14.01 PSet 3 #5 Redo

10/2

Redo for pratice

1. S = Software = Q_S $\quad U=(S,C) = 4\ln(S) + 6\ln(C)$
   C = Clothes = Q_C

copy error

a) $MRS = -\frac{MU_S}{MU_C} = \frac{\partial S / \partial U}{\partial C / \partial U} = \frac{\frac{4}{S} \cancel{+ 6\ln(C)}}{\frac{6}{C} \cancel{+ 4\ln(S)}} = -\frac{4}{S} \cdot \frac{C}{6} = -\frac{4}{6}\frac{C}{S}$ ✓

↑ just that
—remember rules

b) Decreasing as always → tradeoff
2nd less than 1st ...

b) Demand

$MRS = MRT$

$-\frac{4}{6}\frac{C}{S} = -\frac{P_S}{P_C} \rightarrow \frac{4}{6}\frac{C}{S} = \frac{P_S}{P_C}$ ✓

Solve for C

$\frac{6}{4}\frac{P_S}{P_C} = \frac{C}{S}$

$C = \frac{6}{4} S \frac{P_S}{P_C}$ ✓

Plug into constraint

$I = \frac{6}{4} S \frac{P_S}{\cancel{P_C}} \cdot \cancel{P_C} + S P_S$

$I = \left(\frac{6}{4} + 1\right) S P_S = \frac{10}{4} S P_S$

(2)

now what?

- solve for S in terms of P_s, I

$$I = \frac{10}{4} S P_s$$

~~the~~ $S = \frac{4}{10} \frac{I}{P_s}$ ✓ yeah thats it ~~seems~~s weird but correct

Now for C

$$\cancel{\frac{4}{6} \frac{C}{S} = \frac{P_s}{P_c}}$$

$$\cancel{\frac{6}{4} S \frac{P_s}{P_c} = C}$$ did that

So I guess instead

$$\frac{6}{4} \frac{S}{C} = \frac{P_c}{P_s}$$

$$\frac{4}{6} C \frac{P_c}{P_s} = S$$

$$I = \frac{4}{6} C \frac{P_c}{\cancel{P_s}} \cancel{P_s} + C P_c$$

$$I = \left(\frac{4}{6} + 1\right) C P_c$$

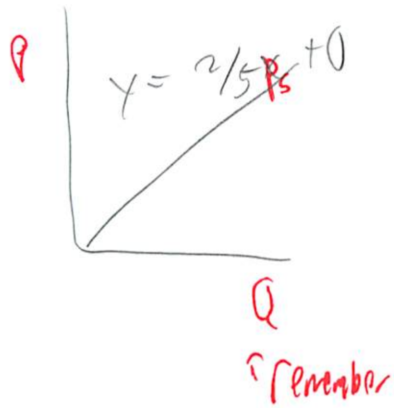
$$I = \frac{10}{6} C P_c$$

solve for C

$$\frac{6}{10} \frac{I}{P_c} = C$$ ✓

③

c) Engall Curve for S



$S = \frac{4}{10}\frac{I}{P_s}$

d) Now plug in and maximize utility

$S = \frac{4}{10}\frac{I}{P_s} = S = \frac{4}{\cancel{10}}\frac{\cancel{10}}{2} = 2$ ✓

$C = \frac{6}{10}\frac{I}{P_c} = C = \frac{6}{\cancel{10}}\frac{\cancel{10}}{3} = 2$ ✓

$U = 4\ln(2) + 6\ln(2) = 10\ln(2) \approx 6.93$

- but how do we know that is max utility?

$2 \cdot 2 + 3 \cdot 2 = 10$

- does happen to meet incom
- some thing about our demand function taking income into effect - what is it on graph?
  - is it even on the graph?

(4)

e) Suppose $p_s = 4$

$S = \frac{4}{10} \cdot \frac{10}{4} = 1$ ✓

$C = \frac{6}{10} \cdot \frac{10}{3} = 2$ ✓

tangent) Compute price elasticity of each

$\frac{dq}{dp} \cdot \frac{p}{q}$ → but can't w/ demand function? What would I need

No $Q_0 = A \cdot p^{\alpha}$ ← think of definition

$\frac{2}{5} I \cdot p^{-1}$

elasticity $= -1$

linear demand curve
?? could not find

$$\boxed{\begin{aligned} Q &= a - bp \\ \epsilon &= -b \frac{p}{q} \end{aligned}}$$

← different format

f) Given price increase, what income for same utility

old utility: $4 \ln(2) + 6 \ln(2) = 10 \ln(2) \approx 6.93$

? calced already

utility w/ price increased
- don't really need

$4 \ln(1) + 6 \ln(2) = 6 \ln(2) \approx 4.15$

5

So need

$$6.93 = 4\ln\left(\frac{4}{10}\frac{i}{4}\right) + 6\ln\left(\frac{6}{10}\frac{i}{3}\right)$$

Solve for i on calc

$i = 13.193$ ✓

Don't forget to answer how many that is!

$S = \frac{4}{10}\frac{13.193}{4} = 1.31$ ✓

$C = \frac{6}{10}\frac{13.193}{3} = 2.63$ ✓

g) Now $I = 10$

- decompose substitution + income effects
- this section need to peek - don't understand what he did
- so this is when the price rose $2 \to 4$ for software

Software

substitution

keeping utility constant, what does change in price do to Qs demanded?

- oh above ans does play a part

~~$1 \to 1.31 = .31$~~

new price old income bad for now

$2 \to 1.31 = -.68$

old price

new price util constant

flip

Income

Total effect - substitution

$(P=2)$ $- -.68 = -.32$

(6)

Clothes try on own

Substitution flip

old price
old income - new price util constant =

2 − 2.63

+.63

income

(new price
old income − old price
old income) − sub effect

2 − 2 − .63

−.63 ✓

So

Substitution

new price
util constant
#3 − Old price
Old income
#1

income

(new price
old income
#2 − Old price
Old income
#1) − Sub effect

#2 redo

- don't have ans sheet, but oh well

a) Same as Before

$$\epsilon_{qa,p} = \frac{\partial q}{\partial p} \frac{p}{q}$$

$$Q = p^{\alpha} f_a(I_a)$$ ← simplified

$$\epsilon = \alpha \frac{p}{q}$$

← oh wow did complex process to find 1st time, but got it

b) if $\epsilon_d < 0$ → not giffen β can find

$\epsilon_d > 0$ giffen α

c) inferior, superior? "normal"

α giffen alway inferior

β not enough info

income $\begin{cases} \frac{dx}{dI} > 0 & \text{normal} \\ \frac{dx}{dI} < 0 & \text{inferior} \end{cases}$

Pratice Exam

10/4

True/False/Uncertain (Total: 20 points)

1. Explain whether each of the following statements is True, False or Uncertain, and provide an explanation. (Note: You will not get points for a correct answer without an explanation.)

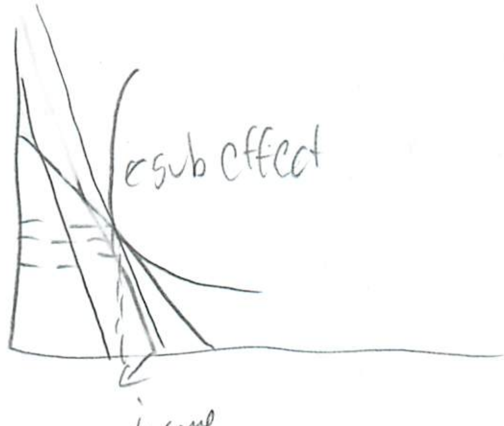
a. (*5 points*) For an inferior good, a price increase has an income effect of opposite sign but greater magnitude than the substitution effect.

- inferior
  - ~~XXX~~ as income ↑ q ↓
- but what happens w/ income effect
- and no wait income < sub = griffin NO

↑ but can be both inferior + griffin

- but can I draw it ?
  - still kinda fuzzy

- (I was thinking kinda of no goods being griffin)



just opposed sign

- ~~X~~ but why ?

normal - both -

↑ and can I do # wise ?

ambigious which is bigger

b. (*5 points*) A rational consumer can choose an optimal bundle of apples (A) and oranges(O) that has $MRS > \frac{P_A}{P_O}$.

MRS = MRT

↑

$\boxed{\text{False}}$

~~don't want~~

~~must be in line~~ w/ prices.

true

corner solution

↑make sure check *

(how?)

MRS ≥ MRT

↑corner

- when 0 of 1 good is purchased

c. (*5 points*) A risk averse person will never pay \$10 for a lottery ticket that pays \$95,000 with probability 0.0001 and nothing otherwise.

