2}}$

(c) $\tan\left(\dfrac{\alpha}{2}\right) = \pm\sqrt{\dfrac{1 - \cos\alpha}{1 + \cos\alpha}} = \dfrac{\sin\alpha}{1 + \cos\alpha}, \quad \cos\alpha \neq -1$

(d) $\tan\left(\dfrac{\alpha}{2}\right) = \dfrac{1 - \cos\alpha}{\sin\alpha}, \quad \sin\alpha \neq 0$

Product/Sum Identities

(a) $2 \cdot \cos\alpha \cdot \cos\beta = \cos(\alpha - \beta) + \cos(\alpha + \beta)$

(b) $2 \cdot \sin\alpha \cdot \sin\beta = \cos(\alpha - \beta) - \cos(\alpha + \beta)$

(c) $2 \cdot \sin\alpha \cdot \cos\beta = \sin(\alpha + \beta) + \sin(\alpha - \beta)$

(d) $2 \cdot \cos\alpha \cdot \sin\beta = \sin(\alpha + \beta) - \sin(\alpha - \beta)$

		sin	cos	tan
$\frac{\pi}{6}$	30	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	45	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	60	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

Formulas

- Law of Sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

- Law of Cosines:

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cdot \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cdot \cos C$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

- "Heading" : Angle made in a clockwise direction from the north
  "Bearing" :

- Area of a Triangle:

$$Area = \frac{1}{2} ab \cdot \sin C$$
$$Area = \frac{1}{2} ac \cdot \sin B$$
$$Area = \frac{1}{2} bc \cdot \sin A$$

$$Area = \frac{1}{2} b \bullet h$$

- Heron's Formula: $K = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{a+b+c}{2}$

- Area of a Sector: $A_{\text{sector}} = \frac{C.A.}{360} \cdot \pi r^2$ or $A_{\text{sector}} = \frac{1}{2} \cdot r^2 \cdot \theta$

Final

Part I EXAM CONCEPTS

Chapters 1, 2, 4, 5, 6

1) Modeling - curve of best fit

2) Functions

Domain, Range, Inverse, Composite Functions

Piecewise Functions

Graphing, translations (shifts), reflections

3) Symmetry

4) Zeros of a function

5) Distance between 2 pts, pt and a line

6) Rational Root Theorem, Factor Theorem

~~Descartes' rule of signs~~

7) Asymptotes

8) Radical Equations

9) Trig (9 questions)

10) Complex numbers

Exam - Part II (Chapts 3, 7, 8, 9)

1) Exponents

2) Logs

3) ~~Polar Equations~~

4) Conic sections

5) Matrices | Systems | Partial Fraction Decomposition

6) Sequences and Limits, Binomial Theorem
series

7) ~~Vectors~~ Parametric Equations

Directions: Welcome to PreCalc POWER. PreCalc POWER is a worksheet that you will help prepare you for the final exam in this course. There will always be 10 questions, and you must try to complete all questions **correctly, with work (when necessary)**. You may use your notes and discuss with others but you may copy from others!! Good Luck!!

$x: x \geq -2$ ↑not = to

1.6

1. Find the inverse of the function $f(x)=\dfrac{1}{\sqrt{x+2}}$. Restrict the domain of the original function, if necessary.

$x = \dfrac{1}{\sqrt{y+2}}$ $\quad y+2 = \left(\dfrac{1}{x}\right)^2$

$x\sqrt{y+2} = 1$ $\quad y = \left(\dfrac{1}{x}\right)^2 - 2$

$\sqrt{y+2} = \dfrac{1}{x}$

Answer: $y = \left(\dfrac{1}{x}\right)^2 - 2$

$x: x \in \mathbb{R}$

2. Find the domain and range of the function $f(x)=\dfrac{3}{x^2-x-20}$. ↓where the horiz asy are

Algebraically Find asymptotes

Domain: $x: x \in \mathbb{R} \neq 5, -4$

Range: $y: \cancel{y \in \mathbb{R}}\ y \neq 0$ ← vert asy

3. Find a real quadratic equation in standard form with real coefficients that has $1+2i$ as a solution.

$1+2i \quad 1-2i$

$[x-(1+2i)][x-(1-2i)]$

$(x-1)+2i \cdot (x-1)-2i$

$x^2-2x+1+(2i \cdot -2i)$ ← $(-4i^2)$

i cancels and (+) $x^2-2x+1+4$

x^2-2x+5

Answer: x^2-2x+5

4. Find the equation of a line parallel to the line $3x-2y+6=0$ and passing through the point (4, 3).

$+2y \quad +2y$

$2y = 3x+6$ $\quad$ Slope $= \dfrac{3}{2}$

$y = \dfrac{3x+6}{2}$ $\quad$ 4, 3 $\quad$ 1, -1.5 $\quad$ 3, 1.5 $\quad$ 0, -3 $\quad$ 2, 0

