12/3

14.01: Final Review

December 3, 2010

Last third of course

1 Lecture 17: Oligopoly Continued (Chapter 12)

– less than an actual third

I. Cournot Math

- Cournot: All firms set quantities at the same time
- Calculate residual demand for a given firm and solve its profit maximization problem to find its best response function to other firms' output decisions.
- Solution is a set of quantities (one for each firm) that solves this system of equations.

II. Cooperative Equilibrium - Cartels

- Firms can form a cartel and behave like a single monopolist, maximizing total industry profits.
- Cartels are unusual because they are fundamentally unstable (incentive to "cheat" and raise own production) and because they are illegal (antitrust laws).

III. Comparing Equilibria

- In terms of welfare, usually Perfect Competition > Oligopoly > Monopoly
- Quantity as an indicator of social welfare
- DWL in welfare analysis comes from trades that aren't made

IV. Many Firms

- In Cournot, as number of firms $\to \infty$, Cournot equilibrium approaches competitive equilibrium
- As number of firms $\to 1$, approaches monopoly
- Markup over competitive price depends on number of firms and elasticity of demand:

$$\frac{p - MC}{p} = -\frac{1}{n\epsilon}$$

V. Price Competition

- Bertrand: firms set prices (instead of quantities) at the same time
- Two firms may be enough to remove market power (i.e. restore competitive outcome) if products are identical
- Recall proof from class that identical Bertrand duopolists drive price down to marginal cost
- Also recall the Stackelberg model where one firm set their quantity before the other firm. In this case, the best response function for the Stackelberg leader takes into account the fact that the follower will respond to the leader's output decision.

2 Lecture 18: Factor Markets (Chapter 15)

I. Competitive Factor Markets

- In SR and in LR, demand for labor will be its marginal revenue product

$$MRP_L = MR \cdot MP_L$$

where MR is marginal revenue from additional unit of output ($MR = p$ if competitive output market). These will differ in LR and in SR because MP_L in LR will take into account optimal capital adjustments.

- LR labor demand more elastic than SR

II. Monopsony

- Monopsony: single buyer in input market
- Barriers to exit (from factor market) create monopsony
- Competitive firms consider price of inputs, but monopsonist looks at *marginal expenditure.*

$$ME(q) = \frac{dTE(q)}{dq} = \frac{d[p(q) \cdot q]}{dq}$$

- Takes "poisoning effect" on inframarginal untis into account
- Increase input usage if ME¡MRP, decrease if ME¿MRP.
- Price comes from supply function
- Monopsonists underhire at lower wages relative to perfect competition

III. Application: Minimum Wage

- Higher minimum wages can restore competitive outcome in a monopsonistic market

3 Lecture 19: International Trade

I. What is International Trade?

- trade deficit = imports-exports

II. Comparative Advantage and Gains from Trade

- Differences in opportunity costs lead to comparative advantage in different goods
- When countries have different comparative advantages in production of different goods, there are potential gains from trade through specialization

III. Welfare Implications from International Trade

- In competitive model, opening to trade increases total welfare but usually at the expense of either consumers or producers
- Practice welfare comparisons using the usual graphs for imports and exports

IV. Trade Policy

- Effects of tariffs and quotas

4 Lecture 20: Uncertainty (Chapter 17, sections 1-3)

I. Decision-Making Under Uncertainty

- Expected utility: probability-weighted average of utility

$$EU = Pr(\text{Lose}) \cdot U(\text{Lose}) + Pr(\text{Win}) \cdot U(\text{Win})$$

- Different than utility of expected value, since utility functions usually concave (diminishing MU of income)

II. Extensions

- Concave utility (decreasing MU income) → risk averse

- Linear utility (constant MU income) → risk neutral
- Convex utility (increasing MU income) → risk loving

III. Applications

- Insurance: risk averse people will pay money to turn a gamble into a certain payoff since they get higher utility from certain income than from a gamble with the same expected value
- Maximum amount they're willing to pay for this is their "risk premium"
- Lottery behavior is a puzzle – maybe risk averse at low incomes and risk loving at high incomes

5 Lectures 21 and 22: Capital Supply and Capital Markets (Chapter 16)

Lecture 21 - Capital Markets

I. Capital and Intertemporal Choice

- Supply of capital from household decisions about how much to save – increasing in interest rate
- Demand for capital comes from firms with potentially productive investments to make
- Intertemporal choice graph over consumption in period one (C_1 on x-axis) and in period 2 (C_2 on y-axis). Slope of the BC is $-(1+r)$
- When r changes, effect on savings depends on relative size of IE and SE

II. Present Value

- Need to translate all future dollars into today's terms
- For a single payment of FV in year t:

$$PV = \frac{FV}{(1+r)^t}$$

- Value of a perpetuity – constant payment of f every period forever:

$$PV = f/r$$

- Importance of compunding
- Real interest rate r is the one we care about – the nominal interest rate i minus inflation π

$$r = i - \pi$$

Lecture 22 - Capital Markets II

I. Choices Over Time

- Choose option with highest present value when choosing between projects or investments

II. Investment Decisions

- Net Present Value (NPV) = PV of revenues - PV of costs
- Rule: Invest if NPV greater than zero
- if revenues R_t in each period and costs C_t, NPV of investment is:

$$NPV = \left[(R_0 - C_0) + \frac{R_1 - C_1}{(1+i)^1} + \frac{R_2 - C_2}{(1+i)^2} + \ldots + \frac{R_t - C_t}{(1+i)^t}\right]$$

- NPV decreasing in interest rate for projects with up-front costs and future payoffs

III. Increasing Savings

- Savings important for economic growth
- Government encourages savings using tax subsidies to retirement savings

6 Lecture 23: Equity and Efficiency (Chapter 10)

I. Equity and Efficiency

II. Choosing the Socially Optimal Allocation

- Social welfare function (SWF) can be though of as a utility function for society taking individual utilities as inputs
$$SWF = f(U_1, U_2, \ldots)$$
- Utilitarian SWF
$$SWF = U_1 + U_2 + \ldots$$
- Rawlsian SWF
$$SWF = min(U_1, U_2, \ldots)$$

III. Inequality in the US and Around the World

- See class handouts on inequality, poverty line, poverty rates over time

IV. Sources of Leakage

- Recall transfer program discussed in class – leads to decrease in labor supply especially among those who qualify or are originally near the cutoff to receive the subsidy.
- Distortionary taxation leads to DWL – this is the cost of redistribution

7 Lecture 24: Taxation and Redistribution

I. Taxation in the U.S.

1. Income tax (progressive, main tax in the U.S.)
2. Payroll tax (flat)
3. Consumption tax (regressive, paid on spending rather than earnings)
4. Property tax (tax on wealth)
5. Corporate tax (tax on businesses, small share of total tax revenue)

II. What Should We Tax?

- European countries raise most revenue through VAT on consumption
- Consumption taxes encourage savings but not progressive
- Excise taxes usually on "sin goods" (goods with negative externalities in consumption)

III. What is the Right Tax Rate?

- Tax revenues = $base \cdot t$
- As tax rate rises, base shrinks
$$\frac{d(\text{tax revenue})}{dt} = base + t \cdot \frac{d(base)}{dt}$$
- Laffer curve → tax revenue initially rising, then falling with tax rate (depends on elasticity of tax base)

IV. Low Income Programs in the U.S.

- Importance of targeting assistance programs
- Earned Income Tax Credit (EITC) is a wage subsidy program that balances targeting and efficiency

Review Session

12/9

Oligopoly

Several firms that interact strategically

Game Theory - solve for Nash =librium

- set of actions for all firms
- it does best it can do
- given what its oponents actually do

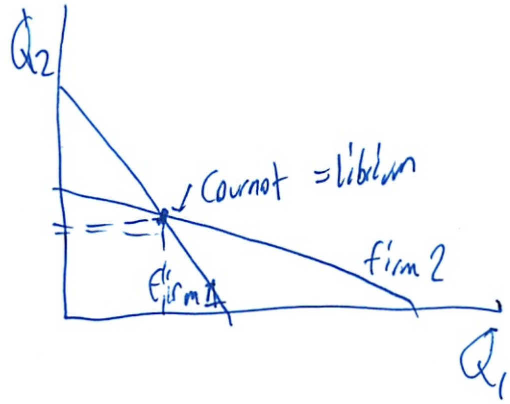
Cournot Duopoly - two firms

- symetric costs
- Compete by choosing output
- Choose output simultaneously
- Cournot =librium = nash =librium (Q_1^*, Q_2^*)
- reaction function - best output, given what other did

  $Q_1(Q_2)$ $Q_2(Q_1)$
- Did a lot of problems on P-set
- Find reaction functions via residual demand

  $D(Q)$ $Q = \sum Q$
- Then find MR
- Set = MC, get =m q

② Look at market demand to find price



Industry ~~profits~~ prices are higher under Cournot

Can do even better by forming a cartel / collude

What if more firms ~10

- becomes more competive
- price + output will converge to perfect outcome

Equalibrium markup $\frac{P-MC}{P} = \frac{1}{n E_D}$

As $n \uparrow$ markup $\to 0$

firms can collude

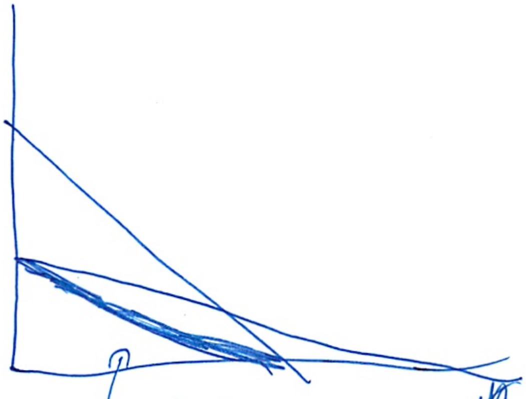
industry will have higher profits

like a monopoly

- except split output and/or profits

but many are unstable - can do better by cheating

③



Remember firms have same cost structure

contract curve

where they would produce as a monoppoly

But always incentive to cheat

Bertand competition — compete on price

- 2 firms
- same cost curves
- constant MC
- no fixed costs
- compete by fixing prices
- nash equalibrium (P_1, P_2)
- firms will compete the price down to perfectly competitive

symetric — # of firms don't matter $(n \geq 2)$

---

asymetric if $MC_1 < MC_2$

$P_1 = MC_2 - \epsilon$

$P_2 = MC_2$

4

Sackelberg — one firm has a first mover advantage

- sets output 1st

Factor Markets ~~#2 Monospony~~

Choice of inputs = firm's demand for input/factors

Firms demand for labor → MRP_L

↳ next workers' MP_L

and sell MR

$$MRP_L = MR \cdot MP_L$$

Firm willing to hire until $MRP_L = \underset{\text{wage}}{W} = MC$

Now can analize like any other market

Monospony

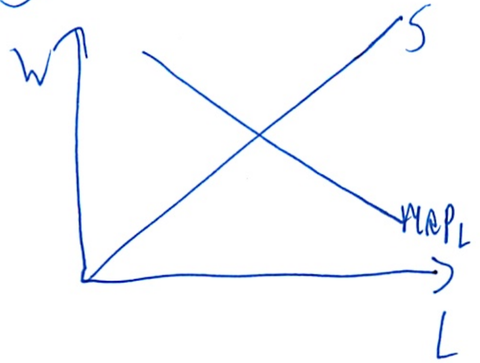
A single firm can be the only employer

The only buyer of a resource

Has market power

Monosponist

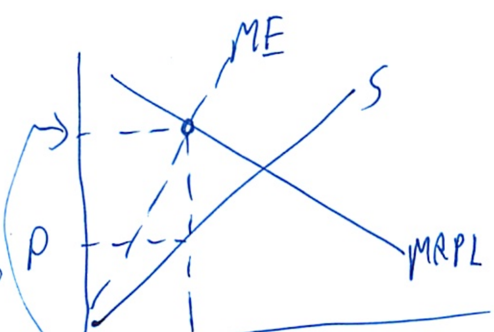
5



If wanted to hire an add worker - would have to give ~~[illegible]~~ everyone else a pay raise

$$TE(L) = S(L) \cdot L$$

$$ME(L) = S(L) + L \frac{dS}{dL} > S(L)$$



$ME(L) = MRPL$

But pay this lower rate (still better off than in competive labor market

~~[illegible]~~ Gov can set min wage

- if sets it right, can eliminate DWL
- Then ME flat till hits supply curve
- ~ then must raise wage, jumps to market wage

(6)

Gov can ~~steps~~ make it worse

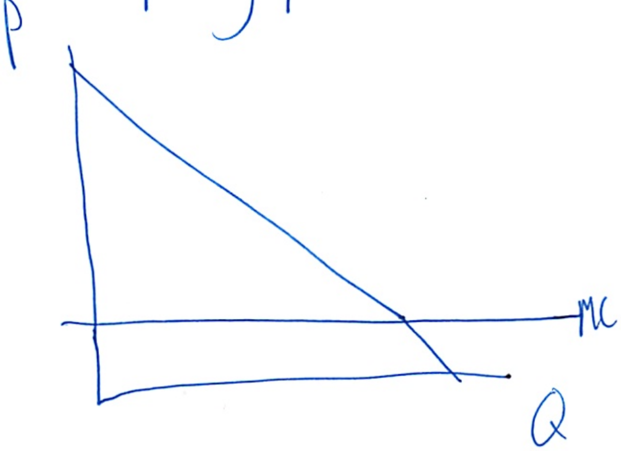
- if sets wage above where ME = S
- monospomist would hire less people

~~International Trade~~

~~Quick review~~ 1/25

Monopoly

- single firm producing a good
- has pricing power



$$\max_{Q} \pi(Q) = TR(Q) - TC(Q)$$

$$MR(Q) = MC$$

$$TR(Q) = D(Q) \cdot Q$$

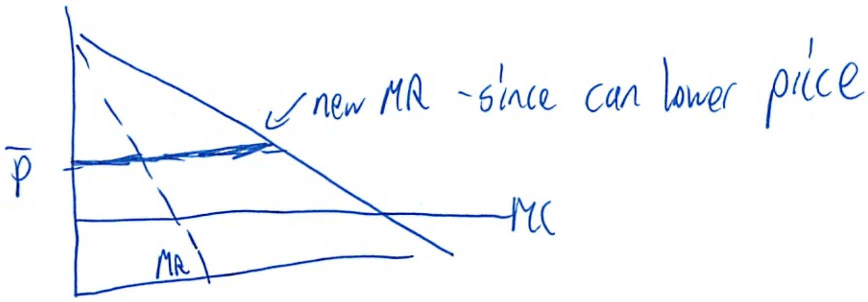
$$= D(Q) \cdot Q + Q \frac{dD}{dQ} < D(Q)$$

