

14.01 Fall 2009: Exam 2 Solutions

Work

November 13, 2009

1. True/False/Uncertain Questions (20 points)

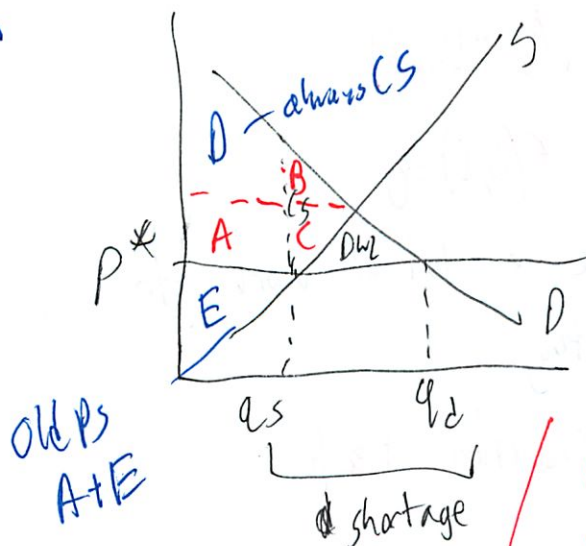
In this section, write whether each statement is True, False or Uncertain. You should fully explain your answer, including diagrams where appropriate. Points will be given based on your explanation.

- (a) (4 points) In a perfectly competitive market, price ceilings ^{top price} make consumers worse off. Please include a graph in your answer.

True. It will cause producers to underproduce leading to a deadweight loss of consumers willing to pay market price, but unable to purchase the good

wait ceiling!
oh yeah - depends where - assuming effective
Uncertain

agrees gain on total, be very clear in thinking



now can I do CS + DWL

confused

- do we need, I would leave blank on real test

just asking welfare

Consumers gain $+A-B$

Producers lose $-A-C$

Net to society $[A-B] - A-C = -B-C$

want in these terms

perhaps I do need to review a bit

(b) (4 points) A firm whose production technology exhibits constant marginal products has constant returns to scale.

True? ✓ by definition

or are they 2 diff things?

Each add. good produced at same cost

$$q = f(k, l)$$

$$f_l = \text{const}$$

$$f_k = \text{const}$$

$$\begin{aligned} f(\lambda k, \lambda l) &= \lambda k \cdot f_k + \lambda l \cdot f_l \\ &= \lambda (k \cdot f_k + l \cdot f_l) \\ &= \lambda f(k, l) \end{aligned} \quad \left. \vphantom{\begin{aligned} f(\lambda k, \lambda l) &= \lambda k \cdot f_k + \lambda l \cdot f_l \\ &= \lambda (k \cdot f_k + l \cdot f_l) \\ &= \lambda f(k, l) \end{aligned}} \right) \text{learn!}$$

Only true if $f(0,0)=0$

↑ That seems like a lot of work to prove something

also mentioned production tech

(c) (4 points) In the short run, a perfectly competitive firm may operate even if it is earning a negative profit.

Yes, if its ~~costs~~ ^{revenue per unit / price} is still above AVC — because of the fixed cost investments it made, which are by definition unrecoverable/ changable in SR.

✓ minimize loss

$$\text{Loss} = \text{price} < \text{ATC}$$

SR can't shut down

— or will it $P < AVC$

LR price

— oh yeah

of course can shutdown

if not making any profit

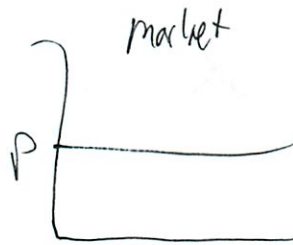
yearly firm

— can close down tmp.

(d) (4 points) In an economy with perfectly elastic demand, any positive tax on the final good will drive the quantity demanded to zero.

~~No - if the tax is evenly applied to all goods~~

- or wait a min



Yes - assuming perfect substitutes, people can escape def of elastic

switch easily to another good

Uncertain - if producers surplus tax comes entirely out of \leftarrow did not think of

if supply curve is perfectly elastic

any tax will prevent market from reaching = equilibrium
can you even have perfectly elastic demand + a monopoly?
- remember market

(c) (4 points) In an Edgeworth Box, indifference curves are always tangent along the contract curve.
wtf?

Did we study this??

false only true when both individuals

are consuming a (+) amt of each good

MAS does not have to be =
↳ indifference curves tangent

← this need to review

Along boundaries of box

2. Production and Costs (16 points)

Suppose that the production function of a firm making burritos is:

$$q(K, L) = K^{\frac{1}{2}} L^{\frac{1}{2}}$$

where q is the number of burritos produced, K is the machines used in production and L is labor used in production.

why capital?

- (a) (4 points) Assume that the wage rate is $w = 4$, and the rental rate on capital is $r = 1$. Derive the long run demand for labor and capital as well as the long-run total cost and average cost functions as a function of output. Does this production function exhibit increasing returns to scale, constant returns to scale, or decreasing returns to scale? Why?

depends on q ?

$$q = k^{1/2} L^{1/2}$$

$$k^{1/2} = \frac{q}{L^{1/2}}$$

$$k = \frac{q^2}{L}$$

$$L = \frac{q^2}{k}$$

then what cost

$$C = r k + w L$$

$$C = 1 \cdot \frac{q^2}{L} + 4 \frac{q^2}{k}$$

now solve both ways

$$\frac{q^2}{L} = C - \frac{4q^2}{k}$$

~~no~~

$$q^2 = C L - \frac{4q^2 L}{k}$$

no-hmm

would this have worked?

-no I remember kinda

ans all parts

constant returns to scale

$$MATS = \frac{k}{L} = \frac{w}{r} = \frac{4}{1} = 4$$

$$k = 4L$$

$$L = \frac{q}{2} \quad k = 2q$$

totally missed this

$$r=1 \quad w=4 \quad C = 4q$$

$$AC = \frac{4q}{q} = 4$$

- (b) (4 points) Now assume that capital is fixed at $\bar{K} = 4$, and the wage rate and the rental rate of capital are as in part (a). Derive the short run total cost, average fixed cost, average variable cost, average total cost, and marginal cost as functions of output.

$$C = K r + L w$$

$$C = 4 \cdot 1 + L \cdot 4$$

$$C = 4 + 4L \quad \begin{array}{l} \leftarrow \text{per unit} \\ \text{no per input} \end{array}$$

$$FC = 4$$

$$AFC = 4$$

$$VC = 4L$$

$$MC = 4$$

but how convert to q

$$q = K^{1/2} L^{1/2}$$

$$q = \sqrt{4} \sqrt{L}$$

$$q = 2\sqrt{L}$$

$$\sqrt{L} = \frac{q}{2}$$

$$L = \frac{q^2}{4} \quad \text{see previous answer}$$

$$C = 4 + 4 \frac{q^2}{4}$$

$$SRTC = 4 + q^2 \quad \checkmark$$

$$AFC = 4/q \quad \checkmark$$

$$AVC = \frac{q^2}{q} = q \quad \checkmark$$

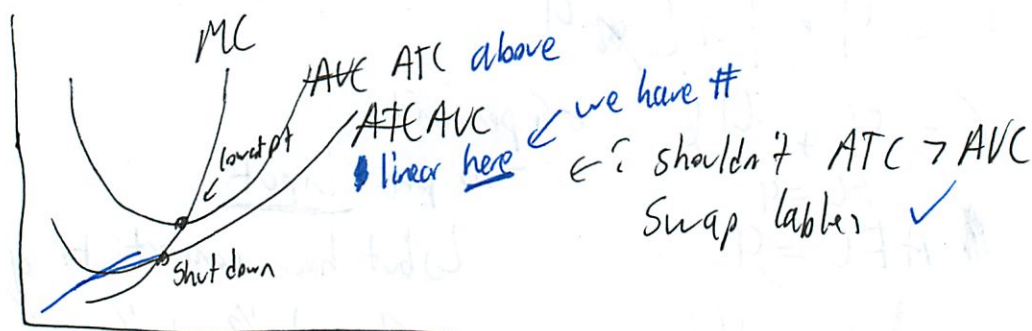
$$MC = 2q \quad \checkmark$$

$$ATC = \frac{4 + q^2}{q} \quad \checkmark$$

$$\leftarrow \text{so } AFC + AVC$$

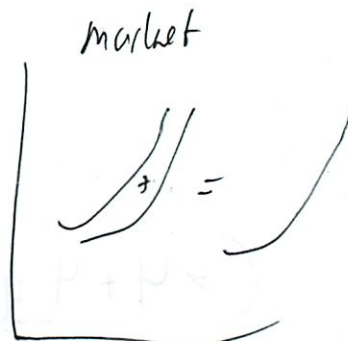
$$\frac{4}{q} + q \quad \odot$$

- (c) (4 points) Draw a diagram plotting MC, AVC, and ATC. At what price should the firm shut down? Explain your intuition. On a separate diagram, graphically show the short run market supply curve assuming that there are two firms with this production function in the market. (If you did not get part (b), then draw a diagram with generic cost curves. Then show at what price the firm should shut down and graph the short run market supply curve assuming that there are two firms in the market and the cost curves you graphed)

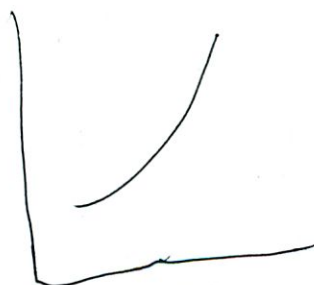


~~Shutdown when $p > MC$~~

Shutdown when $p < AVC$ $MC > AVC$
- w/ these # \rightarrow never shut down



$\pi < 0$ when 1 shut down \rightarrow only 1 firm??



They did not ans all parts
 - I think it was w/ these #
 Some situations would not happen

I always totally fail practice tests

-but I thought I would do better on this
-its the old stuff

3. Competitive Market Supply (24 points)

Assume that the market for tortillas is perfectly competitive. The market demand curve for tortillas is given by:

$$P = 10 - Q$$

Suppose that for each firm, the long run cost of producing q tortillas is:

$$C(q) = q^3 - 2q^2 + 3q$$

(a) (4 points) What are the fixed cost, marginal cost and average cost for each firm?

$$FC = 0 \quad \textcircled{\checkmark}$$

$$MC = 3q^2 - 2 \cdot 2q + 3$$

$$3q^2 - 4q + 3 \quad \textcircled{\checkmark}$$

$$AC = \frac{q^3 - 2q^2 + 3q}{q} = q^2 - 2q + 3 \quad \textcircled{\checkmark}$$

finally got one!

(b) (4 points) Assume that firms are perfectly competitive. What is the market price that each firm faces in the long run?

1. Take market q

$$P = 10 - q$$

2. No start w/ each firm

where $p = MC$

$$10 - q = 3q^2 - 4q + 3$$

solve for q

$$-10 + q$$

$$3q^2 - 3q - 7$$

factor - ? how w/o calc

$$q = 2.10$$

~~3. Now divide by~~

$$\text{market} = 4.2$$

$$4. \text{ Price} = 10 - 4.2 = 5.8$$

price is min of AC

Long run = equilibrium

$$AC = MC$$

$$2q - 2 = 0$$

$$q = 1$$

$$AC = 2$$

$P = 2$ where do they get that?

$$\text{Oh } AC = p = MC$$

I think that's what it is

(c) (4 points) What is the long run equilibrium price, output, and number of firms?

~~is~~ is price above their cost functions?

$$ATC = ~~q^2~~ q^2 - 2q + 3$$

$$= ~~12.24~~$$

Opps I used $q = 4.2$
want $q = 2.1$

~~no p > ATC, right?~~

~~So 1 firm will drop out?~~

$$= 3.21 < \text{price}$$

So firms will enter the market

- but how to figure out LR # firms?

LR $p = 2$ so LR equilibrium $Q = 8$

$$N = \frac{Q}{q} = \frac{8}{1} = 8$$

So I did it totally different

they did quantity led - I did price
lead I believe

- (d) (6 points) Suppose the government imposes an output tax of $t = 3$. Determine the new long run marginal and average costs as well as the new equilibrium price and quantity. How is the output tax affecting the number of firms in the tortilla market?