Answer: $y = \dfrac{3}{2}x - 3$

$0 = 3x - 2y - 6$ ↓ standard form

5. Solve the quadratic equation $8x^2+22x-21=0$ by factoring.

a·c

-168 / 7x

28 / -6

22 / ↓+

b

	$2x$	7
$4x$	$8x^2$	$28x$
-3	$-6x$	-21

$(2x+7)(4x-3)$

$2x = -7$ $\quad$ $4x-3=0$

$x = -\dfrac{7}{2}$ $\quad$ $4x = 3$

$x = \dfrac{3}{4}$

Answer: $-\dfrac{7}{2}, \dfrac{3}{4}$

6. Bo Finkle is at the Phillies game, and he is sitting in the first row of the outfield bleachers (about 26 feet off of the ground). His favorite hitter, Ace Einhorn, has just launched a monster homerun from home plate, and it is heading directly to Bo. If Ace hits the ball at a speed of 105 feet per second and at a height of 3 feet off of the ground, how long will Bo have to react in order to catch the homerun? Use $h(t) = -16t^2 + vt + s$, where *h(t)* is the height of the ball in feet after *t* seconds, *v* is the initial velocity of the ball in feet per second, and *s* is the starting height for the ball in feet.

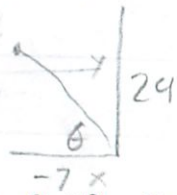
$26 = -16t^2 + 105t + 3$

find intersection

← solve on calc

Answer: 6.33 sec

7. The terminal side of an angle θ in standard position passes through the point (-7, 24). Determine the **exact** values for $\sin\theta$ and $\cos\theta$.



$r^2 = 7^2 + 24^2$

$r = 25$

$\sin = \frac{y}{r}$ $\frac{24}{25}$ $\cos \frac{x}{r} = \frac{-7}{25}$

$\sin\theta = \frac{24}{25}$

$\cos\theta = \frac{-7}{25}$

half

8. For $0 \le \theta < 2\pi$, determine the values for θ when $\sin\theta = -0.4312$.

↓ inverse sin

$-25.54 = \frac{x}{r}$

$+360 = 334.456$

(+) (+) / (−) (−) 3rd

180 + 25.54 → 205.34° or 5.84 r

Answer: 334.456 3.59 r

-25.54

half

9. Determine the zeros of the polynomial function $f(x) = 2x^3 - 17x^2 + 11x + 15$. Show your work! Put in calc to find start

+1.5] 2 −17 11 15

↓ 2 → 3 → −21 → −15

−14 −10 → 0 ✓

$2x^2 - 14x - 10$

roots: ≈7.65

≈.65 → write out + keep roots

$\frac{7 + \sqrt{69}}{2}$

Answer: ~~−.65~~, 1.5, ~~7.65~~

write as roots

10. *Double Bonus:* The dimensions of a rectangular box are given by three consecutive integers, and its volume is 1716 cm³. Determine the dimensions of the box.

2 3 4 ~~5~~ 6 ~~7~~ ~~8~~ 9

↓

858

$429 \cdot 2 \cdot 2 = 1716$

11/12/13

Michael Plasmeier 12/7

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1. Determine all remaining zeros of the following polynomial function given that $-i$ is a zero: $p(x) = x^6 + x^5 - 4x^4 + 2x^3 - 11x^2 + x - 6$.

1st step should be:

$(x-i)(x+i)$ should

$-i$] 1 1 -4 2 -11 1 -6 also

i] 1 $-i$ $-1-i$ $-1+5i$ $5-i$ $-1+6i$ 0

1 $1-i$ $-5-i$ $(1+5i)$ $-6-i$ $(6+i)$ 0

i] 1 i i $-5i$ i $6i$

1 1 -5 1 6 0 ✓

2]

-3]

$x = \pm\sqrt{-1}$

$\pm i$

-3] 1 1 -4 2 -11 1 -6

1 -3 6 -6 12 -3 6

1 -2 2 -4 1 -2 (0)

2] 1 2 0 4 0 2

1 0 2 0 1 0

$x^4 + 2x^2 + 1$

$(x^2+1)(x^2+1)$ → $\pm i$ → $\pm i$ already given 2 -3

Answer: $\cancel{(x-i)(x+3)(x-2)(x^2+1)^2}$

2. Determine the range of the function $f(x) = 1 + \cos x$.

Range $\cos x = (-1, 1)$

$+1$

$\cos x + 1 = (0, 2)$

Answer: $0 \le y \le 2$

3. Write the quadratic function $g(x) = -2x^2 + 12x - 18$ in *vertex form*.

$-2(x^2 - 6x + 9)$ $\left(\frac{b}{2}\right)^2$ $-18 + 18$

$-2(x^2 - 6x + 9)$

$-2(x-3)(x-3)$

Answer: $y = -2(x-3)^2$

4. Solve the polynomial equation $9x^3 - 108x^2 + x - 12 = 0$.

$9x^2(x-12) + (x-12)$

$(9x^2+1)(x-12)$

$\frac{1}{3}i$

$-\frac{1}{3}i$

$x = 12$

Answer: $\frac{1}{3}i, -\frac{1}{3}i, 12$

On Final Review

5. Determine an expression for the distance from the origin to the line $Ax + By = C$

Origin $= (0,0)$

$Ax + By - C = 0$

$\frac{A(x) + B(y) - C}{\sqrt{A^2 + B^2}} \rightarrow \frac{A(0) + B(0) - C}{\sqrt{A^2 + B^2}}$