↑ less than 0

↑ "poisoning effect"

(7)

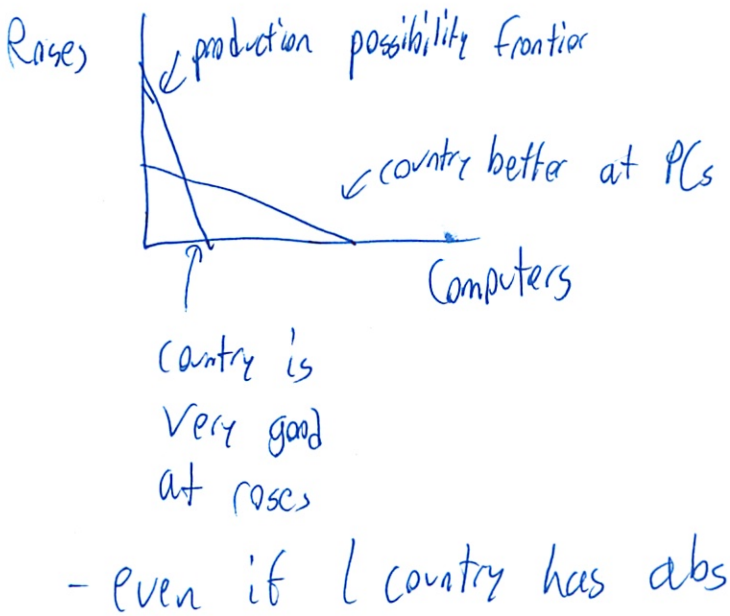
If gov sets a price ceiling



Less DWL Than w/ no price cieling

International Trade

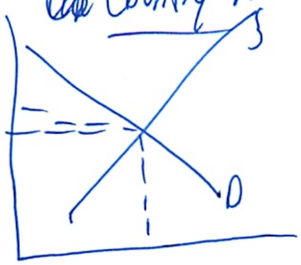
- Countries have a compaitive advantage in producing some goods over another
- lower opportunitiy cost in one Country vs another



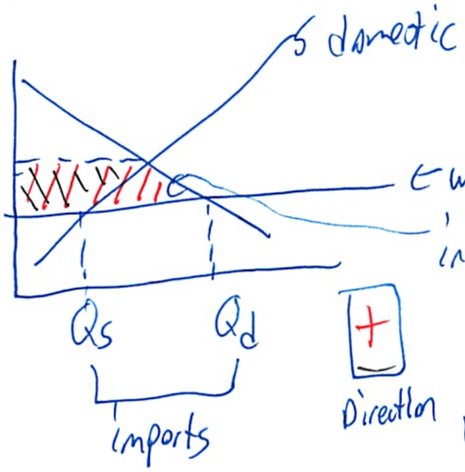
- even if 1 country has abs advantage in both fields - but will still be specialization
- gains in specialization + trade
- increases consumer welfare

⑧

Country A



~~One Country~~ no trade/autarky



increase in consumer welfare

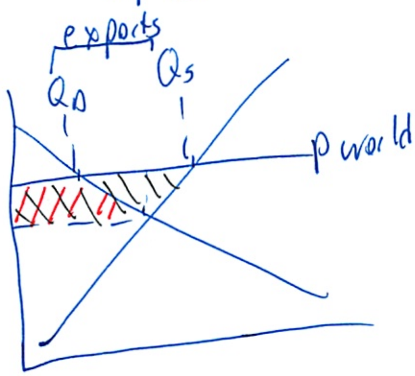
$\boxed{\begin{matrix}+ \\ -\end{matrix}}$

Direction ↳ ΔCS

→ ΔPS

Direction

$\boxed{\begin{matrix}- \\ +\end{matrix}}$



← does not affect world price
price taker

Gov can compesate

- quota
- tariff - will ~~shift~~ shift world price up
  - have weird on DWL

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Uncertainty

- how agents make choices when choices entail risk
- probability distribution
- ie you have a house

tomorrow
- .3 house burns down -$1,000
- .7 house normal 0

- calculate expected value

$$E[\ ] = .3 \cdot -1000 + .7 \cdot 0$$

↳ in general $\sum_i p_i \cdot V_i$

- but that is not certain - is risky
  - gain or loss vs E[ ] risky
- Need to understand what their preferences over risky outcomes are
  - preference function $U(w)$
  - individuals maximize $E[U]$

Example

$W = 1000 +$ value of house

$W_{Burns} = 0$

$U(1000) = 100$

$U(0) = 0$

(this section very easy)

(10)

$E[U] = .3 \cdot U(0) + .7 \, U(1000)$

this is suffecent to describe agent's actions

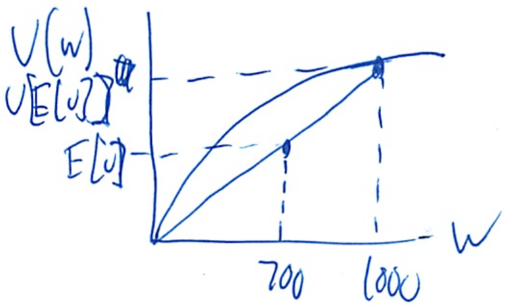
risk adverse -

if $E[U] < U(E[V])$ ← utility of just getting # for sure

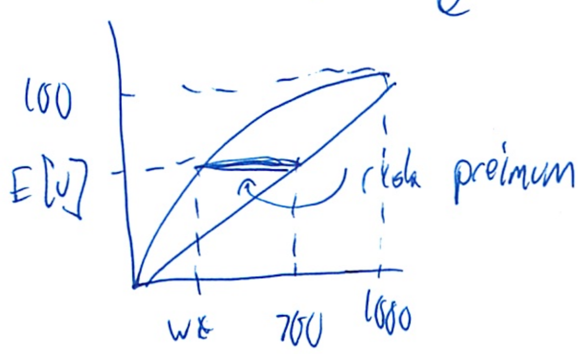
Value less if uncertain

agents have ↓ MU of wealth

- aka $U(w)$ is concave



risk adverse people ↕ willing to buy insurance



Risk neutral

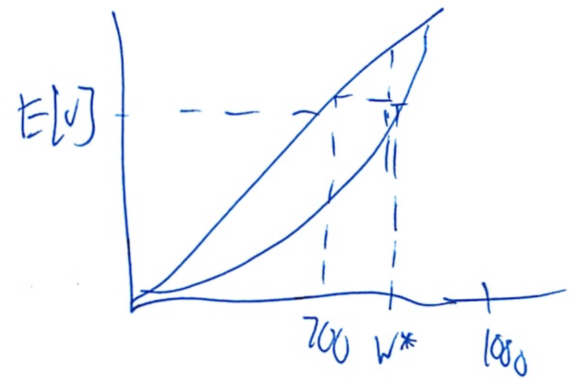
$E[U] = U \, E[V]$

would never buy

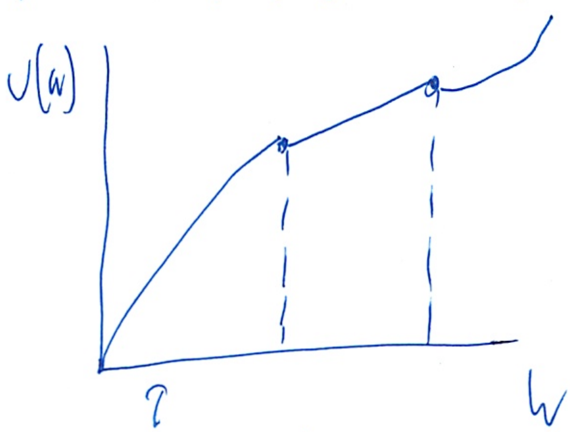
11

Risk loving people

- risk neutral ins co ~~who~~ would ~~not~~ sell ins ← (watch my flawed intuition)
- Since risk loving person would need to be paid $ to buy insurance!



Pref of risk can vary w/ income



If investor wanting to invest $ - where do they

1. Start a firm
   - .7 Succeed $1000
   - .3 Fail $0

(12)

2. Fixed wage job $700 salary

W=0 c ~~m~~ does not cost $ - op cost already built in

What do you choose?

risk adverse person - takes the steady job

risk neutral - indecisive

risk loving - entrepreneur

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had to leave for 6.041 review session

(follow handout)

14.01 Rereading Notes

12/11

Normative – how should be

positive – how is

$$\left.\begin{array}{l} Q(A) = D_A(P) \\ Q(B) = D_B(P) \end{array}\right\}$$ indv demand $\quad Q_M = Q_A + Q_B$

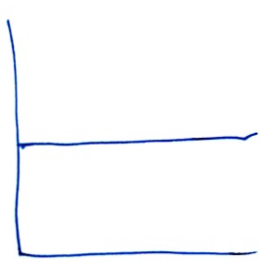
$$Q_S = S(P)$$

$P = S^{-1}(Q)$ ← how we look at it

(I get this much better now)

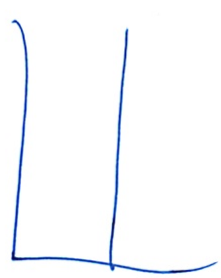
find intersection

$$Q_D = Q_S$$



elastic

any Q – at this price



inelastic

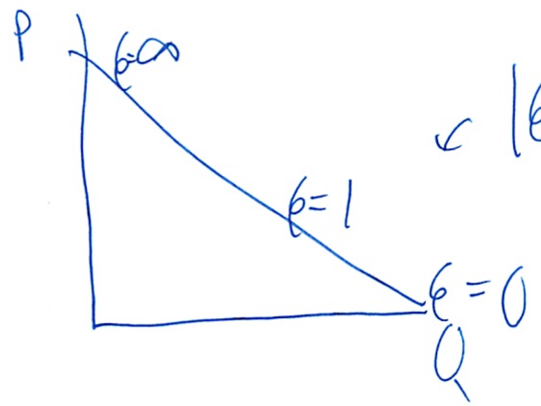
only at this price

$$\varepsilon = \frac{\%\Delta Q}{\%\Delta P} = \frac{\Delta Q / Q}{\Delta P / P} = \frac{P \Delta Q}{Q \Delta P} = \frac{\frac{\partial Q}{Q}}{\frac{\partial P}{P}} = \frac{dP}{dQ} \cdot \frac{Q}{P}$$

> 1 elastic

< 1 inelastic

2



$|\varepsilon|$ - alwas negitive

use avg when have range

or Income

$$\varepsilon_I = \frac{\Delta Q/Q}{\Delta I/I} = \frac{I}{Q}\frac{\Delta Q}{\Delta I}$$

or Cross

$$\varepsilon_{Q_x P_y} = \frac{\frac{\partial Q_x}{Q_x}}{\frac{\partial P_y}{P_y}} = \frac{P_y}{Q_x}\frac{dQ_x}{dP_y}$$

~~Usen~~

Substibes + complements

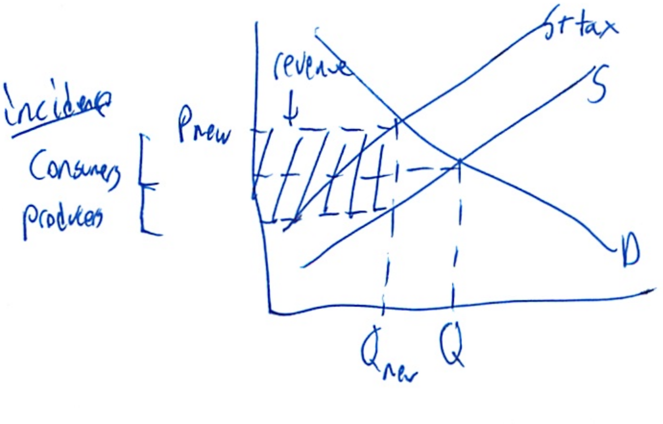
price floors

cielings

subsidies

taxes

- often split ~~the~~ incidence to consmers + producers



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Price elasticity sample (about.com)

$Q_D = 110 - 4P$

Price elasticity at \$5

$$\varepsilon_d = \frac{dQ}{dP} \cdot \frac{P}{Q}$$

$$\frac{dQ}{dP} = -4$$

$$\varepsilon_d = -4 \cdot \frac{P}{110-4P} \leftarrow \text{Plug in}$$

$$= \frac{-4P}{110-4P}$$

← need elasticity at a point!

Now plug in the \$5

$$= \frac{-4(5)}{110-4(5)} = \frac{-20}{90} = -\frac{2}{9}$$ inelastic

– not very sensitive to price change

– So double price, ~~not~~ ~~double change~~ revenue ?
have price, half revenue

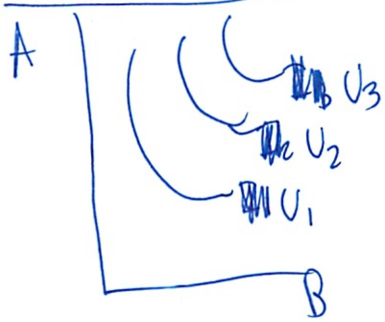
0 ⟶

-1 double price, revenue same) half price, revenue same (depends on direction)

-2 "South west effect" ~~double~~ half price, double revenue

(4)

Indifference curve

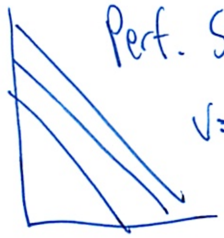


MRTS
MRS

Can't cross
downward sloping
same level of happiness (utility) at any point

Perf Comp.