$\frac{1}{prob}$

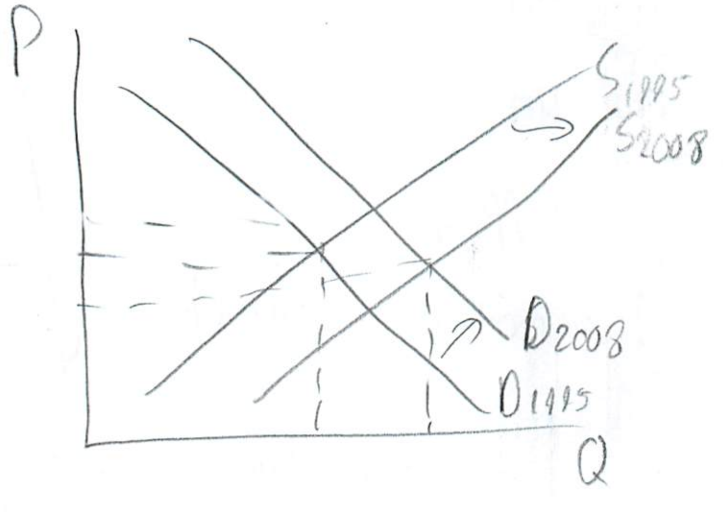
$\frac{10}{95000}$

True - its a bad deal ✓
No one would do it
Supposedly
Everyone is rational

explain

expected value < cost of ticket

d. (5 points) Consider the market for cell phones in 1995 vs. 2008. Take as fact that the income of consumers has increased during this period and cell phones have become cheaper to produce. These two changes imply that the equilibrium price for cell phones has decreased.



False

Demand ↑ would ↑ price
Suppy ↑ would ↓ price) unsure

relative of each

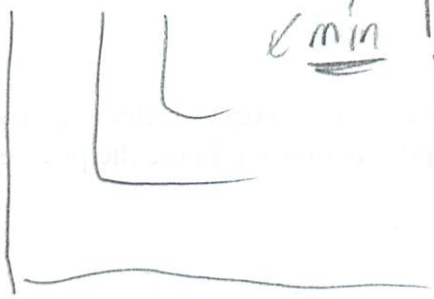
like I said
- they should accept False

$$\frac{d\ \min(x, 2y)}{dx} = \begin{cases} 1 & x - 2y < 0 \\ 2y' & \text{true} \end{cases}$$) don't this problem requires

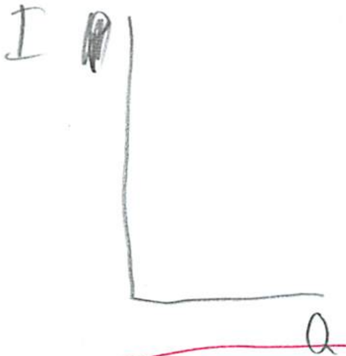
b. (7 points) George has preferences for coffee (C) and teaspoons of sugar (S) according to the utility function

u(C, S) = 2* min{2C, S} + 10

Prices of coffee and teaspoons of sugar are $P_C = 2$, $P_S = 0.5$. Find the equations for the Engel curve for each good.

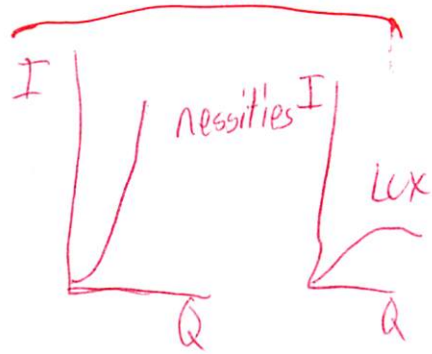


← min ! (Study from P-set)



← ?engall curve what is it again

not sure actually
- not in this class



MRS = MRT ← look what data you have

$$-\frac{\frac{\partial c}{\partial u}}{\frac{\partial s}{\partial u}} = -\frac{P_c}{P_s} = +\frac{2}{.5} = \frac{\cancel{2} \min 2C}{\cancel{2} \min S}$$

← what is derv of min function ??? - never came across before

ans no help

- Slope of it is income elasticity of demand

So $\frac{\Delta Q}{\Delta I}$

$$\boxed{C = \frac{I}{3} \quad S = \frac{2I}{3}}$$

- ? so calc utility now

↓ new sheet

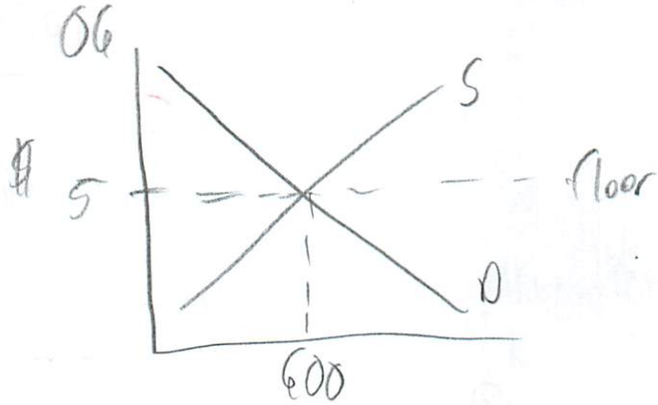
2. Short Questions (Total: 28 points)

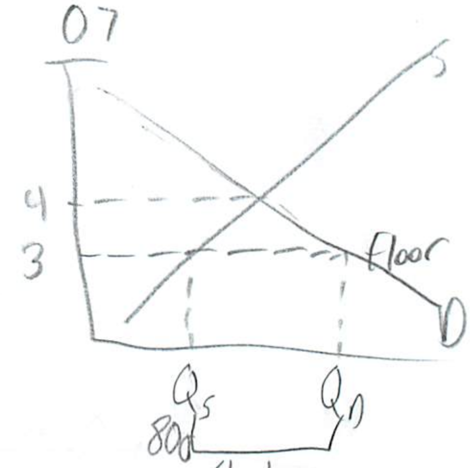
a. (7 points) Between 2006 and 2007, the government changed its price floor on cars. The table below shows the change in the price floor as well as the observed quantity and market price in each year.

Year	Price Floor	Price	Quantity
2006	5	5	600
2007	3	4	800

6 400
7 200
8 0

Assume that neither the demand curve nor the supply curve shifted and that the market demand is linear. Derive the market demand function. Compute the price elasticity of demand in each year.





↑ Shortage
assuming this is Q supplied not Q equalibrium or Q Demanded
-would ask in exam

This calculating is hard part
- did 1x in recitation
- MRS = MRT
(no trade off

then

$$\frac{\Delta Q}{\Delta P} = \frac{-200}{1}$$

(but q is diff here

at each pt

$$\varepsilon = \frac{P06}{Q06}\frac{\Delta Q}{\Delta P} = \frac{5}{600}\cdot -200 = -1.67$$

$$\frac{P08}{Q08}\frac{\Delta Q}{\Delta P} = \frac{4}{800}\cdot -200 = -1$$

$$Q = 200(8-P)$$

$$1600 - 200P$$

(is it right?

(took me some time to rederive

Current util

- but don't know q_c q_s

- let me think through

- we know this is $\frac{\Delta Q}{\Delta I}$

  - or rise over run $\frac{\Delta I}{\Delta Q}$

    or $I = \underline{a} Q + b$

Skip for now

3rd b/

MRS = MRt

$$\frac{\frac{\partial U}{\partial C}}{\frac{\partial U}{\partial S}} \quad \frac{4}{2} = 4$$

- but what does that tell you?
- need variable

b) Matt w/ ans

$2C = S$

↑

know

$P_c C + P_s S = I$. ← but in constraint + solve

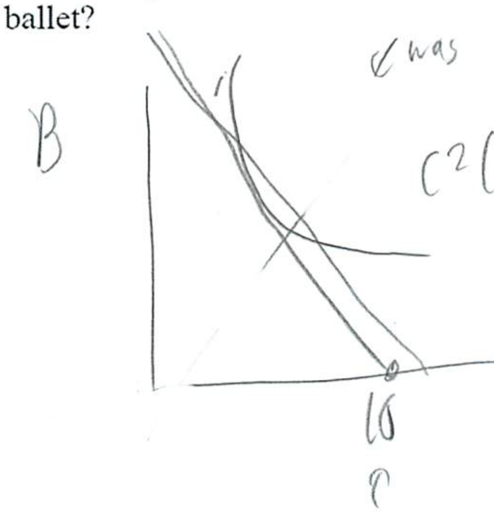
$P_c C + P_s 2C = I$

$C = \frac{I}{P_c + 2P_s} = \frac{I}{3}$

$P_c\left(\frac{S}{2}\right) + P_s S = I$

$S = \frac{I}{\frac{P_c}{2} + P_s} = \frac{2I}{3}$

c. (7 points) Jess has $100 to spend on entertainment this semester. He only cares about going to either concerts (C) or the ballet (B). Jess' utility function over concerts and the ballet is given by: $u(C, B) = C^2(B+2)$. Concert tickets cost $10 each. What is the minimum price of ballet tickets such that Jess will optimally choose to never go to the ballet?