Urr These are always hard

- costs go up

- how much borne by producers, consumers?

$$C = q^3 - 2q^2 + 3q + 3q$$

$$= q^3 - 2q^2 + 6q$$

$$MC = 3q^2 - 4q + 6$$

$$AC = q^2 - 2q + 6$$

what was it $AC = MC$ & oh they simplified before

$$3q^2 - 4q + 6 = q^2 - 2q + 6$$

$$-q^2 + 2q - 6 \quad -q^2 + 2q - 6$$

$$\frac{2q^2 - 2q}{a} = \frac{0}{a}$$

$$2q - 2 = 0$$

$$2q = 2$$

$$q = 1$$

price & they did so weird way I still don't get

$$AC(q^*) = 5 \quad \text{so} \quad P = 5$$

so $Q = 5$ so 5 firms, same ~~again~~

(c) (6 points) Suppose that one specific tortilla firm has a short run marginal cost curve of:

$$MC = 6 + 5q$$

The market supply and demand curves for tortillas are given by:

$$P_S = 4 \cdot Q_S$$

$$P_D = 10 - Q_D$$

For what value of fixed costs, FC , will this firm be making losses in the short run?

$$P = MC$$

$$10 - Q = 6 + 5q$$

$$-10 + q \quad -10 + q$$

$$+4 = 6q$$

$$q = \frac{2}{3}$$

Connected to before $Q = 2 \quad P = 8$

$$MC = MR$$

$$\frac{2}{3} = .4$$

$$\pi = Pq - TC$$

$$= 8 \cdot .4 - (2.8 + FC)$$

$$= .4 - FC$$

$$\begin{aligned} TC &= \int_0^q (6 + 5q) dq \\ &= 6q + \frac{5q^2}{2} + FC \\ &= 2.8 + FC \end{aligned}$$

So makes loss if

$$.4 - FC < 0$$

4. Analysis of Government Policy (20 points)

US steel manufacturers supply steel according to the market supply function:

$$Q_S = 20 + P$$

US steel demand is given by:

$$Q_D = 30 - 4P$$

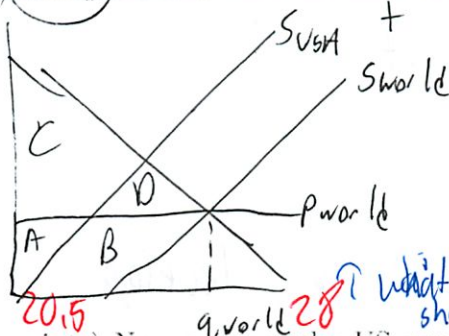
- (a) (2 points) What is the domestic equilibrium quantity and price?

~~just~~ $Q_D = Q_S$ $20 + P = 30 - 4P$ $Q = 22$

-20 $+5P$

$10 = 5P$ $P = 2$

- (b) (6 points) The US government decides to open the steel market to trade. The world price of steel is $P^W = 0.5$. Assume US production is very small relative to world production such that the world price remains $P^W = 0.5$ even after the US opens to trade. What is the change in domestic consumer surplus and producer surplus?



$\Delta CS = (2 - 0.5) \cdot 22 + \frac{1}{2} (28 - 22) 1.5 = 37.5$

Producer $A - \text{no same other effects}$

$\Delta PS = (2 - 0.5) \cdot 20.5 - \frac{1}{2} 1.5 \cdot 1.5 = -31.875$

$CS = \int_{0.5}^{2} (30 - 4P) dP$ - actual expenses

- (c) (4 points) Now suppose the US government decides to restrict imports such that the domestic price is now $P^d = 1$. What quota would the government need to impose?

$Q_D(1) - Q_S(1) = 26 - 21 = 5$ max 5 imports

$Q_S = 20.5$ $Q_D = 26$

So want $30 - 4(1) = 26$

confused $Q_D(1) - Q_S(1) = 26 - 21 = 5$

- (d) (4 points) If the government auctions import permits to foreign producers, what is the maximum amount they would be willing to pay for such permits (in aggregate) given the quota you found in part (c)?

$(1 - 0.5) \cdot 5 = 2.5$ for 5

So .5 each

- the surplus value

Surplus value

of them

(e) (4 points) Think more broadly about the U.S. economy. Would these changes in consumer and producer surplus be the only effects of imposing a quota relative to free trade? How do such restrictions affect the car industry? Would the steel workers union support such an import quota? How about construction workers? What would happen to the incentives to develop steel substitutes?

Would there be DWL?

- well consumers better off

- but should be DWL w/ quota

Car industry obviously hurt \uparrow input prices (vs what?)

Steel union would love it

Construction would not

w/ quota more incentive to develop

Steel substitutes (since price doubled)

✓ all bingo

the qu I am good at!

Remember $\frac{d \ln x}{dx} = \frac{1}{x}$

after review session

5. General Equilibrium (20 points)

from way back

Robinson and Friday live on a desert island. There are two goods on the island that the two agents have utility over: cake (C) and ice-cream (I). Robinson's utility function is:

$$U_R(C_R, I_R) = 2 \cdot \ln(C_R) + \ln(I_R)$$

forgot

while Friday's utility function is:

$$U_F(C_F, I_F) = \ln(C_F) + 2 \cdot \ln(I_F)$$

Robinson has 5 units of cake and 7 units of ice-cream while Friday has 5 units of cake and 6 units of ice cream.

- (a) (4 points) Write an expression for Robinson's and Friday's MRS. Under what condition is the allocation of cake and ice-cream between the two agents Pareto efficient? What is the relationship between the prices for cake, p_C , and ice-cream, p_I , and the MRS of the two agents at a Pareto efficient allocation?

$$MRS_R = \frac{U_C}{U_I} = \frac{2 I_R}{C_R}$$

not take derivative w/ respect to each part! kinda remember

$$MRS_F = \frac{U_C}{U_I} = \frac{C_F}{2 I_F}$$

Efficient allocation $MRS_R = MRS_F$ when did we learn that?

$$\frac{2 I_R}{C_R} = \frac{C_F}{2 I_F} = p_C / p_I$$

- (b) (5 points) Draw an Edgeworth box and mark the initial endowment point and the indifference curves of Robinson and Friday going through that point. Is the initial endowment Pareto efficient? Why or why not?

skip

or in that proportion they did not solve

- (c) (5 points) Derive the ~~contract~~ curve in terms of C_R and I_R - the consumption bundle of Robinson at a Pareto efficient allocation. Using the contract curve write an expression relating the price ratio, $\frac{p_C}{p_I}$, and I_R in a competitive equilibrium.

- (d) (6 points) Compute the price ratio $\frac{p_C}{p_I}$ and the quantities of C and I that Robinson and Friday will consume in a competitive equilibrium of this exchange economy. You can leave your answers as simple fractions.

Tip

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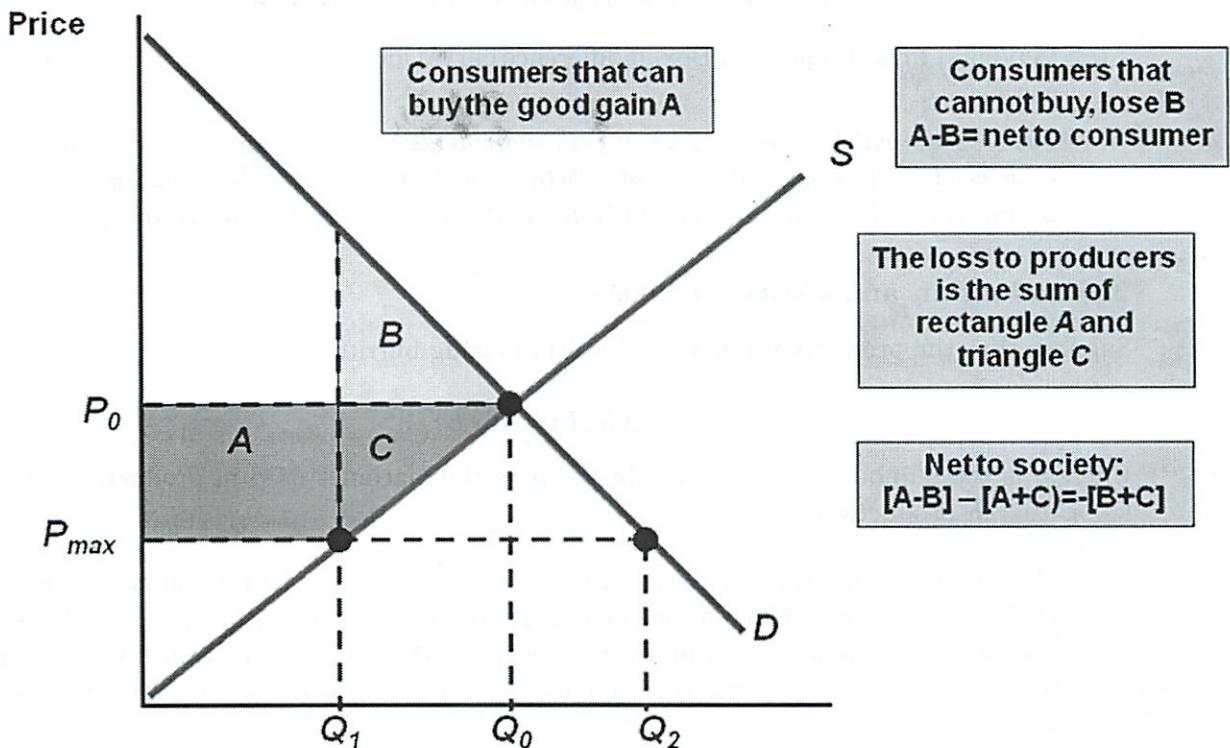
November 13, 2009

1. True/False/Uncertain Questions (20 points)

In this section, write whether each statement is True, False or Uncertain. You should fully explain your answer, including diagrams where appropriate. Points will be given based on your explanation.

- (a) (4 points) In a perfectly competitive market, price ceilings make consumers worse off. Please include a graph in your answer.

Uncertain. This depends on the difference between the loss of consumer surplus due to excess demand at the price ceiling, and the additional consumer surplus received by individuals who are able to purchase the good:



- (b) (4 points) A firm whose production technology exhibits constant marginal products has constant returns to scale.

True.

$$q = f(k, l); f_l = \text{const.}, f_k = \text{const.} \Rightarrow$$

$$f(\lambda k, \lambda l) = \lambda k \cdot f_k + \lambda l \cdot f_l = \lambda(k \cdot f_k + l \cdot f_l) = \lambda f(k, l)$$

Note that this is true only if $f(0, 0) = 0$ as well. Otherwise it will be uncertain.

Some of you interpreted the question to mean that the production technology has non-diminishing marginal products. The correct answer for this interpretation was also accepted since the question was ambiguously stated.

- (c) (4 points) In the short run, a perfectly competitive firm may operate even if it is earning a negative profit.

True. If the average total cost at the profit maximizing level of output is higher than the price, the firm will make a loss. The firm will stay in operation as long as the price is greater than the average variable cost since it's minimizing its losses this way.

- (d) (4 points) In an economy with perfectly elastic demand, any positive tax on the final good will drive the quantity demanded to zero.

Uncertain. If there is a positive amount of producer surplus in equilibrium, it is possible to implement a tax and still have a positive quantity demanded (as the tax revenue comes entirely out of producer surplus). However, if the supply curve is perfectly elastic as well, any tax will prevent the market from reaching equilibrium.

- (e) (4 points) In an Edgeworth Box, indifference curves are always tangent along the contract curve.

False. This will be true only when both individuals are consuming a positive amount of each good. The marginal rates of substitution do not have to be equal (and hence the indifference curves do not have to be tangent) along the boundaries of the box.

2. Production and Costs (16 points)

Suppose that the production function of a firm making burritos is:

$$q(K, L) = K^{\frac{1}{2}} L^{\frac{1}{2}}$$

where q is the number of burritos produced, K is the machines used in production and L is labor used in production.

- (a) (4 points) Assume that the wage rate is $w = 4$, and the rental rate on capital is $r = 1$. Derive the long run demand for labor and capital as well as the long-run total cost and average cost functions as a function of output. Does this production function exhibit increasing returns to scale, constant returns to scale, or decreasing returns to scale? Why?

The production function exhibits constant returns to scale. $MRTS = w/r$ implies $K/L = w/r = 4$. Hence, $K = 4L$ and so $L(q) = q/2$ and $K(q) = 2q$.

Hence, $LRTC = rK(q) + wL(q) = 4q$ and $LAC = 4$.