$U = \min\{ax, by\}$



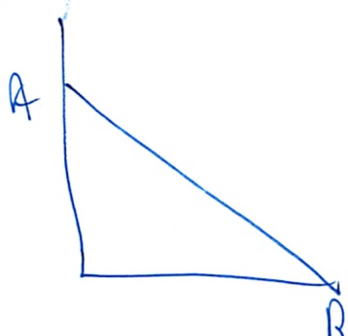
Perf. Sub

$V = ax + by$

What is MRS vs MRTS?
↳ same thing ✓
– MRS = consumer demand
– MRTS = producer input demand

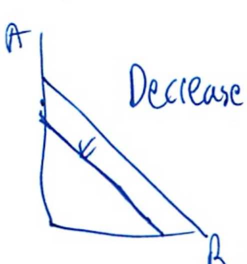
$$MRS = \frac{\Delta A}{\Delta B} = \frac{-MU_B}{-MU_A} \begin{matrix} \leftarrow B \text{ is } x \text{ axis} \\ \leftarrow A \text{ is } y \text{ axis} \end{matrix} = \frac{-\Delta y}{-\Delta x} = \frac{-dy}{dx} = \frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial y}}$$

Budget Constraint

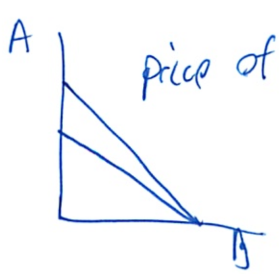


$Y = I = A \cdot p_A + B \cdot P_B$
↑ quantity

MRT = Marginal rate of transformation

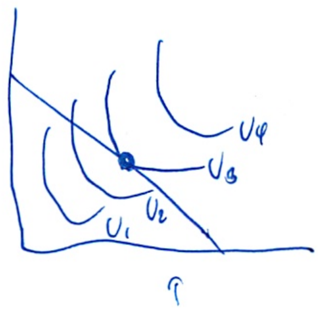


Decrease in income



price of A ↑

(5)



← where budget constraint + indifference curves intersect

$$MRS = MRT$$

$$-\frac{MU_B}{MU_A} = -\frac{P_B}{P_A}$$

Can be corner solutions / interior solutions

If $\frac{P_x}{P_Y} > \frac{U_x}{U_Y}$ than consume all Y

Can also solve w/ lagrange multipliers

The more budget lines you can test, the more you can reveal prefs

(heres where I did the Calculus ~~review~~ book)

Should also do practice tests

---

infrior goods $\frac{\Delta Q/Q}{\Delta I/I} = \gamma < 0$

- move away from as get richer

nessities $\gamma < 1$

- as income double, don't buy double

lux $\gamma > 1$

- as income doubles, buy more than double

6

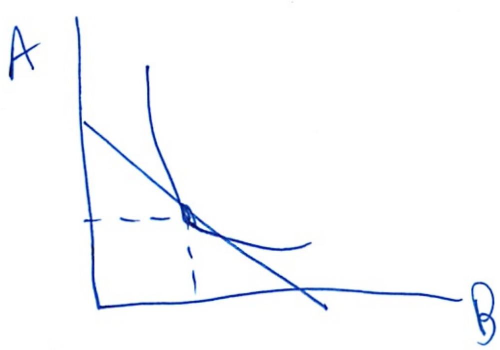
Price elasticity - price of goods change

1. Substitution effect
   - change in demand as $P\uparrow$, holding U constant

$$\left|\frac{\Delta P/P}{\Delta Q/Q}\right|_{\bar{U}}$$

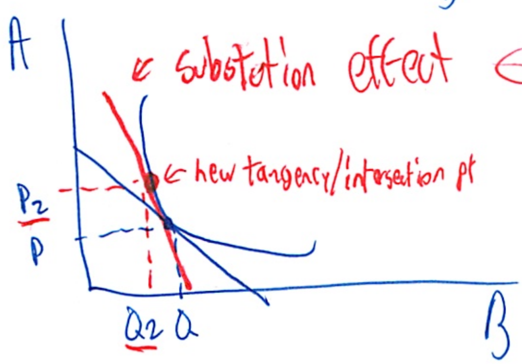
2. Income Effect
   - change in qt demanded as income $\uparrow$, holding prices constant

$$\left|\frac{\Delta Q/Q}{\Delta I/I}\right|_{\bar{P}}$$



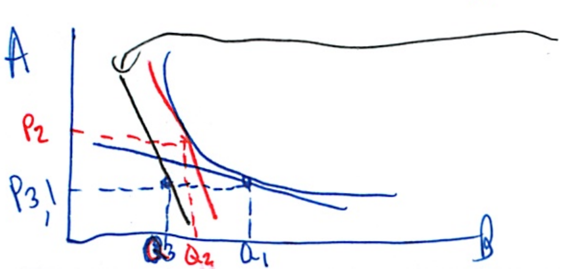
Base

1



substitution effect
- still tangent to utility curve

2



income effect
- you are effectively poorer
  moves in
- price held constant

7

Engel Curve

But income + substitution effect varies

Price ↑

- **Normal Good** — Sub effect Q ↑, income effect Q ↑ — total Q ↑
- **Inferior Good** — Sub effect Q ↑, income effect Q ↓ — total ???

better chart p 10

inferior ↑ in income → ↓ Q consumed of good

Giffin - are inferior
- as price ↑, you want more
- lux status symbols
- ? none exist
- upward sloping demand curve
- income effect > sub effect
- P ↑, Q ↑

(8)

Price increase

Normal $Q\uparrow$

Inferior

sub > income $Q\uparrow$

Giffin

sub < income $Q\downarrow$

Math example

Given

$$U(x,y) = x + 2\sqrt{y}$$

$$P_x = 1$$

$$P_y = 1$$

$$I = 5$$

Current best value

$$MRS = MRT$$

$$5 = x \cdot P_x + y \cdot P_y$$

$$\frac{\frac{dU}{dx}}{\frac{dU}{dp}} = \frac{P_y}{P_x}$$

$$\frac{1}{y^{-1/2}} = \frac{1}{1}$$

$$\sqrt{y}^{\,2} = 1^{2}$$

$$y = 1$$

$$5 = 1 \cdot 1 + x \cdot 1$$

$$x = 4$$

:) Woot figured it out (the basic stuff)

9

Now first Substitution effect

Well first need to be told $P=2$

- hold utility constant

$$\left|\frac{\Delta p/p}{\Delta Q/Q}\right|_{\bar{U}}$$

Income

$I = X\,P_X + Y\,P_Y$

$5 = X \cdot 2 + Y \cdot 1$

$x = \frac{1}{2}$ & " keep every thing else constant and just see how many you can buy

No - run it through again

$MRS = \frac{1}{\frac{1}{\sqrt{Y}}} = \frac{P_x'}{P_Y} = \frac{2}{1}$

$y = 4$

Sub effect

$U(x,y) = x + 2\sqrt{4} = 4 + 2 \cdot \sqrt{1}$

$x = 2$

So sub effect

$(4,1) \to (2,4)$

Income effect

$(2,4) \to (\frac{1}{2},4)$

Yeah problem did it backwards - should do more practice

(10)

For price ↑

⊖ income effect $\frac{dx^*}{I} > 0$ normal: $\frac{dx^*}{I} > 1$ lux; $\frac{dx^*}{I} \leq 1$ nessessity

⊕ income effect $\frac{dx^*}{I} < 0$ inferior: $\frac{dx^*}{I} > 0$ griffin; $\frac{dx^*}{I} < 0$ not griffin

(go over past practice exams + more income/sub effects later)

Sub + income effects - child labor

Producer Theory

k, l $q = (k, l)$

Variable vs Fixed

TSR+LR LR change only

(think got this chap pretty good)

MRTS

$$\left.\frac{\Delta k}{\Delta l}\right|_{\bar{q}} \qquad q = f(k, L) \text{ isoquant}$$

like indifference curve

(Remember this chap harder - no set income limit

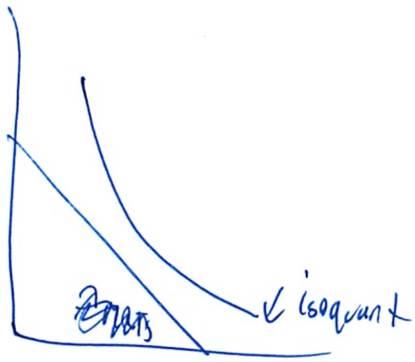
Constant returns to scale

$$f(2L, 2k) = 2f(L, k)$$

IV

Perfectly substituted inputs

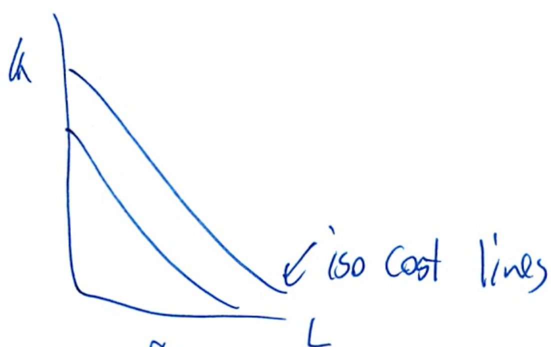
non-substituted inputs



Π = profit

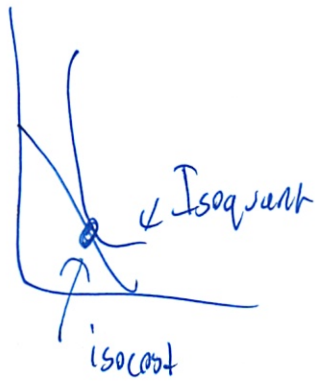
$C(q) = f(wL + rK)$

$MC + \frac{\partial c}{\partial q}$



various because don't have a given income

(12)



Isoquant = iso cost

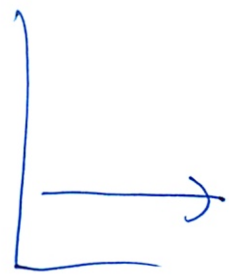
marginal product

$$k = \frac{\bar{C}}{r} - \frac{w}{r} L = \frac{-MP_L}{MP_k}$$

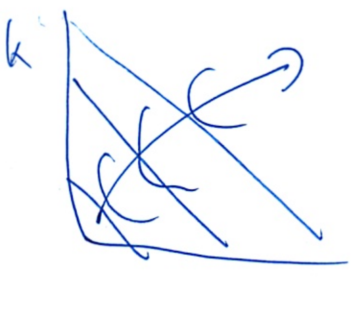
isocost

isoquant

Long run expansion path



SR path
Can only change labor



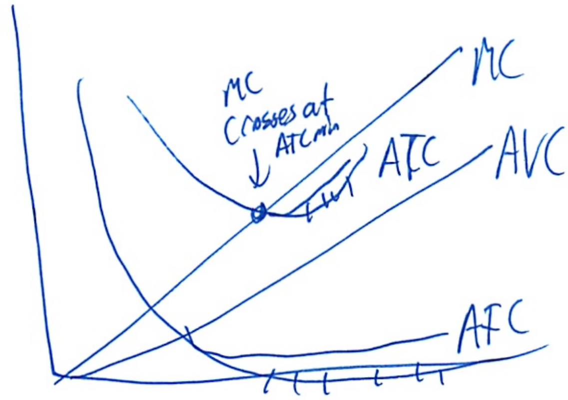
as capital becomes less productive
- needs people
- ie teaching

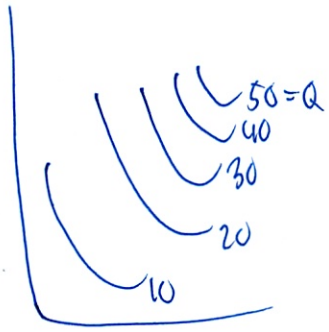
Production Functions

Calculate MP_L, MP_k from it

Then MRTS

↳ isoquants — show what you can produce
- not cost



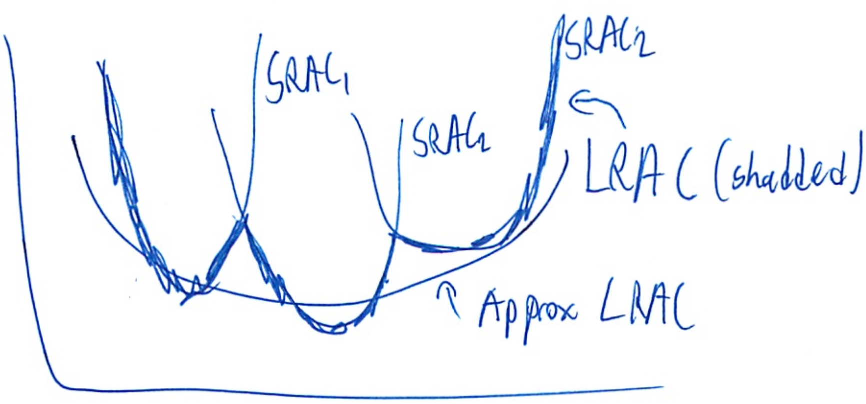


Decreasing returns to scale
(spacing gets smaller)

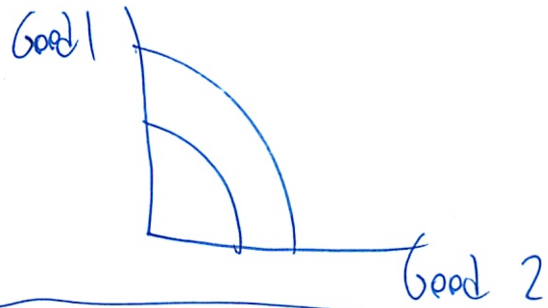
How pick right cost curve again?