Domain: $\frac{|C|}{\sqrt{A^2 + B^2}}$

Answer: $\mathbb{R}$

12/7

6. A company wishes to make boxes with volume 48 in.^{3} from sheets of cardboard 8 in. by 10 in. by cutting squares from each corner. What size squares should they cut? What are the dimensions of the boxes?

$6 \cdot 4 \cdot 2$

Answer: $6 \cdot 4 \cdot 2$ in

7. Determine the zeros and the number of relative extrema of following polynomial function: $h(x) = x^4 + x^3 - 12x^2 - 28x - 16$. ← 4 #, so 3 extrema

−1	1	1	−12	−28	−16
	↓	−1	0	12	−16
	1	0	−12	−16	0 ✓
4	↓	4	16	16	
	1	4	4	0 ✓	

x^2+4x+4 $(x+2)(x+2)$

Answer: $(x+1)(x-4)(x+2)^2$

min $(-2,0)(2.59, \infty)$ ← calc

max $(-1.34, .79)$

8. Evaluate $(g \circ f)(2)$ given $f(x) = x^2 + 1$ and $g(x) = 3x - 4$.

$3(x^2+1) - 4$

11

Answer: 11

9. Divide using synthetic division: $(x^4 + 2x^2 - 10) \div (x+1)$. Give your answer as a quotient only.

−1	1	0	2	0	−10
	↓	−1	1	−3	3
	1	−1	3	−3	−7

Answer: $x^3 - x^2 + 3x - 3 - \frac{7}{x+1}$

10. *Double Bonus:* If $p(x) = 3x^3 + 2x^2 + Kx + M$, $p(-3) = -41$, and $p(2) = 29$, find K and M.

$-41 = 3(-3)^3 + 2(-3)^2 + k(-3) + M$

$-81 + 18 + -3k + M$

$22 = -3k + M$

$29 = 3(2)^3 + 2(2)^2 + k(2) + M$

$24 + 8 + 2k + M$

$-3 = 2k + M$

$K =$ −5

$M =$ 7

calc: $\begin{bmatrix} -3 & 1 \\ 2 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 22 \\ -3 \end{bmatrix} = \begin{bmatrix} -5 \\ 7 \end{bmatrix}$

Michael Plasmeier 12/18

PreCalculus **POWER III**

Directions: Directions: Welcome to PreCalc POWER. PreCalc POWER is a worksheet that you will help prepare you for the final exam in this course. There will always be 10 questions, and you must try to complete all questions **correctly, with work (when necessary)**. You may use your notes and discuss with others but you may copy from others!! Good Luck!!

---

1. Express the following in *expanded form*: $\log_b \sqrt[3]{4x^5y^7}$

$\frac{1}{3}\log(4x^5y^7)$

$\frac{1}{3}\log_b 4 + \frac{1}{3}\log_b x^5 \log_b y^7$

Answer: $\frac{1}{3}\log_b 4 + \frac{5}{3}\log_b x + \frac{7}{3}\log_b y$

2. Find all zeros (including complex zeros) of $f(x) = x^4 + x^2 - 22x - 24$. Show all work algebraically! Seed on calc

3] 1 0 1 -22 -24
3 9 30 +24
1 3 10 +8 0

-1] -1 -2 -8
1 2 8 0 x^2+2x+8

$\frac{-2 \pm \sqrt{-28}}{2}$

$\frac{-2 \pm 2\sqrt{7}i}{2}$

$(-1 \pm \sqrt{7})$

Answer: $(0,3)(0,-1)(0,-1+i\sqrt{7})(0,-1-i\sqrt{7})$

3. Determine the asymptotes for each of the following functions:

(a) $f(x) = \dfrac{2x-1}{4x+3}$

horiz as $y = \frac{2}{4} = \frac{1}{2}$

Vert as $y = 4x+3=0$
$4x=-3$
$x=-3/4$

(slant, degree(top) > degree(bottom))

Answer: $y=\frac{1}{2}$, $x=-3/4$

(b) $g(x) = \log_2(2x-3)$

$\log 2^y = 2x-3$

$2x-3=0$ ← find domain (where log(0))

Answer: $x = \frac{3}{2}$

no horiz as, ← for logs

4. Determine the exact value for each of the following:

(a) $\cos^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$

30°

(+) 1st 30° $\frac{\pi}{6}$
(+) 4th 330° $\frac{11\pi}{6}$

Answer: $\frac{\pi}{6}$, ~~$\frac{11\pi}{6}$~~

b/c domain + range $(0, \pi)$
domain $(-1, 1)$

(b) $\arccos\left(\cos\dfrac{5\pi}{3}\right)$

$\cos^{-1}(.5)$

60°

(+) 1st 60° $\frac{\pi}{3}$
(+) 4th 300° $\frac{5\pi}{3}$

Answer: $\frac{\pi}{3}$, ~~$\frac{5\pi}{3}$~~

outside of $(0, \pi)$

5. A communications system is modeled on the coordinate plane. The top of the transmitting tower is located at (-6, -25), and the receiving dish is located at (4, 11). If the signal travels on a straight path from the tower to the dish, find the distance from an antenna located at (9, 5) to the path of transmission.