- (b) (4 points) Now assume that capital is fixed at $\bar{K} = 4$, and the wage rate and the rental rate of capital are as in part (a). Derive the short run total cost, average fixed cost, average variable cost, average total cost, and marginal cost as functions of output.

$$SRTC = r\bar{K} + wL(q) = 4 + 4L(q) \text{ where we use the production function to find } L(q) = (q^2)/4.$$

$$\text{Hence, } SRTC = 4 + q^2$$

$$AFC = 4/q$$

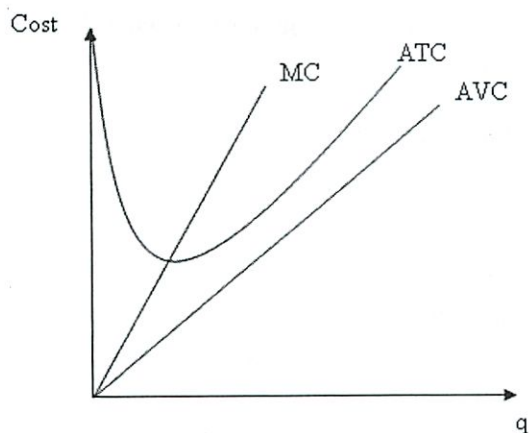
$$AVC = q$$

$$ATC = 4/q + q$$

$$MC = 2q$$

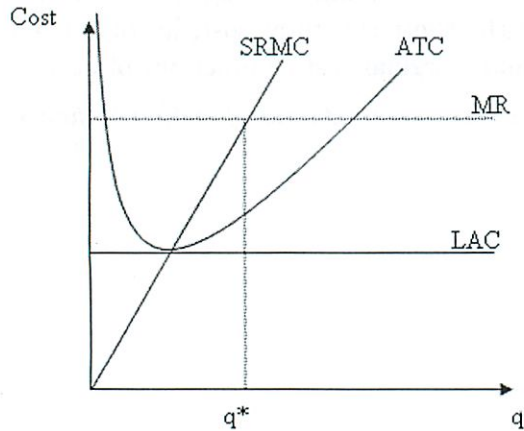
- (c) (4 points) Draw a diagram plotting MC, AVC, and ATC. At what price should the firm shut down? Explain your intuition. On a separate diagram, graphically show the short run market supply curve assuming that there are two firms with this production function in the market. (If you did not get part (b), then draw a diagram with generic cost curves. Then show at what price the firm should shut down and graph the short run market supply curve assuming that there are two firms in the market and the cost curves you graphed)

Since $MC > AVC$ for any quantity, the firm will never shut down in the short run. In other words, for any given price, the firm will be able to cover its variable costs and minimize losses. The market supply curve is given by $P = Q$.



- (d) (4 points) Suppose the price of burritos is $p = 8$. Draw a diagram with both the long-run average cost and the short run average total cost from part (b) (where capital is fixed at $\bar{K} = 4$). Show where short run production is located on this graph, and comment on whether the firm would change its capital level in the long run.

$p = MC = 2q$ implies $q = 4$, which means that short run demand for labor is $L(q) = 4$. Short run average total cost at $q = 4$ equals 5, which is higher than long-run average cost, which is equal to 4 for any q . Hence, in the short run, the levels of both capital and labor equal 4. However, from part (a), we know that the optimal level of capital in the long run is always 4 times the amount of labor. Therefore, the firm would change its capital level in the long run.



3. Competitive Market Supply (24 points)

Assume that the market for tortillas is perfectly competitive. The market demand curve for tortillas is given by:

$$P = 10 - Q$$

Suppose that for each firm, the long run cost of producing q tortillas is:

$$C(q) = q^3 - 2q^2 + 3q$$

- (a) (4 points) What are the fixed cost, marginal cost and average cost for each firm?

There is no fixed cost. The marginal cost is given by

$$MC(q) = 3q^2 - 4q + 3$$

and the average cost is given by

$$AC(q) = q^2 - 2q + 3$$

- (b) (4 points) Assume that firms are perfectly competitive. What is the market price that each firm faces in the long run?

We know that in the long run, the equilibrium is characterized by $AC = MC$ or alternatively the price is at the minimum of the average cost. Hence we have that:

$$2q - 2 = 0$$

and so $q^ = 1$. At $q^* = 1$, we have: $AC = 2$. Hence, the market price that each firm faces in the long run is $P^* = 2$.*

- (c) (4 points) What is the long run equilibrium price, output, and number of firms?

The LR price is given by $P^* = 2$. Hence, the long run equilibrium output is given by $Q^* = 8$ and the number of firms is given by:

$$\begin{aligned} N &= Q^*/q^* \\ &= 8 \end{aligned}$$

- (d) (6 points) Suppose the government imposes an output tax of $t = 3$. Determine the new long run marginal and average costs as well as the new equilibrium price and quantity. How is the output tax affecting the number of firms in the tortilla market?

The new marginal cost is given by

$$MC' = 3q^2 - 4q + 6$$

and the new average cost is given by

$$AC' = q^2 - 2q + 6$$

Hence, in the long run we again have: $q^* = 1$. However $AC^t(q^*) = 5$. Hence, we have that $P^* = 5$. Hence, the price is higher than before. At the new price, $Q^* = 5$ and $N^* = 5$. There are fewer firms in the tortilla industry as a result.

- (e) (6 points) Suppose that one specific tortilla firm has a short run marginal cost curve of:

$$MC = 6 + 5q$$

The market supply and demand curves for tortillas are given by:

$$P_S = 4 \cdot Q_S$$

$$P_D = 10 - Q_D$$

For what value of fixed costs, FC , will this firm be making losses in the short run?

In equilibrium, the price is given by:

$$Q^* = 2 \text{ and } P^* = 8$$

At this price, the firm will produce where $MC = MR$ and so $q^* = \frac{2}{5} = .4$. The profit of the firm is given by:

$$\begin{aligned} \pi &= P^*q^* - TC \\ &= 8 \cdot .4 - (2.8 + FC) \\ &= 0.4 - FC \end{aligned}$$

where the total cost is given by: $TC = \int_0^{q^*} (6 + 5q) dq = 6q + \frac{5q^2}{2} + FC = 2.8 + FC$. Hence, the firm is making a loss if $0.4 - FC < 0$ and so $FC > 0.4$.

4. Analysis of Government Policy (20 points)

US steel manufacturers supply steel according to the market supply function:

$$Q_S = 20 + P$$

US steel demand is given by:

$$Q_D = 30 - 4P$$

- (a) (2 points) What is the domestic equilibrium quantity and price?

In the market equilibrium $P = 2$ and $Q = 22$.

- (b) (6 points) The US government decides to open the steel market to trade. The world price of steel is $P^W = 0.5$. Assume US production is very small relative to world production such that the world price remains $P^W = 0.5$ even after the US opens to trade. What is the change in domestic consumer surplus and producer surplus?

At the world price consumers will demand $Q_D = 30 - 4 \cdot 0.5 = 28$ while domestic producers will supply $Q_S = 20 + 0.5 = 20.5$.

$$\Delta CS = (2 - 0.5) \cdot 22 + \frac{1}{2} \cdot (28 - 22) \cdot 1.5 = 37.5$$

$$\Delta PS = -(2 - 0.5) \cdot 20.5 - \frac{1}{2} \cdot 1.5 \cdot 1.5 = -31.875$$

- (c) (4 points) Now suppose the US government decides to restrict imports such that the domestic price is now $P^d = 1$. What quota would the government need to impose?

$$Quota = Q_D(1) - Q_S(1) = 26 - 21 = 5$$

- (d) (4 points) If the government auctions import permits to foreign producers, what is the maximum amount they would be willing to pay for such permits (in aggregate) given the quota you found in part (c)?

$$Value\ of\ permits = (1 - 0.5) \cdot 5 = 2.5$$

- (e) (4 points) Think more broadly about the U.S. economy. Would these changes in consumer and producer surplus be the only effects of imposing a quota relative to free trade? How do such restrictions affect the car industry? Would the steel workers union support such an import quota? How about construction workers? What would happen to the incentives to develop steel substitutes?

No, raising the price of steel via an import quota would have broader impacts. It would hurt domestic industries that rely on steel as an input like the car and construction industries. Steel worker unions would support the quota, but construction unions would oppose it. By raising the price of steel we would strengthen the incentives to develop alternatives.

5. General Equilibrium (20 points)

Robinson and Friday live on a desert island. There are two goods on the island that the two agents have utility over: cake (C) and ice-cream (I). Robinson's utility function is:

$$U_R(C_R, I_R) = 2 \cdot \ln(C_R) + \ln(I_R)$$

while Friday's utility function is:

$$U_F(C_F, I_F) = \ln(C_F) + 2 \cdot \ln(I_F)$$

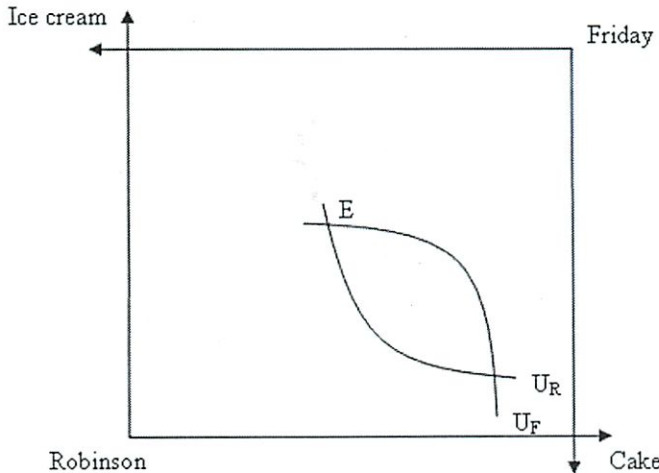
Robinson has 5 units of cake and 7 units of ice-cream while Friday has 5 units of cake and 6 units of ice cream.

- (a) (4 points) Write an expression for Robinson's and Friday's MRS. Under what condition is the allocation of cake and ice-cream between the two agents Pareto efficient? What is the relationship between the prices for cake, p_C , and ice-cream, p_I , and the MRS of the two agents at a Pareto efficient allocation?

We have that $MRS_R = \frac{U_C}{U_I} = 2 \frac{I_R}{C_R}$ and $MRS_F = \frac{U_C}{U_I} = \frac{1}{2} \frac{I_F}{C_F}$. Given the agents' preferences the allocation is Pareto efficient whenever $MRS_R = MRS_F$ or $2 \frac{I_R}{C_R} = \frac{1}{2} \frac{I_F}{C_F}$. At a Pareto efficient allocation we have that $MRS_R = \frac{p_C}{p_I} = MRS_F$.

- (b) (5 points) Draw an Edgeworth box and mark the initial endowment point and the indifference curves of Robinson and Friday going through that point. Is the initial endowment Pareto efficient? Why or why not?

The initial endowment point is not pareto efficient because the MRS for Robinson and Friday are not equal, i.e. we can make both of them better off by giving Robinson more cake and less ice-cream than his initial endowment and giving Friday more ice-cream and less cake than his initial endowment.



- (c) (5 points) Derive the contract curve in terms of C_R and I_R - the consumption bundle of Robinson at a Pareto efficient allocation. Using the contract curve write an expression relating the price ratio, $\frac{p_C}{p_I}$, and I_R in a competitive equilibrium.

We have that at a Pareto efficient allocation $MRS_R = MRS_F$ or $2 \frac{I_R}{C_R} = \frac{1}{2} \frac{I_F}{C_F} = \frac{1}{2} \frac{\bar{I} - I_R}{\bar{C} - C_R}$ where $\bar{I} = 13$ and $\bar{C} = 10$. Then $4(\bar{C}I_R - C_RI_R) = \bar{I}C_R - C_RI_R$ and so $C_R = \frac{4\bar{C}I_R}{\bar{I} + 3I_R} = \frac{40I_R}{13 + 3I_R}$ describes the contract curve. Using MRS_R we get that $2 \frac{I_R}{C_R} = \frac{p_C}{p_I}$ and so $\frac{13 + 3I_R}{20} = \frac{p_C}{p_I}$

- (d) (6 points) Compute the price ratio $\frac{p_C}{p_I}$ and the quantities of C and I that Robinson and Friday will consume in a competitive equilibrium of this exchange economy. You can leave your answers as simple fractions.

Approach 1: Suppose that the price ratio $\frac{p_C}{p_I} = p$. Then from the budget constrain for Robinson we have that $p_C C_R + p_I I_R = p_C 5 + p_I 7$ or dividing by p_I : $p C_R + I_R = 5p + 7$. From part (c) we have that at a competitive equilibrium allocation (which lies on the contract curve) and prices $C_R = \frac{40I_R}{13+3I_R}$ and $p = \frac{13+3I_R}{20}$. Substituting these into Robinson's budget constraint we get an equation for Robinson's consumption of ice-cream in a competitive equilibrium: $3I_R = \frac{13+3I_R}{4} + 7$. Hence, $I_R = \frac{41}{9}$ and so $p = \frac{4}{3}$, $C_R = \frac{41}{6}$, $I_F = \frac{76}{9}$ and $C_F = \frac{16}{9}$.