← was accurate, but partial

$C^2(B+2)$

C eq but $\frac{100}{10}$

$U = 10^2(0+2) = 200$

- so what q of concerts is U also = 200
- and know where $10C + XB = 100$

$200 = C^2(B+2)$ ← 2 eq, 2 unknowns

$100 = C + XB$

$C = 100 - XB$

$200 = (100 - XB)\, B + 2$

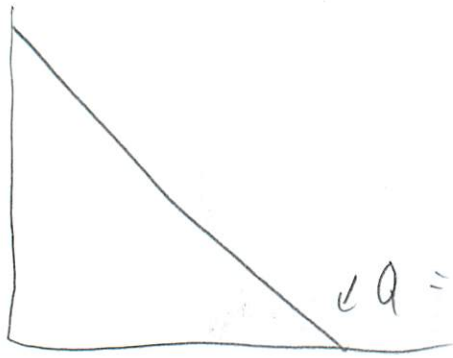
← led 2 unknowns

oh duh never go to ballet

$200 = (100 - X0)(0+2)$

$100 \cdot 2$ ← would be anything!

d. (7 points) Market demand for textbooks is given by $Q = 10 + \frac{600}{P^2}$. What is the gain in consumer surplus from an effective price ceiling on textbooks that reduces prices from \$60 to \$30?



← $Q = 10 + \frac{600}{P^2}$

Market $p = 60$

" $q = 10 + \frac{600}{60^2}$

$60^2 = 60 \times 60 = 3600$

$10 + \frac{600}{3600}$

$10 + \frac{1}{6}$

Old revenue $= 60 \cdot \left(10 + \frac{1}{6}\right)$

610

new revenue $30 \cdot \left(10 + \frac{1}{6}\right)$ ← assuming same supply

305

Close

\$305

← hmm I don't know what I did wrong

they actually did it completly diff.

c) again

$100 = 10C + X \cdot B$ $\swarrow = 0$

want X

$100 = 10C + X \cdot 0$

$C = 10$

what could ballet prices be?

how high " " " " so he would <u>not</u> go

- need to get constraint in here
  - or corner solution?
  - or find for 1 then ↓

$100 = 10C + 1B$

$$C = \frac{100 - B}{10} = 10 - \frac{B}{10}$$

$$U = \left(10 - \frac{B}{10}\right)^2 (B+2)$$

? but compare to what?

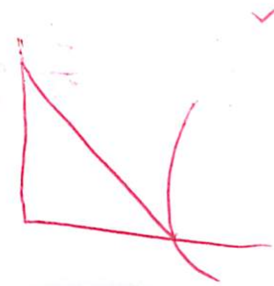
- why do I majorly fail on this!.

Corner solution

- review in book
- if people spend <u>no</u> $ on something

MRS still > Budget line (that T/F qu)

$\boxed{\text{MRS} \geq \text{MRT here}}$



but this tells me nothing about the #s

Ans: $MRS \geq \frac{P_C}{P_B}$

$$MRS = \frac{\frac{\partial U}{\partial C}}{\frac{\partial U}{\partial B}} = \frac{2c(B+2)}{c^2} = \frac{2(B+2)}{C}$$

?! is this backwards?

right $C = 10$

$MRS = \frac{4}{10}$

$MRS \geq \frac{P_C}{P_B} = \frac{10}{P_B}$ so $b_B \geq 25$

So if tickets are more than 25 he will do that

Why do I never do MRS = MRT?

let me try on own

c)

~~3~~2nd try

So MRS $\geq$ MRT

$$|MRS| \geq \left|\frac{P_C}{P_B}\right|$$

is right $\rightarrow$ $$\frac{\frac{\partial U}{\partial C}}{\frac{\partial U}{\partial B}} \geq \frac{10}{P_B}$$

$C^2B + 2C^2$

$$\frac{2\cancel{C}B + 4\cancel{C}}{C^{\cancel{2}}} = \frac{2B+4}{C} \geq \frac{10}{P_B}$$

then what next?

plug in desired B, C #

$$\cancel{\frac{2(10)+4}{0}}$$

flipped it :'(

? divide by 0

$$\frac{2\cancel{B}(0)+4}{10} = \frac{4}{10} \geq \frac{10}{P_B}$$

$$\frac{4}{10} = \frac{10}{P_B}$$ ~~$\frac{4}{10P_B} = \frac{4}{P_B}$~~

? must be 25

Much beter no do it for real

c 4th try

$C P_c + B \cdot P_B = 100$

$U = C^2(B+2)$

Remember its the corner solution

$MRS \geq \frac{P_c}{P_B}$ ← just needed to remember that

So where they are =

$U = C^2B + 2C^2$

$$\frac{\frac{\partial U}{\partial C}}{\frac{\partial U}{\partial B}} = \frac{P_c}{P_B}$$

$$\frac{2CB + 4C}{C^2} = \frac{10}{P_B}$$

$$\frac{2B+4}{C} = \frac{10}{P_B}$$

$$\frac{4}{10} = \frac{10}{P_B} \quad P_B = 25$$

~~14~~

d)

$$\int_{30}^{60} 10 + \frac{600}{p^2} \, dp$$

$$= 10p - \frac{600}{p} \Big]_{30}^{60}$$

$$590 - 280$$

↳ I understand this

- but what was my incorrect solution?

↳ how to represent that on a graph?

↓ retry

$$Q = 10 + \frac{600}{p^2}$$

$$\int_{30}^{60} 10 + \frac{600}{p^2}$$

- remembering how to $\int$!

$-600p^{-1}$

$$\frac{-600p^{-1}}{1}$$

$$\frac{600}{p^{-2}}$$

$$10P - \frac{600}{P} \Bigg|_{30}^{60}$$

$$10(60) - \frac{600}{60} - 10(30) - \frac{600}{30}$$

$$(600 - 10) - (300 - 20)$$

$$590 - 280$$

$$390 - 80 = 310$$

✓ actually right now

not what I did 1st time!

Long Questions:

Reminder: These questions are in parts and are cumulative. If you cannot solve part of the question and you need the results of that part in later parts, you should describe what you would do with the solution from the earlier part to receive partial credit.

3. (25 points) Marco likes both Japanese food (J) and Chinese food (C). His utility function is u (J, C) = J(C+2). Denote the price of Japanese food by P_J, the price of Chinese food by P_C, and his income by I. You can assume that I is sufficiently large such that Marco has no corner solutions in any part of this question.

$JC + 2J$

a. (8 points) Derive Marco's demand functions for Japanese and Chinese food.

-ok see if I can remember

MRS = MRT

$$-\frac{\frac{\partial U}{\partial J}}{\frac{\partial U}{\partial C}} = -\frac{P_J}{P_C} \qquad \frac{-C+2}{J} = \frac{-P_J}{P_C}$$

✓

$$P_J J + P_C C = I$$

solve for C

$$C = \frac{P_J J}{P_C} - 2 \qquad P_J J + P_C\left(\frac{P_J J}{P_C} - 2\right) = I$$

$$P_J J + P_J J - 2P_C = I$$

$$I = 2(P_J J) - 2P_C$$

← duh

$$\boxed{J = \frac{I + 2P_C}{2P_J}}$$

close

$$\frac{J}{C+2} = \frac{P_C}{P_J}$$

$$J = \frac{P_C(C+2)}{P_J} \qquad P_J\left(\frac{P_C(C+2)}{P_J}\right) + P_C C = I$$

$$P_C C + P_C 2 + P_C C = I$$

$$\boxed{C = \frac{I - 2P_C}{2P_C}}$$

✓

note no P_J here

good learned this!

b. (4 points) Use the demand function for Japanese food to determine if Chinese food and Japanese food are substitutes, complements, or independent goods.

$$J = \frac{I + 2P_C}{2P_J} \qquad C = \frac{I - 2P_C}{2P_C}$$

intution: substitute (can only eat one)

- but how to calculate / proove?

- cross elasticity

$$\frac{\Delta Q_J}{\Delta Q_C} \frac{Q}{Q_J}$$

← not it

$$\frac{\frac{\partial J}{\partial C}}{\frac{\partial C}{\partial J}} \quad \frac{C}{J} \quad \leftarrow \quad \frac{1}{2}$$

demand for Jap. food ↑ price of Chinese Food ↑

substitute

Chiese Food ind. of Jap. Food

not includeb

- ok so just look at price

- thrown off other not included

c. (2 points) What are Marco's optimal choices of J and C when $P_J = P_C = 5$ and I=50?

Just plug in

$$J = \frac{50 + 2(5)}{2(5)} = 6$$

$$C = \frac{50 - 2(5)}{2(5)} = 4$$

got one!

d. (2 points) The price of Japanese food is now $P_J = 20$. What are the new optimal choices of C and J for Marco?

$$J = \frac{50 + 2(5)}{2(20)} = 1.5$$

$$C = \frac{50 - 2(5)}{2(5)} = 4$$

e. (9 points) What are the size and signs of the income and substitution effects for the change in Japanese food consumed due to the increase in P_J?

So last part of p-set

Original	5	5	5
	5	20	20
	50	50	?
	6	1,5	?
	4	4	?