- ~~Cost~~ of capital
- ~~Opportunity~~ Cost

No really - need to know how much output you want by looking at markets (other sections)



Productive Transformation Curve



(review comparitive advantage math)

Mary 20 paint jobs or 5 wall paper jobs
John 10 " 4 "

Op cost Mary $\frac{5}{20}$ wall paper jobs for every paint job forgone $= .25$ ← Mary has competitive advantage in painting

John $\frac{4}{10}$ $= .4$

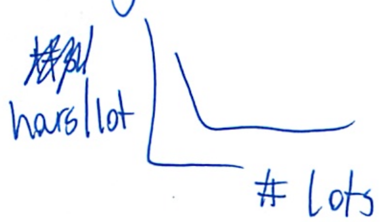
Economies of scale

double inputs, more than double output

Economies of scope

joint output of single firm more than 2 sep. firms

Learning costs



(15)

Competition

- pretty good at this
- getting to recent topics

Residual demand

$$D^r(p) = D(p) - S^o(p)$$

$$\frac{dD^r}{dp} = \frac{dD}{dp} - \frac{dS^o}{dp}$$

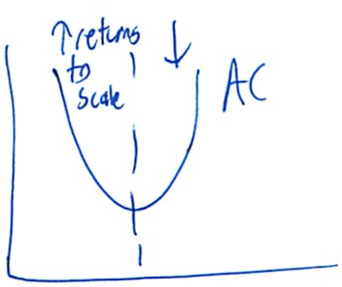
$\uparrow$ $(--)$ $\uparrow$ $(-)$ $\uparrow$ $(+)$

$\uparrow$ very negative

$$q = \frac{Q}{n} \leftarrow \text{market}$$

$MR = MC$

$p = MC$ perfectly competitive



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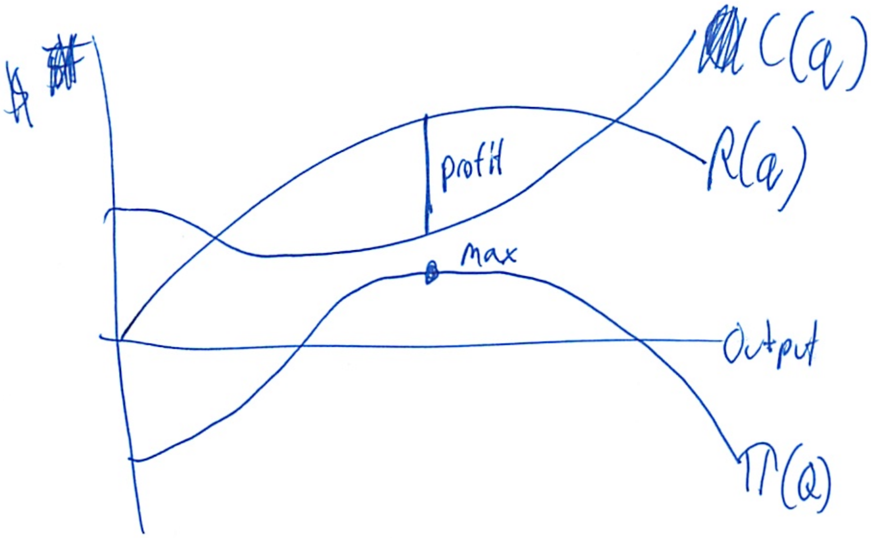
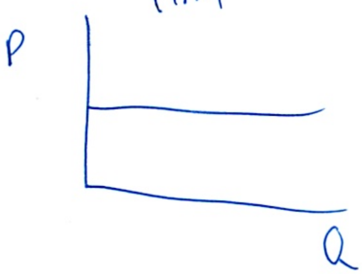


firm

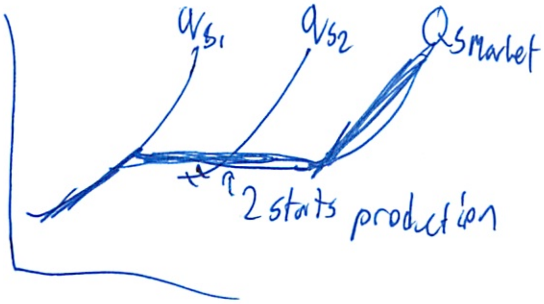


market

perfectly competitive

Supply is MC above AVC in perfectly competitive

Market - add these curves



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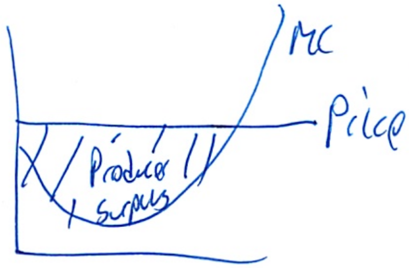
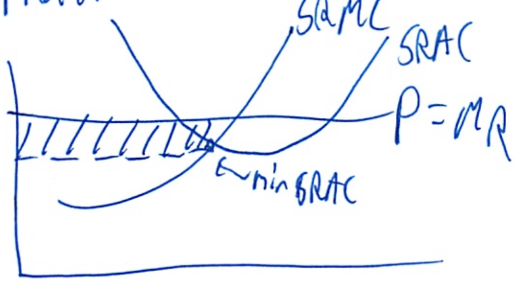
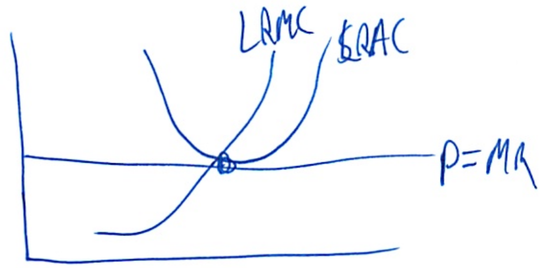