Approach 2: Given prices p_C and p_I from Robinson's utility maximization problem we get that $C_R = \frac{2}{3} \frac{\frac{p_C}{p_I} 5 + 7}{\frac{p_C}{p_I}}$, $I_R = \frac{1}{3} (\frac{p_C}{p_I} 5 + 7)$ and $C_F = \frac{1}{3} \frac{\frac{p_C}{p_I} 5 + 5}{\frac{p_C}{p_I}}$. We also know that $C_R + C_F = 10$ and so we have that $\frac{2}{3} \frac{\frac{p_C}{p_I} 5 + 7}{\frac{p_C}{p_I}} + \frac{1}{3} \frac{\frac{p_C}{p_I} 5 + 5}{\frac{p_C}{p_I}} = 10$ and so $\frac{p_C}{p_I} = \frac{20}{15} = \frac{4}{3}$. This means that $C_R = \frac{41}{6}$, $C_F = \frac{19}{6}$, $I_R = \frac{41}{9}$ and $I_F = \frac{76}{9}$.

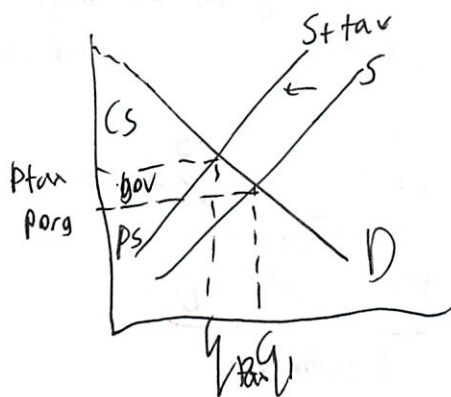
Practice #2

14.01 Fall 2008
Exam #2 ~~Solutions~~ Work
November 4th, 2008

1. (Total: 20 Points) True/False Questions: In this section write whether each claim is True or False. Please fully explain your answer, using a diagram if appropriate. No credit will be awarded to solutions without clear explanations.

- (a) (5 Points) The magnitude of the elasticity of the supply of hamburgers is less than 1, and the government imposes a \$1 tax on hamburgers. *Inelastic*
Claim: The buyer's price of hamburgers will necessarily increase less than 50 cents. (Support your argument with a sketch.) ~ who will assume tax

False - not necessarily



Comes out of consumer surplus - so all from ~~gov~~ consumer with price increase



oh will always \uparrow more than 50 c

\uparrow why, harder to prove

- (b) (5 Points) Claim: Any government intervention in a goods market that increases consumer surplus and generates dead-weight loss in that market must then also reduce producer surplus in that market. (Support your argument with a sketch.)

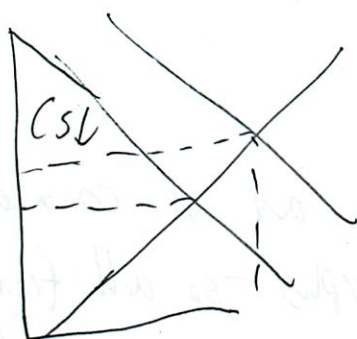
Yes - where else would the \uparrow in CS & DWL come from

Unless we have gov what

- support?
- price ceiling
- floor



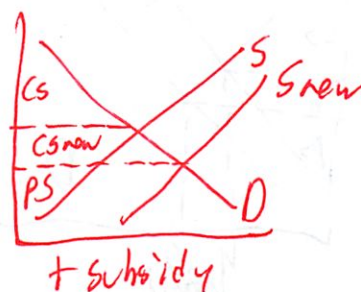
Support



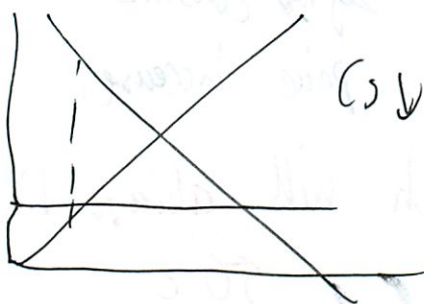
(X)

False
Subsidy

like the one thing I did not try

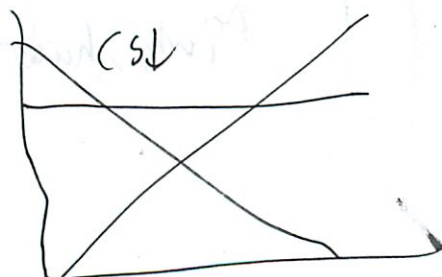


ceiling



CS \downarrow (X)

floor



(X)

True

- (c) (5 Points) Suppose that there are only two goods in the world, cars and wheat. In country X it takes 500 labor hours to produce a car and 0.25 labor hours to produce a pound of wheat. In country Y it takes 600 labor hours to produce a car and 0.35 labor hours to produce a pound of

wheat. These per-unit costs are independent of the quantity produced.

Claim: Country X has a comparative advantage over country Y in the production of cars. (Make explicit any calculations.)

MRTS, is right

1

car
wheat

500
0.25
2000

600
0.35

1714.2

Opps

so how does
this help

I should be able to do this!

entered wrong

False opportunity costs!

had the right #

Almost
had -
just could
not put
it together

2000 $\frac{\text{lb}}{\text{car}}$ > 1714 $\frac{\text{lb}}{\text{car}}$

larger tradeoff
need to give up
2000 lb for a
car

so this better at car

this assumes that each party can only do
1 thing right

- (d) (5 Points) A firm with a production technology that requires a constant ratio of inputs faces constant input prices.



Claim: If the firm exhibits economies of scale in production over some range of output, then it must also exhibit increasing returns to scale. (Make explicit any calculations.)

what is diff again

① economies of scale

~~double~~

~~scale~~ \rightarrow double inputs \rightarrow ^{more than} double output

~~from~~ ^{increasing} ② returns to scale \rightarrow

\rightarrow Cost advantage as business expands
(? so this is over time?)

$$C(aq_0) < aC(q_0), a > 1$$

True

2. (Total: 10 Points) Webcams are produced in a perfectly competitive market. The long run cost function of a webcam manufacturing firm is given by:

$$C(q) = 32 + 2q^3$$

oh let me see
if can do

- (a) (7 Points) Calculate the long run equilibrium price of webcams.

$$p = MC$$

$$MC = 2 \cdot 3q^2 = 6q^2$$

$$p = 6q^2$$

i need a # i

and a quantity - but can't w/o demand

$$AC = \frac{32}{q} + 2q^2$$

oh deh

$$p = AC = MC$$

should make cheat sheet

↑ keep forgetting
- min pt

$$6q^2 = \frac{32}{q} + 2q^2$$

$$4q^2 = \frac{32}{q}$$

$$4q^3 = 32$$

$$q^3 = 8$$

$$q = \sqrt[3]{8} = 2$$

$$p = 6(2)^2 = 24 \quad \checkmark$$

no demand so

no revenue

- just cheapest to
make

(b) (3 Points) Suppose the demand function for webcams is given by:

$$Q_D = 98 - 2p$$

Calculate the number of webcam-producing firms in the long run.

just for this one

Here we go

$$D = 98 - 2 \cdot 24 = 50$$

$$p = MC$$

$$6q^2$$

$$p = 24$$

$$a = 2$$

or is this $Q_D = Q_S$ - I never really know

50 are demanded

~~24~~ is 2 is cheapest to make

$$Q_D = 98 - 2p$$

$$2p = 98 - Q_D$$

$$p = 49 - \frac{Q_D}{2}$$

Price taker

$$24 = 49 - \frac{Q_D}{2}$$

$$-49 - 49 - \frac{Q_D}{2}$$

$$25 = \frac{Q_D}{2}$$

$$Q_D = 50 \leftarrow \text{back}$$

Did not read the ~~xxxx~~ing question # firms

$$\frac{50}{2} = 25$$

I read the qv!

Oh where ATC is lowest, 0? - no that on back

$$24 = 6q^2$$

$$4 = q^2$$

$$q = 2$$

this firm just 2

3. (Total: 10 Points) The production function for constructing skyscrapers is given by:

$$F(k, l) = k + (l + 1)^{\frac{1}{2}}$$

$\hat{Q} = \text{floors}$

where output is measured in number of floors.

The MIT construction company has agreed to construct a 64 floor skyscraper in the US and a 64 floor skyscraper in Farawayland. The firm has the same technology available in both countries. Assume that both capital and labor inputs are variable inputs in each country's production decision.

- (a) (5 Points) In Farawayland, where labor is abundant and capital is scarce, the rental rate of capital is \$60 and wages are \$20. Calculate the quantity of labor and capital inputs used to construct the skyscraper in Farawayland.

MRTS

$$\frac{dQ}{dk} = 1$$

$$\frac{k}{l} = \frac{1}{2(l+1)^{-1/2}}$$

$$\frac{dQ}{dl} = \frac{1}{2}(l+1)^{-1/2}$$

$$\frac{1}{\frac{1}{2}(l+1)^{-1/2}} = \frac{2\sqrt{l+1}}{1}$$

$$\text{now } MRTS = \frac{r}{w} = \frac{60}{20} = \frac{2\sqrt{l+1}}{1}$$

← they did other way
* bond conditions

$$60 = 20 \cdot 2\sqrt{l+1}$$

$$60 = 40\sqrt{l+1}$$

$$1.5 = \sqrt{l+1}$$

~

~

$$2.25 = l+1$$

$$1.25 = l \quad \checkmark$$

what happened to k?

plug into floors to build - my next thought

$$64 = k + (l+1)^{1/2} = k + (2.25)^{1/2}$$

$$k = 62.5 \quad \checkmark$$

- (b) (5 Points) In the US, the rental rate of capital is \$30 and wages are \$30. Calculate the quantity of labor and capital inputs used to construct the skyscraper in the US.

Same as before except diff #

$$\frac{30}{30} = \frac{2\sqrt{l+1}}{1}$$

$$30 = 30 \cdot 2\sqrt{l+1}$$

$$30 = 60\sqrt{l+1}$$

$$\frac{1}{2} = \sqrt{l+1}$$

$$\frac{1}{4} = l+1$$

$$-\frac{3}{4} = l \quad \text{minus 1}$$

① near corner solution

so say $l=0$

$$64 = k + (l+1)^{1/2}$$

$$64 = k + 1^{1/2}$$

$$(63 = k) \quad \checkmark$$

4. (Total: 19 Points) Donuts, D , are made with sugar, S , and machines, M . These inputs can be purchased in competitive markets at prices P_S and P_M respectively. Note that the machines are poorly constructed and fully depreciate after a single production period. Donuts are produced according to the following technology:

$$Q = D = \sqrt{4S + 2M^{1/2}}$$

$$2S^{1/2} + \sqrt{2} M^{1/4}$$

Should not
have
economized
on pg space

- (a) (2 Points) Calculate the firm's MRTS of S for M .

Solve for Q ~~is a~~ ~~need~~ $D^2 = 4S + 2M^{1/2}$

Solve for M ~~is a~~ ~~need~~

Or just S in terms of M ,

$$S = \frac{D^2 - 2M^{1/2}}{4}$$

$$2M^{1/2} = D^2 - 4S$$

$$M^{1/2} = \frac{D^2 - 4S}{2}$$

- (b) (5 Points) Calculate the firm's input demand for S as a function of output (D) and prices (P_M, P_S). For simplicity,

rule out corner solutions by assuming that $D \geq \sqrt{\frac{P_S}{2 \cdot P_M}}$.

$$M = \left(\frac{D^2 - 4S}{2} \right)^2$$

$$= D^4 - 4S^2$$

did kinda

$$S = \frac{D^2 - 2M^{1/2}}{4} \checkmark$$

- but needs to be
function of price - what is it
again?