Original $u = 6(4+2)$ $1{,}5(4+2)$

36 9 36

$36 = J(c+2)$

$$36 = \frac{I+2(5)}{2\cdot 20}\left(\frac{I-2(5)}{2(5)}+2\right)$$

$$36 = \frac{I+10}{40}\left(\frac{I-10}{10}+2\right)$$

$$36 = \left(\frac{I}{40}+\frac{1}{4}\right)\left(\frac{I}{10}-1\right)$$

? how solve w/o calc

? or don't need

– we do need

Calc $x = 120$

Sub Jap did not really memorize

new price util constant #3 – old income old price #1

Income (new price old income #2 – old income old price #1) – sub effect

6 – 1,5

total effect = –4,5

4. (27 points) Audrey and Tim eat both Meat (M) and Vegetables (V). They have utility functions given by:

$$\text{Audrey: } u^A = \min\{M,V\}$$
$$\text{Tim: } u^T = 2M+V$$

a. For Audrey

i) (2 points) Draw an indifference map (put M on the horizontal axis and V on the vertical axis).

(2)

e)

MRS = to new price ration

$\frac{C+2}{J} = \frac{20}{5} = 4$ ← oh just use MRS!

must keep util constant = 36 ← what I did

- so use this to increase

$u(6,4) = 36$

$u(7,4) = 40$

$8,4 \quad 44$

- but what if change C?

- or just guess +J to add to 36?

$36 = J(2+2)$

Or new MRS = MRT

$$\frac{C+2}{J} = \frac{20}{5} = 4$$

or start over

$J(C+2) = 36$ ← new constraint

$J \cdot C + 2J = 36$

$P_J J + P_C C = I$

$P_J \frac{36}{C+2} + P_C C = I$ ← same thing Matt showed me

$\cdot C+2 \quad \cdot C+2 \quad \cdot C+2$

$36 P_J + P_C C(C+2) = I(C+2)$

$36 P_J + P_C C^2 + 2 P_C C = IC + 2I$

$P_C C^2 + 2P_C C - IC = 2I - 36 P_J$

↑ hard to solve

- did I screw up?

$$C = \frac{2I - 36P_J}{(P_C C + 2P_C - I)}$$

; plug in #

$$\frac{2I - 36 \cdot 20}{5 \cdot C + 2 \cdot 5 - I}$$

try w/ other

$$C = \frac{36 - 2J}{J}$$

$$P_J J + P_C \left(\frac{36-2J}{J}\right) = I$$

solve for J

$\cdot J$

$$P_J J^2 + P_C(36 - 2J) = IJ$$

; J in every term

Matt

$$\frac{C+2}{J} \overset{MRS}{=} 4 \rightarrow C+2 = 4J$$

F... Matt solves them so easily!

- need to do this tomorrow
- its just the math!

- Plug into

Constraint $J(C+2) = 36$

$$J(4J) = 36$$

$$J^2 = 9$$

$$J = 3$$

then

$$3(C+2) = 36$$

$$3C + 6 = 36$$

$$C = 10$$

Min cost = $3 \cdot 20 + 5 \cdot 10 = 110$

Still need to solve rest

5	5	5
5	20	20
50	50	110
6	1,5	3
4	4	5
36	9	36

Jap ~~total effect #. - #1~~

Sub #3 - #1

3 - 6 = -3

income (#2 - #1) - sub

1,5 - 6 + 3

-4,5 + 3

-1,5

Chinese

Q) #3 - #1

5 - 4 = 1

(#2 - #1) - sub

4 - 4 - 1

-1

don't care

$$\frac{dM}{dp_m} \cdot \frac{P_m}{M} \qquad M = \frac{I}{P_m + P_v}$$

$$dM = \frac{-I}{(P_m + P_v)^2}$$ ← differeinated wrong slightly

$$\frac{-I}{(P_m + P_v)^2} \cdot \frac{P_m}{\frac{I}{P_m + P_v}}$$

$$\frac{-50}{(10+4)^2} \cdot \frac{10}{\frac{50}{14}}$$

$$\frac{-25}{98} \cdot \frac{14}{5}$$

$$= -\frac{5}{7}$$ ← oh well right idea

---

oh income elasticity

↓

$$\cancel{\frac{dQ}{dI}} \quad \frac{dM}{dI} \cdot \frac{\cancel{P_M}\, I}{M}$$

$$\frac{1}{P_M + P_V} \cdot \frac{I}{\frac{50}{14}}$$

$$\frac{1}{14} \cdot \frac{50}{\frac{50}{14}}$$

$$\frac{1}{14} \cdot \cancel{50}\, 14$$

1 there we go

read the F-ing qu!

ii) (6 points) Derive Audrey's demand functions in terms of the price of meat, P_M, the price of vegetables, P_V, and income, I.

Once again
- one I asked matt
- no original

budget constraint

$UA = \min(M, V) \Rightarrow M = V$

$I = M P_m + \cancel{V} P_v$ (M)

So ~~M = 2I = MP~~

$I = M(P_m + P_v)$

$M = \dfrac{I}{P_m + P_v} = V$ ✓ woot

~~ME~~

iii) (3 points) What is the income elasticity of demand of meat when $P_M = 10$, $P_V = 4$, and income I=50? Is meat a normal, inferior, or neither normal nor inferior good?

don't forget to ans

$$\varepsilon_D = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q}$$

$$\frac{dq}{dp} \rightarrow \frac{dM}{dp_m} \cdot \frac{P_m}{M}$$ ~~no that's not it~~

$\cancel{dq_m}$

↑ don't forget

– look at ~~table~~ chart ~~– perfectly elastic~~ wrong chart

$\varepsilon = 1$

normal → q ↑ as p ↑

– easy

$$\frac{dm}{dp_m} \cdot \frac{10}{M} \qquad M = \frac{I}{P_M + P_V}$$

→ like $d\left(\frac{I}{P_M + P_V}\right)$

$$I(P_m + P_v)^{-1}$$

$$-1 \cancel{P} I P_m^{-2} \cdot \frac{10}{\frac{50}{10+4}}$$

$\cancel{dm}$

$$-\frac{50}{10^2} \cdot \frac{10}{\frac{50}{14}}$$

– come back

$$\frac{-\cancel{50}}{10} \cdot \frac{14}{\cancel{50}} = \frac{-14}{10} = -1.4$$ ε_{II}

b. For Tim $U^T = 2M + V$

i) (2 points) Draw an indifference map (put M on the horizontal axis and V on the vertical axis).

Slope = -2

← draw well

$2M+V$

ii) (6 points) Derive Tim's demand functions in terms of the price of meat, P_M, the price of vegetables, P_V, and income, I. (Be careful to check for different cases.)

- lets see if I can do

$V P_V + M P_M = I$

MRS = MRT

$$\frac{\frac{\partial U}{\partial M}}{\frac{\partial U}{\partial V}} = \frac{P_M}{P_V} \qquad \frac{2}{1} = \frac{P_M}{P_V}$$

$$2 = \frac{P_M}{P_V}$$

now what when no letters
- prob eaiser - but I am confused

Oh Jeeze

$$(M^*, V^*) = \begin{cases} \left(\frac{I}{P_M}, 0\right) & \text{if } 2P_V > P_M \\ \left(0, \frac{I}{P_V}\right) & \text{if } 2P_V < P_M \\ \left\{\left(\frac{x}{P_M}, \frac{I-x}{P_V}\right) \; x \in [0, I]\right\} & \text{if } 2P_V = P_M \end{cases}$$

corner solutions

demand for meat

* if $2 = \frac{P_M}{P_V}$

$M = \frac{x}{P_M}$ ← amt spent, ie income
- spending some amt

$V = \frac{I-X}{P_V}$ ← amt left over

iii) (3 points) What is the income elasticity of demand of meat when $P_M = 10$, $P_V = 4$, and income I=50? Is meat a normal, inferior, or neither normal nor inferior good?

$$e = \frac{\Delta M}{\Delta I} \frac{I}{M}$$

buys more as income ↑

$$M = \frac{x}{P_M} = P_M^{-1}$$

$$-P_M^{-2}$$

$$\frac{1}{-(P_M)^2} \quad \frac{50}{\left(\frac{x}{10}\right)}$$

$$-\frac{1}{100} \cdot \frac{50}{1} \cdot \frac{10}{x}$$

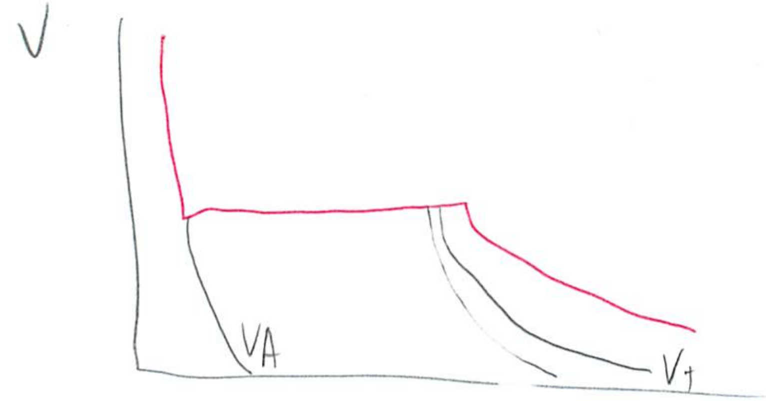
$$-\frac{5}{x}$$ ← negitive is inferior though

0 - neither (sticky)

- look at demand function
  - not based in income
  - like Jap / china food

c. (5 points) Audrey and Tim are the only two people in this economy. What is the total market demand for vegetables when $P_M=10$ and $I=50$ for both consumers (give a mathematical expression of the market demand function for vegetables and draw a picture of the market demand curve)?