SR Profit



market out of balance



Supposed Long run

firms drop in + out of market

even if diff cost structure ~~~~

ie if patent firm can sell patent + exit industry

Agency problem

Welfare economics

- W~~$~~ = CS + PS
  society

- ~~~~ DWL bad

18

Monopolies

Markup power

$$\frac{\text{price} - MC}{\text{price}} = \frac{1}{\epsilon}$$ - .

- was in oligopolies as well
  - on review sheet

Will charge $p > MC$

Sloped MR

Price discrimination

Sources

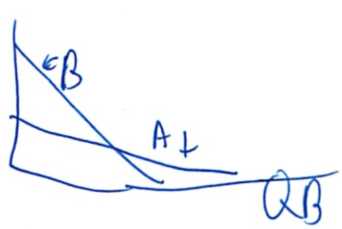
Cost advantage - natural monopolies

Gov actions

Oligopolies

- a few firms w/ market power
- need to think about game theory
- the various models
  - make sure know intracies

Cournot QA



(19)

(Mid term 2 - seemed easy topics - reviewed quickly)

- but was very hard actually ↳ actually took some 10 pages

(Midterm 3 seems even easier)

Recentness also matters

Review Cournot math

$P = 339 - q_A - q_B$ $\quad\quad MC = 147$ for A, B

$R_A = P \cdot q_A = 339 q_A - q_A^2 - q_B q_A$

$\frac{dR_A}{dP} = MR_A = 339 - 2q_A - q_B$

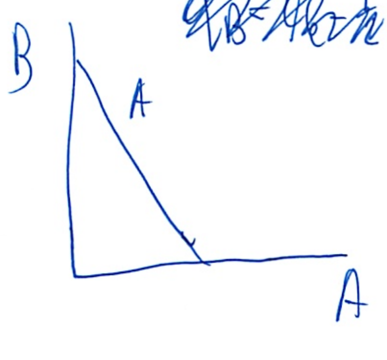
$MR = MC$

$339 - 2q_A - q_B = 147$

$q_A = 96 - \frac{1}{2} q_B$

↑ iso line

need to reverse to graph



$q_A + \frac{q_B}{2} = 96$

$\frac{q_B}{2} = 96 - q_A$

$q_B = 192 - 2q_A$

20

Now do the same For B

- ~~[illegible]~~

- and symetical so same

$q_B = 96 - \frac{1}{2} q_A$

Now solve set = to, to find intersection

$q_A = q_B$

$96 - \frac{1}{2} q_B = 96 - \frac{1}{2} q_A$

$96 - \frac{1}{2} q_A = 96 - \frac{1}{2} q_A$ ← duh replace variable w/ same (something I really only understand this semester)

$+96 \quad + \frac{1}{2} q_A \quad -96 \quad + \frac{1}{2} q_A$

$192 = 0 q_A$

no, of course not

what did they do?

$q_A = 96 - \frac{1}{2} q_B$

$q_B = 96 - \frac{1}{2} q_A$ | solve

$q_A = 96 - \frac{1}{2}(96 - \frac{1}{2} q_A)$ ← duh, only get 1 variable

$q_A = 96 - 48 + \frac{1}{4} q_A$

$\frac{3}{4} q_A = 48$

$\cdot \frac{4}{3} \quad \cdot \frac{4}{3}$

$q_A = 64$

find price now $P = 339 - q_A - q_B = 211$

(21)

(from exam 2, price floors + ceilings w/ respect to welfare was hardest
(not reviewing now
- hope I have figured out well enough to just do

Monopsony

- - \

Production possibility frontier

- this is the int. trade - comparitive advantage stuff

(for the recent section, should do practice problems
(really my challenge is the #s - what to do
- graph secondary)

Expected utility

- out of 6.041

risk loving, etc
(was all in review session)

Capital

The NPV from 15.501

$\frac{1}{(1+r)^t}$

(22)

Perpituity $= \frac{1}{r}$

real = nominal - inflation

↳ rate of return or something

intemoral effects

(uses same math as before)

importance of society savings

Social welfare

Utility functions

Utilitarinism $U = U_1 + \ldots + U_{350 million}$

Rawlsian $U = \min\{U_1, \ldots U_n\}$

Nozichian - dist of opportunities

Commodity equalitaranism - mix of Rawlsian + Nozichian

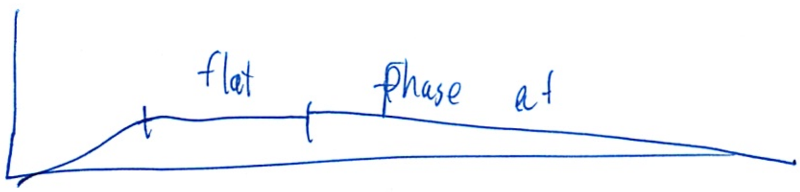
(I think utility will be given by function, not name)

leakage

- rich people work less hard
- poor people quit job to qualify

- need to restrict people from entering
- what do you tax
- laffer curve

- TANF - only 1 parent
- SSI - poor + disabled
- EITC - transfer conditional on work



- Insurance

...

all reading stuff

Ok done

- need to know how to do the math
- watch out for tricks + special cases

Re do past exams + practices

Practice 12/12/10

14.01 Final Exam Fall 2007

1. **True/False Questions** (*TOTAL: 20 points*):

In this section, write whether each statement is True or False. Please fully explain your answer, using a diagram if appropriate. No credit will be given for an answer without an explanation.

(a) (*5 points*) After Professor Wheaton promises that no one would fail, students never study and turn in problem sets again. This is an example of adverse selection problem.

~~True~~ - There is no risk, no punishment for not turning in P-Sets

Moral hazard

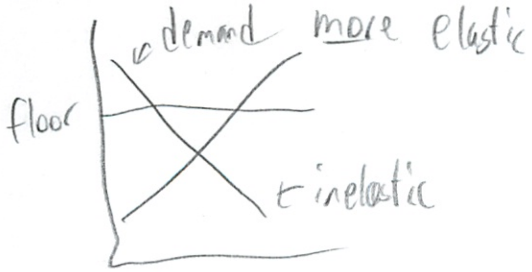
(know diff)

(b) (*5 points*) When a firm chooses among different projects, the one which has the highest present value is always the one with the highest yield rate.

No, the projects can be different sizes. If they were all the same upfront cost, it would be true ✓

or at same time

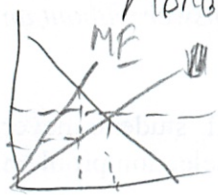
(c) (*5 points*) If the government guarantees a binding price floor for agricultural output by purchasing any surplus, then the demand for farm labor will be more elastic.



True ✓

(d) (5 points) Exploitation Mining Co. is the only employer in the remote town of Uranium City and pays its workers \$10.00/hour. If the government forces the company to raise its wage by a small amount—say to \$10.10/hour—then it will hire more workers.

Monospony



Supply becomes ME until crosses demand

True

✓

(e) (*5 points*) A factory that pollutes a river has negative externalities on residents along the river. If the factory and residents can negotiate, an efficient result can be achieved only if the property right of the river is assigned to the residents.

True has negitive externalities

False - propery rights must be distributed to all

If factory has no rights, would super polute

Coase Theorm

-did not do

Short Question:

2. (*5 points*) A and B play rock, paper, scissors. Their payoffs are as follows (the first number is A's payoff and the second number is B's payoff):

B

A		Rock	Paper	Scissors
	Rock	0,0	-1,1	1,-1
	Paper	1,-1	0,0	-1,1
	Scissors	-1,1	1,-1	0,0

How many Nash equilibria are there in the game? Explain.

Where no player can do better given exactly what other person does

0 - there is always something you ~~ca~~ can change to if you know exactly what other person was doing

Long Questions:

3. *(25 points)* Suppose that Intel has a monopoly in the market for computer chips. In order to produce X computer chips, it costs Intel $C(X) = 2X^2$.

(a) (*2 points*) Find the marginal cost of producing a computer chip for Intel.

$$MC = \frac{\partial C}{dQ} = 2 \cdot 2X = 4X$$

$\uparrow x$

(b) (*6 points*) The demand for computer chips is $X_D = 12 - 0.25P$. Find the level of output that maximizes Intel's profits. What price is Intel charging?

$P = P$

Revenue $= P(12 - .25P)$

$12P - .25P^2$

$MR = 12 - .5P$

Rewrite demand as

$P = 48 - 4X_D$ ← really need to understand this

So revenue $=$

$x(48 - 4x)$

$= 48x\ 4x^2$

$MR = 48 - 8X$

← always $x \cdot P(x)$

need units in quantity

always struggel w/ this

$MC = MR$

? but what units need in ?

$\cancel{12 - .5P} = 4X$

$48 - 8X$

$48 = 12X$

$X = 4$

$P = 48 - 4(4) = 32$ ✓

(c) (*4 points*) What level of output would maximize total surplus in the computer chip market?

↳ no DWL

? Producer + Consumer ?

? If totally discriminated ?

I thought previous thing maximized

that seems to me welfare - not surplus

~~[illegible]~~ when price = MC   oh duh when market is competitive

$p = MC$

$p = 4X$

↑ here wrong units again

no - can converts $48 - 4x = 4x$

$48 = 8x$

$x = 6$

$P = 48 - 4(6)$ ✓

$= 24$

(d) (*4 points*) Suppose the government knew the demand and production functions. Find a price regulation the government could impose that would induce Intel to maximize total surplus, i.e., produce the efficient quantity from part (c).

Set a max. price (price cieling) on Intel at \$24 ✓

and they explained how it would be more effective

(gov not likly to do - since reduces Intel's revenue)

(may not even be legal in US)

(instead of price cieling

(e) (*6 points*) If the government subsidized Intel s for every unit of computer chips produced, what quantity would Intel choose as a function of s? Find the choice of subsidy that maximizes total surplus, i.e., induces Intel to produce the efficient quantity from part (c). — was 6

So now cost function

$C(x) = 2x^2 - sx$

$X_D = 12 - .25P$

$X_D + .25P = 12$

$.25P = 12 - X_D$

$P = 48 - 4X_D$

$R = x(48 - 4X_D)$

$\pi = 48x - 4x^2$

$MR = 48 - 8x$

They put subsidy on rev side - should not matter

$MC = 4x - s$

$MR = MC$

$48 - 8x = 4x - s$

$x = 6$ efficent qu

$48 - 8(6) = 4(6) - s$

$0 = 24 - s$

$s = 24$ ← they got

that seems too high

Try diff math

Old revenue was $4 \cdot 32 = 128$

old cost $= 32$

old $\pi = 96$

new revenue $6 \cdot 24 = 144$

new cost $= 72$

$\pi = 72$

difference $= 24$

$/6 = 4$

inital ans correct ← this is wrong analysis

c w/ subsidy would have greater profit then before

(f) (*3 points*) Both the price regulation policy from part (d) and the subsidy policy from part (e) maximize total surplus. Is there any reason someone might prefer one policy over the other?

$x = 4 + \frac{s}{12}$ I would have gotten

The gov would like the price cieling b/c it does not cost them $

Intels profits higher under e

Gov must raise $ for subsidy leading to loss

Same efficency

diff distribution charasteristics

basically what I said

Cournot - lets see if I can remember

4. (*25 points*) Firm 1 and firm 2 are the only producers of spring water in the market. The market demand for spring water is given by $P = 70 - Q_1 - Q_2$. Firm 1 and firm 2 compete by choosing quantities Q_1 and Q_2 respectively. Each firm has a marginal cost of 10 and no fixed cost.

? I would not know how to do

(a) (*5 points*) Find out firm 1's and firm 2's reaction functions.

Process was what I was thinking - but rechecked

$$R_1 = Q_1(70 - Q_1 - Q_2) \qquad R_2 = Q_2(70 - Q_1 - Q_2)$$

$$= 70Q_1 - Q_1^2 - Q_1Q_2 \qquad = 70Q_2 - Q_1Q_2 - Q_2^2$$

$$MR_1 = 70 - 2Q_1 - Q_2 \qquad MR_2 = 70 - Q_1 - 2Q_2$$

$$MR_1 = MC_1 \qquad MR_2 = MC_2$$

Solve for Q_1 in terms of Q_2

$$70 - 2Q_1 - Q_2 = 10 \qquad 70 - Q_1 - 2Q_2 = 10$$

Solve for Q_2 in terms Q_1

$$-2Q_1 = -60 - Q_2 \qquad -2Q_2 = -60 - Q_1$$

(b) (*5 points*) Suppose the two firms choose quantities simultaneously. What are the equilibrium price, quantities, and profits of the two firms in this market?

$$Q_1 = 30 - \frac{Q_2}{2} \checkmark \qquad Q_2 = 30 - \frac{Q_1}{2} \checkmark$$

remember back to airline example
These are 2 lines
Find intersection / solution

$$Q_1 = 30 - \frac{\left(30 - \frac{Q_1}{2}\right)}{2} \qquad Q_2 = 30 - \frac{20}{2}$$

$$= 20 \checkmark$$

$$Q_1 = 30 - 15 + \frac{Q_1}{4} \qquad P = 70 - 20 - 20$$

$$= 30 \checkmark$$

$$\frac{3}{4}Q_1 = 15$$

$$\cdot\frac{4}{3} \qquad \cdot\frac{4}{3}$$

$$\pi_1 = 20(30) - 10(20) = \pi_2$$

$$600 - 200$$

$$= 400$$

$$Q_1 = 20 \checkmark$$

{this is that Shackfield model - 1 chooses like monopoly
2 uses old model to fill
(?)

(c) (5 points) Suppose only firm 1 has a chance to bribe the government and get the right to choose the quantity first, what is the maximum amount of money that firm 1 is willing to pay? If firm 1 gets to move first, what are the equilibrium quantities and profits of firm 1 and firm 2? [*Hint: if firm 1 does not bribe the government, the two firms will choose quantities simultaneously as in (b).*]

$R = Q_1(70 - Q_1)$
?1 don't care about Q_2
$= 70Q_1 - Q_1^2$
$MR = 70 - 2Q_1$
$MR = MC$
$70 - 2Q_1 = 10$
$-2Q_1 = -60$
$Q_1 = 30$

Firm 2 $R = Q_2(70 - 30 - Q_2)$
$= Q_2 40 - Q_2^2$
$MR = 40 - 2Q_2$
$MR = MC$
$40 - 2Q_2 = 10$
$-2Q_2 = -30$
$Q_2 = 15$

Price $= 70 - 30 - 15$
$= 25$ ← ? lower price ?
$\pi_1 = 30(25) - 30(10)$
$= 750 - 300$
$= 450$ ← higher
$\pi_2 = 15(25) - 15(10)$
$375 - 150$
$= 225$

(d) (5 points) Now back to the situation that the two firms choose quantities simultaneously. Suppose the two firms decide to collude and share the profit equally. Assume that both firms value their reputation and will behave according to their agreement. What are the quantities they will choose for each firm? What is the profit of each firm?

Never ans how much would if bribe
450 - 400
= 50

Like a monopoly
$R = Q(70 - Q)$
$= 70Q - Q^2$
$MR = 70 - 2Q$
$MR = MC$
$70 - 2Q = 10$
$-2Q = -60$
$Q = 30$

$P = 70 - 30$
$= 40$

Firms split production - same cost structure

$Q_1 = 15$
$Q_2 = 15$

$\pi_1 = \pi_2 = 15(40) - 15(10)$
$600 - 150$
$= 450$

very straight forward exam

(e) (*5 points*) Suppose the two firms decide to collude and share the profit equally, but both firms do not care about their reputation and might try to take advantage of the other. Foreseeing this, they make a legally enforceable contract saying that if a firm does not produce the quantity agreed, it has to pay some penalty to the other firm. What is the minimum amount of penalty that ensures each firm producing the right quantity agreed in part (d).

If ~~each~~ a firm cheated, π would not $\uparrow$

450 is already max π under any case

0.

?seems weird

Oh can get 22.5 units
$\pi = 506.25$

* when other firm is producing 15

Penalty ≥ 56.25

(did not remember other firm at 15)

5. (*19 points*) The country of Economia has two industries. In the Clothing industry, the marginal product of labor is always 1. In the Steel industry, the marginal product of labor is $12L_S^{-1/2} - 2$, where L_S is the total number of workers employed in the Steel sector. The total supply of labor in Economia is fixed at $L_C + L_S = 25$, and the output price is 1 for both Clothing and Steel. *[Note on calculus: if you have forgotten, the derivative of aX^b is abX^{b-1}.]*

(a) (*7 points*) Suppose that the labor market is perfectly competitive. How many workers will be employed in the Clothing sector, and how many in the Steel sector? What wage rate will workers in each sector receive? *[Hint: Workers can switch sectors at will. What does that imply about wages in the two sectors?]*

must be same

Will maximize productiveness?

Well depends on each firms demand

Output price = \$1

~~Clothing~~ MP = price

Clothing $1 = W$

Steel

$$\frac{12}{\sqrt{L_S}} - 2 = W$$

$$1 = \frac{12}{\sqrt{L_S}} - 2$$