$\frac{-25-11}{-6-4} = \frac{-36}{-10} = 3.6$

$-3.6x + y + 3.4 = 0$

y intercept

$\frac{\text{abs}(ax+by+c)}{\sqrt{A^2+B^2}}$

$\frac{-3.6x+y+3.4}{\sqrt{(-3)^2+(1)^2}}$

Answer: 6.4

$\frac{|-3.6(9)+(5)+3.4|}{\sqrt{9+1}}$

6.The owner of a small restaurant called, *No Soy Grande,* can prepare a particular gourmet meal at a cost of $10. He estimates that if the menu price of the meal is *x* dollars, then the number of customers who will order that meal at that price in an evening is given by the function $N(x) = 40 - x$. Express his nightly revenue, cost, and profit on this meal as functions of *x*.

multiply it out

Revenue: $N(x)\cdot x \rightarrow 40x - x^2$

Cost: $10x$

Profit: $N(x)x - 10x$

$50x - x^2 - 400$

7.Derive a polynomial function of lowest degree with rational coefficients that has the zeros $2-\sqrt{2}$ and 4.

$(x-4)(x-(2-\sqrt{2}))(x-(2+\sqrt{2}))$

$(x-4)((x-2)^2-2)$

$(x-4)(x^2-4x+4-2)$

$x^3-4x^2+2x-4x^2+16x-8$

Answer $x^3-8x^2+18x-8$

8. Solve the trig equation $2\sin^2 x + 3\cos x - 3 = 0$

$2(1-\cos^2 x)+3\cos x-3$

$2-2\cos^2 x+3\cos x-3$

$-2\cos^2 x+3\cos x-1$

	$-x$	$+1$
$2x$	$-2x^2$	2
-1	1	-1

$(-\sin x+1)(2\sin x-1)$

$-\sin x=-1$

$\sin x=1$

$90°, \frac{\pi}{2}$

$2\sin x=1$

$\sin x=\frac{1}{2}$

$30°$

$150°$

$60°$

$+2n\pi$

no restriction

Answer $\frac{\pi}{3}, \frac{5\pi}{3}$

9. *Solve* $\cot x \cos^2 x = 2\cot x$

$\cot x\cos^2 x - 2\cot x$

$\cot x(\cos^2 x - 2)$

$\cot x = 0$

$\tan x = \frac{1}{0}$

und.

$\cos x = 0$

$\sin x$

$\cos^2 x = 2$

$\cos x = \pm\sqrt{2}$ und

$\frac{\pi}{2}, \frac{3\pi}{2}$

Answer ~~Undefined~~

$\frac{\pi}{2}+n\pi$

10.*Double Bonus:* What is the number of factors when $x^{13} - x$ is factored completely into polynomials and monomials with real integral coefficients? To prove your answer, list all of the factors.

7 factors

Sum of cubes → factoring pattern x^3+y^3 $(x+y)(x^2-xy+y^2)$

difference of cubes → $x^3-y^3=(x-y)(x^2+xy+y^2)$

$x(x^{12}-1)$

$x(x^6-1)(x^6+1)$

$x(x^3-1)(x^3+1)(x^6+1)$

$x(x-1)(x^2+x+1)(x+1)(x^2-x+1)$

$x(x-1)(x^2+x+1)(x+1)(x^2-x+1)(x^2+1)(x^4-x^2+1)$

1 2 3 4 5 6 7

7

Directions: Directions: Directions: Welcome to PreCalc POWER. PreCalc POWER is a worksheet that you will help prepare you for the final exam in this course. There will always be 10 questions, and you must try to complete all questions **correctly, with work (when necessary).** You may use your notes and discuss with others but you may copy from others!! Good Luck!!

4.5 1. State the domain and range of the function $f(x) = \overset{a}{3} \cdot \cos(\overset{b}{2}x) + \overset{d}{2}$.

Domain: $-a, a$

$-1, 1$ *Range:* $(1, 3)$

2. Decompose the following into partial fractions: $\dfrac{6x^2 + 25x - 24}{x^3 + x^2 - 12x}$.

1 $x(x^2 + x - 12)$

Answer: _____

3. Find the equation of the slant asymptote of the graph of this rational function

$$\frac{2x^3 + 5x^2 - 23x - 14}{x^2 - 2x - 3}$$

$x^2 - 2x - 3 \overline{)\, 2x^3 + 5x^2 - 23x - 14}$ quotient $2x - 9$

$-(2x^3 - 4x^2 - 6x)$

$9x^2 - 17x - 14$

$-(9x^2 + 18x + 27)$

$-35x - 41$

$2x - 9 - \dfrac{35x - 41}{x^2 - 2x - 3}$

Answer: _____

4. Prove the following trig identity: $\cos^2\theta - \sin^2\theta = \dfrac{1 - \tan^2\theta}{1 + \tan^2\theta}$. Show all work.