~~now $\frac{dQ}{dS}$~~
no that was wrong
 $\frac{dQ}{dS} = 2 \cdot \frac{1}{2} S^{-1/2}$
 $S^{-1/2}$

then from a)

$$4\sqrt{M} = \frac{P_S}{P_M}$$

plug in

$$S = \frac{D^2 - 2 \left(\frac{16 P_S^2}{P_M^2} \right)^{1/2}}{4}$$

$$\frac{dQ}{dM} = \sqrt{2} \cdot \frac{1}{4} M^{-3/4}$$

$$\frac{\sqrt{2}}{4 M^{3/4}}$$

$$S^{1/2}$$

oppo $\frac{4P_S}{P_M} = \sqrt{M}$

$$\frac{16 P_S^2}{P_M^2} = M$$

$$= \frac{D^2 - 2 \left(\frac{4 P_S}{P_M} \right)}{4}$$

$$\frac{\sqrt{2}}{4 M^{3/4}}$$

$$S^{-1/2} \cdot \frac{4 M^{3/4}}{\sqrt{2}}$$

$$= \frac{D^2 - 2 P_S}{4 P_M}$$

then bounds
if $D > \sqrt{\frac{1}{2} \frac{P_S}{P_M}}$

corner
 $S=0$ if $D < \sqrt{\frac{1}{2} \frac{P_S}{P_M}}$

visualize +
picture

must be same math error - can't trace it back

$(= 4\sqrt{M})$ $\frac{4 M^{3/4}}{\sqrt{2}}$ can't merge right?

(c) (4 Points) Calculate the firm's long run total cost function,

$C(D)$. For simplicity, assume that $D \geq \sqrt{\frac{P_s}{2 \cdot P_m}}$.

$$C(D) = \cancel{wL} + rK$$

$$= \cancel{w}$$

2 weird def.

$$= 5P_s + MP_m \quad \textcircled{1}$$

$$\left(\frac{D^2 - 2M'^2}{4} \right) P_s + (D^2 - 4S^2) P_m$$

in terms of only D .

NO

$$\frac{D^2 P_s}{4}$$

$$- \frac{P_s^2}{16 P_m}$$

but in terms
of only
prices

- just simplification

- (d) (3 points) Suppose that $P_S = P_M = 8$ and the price of donuts is 16. Calculate the quantity of donuts the firm would produce in the long-run.

here we go

$$C(D) = \frac{D^2 P_S}{4} - \frac{P_S^2}{16 P_M} = \frac{D^2 8}{4} - \frac{8^2}{16 \cdot 8}$$

$$= 2D^2 - \frac{1}{2}$$

MC $2 \cdot 2D = 4D$

oops wrong pen

$$4D = P = AC = MC$$

$$4D = 16$$

$$D = 4$$

- (e) (5 Points) Suppose that $P_S = 8$ and that $P_M = 12$ and the price of donuts remains 16. Also, suppose that due to an ordering blunder, the firm purchases and installs 4 machines that cannot be removed in the short run, that may not be rented out to another firm, and that no additional machines may be installed. Calculate the quantity of donuts that the firm produces in the short run if operating. Calculate the firm's resulting accounting profit. Will the firm optimally choose to shut-down in the short run?

So now cost

$$C = \frac{D^2 P_S}{4} - \frac{P_S^2}{16 P_M} + 4M$$

need in terms P, S

$$\frac{D^2 P_S}{4} - \frac{P_S^2}{16 P_M} + 4(D^2 - 4S^2)$$

$$\frac{D^2 P_S}{4} - \frac{P_S^2}{16 P_M} + 4 \left(D^2 - 4 \left(\frac{D^2}{4} - \frac{1}{8} \frac{P_S}{P_M} \right) \right)$$

oh boy!

$$\frac{D^2 P_S}{4} - \frac{P_S^2}{16 P_M} + 4 \left(D^2 - D^2 + \frac{1}{2} \frac{P_S}{P_M} \right)$$

$$\frac{D^2 P_S}{4} - \frac{P_S^2}{16 P_M} + 4D^2 - 4D^2 + 2 \frac{P_S}{P_M}$$

$$C = \frac{D^2 P_S}{4} - \frac{P_S^2}{16 P_M} + 2 \frac{P_S}{P_M}$$

now plug in

$$C = \frac{D^2 \cdot 8}{4} - \frac{8^2}{16 \cdot 12} + \frac{2 \cdot 8}{12}$$

$$= 2D^2 - \frac{1}{3} + \frac{4}{3}$$

$$= 2D^2 + 1$$

$$MC = 4Q \quad Q=4 \quad \checkmark$$

Same quantity

$$AC = \frac{2D^2 + 1}{D} = 2D + \frac{1}{D}$$

shut down if $P < AVC$

$$AVC = 2D = 2 \cdot 4 = 8$$

$P = 6$ not shut down

⊂ but is that per item

- yes

not total

- makes sense w/ 8

$$\pi = 6 \cdot 4 - 2(4)^2 + 1$$

$$64 - 33$$

31 of course stay open

Production Function

$$Q = \sqrt{4S + 2 \cdot 4^{1/2}} \rightarrow S = 3$$

how much sugar to buy

$$\pi = 6 \cdot 4 - 8 \cdot 3 - 12 \cdot 4 = -8$$

guess math was wrong

$$\text{no production } \pi = -48$$

toh could do that too!

Again a basic
or is it?

5. (Total: 20 Points) Consumers A and B consume goods X and Y. There is a fixed total of 10 units of X and 9 units of Y in the world. Consumer A has utility of the form:

$$U_A(X_A, Y_A) = \ln(X_A) + \ln(Y_A)$$

Consumer B has utility of the form:

$$U_B(X_B, Y_B) = \ln(X_B) + 2 * \ln(Y_B)$$

- (a) (5 Points) Suppose that a utilitarian planner allocates the goods such that $\text{SWF} = U_A + U_B$ is maximized. Determine the allocation of goods that maximizes the SWF.

Oh this was on other one + did not get

- is it like that box

will skip - out of context

(b) (2 Points) Calculate each individual's utility level at the allocated consumption levels.

(c) (2 Points) Is this allocation Pareto efficient? Explain.

(d) Suppose that consumers are able to engage in trade, but a law restricts them to trade X for Y at a 1-for-1 ratio. Also, assume the initial allocation of goods is instead $X_A = 5, Y_A = 5, X_B = 5, Y_B = 4$

i. (2 Point) Calculate each individual's budget constraint.

- ii. (2 Points) Determine each consumer's optimal consumption bundle, given his initial endowment and the imposed exchange ratio of 1.
- iii. (3 Points) Continuing to assume that the exchange rate is restricted to be 1-for-1, will trade occur between consumers A and B from the initial allocation?
- iv. (2 points) Is the outcome Pareto efficient? Explain.
- v. (2 points) Suppose that the law restricting the exchange rate of 1 is abolished. Could trading/bargaining from the initial allocation in (d) result in the planner's allocation from (a)? Explain.

6. (Total: 20 Points) In the country of Fritoland domestic demand for corn is given by:

$$Q_D = 120 - p$$

and domestic supply for corn is given by:

$$Q_S = \frac{1}{2}p$$

where quantities are measured in billions of pounds and prices in cents per pound.

- (a) (2 points) Calculate the price of corn and the quantity of corn consumed in Fritoland if the country remains isolated from the rest of the world? (Include units.)

$$Q_D = Q_S$$

$$120 - p = \frac{1}{2}p$$

$$240 - 2p = p$$

$$240 = 3p$$

$$p = 80$$

Cents per
pound

$$Q = 120 - p$$

$$= 120 - 80$$

$$= 40$$

billions of pounds

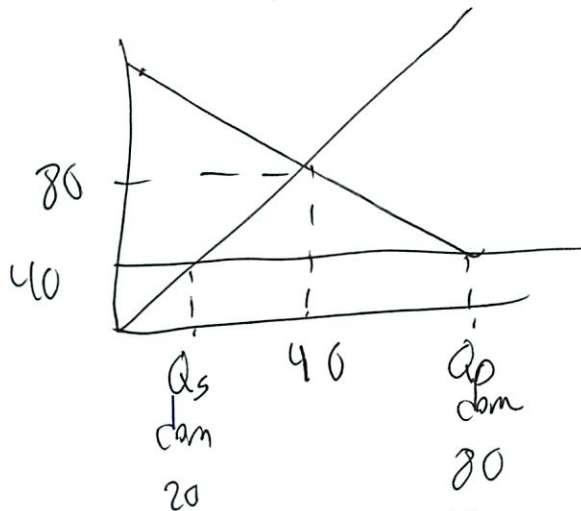
Should be good
at student

we never do that

- (b) (4 points) Suppose that Fritoland opens itself to foreign trade, and imports/exports of corn are now allowed without restriction. Further, suppose that the world price of corn is 40 cents per pound. Calculate the quantity of corn supplied by domestic producers, the quantity of corn

h

consumed domestically, and the net quantity of corn imported/exported by Fritoland. (Include units.)

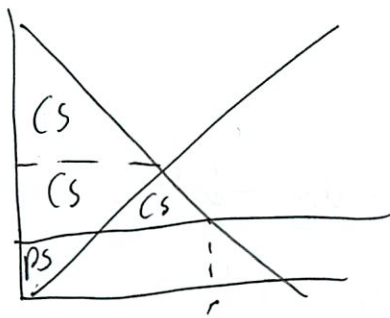


$$Q_{d, \text{dom}} = 120 - 40 = 80 \checkmark$$

$$Q_{s, \text{dom}} = \frac{1}{2} \cdot 40 = 20 \checkmark \text{ billion lbs}$$

$$\text{imported} = 80 - 20 = 60 \text{ billion lbs} \checkmark$$

- (c) (4 points) Calculate the net change in domestic consumer surplus and the net change in domestic producer surplus resulting from opening Fritoland to foreign trade. (Include units.)



$$\int_0^{80} (120 - x) dx - 40 \cdot 80$$

$$120x - \frac{x^2}{2} \Big|_0^{80} - 3200$$

$$6400 - 3200$$

$$\text{new CS} = 3200 \checkmark$$

$$\text{old CS} = \int_0^{40} (120 - x) dx - 80 \cdot 40$$

$$4000 - 3200 = 800 \checkmark$$

$$\text{new CS} = 2400 \text{ billion cents}$$

$$\text{new PS} = \frac{1}{2} \cdot 40 \cdot 20 = 400 \checkmark$$

$$\text{PS loss} = 1200 \checkmark$$

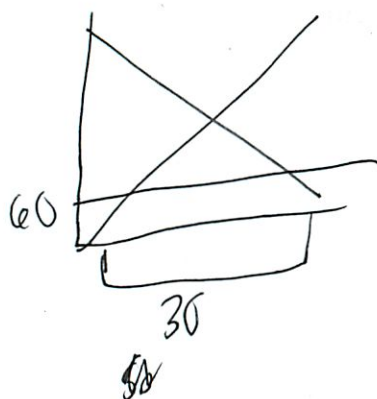
$$- \$12 \text{ billion}$$

$$\text{old PS} = \frac{1}{2} \cdot 80 \cdot 40 = 1600 \checkmark$$

$$\text{Gain to society} = 2400 - 1200 = 1200 \text{ cents}$$

- (d) The domestic producers of corn are not happy with the introduction of free trade and threaten to start a revolution. The government responds by setting a quota that restricts imports to a maximum of 30 billion pounds.

- i. (2 points) Calculate the resulting price of corn in Fritoland. (Include units.)



so what to do?
- set = to

$$Q_D = Q_S + 30$$

$$120 - p = \frac{1}{2}p + 30$$

$$240 - 2p = p + 60$$

$$180 = 3p$$

$$p = 60 \quad \checkmark$$

- ii. (1 points) Calculate the quantity of corn consumed domestically. (Include units.)

$$120 - p$$

$$120 - 60$$

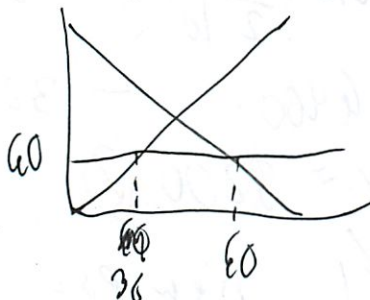
$$60 \quad \checkmark$$

$$Q_S = \frac{1}{2} \cdot 60$$

$$= 30$$

not change!