$$3 = \frac{12}{\sqrt{L_S}}$$

$$3\sqrt{L_S} = 12$$

$$\sqrt{L_S} = 4$$

$$L_S = 16$$ ✓

Clothing $25 - 16 = \cancel{5}\ 9 = L_C$

math error

wage?

$$\frac{12}{\sqrt{16}} - 2 = W \qquad 1 = W$$ ✓

$$\frac{12}{4} - 2$$

$1 = W$ ✓

figured it out - except for $25 - 16 = 5$ math error

But has a diff way of thinking

(*b*) (8 points) Suppose that workers in the Steel sector form a union, which acts as a monopolist in supplying labor to the Steel industry. The union chooses a level of employment that maximizes the total wages of its members (i.e., it maximizes $w_S L_S$). How many workers will the union allow to be employed in the Steel sector? How many will now be employed in the Clothing sector? What wage rate will workers in each sector receive? [*You should assume that the prices of Clothing and Steel remain 1. The labor market for Clothing workers is still perfectly competitive.*]

~~MR = MC~~

Winnings $= w\left(\frac{12}{\sqrt{L_S}} - 2\right)$

? get it terms of w

$w = \frac{12}{\sqrt{L_S}} - 2$

$w + 2 = \frac{12}{\sqrt{L_S}}$

$(w+2)\sqrt{L_S} = 12$

$\sqrt{L_S} = \frac{12}{w+2}$

$L_S = \frac{144}{(w+5)^2}$

$= w\left(\frac{144}{(w+5)^2}\right)$

Take deriv, set $= 0$ to maximize

Calc

$-\frac{288}{(w+2)^3} = 0$

No Solution exists

Wage rate still = MRPL

Maximize $L_S\left(\frac{12}{\sqrt{L_S}} - 2\right)$ ← So it waste the other variable

$= 12\sqrt{L_S} - 2L_S$

– How supposed to know ??

deriv, $= 0$

– ?, or should still come out – just I did math wrong?

$\frac{6}{\sqrt{L_S}} - 2 = 0$

$L_S = 9$

Clothing $25 - 9 = 16$

Wage still product

$w_S = \frac{12}{\sqrt{9}} - 2 = 2$ ✓

Clothing still #1 still MRP

(*c*) (4 points) If workers in Clothing had unionized in order to increase their wage rate instead of workers in Steel, what would have happened to employment and wages in each sector? Explain why. [*No calculations necessary; just describe the outcome qualitatively.*]

?, No change – Since MRP always same

no economies of scale

mills have perfectly elastic demand for labor

– how do you see?

If $w > 1$ – no one would hire

$w < 1$ every one would hire, wage would be competition

Massachusetts Institute of Technology
Department of Economics

14.01 Principles of Microeconomics
Final Exam
Wednesday, December 19th, 2007

Last Name (Please print): ____________

First Name: ____________

MIT ID Number: ____________

Instructions. Please read carefully.

The exam has a total of 100 points. Answers should be as concise as possible. This is a closed book exam. You are not allowed to use notes, equation sheets, books or any other aids. You are not allowed to use calculators. You must write your answers in the space provided between questions. DO NOT attach additional sheets of paper. This exam consists of (18) sheets (13 pages + 5 blank pages for scratch work).

0. Circle Your Section/Recitation (1 point):

Please circle the section or recitation which you are attending below. The marked exam will be returned to you, in the section or recitation that you indicate. You will loose 1 point if you leave it unselected.

S01: MWF9	(Peter Schnabl)	R01: F10	(Rongzhu Ke)
S02: MWF10	(Chia-Hui Chen)	R02: F11	(Rongzhu Ke)
S03: MWF11	(Chia-Hui Chen)	R03: F2	(Rongzhu Ke)
S04: MWF1	(Monica Martinez-Bravo)	R04: F12	(Marco Migueis)
		R05: F1	(Marco Migueis)
		R06: F2	(Marco Migueis)

DO NOT WRITE IN THE AREA BELOW:

Question 1 __/25	Question 2 __/5
Question 3 __/25	Question 4 __/25
Question 5 __/19	Question 0 _/1

Total __/100

1. **True/False Questions** (*TOTAL: 20 points*):

In this section, write whether each statement is True or False. Please fully explain your answer, using a diagram if appropriate. No credit will be given for an answer without an explanation.

(a) (*5 points*) After Professor Wheaton promises that no one would fail, students never study and turn in problem sets again. This is an example of adverse selection problem.

False. It is an example of moral hazard.

(b) (*5 points*) When a firm chooses among different projects, the one which has the highest present value is always the one with the highest yield rate.

False. A big project with small yield rate might have higher present value than a small project with high yield rate.

(c) (*5 points*) If the government guarantees a binding price floor for agricultural output by purchasing any surplus, then the demand for farm labor will be more elastic.

True. With the price guarantee, firms move along their conditional demand curve, which is more elastic than the unconditional demand for labor.

(d) (5 points) Exploitation Mining Co. is the only employer in the remote town of Uranium City and pays its workers \$10.00/hour. If the government forces the company to raise its wage by a small amount—say to \$10.10/hour—then it will hire more workers.

True. This is just like the price regulation example we saw for monopolists, except that here the firm is a monopsonist in the market for labor. A regulated small increase in the wage will increase the quantity of labor used above the monopsony level.

(e) (*5 points*) A factory that pollutes a river has negative externalities on residents along the river. If the factory and residents can negotiate, an efficient result can be achieved only if the property right of the river is assigned to the residents.

False. According to the Coase Theorem, as long as the property right is well specified, efficiency can be achieved no matter who owns the river.

Short Question:

2. (*5 points*) A and B play rock, paper, scissors. Their payoffs are as follows (the first number is A's payoff and the second number is B's payoff):

B

A	Rock	Paper	Scissors
Rock	0,0	-1,1	1,-1
Paper	1,-1	0,0	-1,1
Scissors	-1,1	1,-1	0,0

How many Nash equilibria are there in the game? Explain.
0 if only pure strategy Nash equilibria are considered.

Long Questions:

3. *(25 points)* Suppose that Intel has a monopoly in the market for computer chips. In order to produce X computer chips, it costs Intel $C(X) = 2X^2$.

(a) (*2 points*) Find the marginal cost of producing a computer chip for Intel.

Solution: $MC(X) = 4X$.

(b) (*6 points*) The demand for computer chips is $X_D = 12 - 0.25P$. Find the level of output that maximizes Intel's profits. What price is Intel charging?

Solution: We can rewrite the demand function as $P = 48 - 4X_D$, *so total revenue is* $R(X) = X(48-4X) = 48X - 4X^2$ *and marginal revenue is* $MR(X) = 48 - 8X$. *Equating marginal revenue to marginal cost gives*

$4X = 48 - 8X$
$X_m^* = 4$

The price is $P = 48 - 4(4) = 32$.

(c) (*4 points*) What level of output would maximize total surplus in the computer chip market?

Solution: Total surplus is maximized when price equals marginal cost. That is,

$48 - 4X = 4X$
$X_{TS}^* = 6$

(d) (*4 points*) Suppose the government knew the demand and production functions. Find a price regulation the government could impose that would induce Intel to maximize total surplus, i.e., produce the efficient quantity from part (c).

Solution: The price that gets consumers to demand 6 units is 24. If the government set a regulated price cap at $P = 24$, *Intel would face a constant marginal revenue function and would choose to produce where* $MC = 24$, *i.e., at the efficient level from part (c).*

(e) (*6 points*) If the government subsidized Intel s for every unit of computer chips produced, what quantity would Intel choose as a function of s? Find the choice of subsidy that maximizes total surplus, i.e., induces Intel to produce the efficient quantity from part (c).

Solution: Now marginal revenue is $MR(X) = 48 - 8X + s$. *If Intel equates marginal revenue to marginal cost, then*

$$4X = 48 - 8X + s$$

$$X^*(s) = 4 + \frac{s}{12}$$

The surplus-maximizing choice of subsidy induces Intel to produce the efficient number of chips from part (c). Solving for $X^*(s) = 6$, *we find that a subsidy of 24 per unit is required.*

(f) (*3 points*) Both the price regulation policy from part (d) and the subsidy policy from part (e) maximize total surplus. Is there any reason someone might prefer one policy over the other?

Solution: While the policies have the same efficiency characteristics, they have very different distributional consequences. The subsidy increases Intel's profits at the expense of taxpayers; the price regulation redistributes some of Intel's profits to consumers.

4. (*25 points*) Firm 1 and firm 2 are the only producers of spring water in the market. The market demand for spring water is given by $P = 70 - Q_1 - Q_2$. Firm 1 and firm 2 compete by choosing quantities Q_1 and Q_2 respectively. Each firm has a marginal cost of 10 and no fixed cost.

(a) (*5 points*) Find out firm 1's and firm 2's reaction functions.

Solution: $Q_1 = \frac{60 - Q_2}{2}, Q_2 = \frac{60 - Q_1}{2}$.

(b) (*5 points*) Suppose the two firms choose quantities simultaneously. What are the equilibrium price, quantities, and profits of the two firms in this market?

Solution: $Q_1 = Q_2 = 20, P = 30$

(c) (*5 points*) Suppose only firm 1 has a chance to bribe the government and get the right to choose the quantity first, what is the maximum amount of money that firm 1 is willing to pay? If firm 1 gets to move first, what are the equilibrium quantities and profits of firm 1 and firm 2? [*Hint: if firm 1 does not bribe the government, the two firms will choose quantities simultaneously as in (b).*]

Solution: The quantity of the first mover is 30 and the profit is 450. The quantity of the second mover is 15 and the profit is 225. If firm 1 does not get the right, its profit is 400. Therefore, firm 1 is willing to pay is 450-400=50.

(d) (*5 points*) Now back to the situation that the two firms choose quantities simultaneously. Suppose the two firms decide to collude and share the profit equally. Assume that both firms value their reputation and will behave according to their agreement. What are the quantities they will choose for each firm? What is the profit of each firm?

Solution: Each firm produces 15 units and gets profit 450.

(e) (*5 points*) Suppose the two firms decide to collude and share the profit equally, but both firms do not care about their reputation and might try to take advantage of the other. Foreseeing this, they make a legally enforceable contract saying that if a firm does not produce the quantity agreed, it has to pay some penalty to the other firm. What is the minimum amount of penalty that ensures each firm producing the right quantity agreed in part (d).

Solution: Given the other firm produces 15 units, the best response of a firm is to produce 22.5 units and it gets profit 506.25. Therefore the penalty should be at least 56.25 so that no one would deviate.

5. (*19 points*) The country of Economia has two industries. In the Clothing industry, the marginal product of labor is always 1. In the Steel industry, the marginal product of labor is $12L_S^{-1/2} - 2$, where L_S is the total number of workers employed in the Steel sector. The total supply of labor in Economia is fixed at $L_C + L_S = 25$, and the output price is 1 for both Clothing and Steel. *[Note on calculus: if you have forgotten, the derivative of aX^b is abX^{b-1}.]*

   (a) (*7 points*) Suppose that the labor market is perfectly competitive. How many workers will be employed in the Clothing sector, and how many in the Steel sector? What wage rate will workers in each sector receive? *[Hint: Workers can switch sectors at will. What does that imply about wages in the two sectors?]*

   Solution: Workers in each sector will receive the same wage (equal to their marginal revenue product), so $w_C = w_S = 1$. This implies that $L_C = 9$ and $L_S = 16$.

   (b) (8 points) Suppose that workers in the Steel sector form a union, which acts as a monopolist in supplying labor to the Steel industry. The union chooses a level of employment that maximizes the total wages of its members (i.e., it maximizes $w_S L_S$). How many workers will the union allow to be employed in the Steel sector? How many will now be employed in the Clothing sector? What wage rate will workers in each sector receive? [*You should assume that the prices of Clothing and Steel remain 1. The labor market for Clothing workers is still perfectly competitive.]*

   Solution: The wage rate will still be the marginal revenue product of labor, so the union maximizes $L_S(12L_S^{-1/2} - 2) = 12L_S^{1/2} - 2L_S$. Setting the derivative of total wages equal to 0, we find that

   $$6L_S^{-1/2} - 2 = 0$$
   $$L_S = 9$$

   That is, now 16 workers will be employed in Clothing and only 9 in Steel. Workers in each sector still receive their marginal revenue products, so $w_C = 1$ and $w_S = 2$.

   (c) (4 points) If workers in Clothing had unionized in order to increase their wage rate instead of workers in Steel, what would have happened to employment and wages in each sector? Explain why. [*No calculations necessary; just describe the outcome qualitatively.*]

   Solution: Nothing. Workers in the Clothing sector can't raise wages at all—Clothing firms have a perfectly elastic demand for labor!

no cheat cheat etc

~~14.01~~ 14.01 Midterm 2 mine

Redo

12/12

1. ~~Same econ~~

False - by definition

True - Same line

but points will be diff distance apart

Very tricky

b) Negitive profit - shut down SR?

No - if ~~cost~~ price is > AVC - still better

to make some ~~$~~ back on fixed ✓

c) Dog market at min AC

No monopoly were MR = MC

- Competive would have been yes? ✓

Uncertain - market barriers

But I think market barriers would have been sufficent

d) True - I explained well on exam w/ cheating ✓

②

2. Cost + profit maximization