-we did not really cover



$$VD = \begin{cases} \frac{50}{10+P_V} & \text{if } P_V > 5 \\ \left\{ \frac{5}{10+P_V} + \frac{x}{P_V} : x \in [0, 50] \right\} & \text{if } P_V = 5 \\ \frac{50}{10+P_V} + \frac{50}{P_V} & \text{if } P_V < 5 \end{cases}$$

look at their edge effects + how they line up

Pratice

10/2

Massachusetts Institute of Technology
Department of Economics

14.01 Principles of Microeconomics
Midterm Exam #1
Tuesday March 4th, 2008

Last Name (Please print): Michael

First Name: ____________________

MIT ID Number: ____________________

Instructions. Please read carefully.
The exam has a total of 100 points. You have 2 hours. Answers should be as concise as possible. This is a closed book exam. You are not allowed to use notes, equation sheets, books or any other aids. You are not allowed to use calculators. You must write your answers in the space provided between questions. DO NOT attach additional sheets of paper. This exam consists of 25 sheets (21 pages + 4 blank pages for scratch work). Please write your name on the blank pages if you use them.

Please circle the section or recitation which you are attending below. The marked exam will be returned to you in the section or recitation that you indicate.

R01: F10	(Jeanne LaFortune)	S01: MWF9	(Brandon Lehr)
R02: F11	(Jeanne LaFortune)	S02: MWF10	(Brandon Lehr)
R03: F12	(Monica Martinez-Bravo)	S03: MWF10	(David Walton Brown)
R04: F1	(Jeanne LaFortune)	S05: MWF11	(David Walton Brown)
R05: F1	(Monica Martinez-Bravo)	S06: MWF12	(Roger Ke)
R06: F2	(Monica Martinez-Bravo)	S07: MWF2	(HeiWai Tang)
		S09: MWF3	(HeiWai Tang)

DO NOT WRITE IN THE AREA BELOW:

Question 1 __ /20 Question 3 __ /25

Question 2 __ /28 Question 4 __ /27

Total __ /100

True/False/Uncertain (Total: 20 points)

1. Explain whether each of the following statements is True, False or Uncertain, and provide an explanation. (Note: You will not get points for a correct answer without an explanation.)

a. (*5 points*) For an inferior good, a price increase has an income effect of opposite sign but greater magnitude than the substitution effect.

False. Although it is true that the income and substitution effects have opposite signs for inferior goods, it is ambiguous which effect will be greater in magnitude. For inferior goods that are Giffen, it is true that the income effect is greater in magnitude than the substitution effect, but it is not necessarily true for the rest of the inferior goods.

~~Inf~~ inferior - as price ↑ you buy less

$\frac{dQ}{dp} > 0$

income > substitution

true I think - but why?

b. (*5 points*) A rational consumer can choose an optimal bundle of apples (A) and oranges(O) that has $MRS > \frac{P_A}{P_O}$.

True. Her optimum can be at a corner solution.

c. (*5 points*) A risk averse person will never pay \$10 for a lottery ticket that pays \$95,000 with probability 0.0001 and nothing otherwise.

True. The expected value of this lottery is 95,000(1/10,000)=9.5 that is lower than the cost of the ticket.*

d. (5 points) Consider the market for cell phones in 1995 vs. 2008. Take as fact that the income of consumers has increased during this period and cell phones have become cheaper to produce. These two changes imply that the equilibrium price for cell phones has decreased.

Uncertain. The increase in income indicates an outward shift in the demand curve and the lower input prices indicate an outward shift in the supply curve. The effect on the equilibrium price is ambiguous, as it depends on the magnitudes of the shifts and the elasticities of demand and supply.

2. Short Questions (Total: 28 points)

a. (7 points) Between 2006 and 2007, the government changed its price floor on cars. The table below shows the change in the price floor as well as the observed quantity and market price in each year.

Year	Price Floor	Price	Quantity
2006	5	5	600
2007	3	4	800

Assume that neither the demand curve nor the supply curve shifted and that the market demand is linear. Derive the market demand function. Compute the price elasticity of demand in each year.

Solution:

$\Delta Q/\Delta P=-200$

$E_{p}^{2006}=(P^{2006}/Q^{2006})(\Delta Q/\Delta P)=(5/600)(-200)=-1.67$

$E_{p}^{2007}=(P^{2007}/Q^{2007})(\Delta Q/\Delta P)=(4/800)(-200)=-1$

The linear demand curve is: $Q=1600-200P$

b. (7 points) George has preferences for coffee (C) and teaspoons of sugar (S) according to the utility function

$u(C, S) = 2 * \min\{2C, S\} + 10$

Prices of coffee and teaspoons of sugar are $P_C = 2$, $P_S = 0.5$. Find the equations for the Engel curve for each good.

Solution:
$C=I/3$, $S=2I/3$

c. (7 points) Jess has \$100 to spend on entertainment this semester. He only cares about going to either concerts (C) or the ballet (B). Jess' utility function over concerts and the ballet is given by: $u(C, B) = C^2(B+2)$. Concert tickets cost \$10 each. What is the minimum price of ballet tickets such that Jess will optimally choose to never go to the ballet?

Solution:

The condition for a corner solution in which Jess never goes to the ballet is that the marginal rate of substitution of concerts for ballet be at least P_C/P_B. We can compute the MRS by taking the ratio of partial derivatives:

$$MRS=(\partial u/\partial C)/(\partial u/\partial B)=2C(B+2)/C^2=2(B+2)/C$$

If we want $B=0$, then the budget constraint implies that $C=10$. Hence, the $MRS = 4/10$. This is optimal if $MRS \geq P_C/P_B = 10/P_B$. Thus, $P_B \geq 25$. So if Ballet tickets are at least \$25, Jess will optimally choose a corner solution in which he never goes to the Ballet.

d. (7 points) Market demand for textbooks is given by $Q = 10 + \frac{600}{P^2}$. What is the gain in consumer surplus from an effective price ceiling on textbooks that reduces prices from \$60 to \$30?

Solution:

$$\textit{Consumer Surplus Gain} = \int_{30}^{60} 10 + \frac{600}{P^2}\,dP = \left[10P - \frac{600}{P}\right]_{30}^{60} = 310$$

Long Questions:

Reminder: These questions are in parts and are cumulative. If you cannot solve part of the question and you need the results of that part in later parts, you should describe what you would do with the solution from the earlier part to receive partial credit.

3. (25 points) Marco likes both Japanese food (J) and Chinese food (C). His utility function is u (J, C) = J(C+2). Denote the price of Japanese food by P_J, the price of Chinese food by P_C, and his income by I. You can assume that I is sufficiently large such that Marco has no corner solutions in any part of this question.

a. (8 points) Derive Marco's demand functions for Japanese and Chinese food.

Solution:

We first set MRS equal to the price ratio.

$$P_J/P_C = MRS = (\partial u/\partial J)/(\partial u/\partial C) = (C+2)/J$$

Plugging this condition into the budget constraint and solving yields:

$C=(I-2P_C)/2P_C$

$J=(I+2P_C)/2P_J$

b. (4 points) Use the demand function for Japanese food to determine if Chinese food and Japanese food are substitutes, complements, or independent goods.

Solution:

Demand for Japanese food is increasing in the price of Chinese food. This means that Japanese food is a substitute for Chinese food. Note that the demand for Chinese food does not depend on the price of Japanese food, so Chinese food is independent of Japanese food.

c. (2 points) What are Marco's optimal choices of J and C when $P_J = P_C = 5$ and I=50?

Solution:
Plugging into the demand functions from above, J=6 and C=4.

d. (2 points) The price of Japanese food is now $P_J = 20$. What are the new optimal choices of C and J for Marco?

Solution:
Plugging into the demand functions from above, J=1.5 and C=4.

e. (9 points) What are the size and signs of the income and substitution effects for the change in Japanese food consumed due to the increase in P_J?