- iii. (2 points) Calculate the resulting domestic consumer surplus and domestic producer surplus. (Include units.)



$$\int_0^{60} (120 - x) dx - 60 \cdot 60$$

$$5400 - 3600$$

$$CS = 1800 \quad \checkmark$$

$$PS = \frac{1}{2} \cdot 30 \cdot 60 = 900 \quad \checkmark$$

- iv. (2 points) Calculate the deadweight loss associated with the quota relative to the free trade state. (Ignore the surplus of foreign producers in calculating social surplus). (Include units.)

$$DWL = \text{Old Society welfare}^{\text{free trade}} - \text{new}^{\text{quota}}$$

~~2400~~

~~2400~~

$$3200 + 400$$

$$- (1800 + 400)$$

$$3600$$

$$- 2200$$

$$900$$

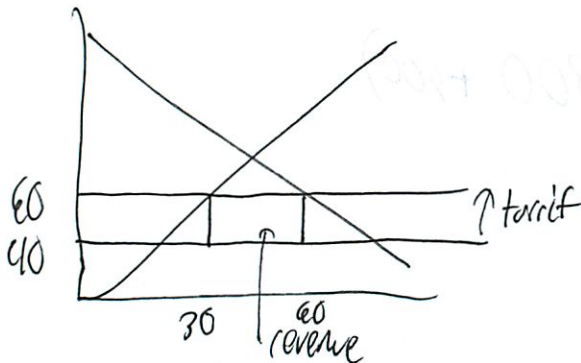


I think I am doing
really well here

- (e) (3 points) Suppose that to further appease domestic producers, the government decides to eliminate the quota in favor of a tariff that will leave imports at the current level

hmm

as under the quota, and distributes the revenues of the tariff among domestic producers. Will this policy fully compensate the producers for the loss of surplus due to imports (compared to the isolated economy in (a))?



$$\begin{array}{cc} (60 - 40) & (60 - 30) \\ 20 & 30 \\ 600 \end{array}$$

$$900 + 600 = 1500 \neq 1600 \text{ as before}$$

✓

perfect - got this entire section
 - now just need to do this for rest
 of the exam

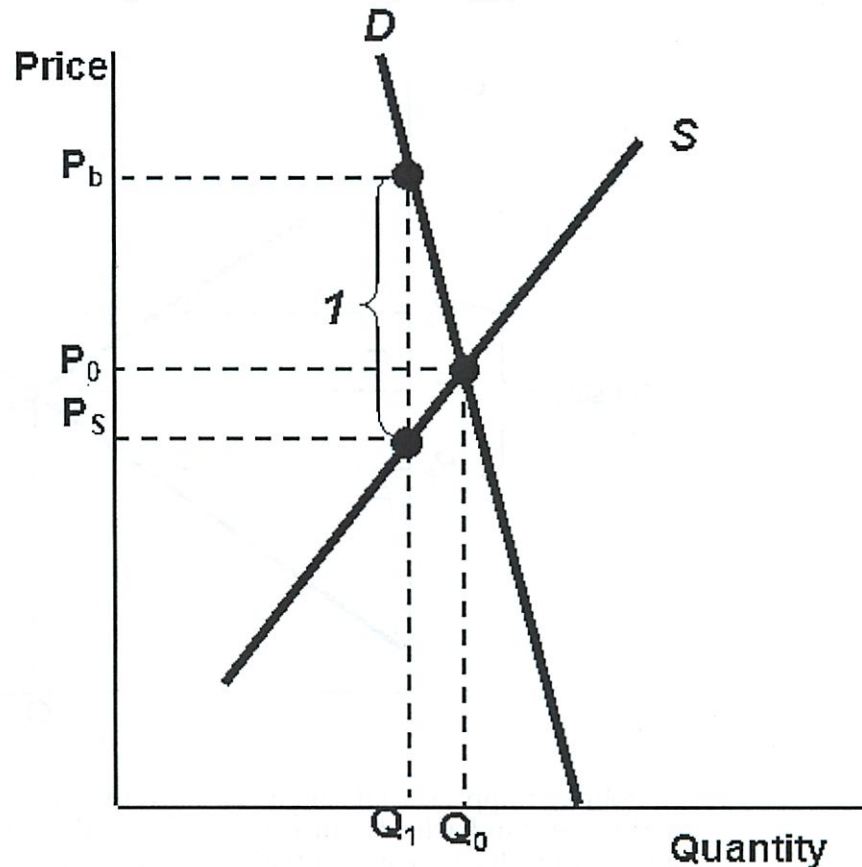
14.01 Fall 2008
Exam #2 Solutions
November 4th, 2008

1. (Total: 20 Points) True/False Questions: In this section write whether each claim is True or False. Please fully explain your answer, using a diagram if appropriate. No credit will be awarded to solutions without clear explanations.

- (a) (5 Points) The magnitude of the elasticity of the supply of hamburgers is less than 1, and the government imposes a \$1 tax on hamburgers.

Claim: The buyer's price of hamburgers will necessarily increase less than 50 cents. (Support your argument with a sketch.)

False, if $|E_D| < |E_S|$ the price of hamburgers will increase more than 50 cents regardless of the elasticity of supply.



- (b) (5 Points) Claim: Any government intervention in a goods market that increases consumer surplus and generates deadweight loss in that market must then also reduce producer surplus in that market. (Support your argument with a sketch.)

False: a subsidy makes both producers and consumer better off, but there is deadweight loss since the government expenditure is greater than the sum of the gains in surplus.

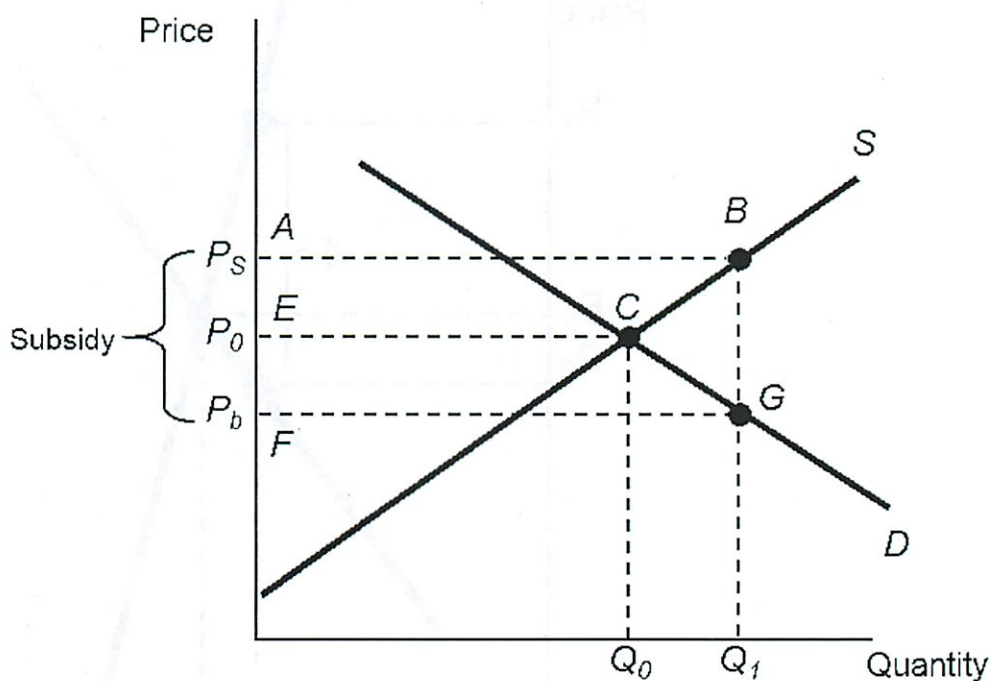
In the diagram:

$$\Delta CS = ECGF > 0$$

$$\Delta PS = ABCE > 0$$

$$\text{Govt. Expenditure} = ABGF$$

$$\begin{aligned} DWL &= |\Delta CS + \Delta PS - \text{Govt. Expenditure}| \\ &= CBG \end{aligned}$$



- (c) (5 Points) Suppose that there are only two goods in the world, cars and wheat. In country X it takes 500 labor hours to produce a car and 0.25 labor hours to produce a pound of wheat. In country Y it takes 600 labor hours to produce a car and 0.35 labor hours to produce a pound of

wheat. These per-unit costs are independent of the quantity produced.

Claim: Country X has a comparative advantage over country Y in the production of cars. (Make explicit any calculations.)

False. Opportunity cost of a car in terms of wheat in country X is $\frac{500 \frac{lb}{car}}{.25 \frac{lb}{lb}} = 2000.0 \frac{lb}{car} > \frac{600}{0.35} = 1714.3 \frac{lb}{car}$ = opportunity cost of a car in terms of wheat in country Y. This means Y has a comparative advantage over X in the production of cars since has lower opportunity cost. (Even though X has absolute advantage).

- (d) (5 Points) A firm with a production technology that requires a constant ratio of inputs faces constant input prices.

Claim: If the firm exhibits economies of scale in production over some range of output, then it must also exhibit increasing returns to scale. (Make explicit any calculations.)

True. Intuitively, let $q_0 = F(K_o, L_o) \rightarrow C(q_o) = wL_o + rK_o$.

Consider increasing inputs by a scalar $a > 1$. We know by Economies of Scale:

$$C(aq_o) < aC(q_o), a > 1$$

Let $q_1 = F(aK_o, aL_o)$. By constant ratio of inputs we know cost increases by a . If $q_1 = aq_o$ then increasing output by a is a times the initial cost, but we know the cost is strictly less. Therefore it must be that $q_1 > aq_o$ and we have increasing returns to scale.

$$F(aK_o, aL_o) > aF(K_o, L_o)$$

Alternatively note

$$C(aF(K_o, L_o)) = \underbrace{C(aq_o) < aC(q_o)}_{\text{By economies of scale}} = \underbrace{aC(F(K_o, L_o))}_{\text{By constant ratio of inputs}} = C(F(aK_o, aL_o))$$

$$\rightarrow aF(K_o, L_o) < F(aK_o, 2L_o) \rightarrow \text{Increasing Returns to Scale}$$

2. (Total: 10 Points) Webcams are produced in a perfectly competitive market. The long run cost function of a webcam manufacturing firm is given by:

$$C(q) = 32 + 2q^3$$

- (a) (7 Points) Calculate the long run equilibrium price of webcams.

We have that:

$$\begin{aligned} MC &= 6q^2 \\ AC &= \frac{32}{q} + 2q^2 \\ TVC &= 2q^3 \\ AVC &= 2q^2 \end{aligned}$$

In the long run profits must be zero, so $p = AC$ and profit maximization implies $p = MC$ therefore:

$$\begin{aligned} AC &= MC \\ \Rightarrow \frac{32}{q} + 2q^2 &= 6q^2 \\ \Rightarrow \frac{32}{4} &= q^3 \\ \Rightarrow q &= 2 \end{aligned}$$

so the long run equilibrium price will be

$$MC(2) = 6(2^2) = 24$$

- (b) (3 Points) Suppose the demand function for webcams is given by:

$$Q_D = 98 - 2p$$

Calculate the number of webcam-producing firms in the long run.

Given the long run equilibrium price, we know the quantity demanded will be $98 - 2(24) = 50$. Since in the long run each firm produces 2 webcams, the number of firms will be 25

3. (Total: 10 Points) The production function for constructing skyscrapers is given by:

$$F(k, l) = k + (l + 1)^{\frac{1}{2}}$$

where output is measured in number of floors.

The MIT construction company has agreed to construct a 64 floor skyscraper in the US and a 64 floor skyscraper in Farawayland. The firm has the same technology available in both countries. Assume that both capital and labor inputs are variable inputs in each country's production decision.

- (a) (5 Points) In Farawayland, where labor is abundant and capital is scarce, the rental rate of capital is \$60 and wages are \$20. Calculate the quantity of labor and capital inputs used to construct the skyscraper in Farawayland.

Given the production function, we have:

$$\begin{aligned} MP_k &= 1 \\ MP_l &= \frac{1}{2} (l+1)^{-\frac{1}{2}} \end{aligned}$$

then

$$MRTS = \frac{1}{2\sqrt{l+1}} \in \left(0, \frac{1}{2}\right)$$

So there will be an interior solution iff $\frac{w}{r} < \frac{1}{2}$. Therefore to find the amount of capital and labor used in Faraway land we set:

$$\begin{aligned} MRTS &= \frac{w}{r} \\ \frac{20}{60} &= \frac{1}{3} = \frac{1}{2\sqrt{l+1}} \\ l &= \frac{9}{4} - 1 \\ l &= \frac{5}{4} \end{aligned}$$

Therefore, since 64 floors need to be constructed:

$$64 = k + \sqrt{\frac{5}{4} + 1}$$

so

$$\begin{aligned} k &= 64 - \frac{3}{2} \\ &= 62.5 \end{aligned}$$

- (b) (5 Points) In the US, the rental rate of capital is \$30 and wages are \$30. Calculate the quantity of labor and capital inputs used to construct the skyscraper in the US.