- price taking firm

In	Out
0	0
4	8
7	9
9	15

p input = 1

This was a total mess - go off solution sheet

a) Cost

$$C(\cancel{x}) = \begin{cases} 0 & \text{if } \cancel{x=0} \;\; y=0 \\ 4 & \cancel{4} \;\; 8 \\ 7 & \cancel{7} \;\; 9 \\ 12 & \cancel{12} \;\; 15 \end{cases}$$

is a function of output (asked for in qu as well)

b) Find firm's supply function

$$y(p) = \begin{cases} 0 & p < 1/2 \\ 8 & 1/2 \le p < 1 \\ 16 & p \ge 1 \end{cases}$$

function of p

So why is this?

③

If output $0 \to 8$ cost $\uparrow .50$/unit

so will do till $p > \frac{1}{2}$

$8 \to 9$ the extra cost is \$3

← will never do ?

$8 \to 16$ \$1/unit

8 units when price $.50 < p < 1$

16 units when $p > 1$

Look at what function asking for

Does make sense

↓ try on own 1st!

c) Suppose 5 price taking firms - no entry

$Q_D = 100 - 10p$ =librium p/q

? are all firms same ?

$$\frac{Q_D}{5} = \frac{100 - 10p}{5}$$

$$q = \frac{100 - 10p}{5}$$

When $p = .50$

$q = 19$

$p = 1$

$q = 18$

? how else but discrete not even thinking how I would do w/ continuous

Will produce 16 at

$$16 = \frac{100 - 10p}{5}$$

(4)

$80 = 100 - 10p$

$-20 = -10p$

$p = 2$

✓ actually got

Didn't do where market "clears" at - but can skip

---

d) Now 1 monopolist

$MR = MC$ - but how to do here

- calculate π at each pt
- find max π

$x_m = 0$ $\pi = 0$

$x_m = 20$

flip $Q_D = 100 - 10p$

$Q_D + 10p = 100$

$10p = 100 - Q_D$

$p = 10 - \frac{Q_D}{10}$

$p = 10 - \frac{20}{10} = 8$

$\pi = 8 \cdot 20 - 20 \cdot 1$

$= 140$

(5)

$x_m = 45$

$$p = 10 - \frac{45}{10} = 5.5$$

$$\pi = 45 \cdot 5.5 - 35$$
$$= 212.5$$ ← best ✓

$x_m = 80$

$$p = 10 - \frac{80}{10} = 2$$

$$\pi = 80 \cdot 2 - 60$$
$$= 100$$

- But can it be in middle?

No will only produce at that

- Ah but price can be higher?

Actually in a way this piecewise was easier

- The way you learn it in early HS

  - Oh got this one

  - did a bunch of surplus work though)

(6)

3. Monopoly + Oligopoly

$C(Q) = 20Q$

$P = 100 - Q$

Find competive + monopoly amt

Monopoly

$MR = MC$ $\quad$ $MC = 20$

$R = Q(100 - Q)$

↑ whant Q here

$MR = 100 - 2Q$

$100 - 2Q = 20$

$-2Q = -80$

$Q = 40$

$P = 100 - 40 = 60$

$\pi = 60 \cdot 40 - 20(40)$

$= 1600$

getting good at this

Perfectly competive

$P = MC$

$P = 20$

$Q = ?$

(1) Reverse

$P = 100 - Q$

$P + Q = 100$

↑ seems so weird w/ units

$Q = 100 - P$

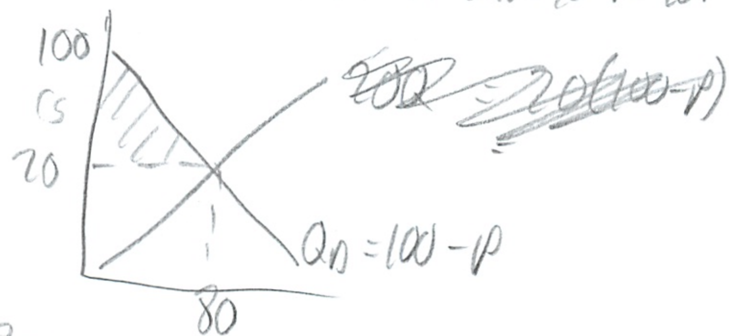
$Q = 100 - 20 = 80$

$\pi = 80(20) - 80(20) = 0$ ✓

~~Dwh~~ Now find DWL

– do graphically

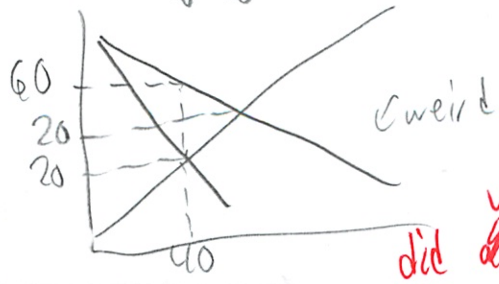
– used to always do 1st



$CS = \frac{1}{2} \cdot 80 \cdot 80 = 3200$

$PS = 0$

drew monsopony



$CS_{new} = \frac{1}{2} \cdot 40 \cdot 40 = 800$

PS = ? what again – profit = 1600

welfare = 800 + 1600 = 2400

DWL = 3200 − 2400 = 800

did a lot more work now

(8)

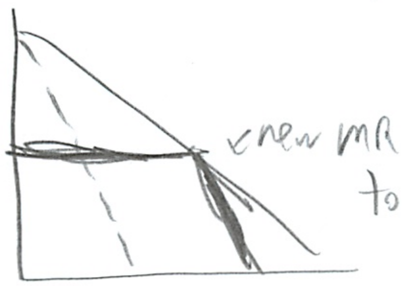
b) Two scenarios

- price cieling
- ~~Gov~~ R+D so MC = 0

(I remember the R+D was better)

Remember asks for consumer + total seperatly

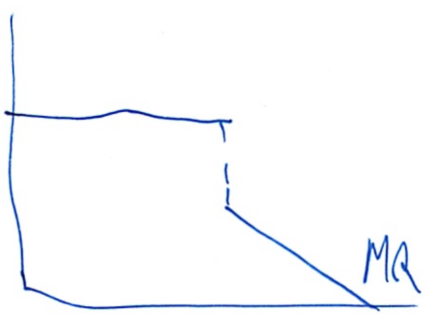
In any case I just skipped this problem — should not have



to the demand curve, right?

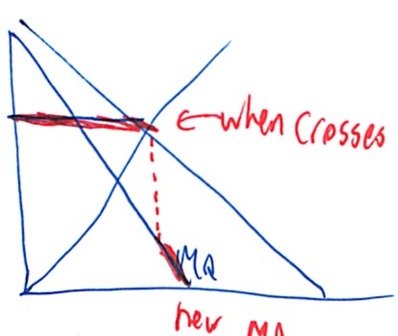
— they did not fix my drawing

let me check



So for us on exam

— why did grader not mark it wrong



9

d I thought my ans was right

On inelastic part of demand curve MR ⊖

So always raise price

- could have explained better but should have been -2

e) Oligopoly now

Oh can they collude - no can't be sustained ✓

kinda peaked

economically will always cheat w/ certainty

- again in some qu they say

" assume cooperate b/c of reputation "

but here they don't give any pts even if you explain

10

4. Sweat-trapping headbands

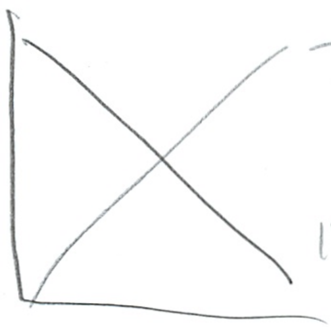
- remember this was difficult

- or weird w/ price

$q_D = 1200 - p$

$TC(q) = 72 + .5q^2$ each of 49 firms

World $p = 10$



Supply curve of US producer

$p = 10$

$p = MC$

$MC = q$

$10 = q$ each make 10

490 sold in US by US producers

$q_d = 1200 - 10$

$= 1190$

$1190 - 490 = 700$ imported

✓ wow

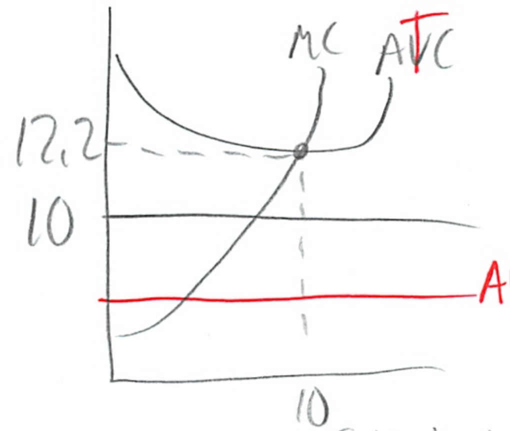
Got it right now w/o looking at solutions since this - unless did in recitation

(11)

b) Profits by US producers

$$\pi = 10 \cdot 10 - (72 + .5(10)^2)$$
$$= -22$$ ✓

Total profits $= -22 \cdot 49 = -1078$ ✓



↑ 10 is best q at this price

If no set price

$$10 \cdot p = 72 + .5(10)^2$$
$$10p = 122$$
$$p = 12.2$$

not valid
– not asked for here

$$AVC = \frac{.5q^2}{q} = .5q$$

(12)

c) What happens to industry LR

Firms drop out

All will drop out – because below ATC for each

No more domestic industry!

d) Now autarky – regular market

each firm still competive

– but price not set

– where is ATC min?

– no # firms fixed

– so w/ demand

– where to start???

$Q_D = 1200 - p$

why did I have trouble w/ this befor

$q = \frac{Q_D}{49} = \frac{1200 - p}{49}$

want in q

$49q = 1200 - p$

$49q + p = 1200$

$p = 1200 - 49q$

$R = 1200q - 49q^2$

– well what form is this

– perfectly competive

(13)

$MR = 1200 - \cancel{\ }98q \quad MC = q$

$1200 - 98q = q$

$1200 = 99q$

$q = 12.12$ ← oh yeah my perfectly competitive price
I found before worked

$Q = 49 \cdot 12.12 = 594$

$P = 1200 - 594 = 606$

← ?? wtf

intersection domestic demand + supply

$P = P$

$1200 - Q = \frac{Q}{49}$ ← I think I missed this before?

← why??? oh when split up
- each makes $\frac{1}{49}$ more
- or each time another turn to add 1

$Q = 1200 \cdot \frac{49}{50} = 1176$

$P = 24$

Domestic supply is MC above AVC

? Q

$Supply = 49Q = Q + Q + Q + Q \ldots$

or is it $\frac{Q}{49}$ ← should be, sum of individual curves

← so don't get

(14)

what is π

$\pi =$

$q = \frac{1176}{49} = 24$

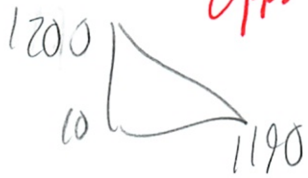
? yeah $p = q$
w/ these weird #s

$\pi = 24 \cdot 24 - [72 + .5(24)^2)$
$= 576 - 360$
$= 216$ ✓

Industry profits $= 216 \cdot 49 = 10{,}584$ ✓

Opps change in industry profit

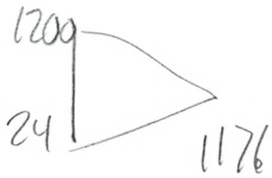
Before CS =



✓

708050

After CS =



↓ Drop $= 16562$

691 488

f) How many firms
- Urg - what to do now
- Find min ATC

$ATC = \frac{72 + .5q^2}{q} = \frac{72}{q} + .5q$

$\text{deriv} = 72 \cdot -1 q^{-2} + .5$
$-\frac{72}{q^2} + .5$

(15) set $= 0$

$$0 = .5 - \frac{72}{q^2}$$

$$.5 = \frac{72}{q^2}$$

$$.5q^2 = 72$$
$$\cdot 2 \quad \cdot 2$$

$$q^2 = 144$$
$$\sqrt{} \quad \sqrt{}$$

$$q = 12$$

Price is 12 - free entry + exit

demand $= 1200 - 12 = 1188$

$1188/12 = 99$ firms ✓

all make $0\ \pi$

need to figure out for each thing

g) Suppose gov taxes headbands

- san must be $12.\cancel{2}$ oh was 12 in proper figure

$$\frac{12.\cancel{2} - 10}{10} = \frac{2.\cancel{2}}{10} = 2\cancel{2}0\% \text{ tax}$$

Solutions

Massachusetts Institute of Technology

Department of Economics

14.01 Principles of Microeconomics

Exam 2

Tuesday, November 9th, 2010

Last Name (Please print): _______________

First Name: _______________

Kerberos ID: _______________

Instructions. Please read carefully.

The exam has a total of 100 points. Answers should be as concise as possible. This is a closed book exam. You are not allowed to use notes, equation sheets, books or any other aids. You are not allowed to use calculators. You must write your answers in the space provided between questions. Fractional answers are permissible in any part of this exam. DO NOT attach additional sheets of paper. This exam has 18 pages (13 pages + 5 blank pages for scratch work)

Circle Your Section/Recitation:

Please circle the section or recitation, which you are attending below. The marked exam will be returned to you in the section or recitation that you indicate.

MWF 9AM	**F 10AM**
MWF 11AM	**F 11 AM**
MWF 1PM	**F 1PM**
MWF 2PM	**F 3PM**

DO NOT WRITE IN THE AREA BELOW:

Question 1	__/16	Question 2	__/22
Question 3	__/27	Question 4	__/35
Total	__/100		

1. True/False/Uncertain Questions (16 points)

In this section, write whether each statement is True, False or Uncertain. You should fully explain your answer, including diagrams where appropriate. Points will be given based on your explanation.

(a) (4 points) Two firms are producing similar goods, but one enjoys economies of scale and one has diseconomies of scale. Claim: both firms can have an identical long-run linear expansion path.

True. A linear expansion path simply indicates that the optimal mix of labor and capital does not change, but there can be either economies or diseconomies of scale depending on the production function. These firms could have the same expansion path, but output using the same quantity of inputs will not be the same.

(b) (4 points) A firm is currently earning negative profit on each good it produces. Claim: it is always optimal for this firm to shut down in the short run.

False. A firm will not shut down in the short run as long as its revenue is greater than average variable cost. If revenue is less than average variable cost (i.e., the absolute value of negative profits is larger than the fixed costs), the firm will shut down. In the long run, of course, firm exit will occur and profits will go to zero.

(c) (4 points) The market for drug production is characterized by both lengthy periods of patent protection and the need for FDA permission to market products. Claim: this market will be characterized by production at the minimum level of average cost in the long run. *Uncertain. Both patent protection and government regulation constitute market barriers in this case. In a market with barriers to entry, the long-run supply curve will not necessarily be horizontal and average cost will not necessarily be minimized. To guarantee a horizontal supply curve, there must be be no barriers to entry, all firms must be identical, and there can be no variation in input costs.*

(d) (4 points) Cooperative behavior between two oligopolists is impossible if they know that they will be competing for a limited number of years. *True. A cooperative equilibrium can be sustained only if the competition game will continue forever. If they know the period of competition is finite, both will cheat.*

2. Costs and profit maximization (22 points)

A profit-maximizing, price-taking firm produces output Y using a single input X. The firm can produce 0, 8, 9, or 15 units of Y by using 0, 4, 7, or 9 units of X, respectively. There are no other possible input-output combinations. The firm's production function is therefore given by:

$$Y = f(X) = \begin{cases} 0 & \text{if } X = 0 \\ 8 & \text{if } X = 4 \\ 9 & \text{if } X = 7 \\ 16 & \text{if } X = 12 \end{cases}$$

The price of input X is 1 dollar per unit.

(a) (4 points) Write the firm's total cost as a function of output.

$$C(Y) = \begin{cases} 0 & \text{if } Y = 0 \\ 4 & \text{if } Y = 8 \\ 7 & \text{if } Y = 9 \\ 12 & \text{if } Y = 16 \end{cases}$$

(b) (6 points) Find the firm's supply function $y(p)$. Explain your answer.

$$y(p) = \begin{cases} 0 & \text{if } p < 1/2 \\ 8 & \text{if } 1/2 \leq p < 1 \\ 16 & \text{if } p \geq 1 \end{cases}$$

If output increases from 0 to 8, costs per unit increase by \$0.50/unit. Therefore the firm will supply 0 units if $p < \frac{1}{2}$.

If the firm increases output from 8 to 9, the cost to produce the additional unit is \$3. However, if the firm increases output from 8 to 16, the additional cost per unit of the additional units 8 is \$1/unit. Therefore the firm will produce 8 units when the price is above \$0.50 and below \$1, and 16 units if the price is above \$1.

(c) (5 points) Suppose that there are 5 such price-taking firms in the market, and that there is no entry. Market demand is given by $Q_D = 100 - 10p$. What is the equilibrium price and quantity in this market?

Total market supply with 5 firms is:

$$5 \cdot y(p) = \begin{cases} 0 & \text{if } p < 1/2 \\ 40 & \text{if } 1/2 \leq p < 1 \\ 80 & \text{if } p \geq 1 \end{cases}$$

With this level of demand, the market clears at $p_M = 2$ and $Q_M = 80$.

(d) (7 points) Suppose now that instead of the 5 firms in part (c) there is actually a single monopolist that is five times as large as one of these individual firms. The monopolist's production function is therefore:

$$Y_M = 5 \cdot f(X_M) = \begin{cases} 0 & \text{if } X_M = 0 \\ 40 & \text{if } X_M = 20 \\ 45 & \text{if } X_M = 35 \\ 80 & \text{if } X_M = 60 \end{cases}$$

Demand is given by $Q_D = 100 - 10p$ as before. Assume now that entry is once again impossible. What is the equilibrium price and quantity in this market? Compare this outcome with the equilibrium in (c) and explain why they are the same / differ.

The monopolist compares marginal revenue to marginal cost when deciding whether or not to increase production:

Q	$P = 10 - Q/10$	TR	TC	MR	MC	Profit
0	10	0	0			0
40	6	240	20	240	20	220
45	5.5	247.5	35	7.5	15	212.5
80	2	160	60	-87.5	25	100

The monopolist will not increase production beyond 40 because MR<MC for all quantities greater than 40. As a result, profits are maximized at $Q = 40$. The equilibrium price at this quantity is \$6.

This is a lower quantity and higher price than in (d) because the large monopoly firm has market power and the five smaller firms are assumed to be price takers.

3. Monopoly and oligopoly (27 points)

A uniform pricing monopolist has the following cost function and faces the following demand curve for its product

$$\begin{aligned} C(Q) &= 20Q \\ P &= 100 - Q \end{aligned}$$

(a) (3 points) Find the monopolist quantity (Qm), price (Pm), and deadweight loss relative to the perfectly competitive outcome. Draw a diagram labeling the perfectly competitive outcome as A, and the monopolist outcome as B. Be sure to include the marginal cost and marginal revenue curves in your diagram.