1 $(1 - \sin^2 x) - \sin^2 x$

$1 - 2\sin^2 x$

$\dfrac{\csc^2 x - 2}{\csc 2x}$

$\dfrac{\csc^2 x}{\csc^2 x} - \dfrac{2}{\csc^2 x}$

Answer: _____

5. Find the real zeros, algebraically, of the following polynomial function $f(x) = x^4 + x^3 - x - 1$.

1 | 1 1 0 −1 −1
↓ 1 2 2 1
1 2 2 1 0 ✓

$\dfrac{-1 \pm \sqrt{-3}}{2}$

$\dfrac{-1 \pm \sqrt{3}i}{2}$

Answer: $x = 1,\ x = -1,\ x = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

−1 | 1 −1 −1 −1
1 1 1 0 ✓
$x^2 + x + 1$

6. Divide $\frac{6-i}{8+3i}$ and write the result in standard form.

$\frac{6-i}{8+3i} \times \frac{(8-3i)}{(8-3i)}$ $\frac{48-8i-18i+3i^2}{64-24i+24i^3-9i^2}$ Answer: $\frac{45}{73} - \frac{26i}{73}$

$\frac{3i^2-26i+48}{-9i^2+64}$ $\frac{-3-26i+48}{9+64}$ $\frac{45-26i}{73}$ $\frac{45}{73} - \frac{26i}{73}$

7. Expand the following binomial $(3x^2-4y^3)^5$. Show all steps.

$(3x^2)^5 + 5(3x^2)^4(-4y^3) + 10(3x^2)^3(-4y^3)^2 + 10(3x^2)^2(-4y^3)^3 + 5(3x^2)(-4y^3)^4 + (-4y^3)^5$

$243x^{10} + 5 \cdot 81x^8 \cdot -4y^3 + 10 \cdot 27x^6 \cdot 16y^6 + 10 \cdot 9x^4 \cdot -64y^9 +$
$+ 5 \cdot 3x^2 \cdot 256 \cdot y^{12} - 1024 \cdot y^{15}$

Answer: 2

$243x^{10} - 1620x^8y^3 + 4320x^6y^6 + 5760x^4y^9 + 3840x^2y^{12} - 1024y^{15}$

8. If $f(x) = 3x^2$ and $g(x) = 7 - x$, determine $(f \circ g)(-x-4)$.

$7-(-x-4)$ $3(x+35)^2$

$7+x+28$ $3(x^2+35x+35x+1225)$

$x+35$ $3x^2+210x+3675$

Answer: $3x^2+210x+3675$

9. Determine the domain of the logarithmic equation $\log_2(x-4) + \log_2(5) = 3$, and then solve for **x**.

$x>4$ $5>0$

$\log_2((x-4)(5)) = 3$

$\log_2(5x-20) = 3$

$2^3 = 5x-20$ $28 = 5x$

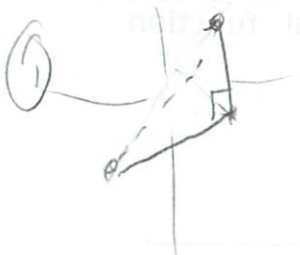
$8 = 5x-20$ $x = 5.6$

$+20$ $+20$

Domain: $x>4$

Answer: $x = 5.6$

10. *Double-Bonus:* Given *E*(3, 4) and *F*(-3, -4), write an equation in terms of *x* and *y* for all points *P*(*x*, *y*) such that segment *PE* is perpendicular to segment *PF*. Simplify the equation and explain your result.

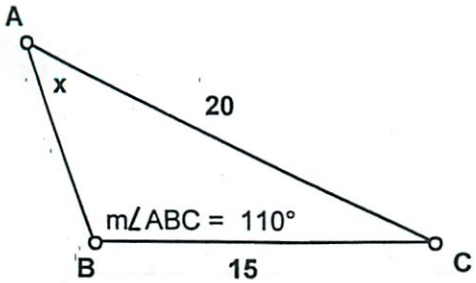


Michael Plasmeier

Directions: Welcome to PreCalc POWER. This POWER is worth **20 points**, with each question being worth **2 points**. Read each question carefully and show all work necessary. Good Luck.