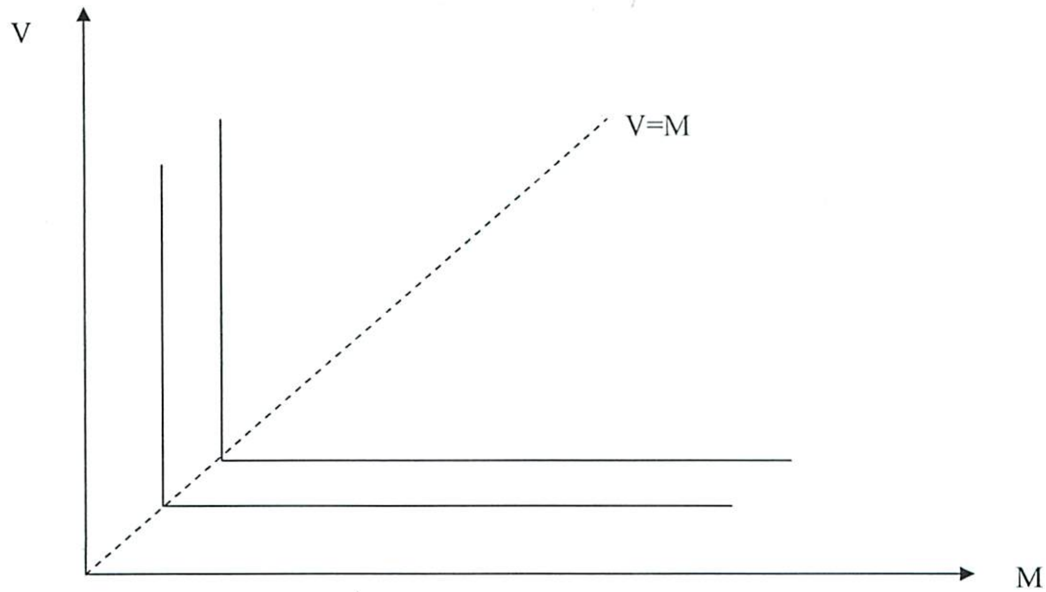
Solution:
The total effect of the change in Japanese food is -4.5. Setting MRS equal to the new price ratio, we have $(C+2)/J=20/5=4$. We also know that the market basket selected by Marco after the price change but holding initial utility constant must satisfy $u(J,C)=u(6,4)=36$. Combining these two conditions gives us $J=3$ and $C=10$. Thus, the substitution effect for Japanese food equals -3 and the income effect equals -1.5.

4. (27 points) Audrey and Tim eat both Meat (M) and Vegetables (V). They have utility functions given by:

$$\text{Audrey: } u^A = \min\{M, V\}$$
$$\text{Tim: } u^T = 2M + V$$

a. For Audrey

i) (2 points) Draw an indifference map (put M on the horizontal axis and V on the vertical axis).



ii) (6 points) Derive Audrey's demand functions in terms of the price of meat, P_M, the price of vegetables, P_V, and income, I.

Solution:
The optimal market basket for Audrey will occur at the kink points, so M=V. Plugging this into the budget constraint yields:

$$M^A = I/(P_M + P_V)$$

$$V^A = I/(P_M + P_V)$$

iii) (3 points) What is the income elasticity of demand of meat when $P_M = 10$, $P_V = 4$, and income I=50? Is meat a normal, inferior, or neither normal nor inferior good?

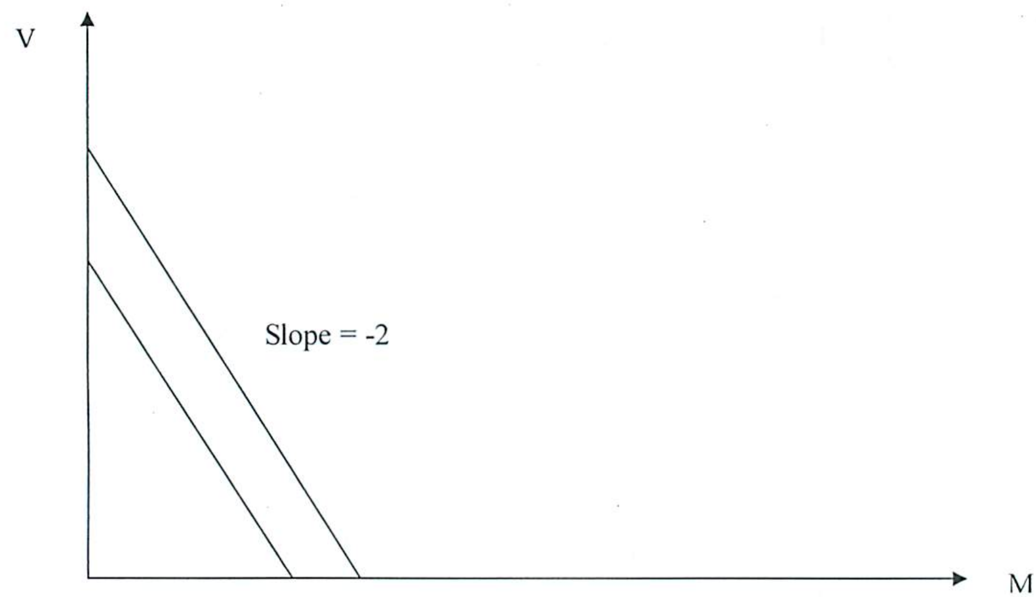
Solution:

Meat is a normal good for Audrey at these prices since:

$$\varepsilon_I^{M^A} = \frac{\partial M^A}{\partial I}\frac{I}{M^A} = 1$$

b. For Tim

i) (2 points) Draw an indifference map (put M on the horizontal axis and V on the vertical axis).



ii) (6 points) Derive Tim's demand functions in terms of the price of meat, P_M, the price of vegetables, P_V, and income, I. (Be careful to check for different cases.)

$$(M^T, V^T) = \begin{cases} (\frac{I}{P_M}, 0) & \text{if } 2P_V > P_M \\ (0, \frac{I}{P_V}) & \text{if } 2P_V < P_M \\ \{(\frac{x}{P_M}, \frac{I-x}{P_V}) : x \in [0, I]\} & \text{if } 2P_V = P_M \end{cases}$$

iii) (3 points) What is the income elasticity of demand of meat when $P_M = 10$, $P_V = 4$, and income I=50? Is meat a normal, inferior, or neither normal nor inferior good?

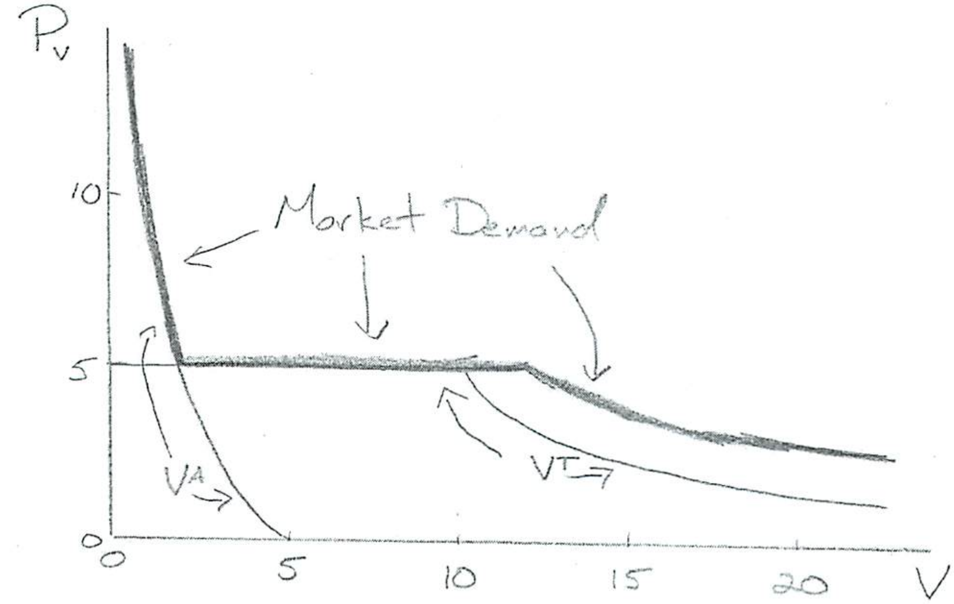
Solution:

Meat is neither a normal nor inferior good for Tim at these prices since:

$$\varepsilon_I^{M^A} = \frac{\partial M^A}{\partial I}\frac{I}{M^A} = 0$$

c. (5 points) Audrey and Tim are the only two people in this economy. What is the total market demand for vegetables when P_M=10 and I=50 for both consumers (give a mathematical expression of the market demand function for vegetables and draw a picture of the market demand curve)?

$$V^D = \begin{cases} \frac{50}{10+P_V} & \text{if } P_V > 5 \\ \{\frac{50}{10+P_V} + \frac{x}{P_V} : x \in [0,50]\} & \text{if } P_V = 5 \\ \frac{50}{10+P_V} + \frac{50}{P_V} & \text{if } P_V < 5 \end{cases}$$



END OF EXAM

date?

14.01 Fall 2008
Midterm Exam #1
Solutions

1. (Total: 20 Points) True/False Questions: In this section write whether each claim is True or False. Please fully explain your answer, using a diagram if appropriate. No credit will be awarded to solutions without clear explanations.

   (a) (5 Points) The price elasticity of demand for good x is constant. Claim: Total consumer expenditure on that good will be held constant as the price changes.

   False. This will be true only if the constant price elasticity is equal to -1. Otherwise revenue can either increase or decrease with price.

   (b) (5 Points) Erin buys light bulbs (L) and candles (C). Her utility is of the form

   $$U(L, C) = aL + bC$$

   Claim: Given an observed price price ratio, we can recover the parameters a and b.