Now, in the US $\frac{w}{r} = 1$ so there will be a corner solution and only capital will be used. Therefore:

$$l = 0$$

which implies:

$$\begin{aligned} 64 &= (0+1)^{\frac{1}{2}} + k \\ k &= 63 \end{aligned}$$

4. (Total: 19 Points) Donuts, D , are made with sugar, S , and machines, M . These inputs can be purchased in competitive markets at prices P_S and P_M respectively. Note that the machines are poorly constructed and fully depreciate after a single production period. Donuts are produced according to the following technology:

$$D = \sqrt{4S + 2M^{1/2}}$$

- (a) (2 Points) Calculate the firm's MRTS of S for M .

$$\frac{\frac{\partial D}{\partial S}}{\frac{\partial D}{\partial M}} = \frac{\frac{1}{2}(4S+2M^{1/2})^{-1/2} \cdot 4}{\frac{1}{2}(4S+2M^{1/2})^{-1/2} M^{-1/2}} = 4\sqrt{M}$$

- (b) (5 Points) Calculate the firm's input demand for S as a function of output (D) and prices (P_M, P_S). For simplicity, rule out corner solutions by assuming that $D \geq \sqrt{\frac{P_S}{2P_M}}$.

$$D = \sqrt{4S + 2M^{1/2}}$$

$$S = \left(\frac{D^2}{4} - \frac{1}{2} M^{1/2} \right)$$

From (a)

$$4\sqrt{M} = \frac{P_S}{P_M} \rightarrow M = \frac{1}{16} \left(\frac{P_S}{P_M} \right)^2$$

$$S(D, P_S, P_M) = \left(\frac{D^2}{4} - \frac{1}{8} \frac{P_S}{P_M} \right)$$

$$\text{if } D > \sqrt{\frac{1}{2} \frac{P_S}{P_M}}; S = 0 \text{ if } D < \sqrt{\frac{1}{2} \frac{P_S}{P_M}}$$

- (c) (4 Points) Calculate the firm's long run total cost function, $C(D)$. For simplicity, assume that $D \geq \sqrt{\frac{P_S}{2P_M}}$.

$$C = P_S S + P_M M$$

$$C(D)_{LR} = P_S \left(\frac{D^2}{4} - \frac{1}{8} \frac{P_S}{P_M} \right) + P_M \frac{1}{16} \left(\frac{P_S}{P_M} \right)^2$$

$$\frac{D^2 P_S}{4} - \frac{P_S^2}{16 P_M}$$

- (d) (3 points) Suppose that $P_S = P_M = 8$ and the price of donuts is 16. Calculate the quantity of donuts the firm would produce in the long-run.

$$MC = P_D = 16$$

$$P_S \frac{D}{2} = 4D = 16 \rightarrow D = 4$$

Note that the LRAC is

$$\frac{P_S \left(\frac{D^2}{4} - \frac{1}{8} \frac{P_S}{P_M} \right) + P_M \frac{1}{16} \left(\frac{P_S}{P_M} \right)^2}{D} = \frac{8 \left(\frac{4^2}{4} - \frac{1}{8} \right) + 8 * \frac{1}{16}}{4} = 7.875$$

Since $P_D = 16 > LRAC = 7.875$ this is a stable production choice.

- (e) (5 Points) Suppose that $P_S = 8$ and that $P_M = 12$ and the price of donuts remains 16. Also, suppose that due to an ordering blunder, the firm purchases and installs 4 machines that cannot be removed in the short run, that may not be rented out to another firm, and that no additional machines may be installed. Calculate the quantity of donuts that the firm produces in the short run if operating. Calculate the firm's resulting accounting profit. Will the firm optimally choose to shut-down in the short run?

$$MC_{SR} = P_D = 16$$

$$P_S \frac{D}{2} = 4D = 16 \rightarrow D = 4$$

From the production function

$$4 = \sqrt{4S + 2 * 4^{1/2}} \rightarrow S = 3$$

$$\begin{aligned} \text{Profit} &= P_D D - P_S S - P_M M \\ &= 16 * 4 - 8 * 3 - 12 * 4 = -8 \end{aligned}$$

Even with negative accounting profit, firm continues to produce since absent production accounting profit would be -48.

5. (Total: 20 Points) Consumers A and B consume goods X and Y. There is a fixed total of 10 units of X and 9 units of Y in the world. Consumer A has utility of the form:

$$U_A(X_A, Y_A) = \ln(X_A) + \ln(Y_A)$$

Consumer B has utility of the form:

$$U_B(X_B, Y_B) = \ln(X_B) + 2 * \ln(Y_B)$$

- (a) (5 Points) Suppose that a utilitarian planner allocates the goods such that $SWF = U_A + U_B$ is maximized. Determine the allocation of goods that maximizes the SWF.

$$\max \ln(X_A) + \ln(Y_A) + \ln(X_B) + 2\ln(Y_B)$$

subject to

$$Y_A + Y_B = 9; X_A + X_B = 10$$

$$\ln(10 - X_B) + \ln(9 - Y_B) + \ln(X_B) + 2\ln(Y_B)$$

$$X_B] \frac{-1}{10 - X_B} + \frac{1}{X_B} = 0$$

$$X_B = 5 \rightarrow X_A = 5$$

$$Y_B] \frac{-1}{9 - Y_B} + \frac{2}{Y_B} = 0$$

$$Y_B = 6 \rightarrow Y_A = 3$$

- (b) (2 Points) Calculate each individual's utility level at the allocated consumption levels.

$$\ln(X_A) + \ln(Y_A); \ln(X_B) + 2\ln(Y_B)$$

$$U_A(X_A, Y_A) = \ln(5) + \ln(3) = 2.708$$

$$U_B(X_B, Y_B) = \ln(5) + 2 * \ln(6) = 5.193$$

- (c) (2 Points) Is this allocation Pareto efficient? Explain.

Yes. *MRS's are equal at this allocation.*

$$\frac{\frac{1}{X_A}}{\frac{1}{Y_A}} = \frac{3}{5} = \frac{\frac{1}{X_B}}{\frac{2}{Y_B}} = \frac{3}{5}$$

- (d) Suppose that consumers are able to engage in trade, but a law restricts them to trade X for Y at a 1-for-1 ratio. Also, assume the initial allocation of goods is instead $X_A = 5, Y_A = 5, X_B = 5, Y_B = 4$

- i. (2 Point) Calculate each individual's budget constraint.
 $\frac{P_X}{P_Y} = 1 \rightarrow P_X = P_Y = P$

$$PX_A + PY_A = 5P + 5P$$

$$X_A + Y_A = 10$$

$$PX_B + PY_B = 5P + 4P$$

$$X_B + Y_B = 9$$

- ii. (2 Points) Determine each consumer's optimal consumption bundle, given his initial endowment and the imposed exchange ratio of 1.

$$\ln(X_A) + \ln(Y_A); \ln(X_B) + 2\ln(Y_B)$$

$$MRS_A = \frac{X_A}{Y_A} = \frac{X_A}{10 - X_A} = \frac{1}{1} \rightarrow X_A = 5, Y_A = 5$$

$$MRS_B = \frac{2X_B}{Y_B} = \frac{2X_B}{9 - X_B} = \frac{1}{1} \rightarrow X_B = 3, Y_B = 6$$

- iii. (3 Points) Continuing to assume that the exchange rate is restricted to be 1-for-1, will trade occur between consumers A and B from the initial allocation?

No. Person A has their optimal allocation initially, so they will not trade. Person B wants to trade Y for X but there is no one to trade with.

- iv. (2 points) Is the outcome Pareto efficient? Explain.

No. Their MRS's when consuming the initial endowment are not equal

$$MRS_A = \frac{X_A}{Y_A} = \frac{5}{5} \neq \frac{2X_B}{Y_B} = \frac{10}{4} = MRS_B$$

For example A and B are better off consuming (6,4.5) and (4,4.5) than (5,5) and (5,4)

- v. (2 points) Suppose that the law restricting the exchange rate of 1 is abolished. Could trading/bargaining from the initial allocation in (d) result in the planner's allocation from (a)? Explain.

Consider the utility levels at the endowment in part d)

$$\ln(X_A) + \ln(Y_A); \ln(X_B) + 2\ln(Y_B)$$

$$U_A(X_A, Y_A) = \ln(5) + \ln(5) = 3.219$$

$$U_B(X_B, Y_B) = \ln(5) + 2 * \ln(4) = 4.382$$

Compare these to the utilities attained when the planner allocates

$$\ln(X_A) + \ln(Y_A); \ln(X_B) + 2\ln(Y_B)$$

$$U_A(X_A, Y_A) = \ln(5) + \ln(3) = 2.708$$

$$U_B(X_B, Y_B) = \ln(5) + 2 * \ln(6) = 5.193$$

Note that Person A is strictly better off with the allocation at in part d. Therefore the planners allocation does not lie in the "gains from trade" region from the initial allocation and the bargaining/trading equilibrium can never reach the planners allocation.

6. (Total: 20 Points) In the country of Fritoland domestic demand for corn is given by:

$$Q_D = 120 - p$$

and domestic supply for corn is given by:

$$Q_S = \frac{1}{2}p$$

where quantities are measured in billions of pounds and prices in cents per pound.

- (a) (2 points) Calculate the price of corn and the quantity of corn consumed in Fritoland if the country remains isolated from the rest of the world? (Include units.)

Setting $Q_D = Q_S$:

$$\frac{1}{2}p = 120 - p$$

$$p = 80 \text{ cents per pound}$$

and

$$Q = 40 \text{ billion pounds}$$

- (b) (4 points) Suppose that Fritoland opens itself to foreign trade, and imports/exports of corn are now allowed without restriction. Further, suppose that the world price of corn is 40 cents per pound. Calculate the quantity of corn supplied by domestic producers, the quantity of corn

consumed domestically, and the net quantity of corn imported/exported by Fritoland. (Include units.)

$$Q_S = \frac{1}{2} (40) = 20$$

So the quantity supplied by domestic producers will be 20 billion pounds.

$$Q_D = 120 - 40 = 80$$

So the total quantity consumed will be 80 billion pounds:

Then imports will be 60 billion pounds

- (c) (4 points) Calculate the net change in domestic consumer surplus and the net change in domestic producer surplus resulting from opening Fritoland to foreign trade. (Include units.)

Before opening the economy:

$$CS = \frac{1}{2} (120 - 80) (40) = 800$$

$$PS = \frac{1}{2} (80) (40) = 1600$$

After opening the economy:

$$CS = \frac{1}{2} (120 - 40) (80) = 3200$$

$$PS = \frac{1}{2} (40) (20) = 400$$

Therefore:

$$\Delta CS = 3200 - 800 = 2400 \text{ billion cents} = \$24 \text{ billion}$$

$$\Delta PS = 400 - 1600 = -1200 \text{ billion cents} = -\$12 \text{ billion}$$

- (d) The domestic producers of corn are not happy with the introduction of free trade and threaten to start a revolution. The government responds by setting a quota that restricts imports to a maximum of 30 billion pounds.
- i. (2 points) Calculate the resulting price of corn in Fritoland. (Include units.)

We have that imports now have to be 30 billion. Therefore, the difference between domestic consumption and domestic supply will be 30:

$$Q_D - Q_S^d = 30$$

$$120 - p - \frac{1}{2}p = 30$$

$$90 = \frac{3}{2}p$$

$$p = 60$$

Then $p = 60$ cents per pound.

- ii. (1 points) Calculate the quantity of corn consumed domestically. (Include units.)

Substituting in the demand:

$$Q = 120 - 60 = 60$$

so the new quantity consumed will be 60 billion pounds.

- iii. (2 points) Calculate the resulting domestic consumer surplus and domestic producer surplus. (Include units.)

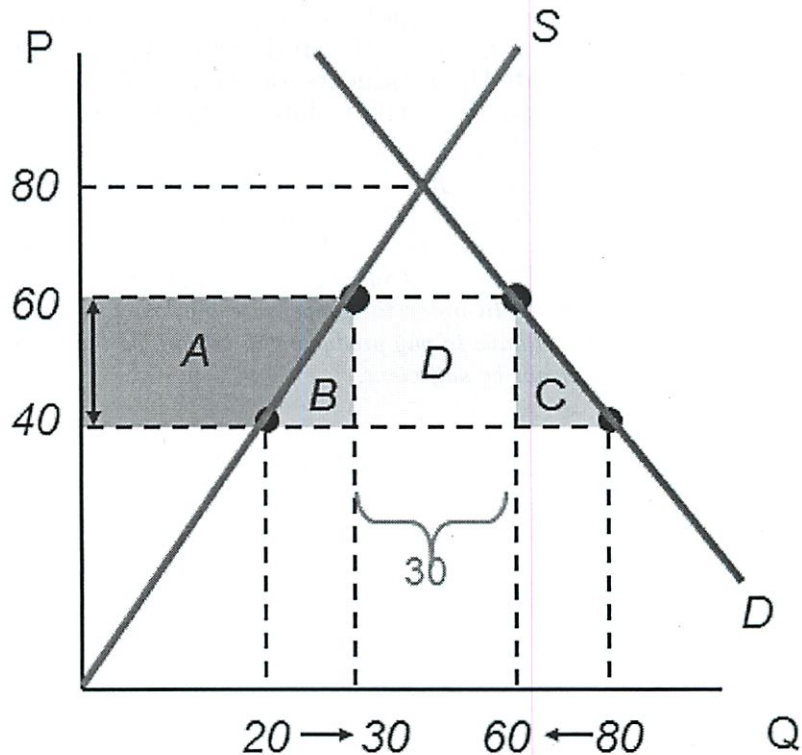
What will be the resulting consumer surplus and domestic producer surplus?

$$\begin{aligned} CS &= \frac{1}{2} (120 - 60) 60 = 1800 \text{ billion cents} \\ &= \$18 \text{ billion} \end{aligned}$$

The quantity supplied by the domestic producers will be 30, so

$$\begin{aligned} PS &= \frac{1}{2} (60) (30) = 900 \text{ billion cents} \\ &= \$9 \text{ billion} \end{aligned}$$

- iv. (2 points) Calculate the deadweight loss associated with the quota relative to the free trade state. (Ignore the surplus of foreign producers in calculating social surplus). (Include units.)