NAME:_____

1. Solve the triangle below for *x*.



Answer: 44.81°

$\frac{\sin 110}{20} = \frac{\sin x}{15}$ $20\sin x = 14.10$ 44.81°

2. Determine the period and phase shift of the function $f(x) = 2 \cdot \tan\left(-\frac{3}{4}x - 5\right)$.

$y = ax$

$\frac{2\pi}{b}$ sin cos

Period: π always $\frac{\pi}{b}$

Phase Shift: $\frac{|c|}{b}$ $\frac{5}{.75} = -\frac{20}{3}$

3. Find the inverse of the function $f(x) = x^2 - 6x + 9$. Restrict the domain of the original function, if necessary, in order to make the inverse a function.

$x = y^2 - 6y + 9$

$x = (y-3)^2$

$\sqrt{x} = y - 3$

$x > 0$

Answer: $\sqrt{x} + 3$

4. Find the general equation of a line that is perpendicular to $4x - 9y - 4 = 0$ and passes through the point (9, -2).

$y - y_1 = m(x - x_1)$

$y - -2 = \frac{9}{4}(x - 9)$

$y + 2 = \frac{9}{4}x + 20.25$

$4y = -9x + 73$

$Ax + By + C = 0$

right angle

negitive reciprical

Solve for y

$4y - 4 = 9y$

$\frac{4x - 4}{9} = y$

$-\frac{4}{9} = m$

Answer: $9x + 4y - 73 = 0$

5. Solve the exponential equation $x^2e^{x+1} + 4xe^{x+1} - 5e^{x+1} = 0$.

$y = \log_a x$

$x = a^y$

$e^{x+1}(x^2 + 4x - 5)$

$x + 1 = e^0$

-5

Answer: $x = 1$

no solution

$x = -5$

6. Sketch the graph of $f(x) = -3\csc(\frac{4}{5}x)$ Label your axes and scale completely.

Sketch:

7. Solve the equation $\sqrt{x+30} = x+10$.

$x+30 = x^2+100$

$x^2 - x - 70 = 0$

Answer: _____

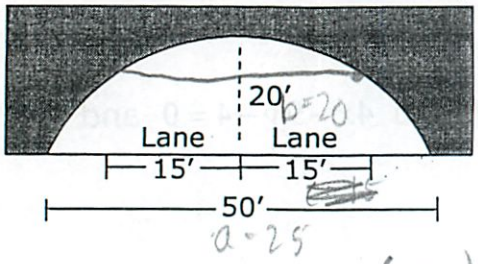
8. Determine all remaining zeros of the polynomial $f(x) = 3x^4 - 16x^3 + 24x^2 + 44x - 39$ given $3+2i$ is a zero.

$3+2i$] 3 -16 24 44 -39

3

Answer: _____

9. A bridge has a parabolic arch that is 20 feet high in the center and 50 feet wide at the bottom. If a two-lane highway passes underneath the bridge, and the end of each lane is 15 feet from the center of the arch, how much clearance would a truck or large vehicle have for passing under the bridge (assuming that the driver of the vehicle obeys the law and stays in his or her proper driving lane)?



Answer: 13.28 feet

$\frac{x^2}{25^2} + \frac{y^2}{20^2}$

$(15, y)$

$\frac{15^2}{25^2} + \frac{y^2}{20^2} = 1$

$14375 = \frac{y^2}{20^2}$

$\sqrt{175}$

13.28

10. Double-Bonus: Determine the **two** equations, *in conic form (Chapter 9 Style)*, for a parabola containing the points (1, 3), (-1, 9), and (5, 15).

Equation #1: _____

Equation #2: _____

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1. Determine the area of the segment of the circle with arc degree measure 80° and radius 26 cm.

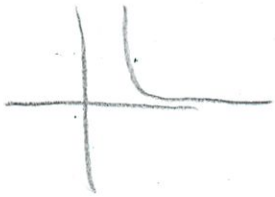
$A = \frac{\theta r^2}{2}$ $\frac{(26)^2}{2} =$

$80° \to 1.39r$

$s = r\theta$ r = radians

Answer: 471.93 cm²

2. Determine the domain and range of the function $f(x) = \frac{3}{\sqrt{x-2}}$.



Domain: $x > 2$

Range: $y > 0$

3. What is the equation of the function that results when $f(x) = \log_7 x$ is reflected through the line $y = x$?

$x = 7^{f(x)}$

Answer: _____

4. Determine the rational zeros of the function $f(x) = x^3 - 3x^2 - 6x + 8$. Show all of your work.

1 | 1 -3 -6 8
↓ 1 -2 -8
1 -2 -8 0 ✓
$x^2 - 2x - 8$
$x = 4$
$x = -2$

Answer: 1, 4, -2

5. Simplify the expression : $\left(\frac{x^{\frac{11}{2}} \cdot y^{\frac{-1}{2}}}{x^{\frac{2}{3}} \cdot y^{\frac{5}{3}}}\right)^5$.