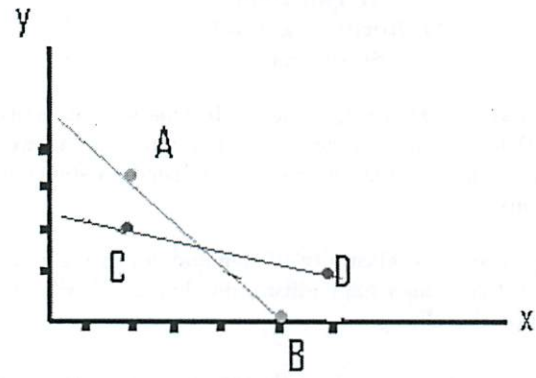
   False. If the optimal bundle is an interior solution, we know that $MRS = \frac{U_L}{U_C} = \frac{a}{b} = \frac{P_L}{P_C}$ *but this does not pin down* a *and* b *uniquely, only the ratio. If the solution is at a corner, we know nothing about* a *or* b *except that* $\frac{a}{b} > \frac{P_L}{P_C}$ *or* $\frac{a}{b} < \frac{P_L}{P_C}$.

   (c) (5 Points) There are two goods in the economy, and Paul buys 3 units of good x and 4 units of good y. Claim: Goods x and y cannot be perfect complements.

   False. The ratio of quantities consumed for perfect compliments does not have to be 1 for 1 (e.g. 8 ounces of water and 1 water glass).

   (d) (5 Points) A consumer with linear indifference curves is indifferent between (5 units of x coupled with 0 units of y) and (2 units of x coupled with 3 units of y). She is also indifferent between (2 units of x coupled with 2 units of y) and (6 units of x coupled with 1 units of y). Claim: This consumer has rational preferences.

   False. The IC's cross $\rightarrow$ violates transitivity.



$A \simeq B \;\; A \succ C \rightarrow B \succ C \simeq D$ but $D \succ B$

2. (Total: 12 Points) Jerry's demand for Honey Nut Cheerios is

$$Q_D = 100 - p^2 + \frac{I}{20}$$

(a) (5 Points) At an income level of 100, Jerry buys 80 units of Honey Nut Cheerios. At this consumption level, what is his price elasticity of demand (E_P^D)?

$80 = 100 - P^2 + \frac{100}{20} \rightarrow P = 5$
$E_P^D|_{P=5} = \frac{\partial Q}{\partial P}\frac{P}{Q} = -2P\frac{P}{Q} = -\frac{5}{8}$
Or $|E_P^D| = \frac{5}{8}$ *(your answer should have made clear you were considering price elasticity of demand a positive quantity if you express it as one)*

(b) (7 Points) Jerry's income increases. You observe at a price of $\sqrt{20}$ his income elasticity of demand is $\frac{1}{2}$. What is his new income?

$E_I^D = \frac{\partial Q}{\partial I}\frac{I}{Q} = \frac{1}{20}\frac{I}{100-P^2+\frac{I}{20}}$
$\rightarrow \frac{1}{2} = \frac{1}{20}\frac{I}{100-20+\frac{I}{20}} \rightarrow I = 1600$

3. (Total: 15 Points) In the small country of Smogsville hybrid cars have recently been introduced. Only two people, Ann (A) and Bob (B), live in this country. Their individual demand functions, evaluated at fixed levels of income (\$ USD), are given by:

$$Q_D^A = 70 - p$$

$$Q_D^B = 60 - p$$

The supply of hybrid cars to the market is given by:

$$Q_s = 70 + 2p$$

(a) (3 Points) Determine the market demand function for hybrid cars.
Adding the demands of Ann and Bob we have:

$$Q_D(p) = \begin{cases} 130 - 2p & 0 \le p \le 60 \\ 70 - p & 60 < p \le 70 \\ 0 & 70 < p \end{cases}$$

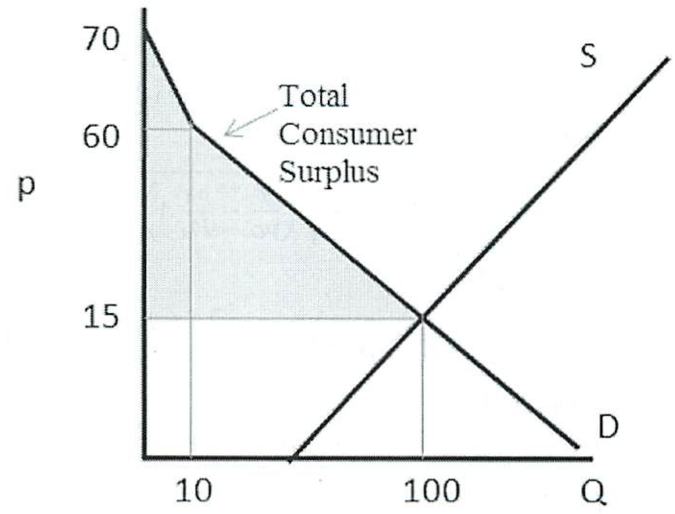
(b) (2 Points) Find the equilibrium price and the quantity of hybrid cars purchased in the market.
Setting $Q_D = Q_s$

$$\begin{aligned} 130 - 2p &= 70 + 2p \\ &\implies 60 = 4p \\ &\implies p = 15 \end{aligned}$$

which is consistent with $0 \le p \le 60$
then substituting back in the supply curve:

$$\begin{aligned} Q &= 70 + 2(15) \\ &= 100 \end{aligned}$$

(c) (3 Points) Sketch the market supply and market demand curves in the same axis, indicating the total consumer surplus.



(d) (4 Points) Calculate the total consumer suplus. (Be sure to include units)

The area of the shaded region is:

$$\frac{1}{2}(10)(70-60)+(60-15)(10)+\frac{1}{2}(100-10)(60-15)=2525$$

and since the horizontal axis is measured in \$/car and the vertical in cars, the area will be measured in \$ so the surplus will be:

$$\$2525$$

(e) (3 Points) Without calculation, evaluate the following claim: If Ann's sister moves to Smogsville (and has the same demand function as Ann), then both Ann and Bob would obtain less consumer surplus from the hybrid car market. Justify your solution.

When Ann's sister moves to Smogsville demand for hybrid cars rises, causing the equilibrium price level to rise as well. Since Bob and Ann's individual demands do not change, each of them will now consume less cars at a higher price. Therefore, both Bob's and Ann will obtain less consumer surplus.

4. (Total: 28 Points) Paul can spend his money on orange juice (O) and apple juice (A). His utility function is given by:

$$u(O,A)=\sqrt{O\times A}$$

(a) (5 Points) Derive Paul's demand function for orange juice $Q_O(I,p_O,p_A)$ and apple juice $Q_A(I,p_O,p_A)$.

Paul solves:

$$\max_{O,A}\sqrt{O\times A}$$
$$s.t.\ \ I=p_O O+p_A A$$

which is equivalent to:

$$\max\sqrt{\left(\frac{I}{p_O}-\frac{p_A}{P_O}A\right)A}$$

which has FOC:

$$0=\frac{1}{2}\left[\left(\frac{I}{p_O}-\frac{p_A}{P_O}A\right)-\frac{p_A}{P_O}A\right]\left[\left(\frac{I}{p_O}-\frac{p_A}{P_O}A\right)A\right]^{-1/2}$$

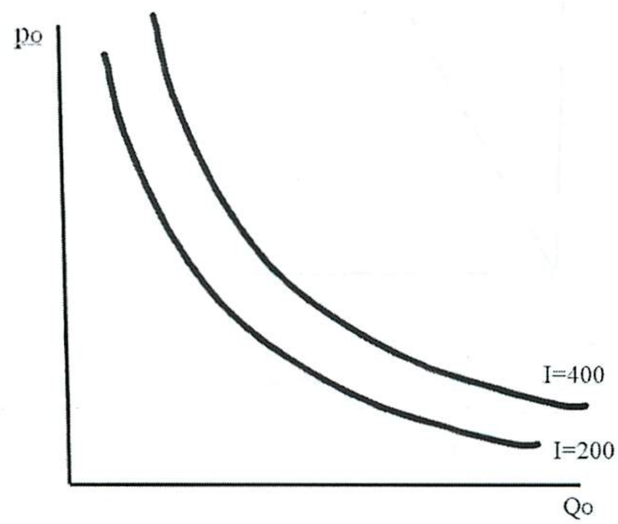
solvng for A:

$$A=\frac{1}{2}\frac{I}{p_A}$$

and analogously:

$$O=\frac{1}{2}\frac{I}{p_O}$$

(b) (2 Points) On the same set of axis, sketch the demand curves for orange juice corresponding to $I = 200$ and $I = 400$. Clearly label which curve corresponds to each income level.



(c) (2 Points) Sketch the Engel curve for orange juice (let $p_A = 4$ and $p_O = 1$).