We have that the ΔPS will be the region A :

$$\Delta PS = \frac{1}{2} (30 + 20) (60 - 40) = 500$$

The loss in consumer surplus will be the sum of the regions A, B, C and D :

$$\Delta CS = -\frac{1}{2} (60 + 80) (60 - 40) = -1400$$

Since there is no government revenue or expenditure and we ignore the surplus of foreign producers, the loss of social surplus will be the sum of the regions denoted as B, C and D in the diagram :

$$\begin{aligned} DWL &= -(\Delta PS + \Delta CS) \\ &= 900 \text{ billion cents} \\ &= \$9 \text{ billion} \end{aligned}$$

- (e) (3 points) Suppose that to further appease domestic producers, the government decides to eliminate the quota in favor of a tariff that will leave imports at the current level

as under the quota, and distributes the revenues of the tariff among domestic producers. Will this policy fully compensate the producers for the loss of surplus due to imports (compared to the isolated economy in (a))?

Since the quota rose the price from 40 to 60 cents/lb, the same effect would result from a 20 cent/lb quota. Imports would be 30 billion pounds, but now there would be a government revenue of $(20)(30) = 600$ billion cents = \$6 billion (region D in the diagram). However PS remains at \$9 billion, whereas before there were imports it was \$16 billion. In order to compensate the loss of surplus, the government would have to pay producers \$7 billion, so the revenue from the tariff will not be sufficient.

$P = AC = MC$ = long run = librium price
w/ just cost function

-cheapest # to make

$$\text{MRTS } \frac{\frac{\partial F}{\partial K}}{\frac{\partial F}{\partial L}} = \frac{r}{w} \quad \text{) or flip}$$

solve for ~~the~~ one
plug into quantity for other

$$\text{MRTS} = \frac{P_{\text{top}}}{P_{\text{bottom}}}$$

*Exam Redo

11/9

$$2. \quad q(k, L) = k^{1/2} L^{1/2} \quad w=4 \quad r=1$$

LR demand for L, k

$$\begin{aligned} \text{MRTS} &= \frac{\partial q}{\partial L} = \frac{1}{2} L^{-1/2} k^{1/2} = \frac{\sqrt{k}}{2\sqrt{L}} \\ \frac{\partial q}{\partial k} &= \frac{1}{2} k^{-1/2} L^{1/2} = \frac{\sqrt{L}}{2\sqrt{k}} \end{aligned}$$

note reversing double deriv for marginal product

MATS = input prices $\frac{w}{r}$ ← labor was on top

$$\frac{k}{L} = \frac{w}{r} = \frac{4}{1} \quad | k = 4L \quad \text{✓}$$

$$\text{Cost: } k r + L w$$

$$= 4L \cdot 1 + L \cdot 4$$

$$= 4L + 4L$$

$$= 8L$$

want in terms of q

$$L^{1/2} = \frac{q}{\sqrt{k}}$$

$$L = \frac{q^2}{k} \quad k = 2q$$

$$k^{1/2} = \frac{q^2}{L}$$

$$L = \frac{q^2}{k}$$

where did the get this?

k isoquant - (usually k in terms of L)

②

$$\text{Cost} = \frac{8q^2}{k}$$

k ← still has k in there!
- circular

$$AC = \frac{\text{Cost}}{q}$$

Returns to scale doubling input \Rightarrow double output

Oh, right, right, right

- always have an issue w/ fn's

$$L = \frac{q^2}{k}$$

$$k = 4L$$

does help

$$L = \frac{q^2}{4L}$$

$$4L \cdot 4L$$

$$4L^2 = q^2$$

$$2L = q$$

$$L = \frac{q}{2}$$

$$k = \frac{q^2}{L}$$

$$L = \frac{k}{4}$$

$$k = \frac{q^2}{\frac{k}{4}}$$

$$k = \frac{q^2 \cdot 4}{k}$$

$$k^2 = q^2 \cdot 4$$

$$k = q^2$$

③ $\text{Cost} = 8l$ ← copy error

$= \frac{8a}{2} = 4a$ ~~← should be~~

$AC = \frac{4a}{a} = 4$

$MC = 4$

Ok now I get

One more time to be sure

$q(k, L) = k^{1/2} L^{1/2}$

really watch cap/lower letters

$$MRTS = \frac{\frac{\partial q}{\partial L}}{\frac{\partial q}{\partial k}} = \frac{\frac{1}{2} k^{1/2} L^{-1/2}}{\frac{1}{2} L^{1/2} k^{-1/2}} = \frac{\frac{\sqrt{k}}{2\sqrt{L}}}{\frac{\sqrt{L}}{2\sqrt{k}}} = \frac{\sqrt{k}}{2\sqrt{L}} \cdot \frac{2\sqrt{k}}{\sqrt{L}} = \frac{k}{L}$$

MRTS = ratio of input prices
 ← in order of above

$\frac{k}{L} = \frac{w}{r} = \frac{4}{1}$

$4L = k$

④ Now what is next

$$\text{Cost} = kr + wL$$

$$4L \cdot 1 + 4 \cdot L \\ = 8L$$

$$\sqrt{k} = \frac{q}{\sqrt{L}}$$

↑
sub in

$$\sqrt{L} = \frac{a}{\sqrt{k}}$$

$$L = \frac{q^2}{k}$$

$$\sqrt{k} = \frac{q}{\sqrt{\frac{k}{4}}}$$

$$k = \frac{q^2}{\frac{k}{4}}$$

~~$$k \sqrt{\frac{k}{4}} = q$$~~

$$k = \frac{q^2 \cdot 4}{k}$$

~~$$k^2 \frac{k}{4} = q^2$$~~

$$k^2 = \frac{q^2 \cdot 4}{k}$$

$$k = 2q$$

$$L = \frac{q^2}{4k}$$

$$4L^2 = q^2$$

$$2L = q$$

$$L = \frac{q}{2}$$

$$\text{So Cost} = 8L = \frac{8q}{2} \\ = 4q$$

$$AC = \frac{4q}{q} = 4$$

$$MC = 4$$

← relate back to q
(what is each step called again???)
↑ I think that was my problem

5)

b) K fixed at 4

$$SRTC = 4.1 + 4L$$

$$C(q) = 4 + 4L$$

$$MC = 4$$

$$AC = \frac{4 + 4L}{q} = \frac{4 + 4q}{2} = \frac{4 + 2q}{1}$$

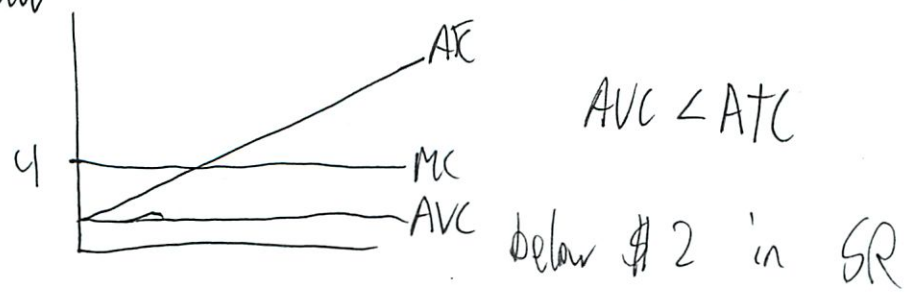
$$AC = \frac{4}{q} + 2$$

$$AFC = \frac{4}{q}$$

$$AVC = \frac{4q}{2} = 2$$

$$ATC = \frac{4}{q} + 2 \quad \textcircled{v}$$

c) draw



LR $P < ATC$ so (what is this?) - had trouble w/ before
- since depends on q

6

but where lowest - when $AC = MC$

$$\frac{4}{q} + 2 = 4$$

-2 -2

$$\frac{4}{q} = 2$$

$$4 = 2q$$

$$2 = q$$

↑ will produce

$$ATC = \frac{4}{q} + 2$$

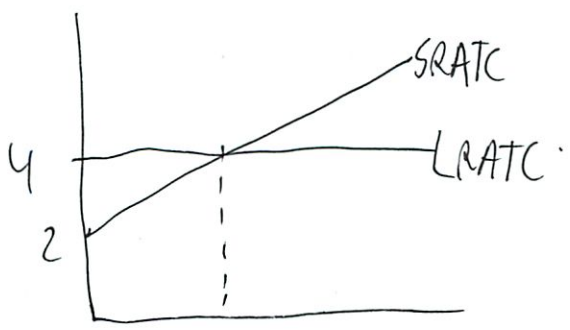
$$= \frac{4}{2} + 2$$

$$= 2 + 2$$

$$= 4$$

so at price 4 or below will
shut down in LR
✓ makes sense

d) price = 8



2
PSR optimal amt

yes above this ~~optimal~~ output level will
increase ~~output~~ capital
(but by how much)

⑦

Opps did not do long run = equilibrium
That's why last part was an open qv

also weird
d'd LR before
SN

$$LRATC = LRMC$$

$$4q = 4$$

$$q = 1$$

$$p = 8$$

$$ATC = 4 \cdot 4 = 16$$

Then they found SRATC at that output (4)

was = 5 - so more capital would ↓

but why use that reasoning?

3. ~~P=10-Q~~ New Qv

$$P = 10 - Q$$

$$C(q) = q^3 - 2q^2 + 3q$$

a) $FC = 0$

$$MC = 3q^2 - 2 \cdot 2q + 3$$

$$AC = \frac{q^3 - 2q^2 + 3q}{q} = q^2 - 2q + 3$$

(8)

1. Find each firm's cheapest pt

$$MC = AC$$

$$3q^2 - 4q + 3 = q^2 - 2q + 3$$

$$2q^2 - 2q + 0 = 0$$

$$2q^2 - 2q = 0$$

$$q$$

$$2q - 2 = 0$$

$$2q = 2$$

$$q = 1$$

~~2. Market price = 10 - Q~~

(always under on steps)

~~hmm - depends on #~~

wait Find ATC at $q = 1$

$$q^3 - 2q^2 + 3q$$

$$1 - 2 + 3$$

2 so p they will sell at is 2

Demand $Q = 10 - p = 8$ each firm does 1, so 8 firms

not ~~yet~~ yet \rightarrow LR

✓ profit this time

9

c) What is different in LR?

- firms in and at

- oh right did this wrong before

- here $p = 2$, 8 firms

d) Tax = 3

So $ATC = 2 + 3 = 5$

So they will only supply at price = 5

$$Q = 10 - 5 = 5$$

So still $q = 1$, 5 firms ✓ finally getting it

e) $MC = 6 + 5q$

$$P_s = 4Q_s$$

$$P_D = 10 - Q_D$$

) supply + demand functions

What FC will they break even

$$4Q = 10 - Q$$

$$5Q = 10$$

$$Q = 2$$

$$P = 8 = 4 \cdot 2$$

$$MC \text{ for this unit} = 6 + 5(2)$$

$$6 + 10 = 16$$

making a loss?

- no, I'm not seeing it

(10)

$MC = MR$ ← but I thought this was perfectly competitive?

$$\frac{2}{5} = .4$$

$$\pi = p \cdot q - TC$$

$$= 8 \cdot .4 - TC$$

Why .4?

=

$$\leftarrow TC = \int_0^q (6 + 5q) dq$$

$$= 6q + 5\frac{q^2}{2} + FC$$

$$= 2.8 + FC$$

remember

TC is \int of $MC + FC$

thought of this but did not try it - why???