$\frac{x^{\frac{11}{2} \cdot 5} \cdot y^{\frac{-1}{2} \cdot 5}}{x^{\frac{2}{3} \cdot 5} \cdot y^{\frac{5}{3} \cdot 5}} \to \frac{x^{\frac{55}{2}} \cdot y^{-\frac{5}{2}}}{x^{\frac{10}{3}} \cdot y^{\frac{25}{3}}}$ $x^{\frac{145}{6}} \cdot y^{-\frac{125}{6}}$

Answer: $x^{\frac{145}{6}} \cdot y^{-\frac{125}{6}}$

6. Determine the distance between the graphs of the equations $2x-5y=9$ and $-10x+25y=14$.

$+10x \quad +10x$

$25y = 10x + 14$

$y = 2/5x + 14/25$

$-2x$

$-5y = -2x + 9$

$y = \frac{2}{5}x - 9/5$

$-\frac{9}{5} - \frac{14}{25}$

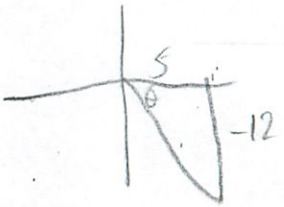
$-59/25$

Answer: _____

7. If $\tan\theta = -\frac{12}{5}$ and θ is in Quadrant IV, find the exact value of $\cos\theta$.

$\sqrt{5^2+12^2}$

13



$\cos = \frac{5}{13}$

Answer: $\frac{5}{13}$

8. Determine the exact value of $\sin\left(\tan^{-1}\sqrt{3}\right)$.

$\sin(60^\circ)$

$\frac{\sqrt{3}}{2}$

Answer: $\frac{\sqrt{3}}{2}$

9. Express $\log_b\left(\frac{\sqrt[x]{mt}}{w^k}\right)$ in expanded form.

$\log_a a^x = x$

$a^{\log_a x} = x$

$y = \log_3 1$

$3^y = 1$

$\log_b\left(\sqrt[x]{mt}\right) - \log_b\left(w^k\right)$

$\frac{1}{x}\log_b(mt) - k\log_b w$

$\frac{1}{x}\left[\log_b(m) + \log_b(t)\right] - k\log_b w$

Answer: _____

10. *Double-Bonus:* In ΔABC, $\angle A = 120^\circ$, $c+a=21$, and $b+a=20$. Find *a*. (Don't cry; you need to know the law if you want this one).

Answer: _____

Directions: Welcome to PreCalculus POWER. PreCalculus POWER is a worksheet that you will complete once a week to prepare you for the final exam in this course. There will always be 10 questions, and you must complete all questions **correctly, with work (when necessary)**. IA POWER will always be worth **20 points**, with each question being worth **2 points**. Finally, some weeks there will be a final question worth double the points if you get it correct (but not a double loss of points if wrong). Good Luck..

1. Find the inverse of the function $f(x)=\frac{\sqrt{x}}{2}$. Restrict the domain of the original function, if necessary.

$x=\frac{\sqrt{y}}{2}$

$2x=\sqrt{y}$

$(2x)^2=y$

Answer: $y=4x^2$

2. The relation f is : $\{(2,-3),(5,-4),(8,-2)\}$. Give the domain and determine what the inverse of this relation is. In Out

input in stat + find function

$y=.164x^2-1.5x-.667$

Domain: _____

Inverse: _____

3. Find the minor M_{32} of matrix A: $A=\begin{bmatrix} 7 & -1 & -2 \\ -4 & 8 & 1 \\ 9 & 5 & 6 \end{bmatrix}$.

determinent of only certain columns

row 3 and col 2 removed

Answer: -1

4. Find the equation of a line perpendicular to the line $3x-2y+6=0$ and passing through the point (4, 3).

$y-3=-\frac{2}{3}(x-4)$

$y-3=-2/3x+8/3$

$y=-2/3x\ 17/3$

$3x+6=2y$

$\frac{3x+6}{2}=y$

$\frac{3}{2}x$

Answer: _____

5. Given $1^3+2^3+3^3+4^3+$....., give the sigma notation for the first 20 terms of the series.

Answer: $\sum_{i=1}^{20} i^3$

6. Find the limit of the sequence $a_n = \dfrac{2n^2+1}{5n^2-3n+5}$.

Answer: $= \frac{2}{5}$

7. Determine whether this graph $y=-\sqrt{16-x^2}$ is symmetric with respect to the x-axis, y axis , origin or none of these.

calc 

Answer: y axis

8. What is the sixth term in the expansion of $(3x+5y)^7$.

$_7C_6 = 7$

$7(3x)^1(5y)^6$

$7 \cdot 3 \cdot x \cdot 30 \cdot y^6$

$630xy^6$

Answer: $630xy^6$

9. A ball is dropped from a height of 36 meters.Each time it strikes the ground it rebounds to a height of 70% of the distance it fell. Find the total distance the ball travels.

$y = 36 + 2(.7)36 + 2(.7)^2 36 + 2(.7)^3 36$

$6 + 2\left(\frac{a_1}{1-r}\right)$ $6 + 2\left(\frac{36}{1-.7}\right)$

Answer: 77.4

*10.DoubleBonus:*Determine the remainder on dividing $x^8+x^6+x^4+x^2+x+1$ by $x-1$. Show all work!!