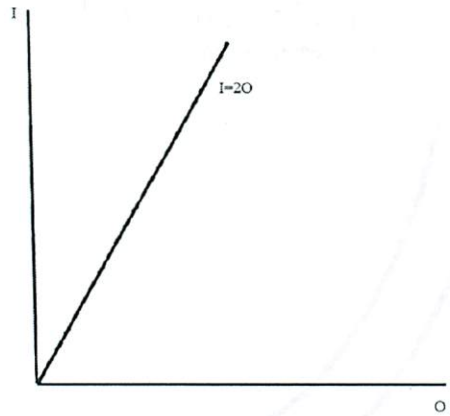
We have

$$O = \frac{1}{2}\frac{I}{p_O}$$

so if $p_O = 1$ then:

$$I = 2O$$

so the Engel curve will be a linear function that goes through the origin and with slope = 2:



(d) (2 Points) Let $I = 200$, $p_A = 4$ and $p_O = 1$. Determine Paul's utility maximizing consumption bundle of orange juice and apple juice. Also, calculate the corresponding utility level achieved.

Substituting in the demand functions:

$$\begin{aligned} A &= \frac{200}{2(4)} \\ &= 25 \end{aligned}$$

$$\begin{aligned} O &= \frac{200}{2} \\ &= 100 \end{aligned}$$

and his utility level will be:

$$\begin{aligned} u &= \sqrt{(100)(25)} \\ &= 50 \end{aligned}$$

(e) Now consider a rise in the price of orange juice ($p_O = 4$).

i. (2 Points) Determine the revised optimal consumption bundle.
Substituting again in the demand function for orange juice:

$$\begin{aligned} O &= \frac{200}{2(4)} \\ &= 25 \end{aligned}$$

and the consumption of apple juice will remain unchanged since its demand function does not depend on p_O.

ii. (3 Points) Calculate the income level necessary to achieve the same utility level as in part (d).

Substituting the demand functions in the utlity function we have:

$$u = \sqrt{\left(\frac{1}{2}\frac{I}{p_O}\right)\left(\frac{1}{2}\frac{I}{p_A}\right)}$$

therefore, the income necessary to achieve the original utility level must satisfy:

$$\begin{aligned} 50 &= \frac{I}{2}\sqrt{\frac{1}{p_O}\frac{1}{p_A}} \\ I &= \frac{100}{1/4} = 400 \end{aligned}$$

iii. (1 Points) Determine the optimal consumption bundle with the revised prices and the theoretical income level calculated in part (e-ii).
Substituting again in the demand functions

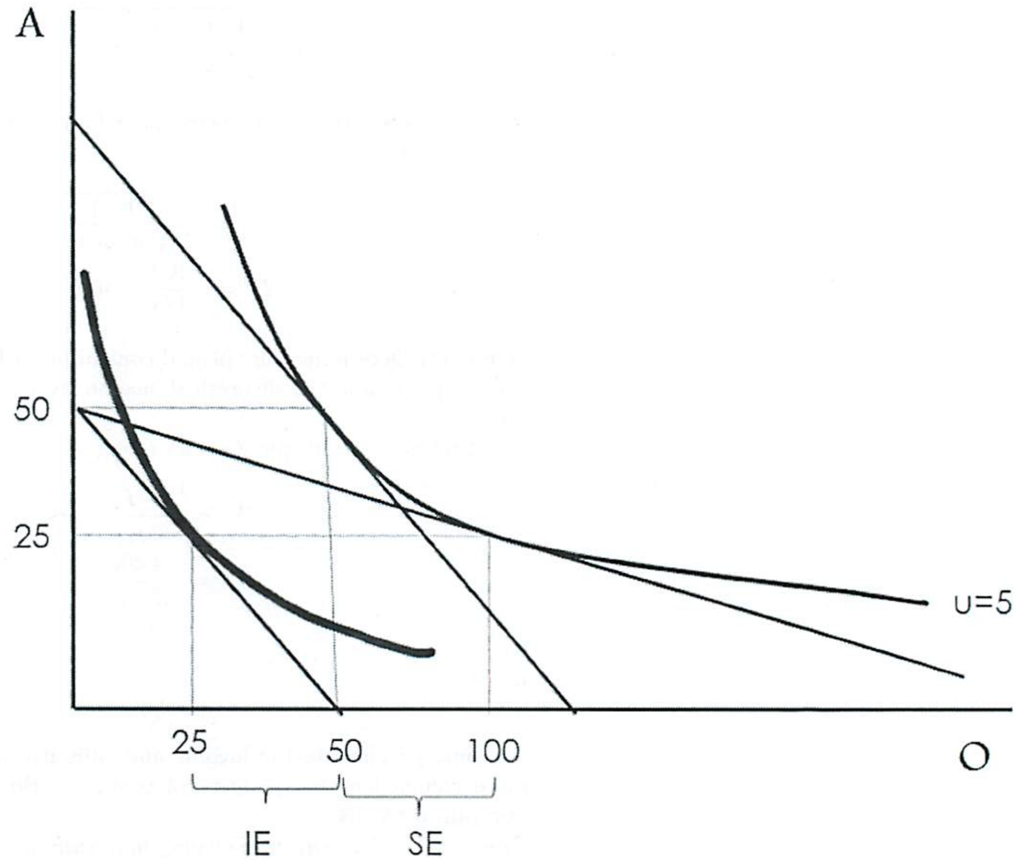
$$\begin{aligned} O &= \frac{1}{2}\frac{I}{p_O} \\ &= \frac{1}{2}\frac{400}{4} \\ &= 50 \end{aligned}$$

and

$$A = 50$$

iv. (5 Points) Calculate the income and substitution effects of the price change on the quantity of oranges. Be sure to include appropriate signs.
The substitution effect =consumption with new relative prices keeping utility constant − *original consumption*= $50 - 100 = -50$
The income effect = *new level of consumption* − *consumption with new relative prices keeping utility constant* = $25 - 50 = -25$.

v. (4 Points) Sketch a diagram detailing the income and substitution effects (label both effects on the same diagram).



vi. (2 Points) Explain whether orange juice is a normal good , an inferior good or neither.
Orange juice is a normal good since consumption increases with income.

5. **Note: The attached table may be helpful in these calculations.** (Total: 25 Points) A student comes to MIT with \$10,000 and a bike. With probability $\frac{1}{5}$ her bike will be vandalized within one year. If it is, the student must replace the bike (she needs it to get around), and bikes cost \$1,900. Assume that vandalized bikes can not be repaired and must be replaced and furthermore that the student can have her bike vandalized at most 1 time in the year (her replaced bike will not be vandalized). The student has utility over final wealth at the end of the year $U(w) = \sqrt{w}$, where w is monetary wealth (bikes provide no direct utility).

   (a) (5 Points) Calculate the student's expected final wealth at the end of the year.

$$E[w] = 0.2 * (10,000 - 1,900) + 0.8(10,000) = 9,620$$

(b) (3 Points) What is the standard deviation of wealth (write out the expression, but do not calculate)? In this context, what does the sd measure?

$$sd = \sqrt{0.2\,(8,100 - 9,620)^2 + 0.8\,(10,000 - 9,620)^2}$$

In this context sd measures the risk to wealth associated with the "gamble" that the student's bike will be stolen. Note that is does not measure the risk of bike being stolen (that's $\frac{1}{5}$). One might say it measures the variability of final wealth or spread of final wealth, but it is not entirely correct to say that it gives the probability of ending up with a certain final wealth or that $\frac{2}{3}$ of the time wealth will be within + or - 1 sd since this is a binary outcome; wealth is 10,000 with probability $\frac{4}{5}$ and 8,100 with probability $\frac{1}{5}$.

(c) (5 Points) How can the student's risk preferences be described? Justify your answer by calculating and comparing the quantities $E[U(w)]$ (expected utility) and $U(E[w])$ (utility of expected wealth).

Utility of expected income:

$$U(E[w]) = \sqrt{9,620} = 98.08$$

Student's expected utility:

$$E[U(w)] = 0.2\sqrt{8,100} + 0.8\sqrt{10,000} = 98$$

Risk averse since $E[U(w)] < U(E[w])$. Moreover, the concave utility function tells us this person is risk averse. Note: saying "almost risk neutral" or "slightly" risk averse is not correct. Suppose $U(w) = 1000000\sqrt{w}$. Then the difference $U(E[w])$ - $E[U(w)]$ is 80000, scale doesn't matter: utility is ordinal.

(d) (7 Points) Now suppose that a student can buy a personal bicycle guard dog, which reduces that student's probability of vandalism to $\frac{1}{10}$. Suppose that a guard dog costs \$195. Will the student buy a guard dog?

Buy the dog if

$$0.1\sqrt{8,100 - 195} + 0.9\sqrt{10,000 - 195} > 0.2\sqrt{8,100} + 0.8\sqrt{10,000}$$

$$0.1(88.91) + 0.9(99.02) > 98$$

$$98.00919 > 98$$

so she would buy the dog since her expected utility is higher buying the dog than not buying the dog.

(e) (5 Points) Suppose there is another student who has utility $U(w) = w$. Would this student buy the guard dog for \$195? Provide an intuitive explanation for your solution.
Buy the dog if

$$0.1(8,100 - 195) + 0.9(10,000 - 195) > 0.2(8,100) + 0.8(10,000)$$

$$0.1(7905) + 0.9(9805) > 0.2(8,100) + 0.8(10,000)$$

$$9615 > 9620$$

so he would not buy the dog. The intuition is that this utility function implies that the student is risk neutral; so $EV = U(EV)$ and buying the dog must reduce EV