$x^7 - x^6 + 2x^5 - 2x^4 + 3x^3 - 3x^2 + 4x + 5$

$x-1 \overline{)\, x^8 + x^6 + x^4 + x^2 + x + 1}$

$-(x^8 - x^7)$

$-x^7 + x^6 + x^4 + x^2 + x + 1$

$-(-x^7 + x^6)$

$2x^6 + x^4 + x^2 + x + 1$

$-(2x^6 - 2x^5)$

$-2x^5 + x^4 + x^2 + x + 1$

$-(-2x^5 + 2x^4)$

$3x^4 + x^2 + x + 1$

$-(3x^4 - 3x^3)$

$-3x^3 + x^2 + x + 1$

$-(-3x^3 + 3x^2)$

$4x^2 + x + 1$

$-(4x^2 - 4x)$

$5x + 1$

$-(5x - 5)$

6

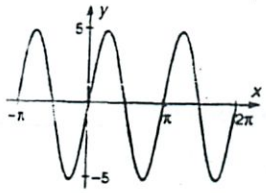
$+ \dfrac{6}{x^8+x^6+x^4+x^2+x+1}$

SHOW ALL WORK! 20 points NAME: _____

1. Determine a function represented by this graph:

(T5 4-2)

amp = 4
pd = $\frac{2\pi}{6}$



Answer: $y = 5\sin\left(\frac{1}{2}x\right)$

2. A record store sells CDs at $9.50 each. For a CD on the top of the chart, the store expects to sell 150 CDs per week. For each 50 cent decrease in the price, the store will sell approximately 20 more CDs per week. The store wants to make a profit of $1050. If each CD costs the $3.00, determine the price at which they should sell the CDs.

price · people

(9.50 − .50(x)) · (150 + 20(x)) = 1050

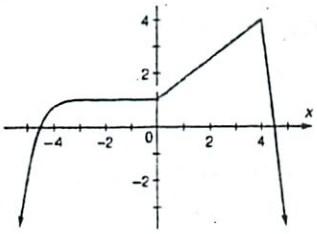
975 + 130x − 75x − 10x² = 1050

−10x² + 55x − 75 factor

Answer: $8.25 or A8

3. From this graph, determine the open intervals for which f(x) is increasing, decreasing, or constant.

format



Increasing: A ∈ x < −3, A ∈ 0 < x < 4
Decreasing: A ∈ x > 4
Constant: A ∈ −3 < x < 0

4. Determine k so that $h(x) = 2x^3 + 5x^2 + \boxed{kx} - 16$ has $x - 2$ as a factor.

2 | 2 5 k −16
↓ 4 18 36+2k
2 9 18+k 20+2k−16

Answer: _____

5. Solve this equation $4^{2x-1} = 5^{x+2}$.

$\log_{10}(4^{2x-1}) = \log_{10}(5^{x+2})$

$2x - 1(\log 4) = x + 2(\log 5)$

Answer: _____

6. In a geometric series if $|r|<1$ the series converges (has a definite limit or sum). If $|r|>1$ the geometric series diverges or has no sum. Does this series $6+2+\frac{2}{3}+\frac{2}{9}+.....$ converge or diverge. If it converges find the sum. Please show all work.

$\frac{2}{15}+\frac{2}{1}+\frac{2}{3}+\frac{2}{9}+\frac{2}{\infty}$ $\quad \frac{a_1}{1-r} = \frac{6}{1-\frac{1}{3}} \rightarrow$

$r=\frac{1}{3}$ Converge

Answer: 9

7. Find all solutions of sin 5x + sin 3x =0 in the interval $[0,2\pi]$.

Calc
Zoom trig
find 0s

? algebra

Answer: 0, 45°, 90°, 135°, 180°, 225°, 270, 315°

8.Simplify this expression $\frac{(xy)^{-2}(yz)^{-3}}{(xz)^{3}(zw)^{-2}}^{\frac{-1}{2}}$. Show all work!

??

Answer: _____

9.Determine all asymptotes of this function $f(x)=\frac{2x^2-3x}{x+1}$. Show your work!

(2.6)

horiz asy → degree (top) < bottom
$y=0$
degree = → leading coefficients
* top > bottom → none

vert asy = $x+1=0$
$x=-1$

Slant $2x-5$
$x+1\overline{)2x^2-3x}$
$-(2x^2+2x)$
$-5x+0$
$-(-5x-5)$
5 ← remainder

Answer: $x=-1$, $y=2x-5$

10.*Solve this equation* $665,400=560,774e^{7r}$ *for r. Show all work.*

Answer: _____

Final Review

Extra Problem

1/18

1. $x - 2y + 3z = 9$
$-x + 3y - z = -6$
$2x - 5y + 5z = 17$

Calc

$$\begin{bmatrix} 1 & -2 & 3 \\ -1 & 3 & -1 \\ 2 & -5 & 5 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ -6 \\ 17 \end{bmatrix}$$

$$[A]^{-1}[B]$$