The monopolist sets $MR = MC \longrightarrow 100 - 2Q = 20 \longrightarrow Q_m = 40, P_m = 60$. $DWL = \frac{1}{2}40 * 40 = 800$.

(b) (6 points) There are two possible scenarios for the monopolist:

(i). The government set a price ceiling of \$ 40/unit in which case the monopolist does not invest in any R & D because it is wary of future government regulation.

(ii). There is no government regulation, so then the monopolist invests in R & D which then changes the cost function so that $MC = 0$.

Which scenario has higher welfare (ignore the cost of R & D for producer surplus)? Which scenario do the consumers prefer? Explain.

(i). *At* $P = 40, Q = 60$, *we have* $CS = \frac{1}{2}BH = \frac{1}{2} * 60 * 60 = 1800$.
$PS = BH = 60 * (40 - 20) = 1200$.
$W = CS + PS = 3000$.

(ii). $MC = 0, MR = MC \longrightarrow 100 - 2Q = 0 \longrightarrow Q_m = 50, P_m = 50$. *At* $P_m = 50$, $Q_m = 50$, *we have* $CS = \frac{1}{2}BH = \frac{1}{2} * 50 * 50 = 1250$
$PS = BH = 50 * 50 = 2500$,
$W = CS + PS = 3750$.

Consumers would prefer scenario (i) since consumer surplus is higher because they pay a lower price. Welfare is higher under scenario (ii) because the cost saving R & D generates large producer surplus and also limits the loss to consumer surplus because output expands.

(c) (6 points) For plan (i), the MR curve features a discontinuity at some Q'. Explain intuitively why the MR curve has this discontinuity.

Generally there are two effects for a monopolist when you increase Q by one unit: the "output effect" when you get to sell the additional unit you produce, and the "poisoning the well" effect when you receive a lower price on the inframarginal units (units you continue to sell). When $Q < Q'$, the MR is constant at the price ceiling of 40 because the monopolist is a price taker. While the monopolist would like to charge more, it is constrained by the government regulation and only has the "output effect". When Q >Q', the MR suddenly drops because you are already selling a lot of inframarginal units, and now the government price ceiling no longer binds, which implies that the "poisoning the well" effect is very large , leading to this discontinuity.

(d) (6 points) In scenario (ii), when $MC = 0$, the monopolist chooses (Q_m, P_m) such that $|\epsilon^D| = 1$. Will an unregulated uniform pricing monopolist ever choose (Q_m, P_m) such that $|\epsilon^D| < 1$? Explain intuitively.

On the inelastic portion of the demand curve, the marginal revenue is negative. If the monopolist is ever on this portion of the demand curve, you can increase profits unambiguously by decreasing quantity by one unit. It is able to save the marginal cost, and its revenues also increase since $MR < 0$.

(e) (6 points) Go back to your solution in (a). Suppose now the government allows one other identical firm to enter this market and firms compete on quantity. Let x = the value of the MR at the monopolist output when there's only one firm. Claim: If the two firms each produce half the Monopoly quantity, then $MR = x$ for both firms at current levels of output. Is this claim true, false, or uncertain? Please explain your reasoning.

False. $MR > x$ for each of the firms, because when the quantity is split between two firms, there are fewer inframarginal units so that the "poisoning the well" effect is smaller from the perspective of one firm. The "output effect" is the same for either the duopoly case or monopoly case.

4. Trade in sweat-trapping headbands (35 points)

The U.S. demand for sweat-trapping headbands is summarized by the function $q_d = 1200 - p$, where p is the market price of a headband. There are currently 49 identical, profit- maximizing domestic headband producers in the U.S., each with the cost function $TC(q) = 72 + 0.5q^2$. This cost function is identical in the short run and the long run. U.S. consumers consider domestically produced headbands to be identical to foreign-produced headbands, and currently have access to a huge supply of foreign-made handbands at a constant price of \$10 per band (that is, we can think of the worldwide supply of sweat-trapping headbands as a horizontal curve at $p = 10$).

(a) (5 points) Write down an expression for the supply curve for an individual domestic headband producer. If there is perfectly free international trade in sweat-trapping headbands, what will be the market price of a headband in the U.S.? How many headbands will be purchased in the U.S.? How many of those will be imported from abroad?

(b) (5 points) What will be the profits of each domestic headband producer? What will be the total profits earned by firms in the U.S. domestic headband industry? Assume, for now, that the number of domestic producers remains fixed at 49. Illustrate your answer with two diagrams: one showing the profit-maximization decision of an individual domestic firm, the other showing the entire U.S. headband industry.

(c) (2 points) What do you predict would happen to the U.S. domestic headband industry in the long run?

(d) (3 points) Suppose the U.S. government passes a dramatic new trade bill, the Sweat-Trapping Headbands Industry Protection Act of 2009, outlawing all imports of foreign-made head- bands. If the number of domestic headband producers remains fixed at 49, what will be the new market price of a headband? How many headbands will be purchased in the U.S.?

(e) (6 points) What will be the profits of each domestic producer after the new law is passed? What is the total increase in profits in the domestic headband industry as a result of this new trade law? Depict graphically the change in consumer surplus for headband buyers in the United States as a result of this policy as compared to your answers in (a). Under which scenario will consumer surplus be larger?

(f) (6 points) Assuming that the law described in (b) remains in force, but that there is free entry and exit in the domestic headband industry over time, what do you predict will be the long-run market equilibrium price? How many sweat-trapping headbands will be sold in the U.S.? How many headband-producing firms will there be in the U.S. in the long run?

(g) (8 points) Suppose that instead of the bill described in (b), the U.S. government decides to pass a less extreme law, assessing a $t\%$ tax on all imports of foreign headbands. How large must t be (5%? 10%? etc.) in order to ensure that the 49 existing domestic producers can remain in business in the long run?

Problem 4 - solution

(a) Individual domestic producers' supply function can be found by setting $P = MC = Q^{i,\text{dom}}$. Since there are 49 domestic producers, the total domestic supply will be $Q^{\text{dom}} = 49P$ or $Q^{dom}/49 = P$. The market price will be equal to the world price of 10. Since the market demand is $P = 1200 - Q$, this implies $Q^{total} = 1200 - 10 = 1190$. Domestic producers supply $Q^{\text{dom}} = 49 \cdot 10 = 490$ units and foreign producers supply the remainder, $Q^{\text{foreign}} = 1190 - 490 = 700$.

(b) Since $P = 10$ and each domestic firm produces 10 units, individual profits are: $\pi = P \cdot Q - TC(Q) = 10 \cdot 10 - 0.5 \cdot (10)^2 - 72 = -22$. Industry profits are $49 \cdot (-22) = -1078$.

(c) As short run profits are negative, we expect firms to exit this market in the long run

(d) When imports are banned, the new market equilibrium is the intersection of demand and domestic supply: $1200 - Q = Q/49 \Longrightarrow Q = 1200 \cdot 49/50 = 1176$, so $P = 24$.

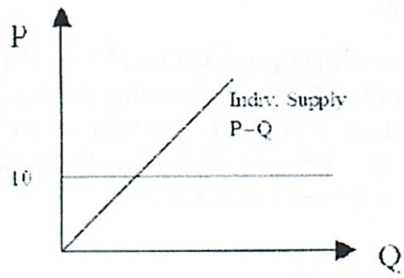
(e) Individual profits are: $\pi = 24 \cdot 24 - 0.5 \cdot (24)^2 - 72 = 216$, and industry profits are $49 \cdot 216 = 10584$. The total change in the industry profits is thus $10584 - (-1078) =$

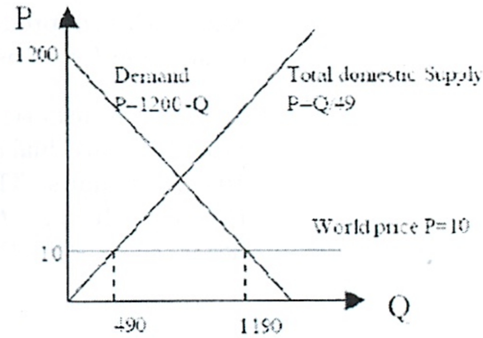
11662. Consumer surplus is larger in the free trade case by the triangle indicated on the graph below.

(f) Since profits are positive in the short run equilibrium without imports, firms will enter until $MC = ATC$, such that individual profits are zero. Solving for the individual quantities: $Q = 72/Q + 0.5 \cdot Q \Longrightarrow Q^2 = 72 \cdot 2 = 144 \Longrightarrow Q = 12$ With $P = MC$ this implies $P = 12$. The total quantity in the market is thus $Q^{\text{total}} = 1200 - 12 = 1188$. And since each firm produces 12 units, we have $99 = 1188/12$ firms in the long run market equilibrium (or 50 new firms entering).

(g) If the government sets a tax t, then the market price will be $P^{\text{tax}} = P \cdot (1+t) = 10 \cdot (1+t)$. From the individual supply functions $Q = P$, this implies that producers are each selling $10 \cdot (1+t)$ units. Their profits are thus: $\pi = P \cdot Q - TC(Q) = (10 \cdot (1+t))(10 \cdot (1+t)) - 0.5 \cdot (10 \cdot (1+t))^2 - 72 = 50 \cdot (1+t)^2 - 72$. For individual profits to be zero, we need $72/50 = (1+t)^2 \Longrightarrow 1.44 = (1+t)^2 \Longrightarrow t = 0.2 = 20\%$

END OF EXAM


P
Indiv. Supply
P=Q
10
Q


P
1200
Demand
P=1200-Q
Total domestic Supply
P=Q/49
World price P=10
10
490
1190
Q

Massachusetts Institute of Technology
Department of Economics

14.01 Principles of Microeconomics
Exam 1
Tuesday, October 5th, 2010

Last Name (Please print): Plasmeier

First Name: Michael

Kerberos ID: theplaz

Instructions. Please read carefully.

The exam has a total of 100 points. Answers should be as concise as possible. This is a closed book exam. You are not allowed to use notes, equation sheets, books or any other aids. You are not allowed to use calculators. You must write your answers in the space provided between questions. DO NOT attach additional sheets of paper. This exam has 18 pages (13 pages + 5 blank pages for scratch work)

Circle Your Section/Recitation:

Please circle the section or recitation, which you are attending below. The marked exam will be returned to you in the section or recitation that you indicate.

MWF 9AM	**F 10AM**
MWF 11AM	**F 11 AM**
MWF 1PM	**F 1PM**
MWF 2PM	**F 3PM**

DO NOT WRITE IN THE AREA BELOW:

Question 1	13/16	Question 2	10/10
Question 3	21/23	Question 4	20/25
Question 5	20/26		
Total	84/100		

pretty good
a few pts off in each

B

Mean 74 - flat distribution
- A/B division small

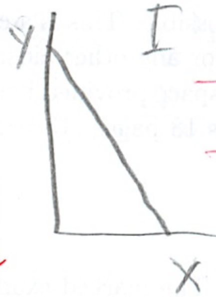
1. True/False/Uncertain Questions (16 points)

In this section, write whether each statement is True, False or Uncertain. You should fully explain your answer, including diagrams where appropriate. Points will be given based on your explanation.

(a) (4 points) A consumer finds two goods to be perfectly substitutable. Claim: The optimal bundle for this consumer will always be a corner solution.

ABA

True. If the goods are perfect substitutes, their is no preference one way or another so the person would just buy which ever is cheaper. If they were the exact same price, I guess he would pick some random number of each - but their is no provision in Economics for this randomness, so assume 1 is always cheaper

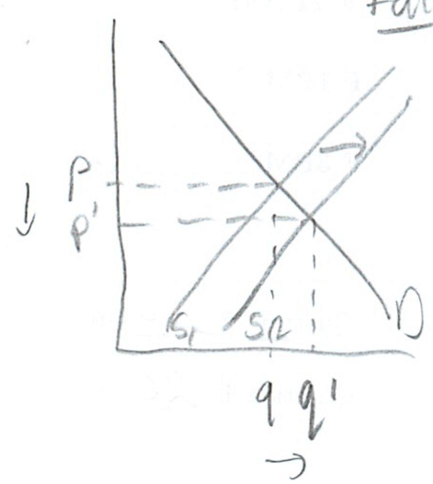


Not true all the time
$U(x,y) = x + y$
$P_x = P_y = 1$
$I = 10$
- any combo of x, y feasable
knife edge result

(b) (4 points) Innovations in the production of batteries lead to a rightward shift in the market supply for hybrid cars while demand stays the same. Since this leads to a decrease in the equilibrium price and an increase in the equilibrium quantity, demand is more inelastic at the new equilibrium.

−1

False

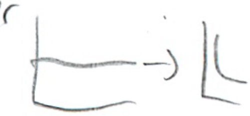


$e = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q}$

demand even more elastic there

- Depends on slope of demand
- If the curve becomes steeper it will become more inelastic

- If the curve becomes flatter more elastic



- If ~~linear~~, no change) this is more likely to happen ? So elastic prob.

if demand linear elasticity stays the same

(c) (4 points) A consumer has selected an optimal bundle of two goods that includes some of each good. The price of one good increases. Claim: her utility is lower after the price increase compared to before it.

-2

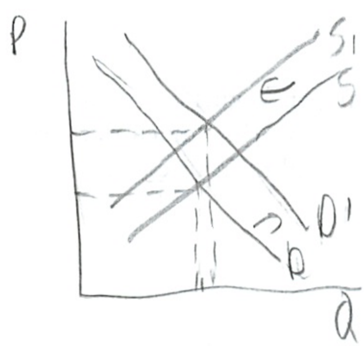
True, as long as it is not a corner solution, where utility would be same,
MRS ≥ MRT

Consumer already has their optimal bundle. If the price of a normal good increases, demand for it will decrease, This will lead to a substitution to a different good as well as making the person feel poorer, decreasing consumption, thus utility (in this case)

Counter example - part a
Consume 1 good - utility remains same

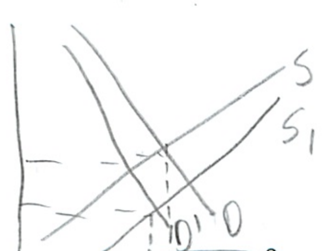
(d) (4 points) When market demand and supply shift in opposite directions we can unambiguously say how the equilibrium price and quantity change.

D ↑
S ↓



False, you need to know the relative effects of each change. Depending on which is larger will determine which way Q will change. Price moves knowable.

D ↓
S ↑



$P + \frac{1}{2}Q = 5 \qquad P = 20 - Q$

$\frac{1}{2}Q = 5 - P \qquad P + Q = 20$

$Q = 10 - 2P \qquad Q = 20 - P$

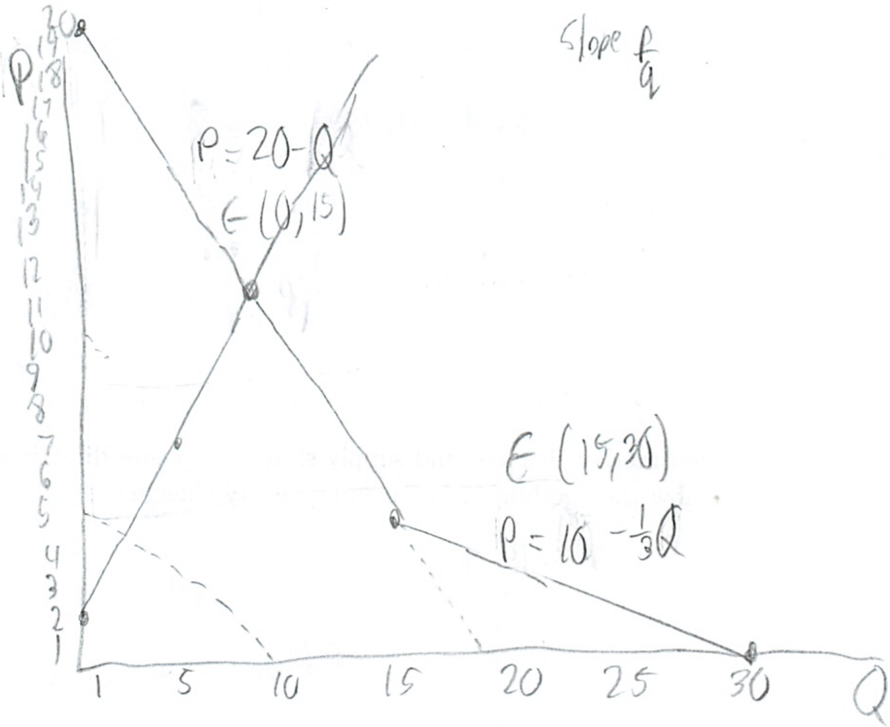
2. Market demand for frozen yogurt (10 points)

Market surveys show that there are two types of consumers for frozen yogurt. The first type like frozen yogurt and have an inverse demand curve of $P = 5 - \frac{1}{2}Q$. The second type are crazy about frozen yogurt and have an inverse demand curve of $P = 20 - Q$. In the town of Smallville there are only 2 consumers: one of them likes frozen yogurt and the other is crazy about frozen yogurt.

(a) (5 points) Using the individual demand curves above derive the market demand for frozen yogurt in Smallville. Plot the market demand curves.

never really looked at this

P	Q_1	Q_2	ΣQ	S
0	10	20	30	2
1	8	19	27	3
2	6	18	24	4
3	4	17	21	5
4	2	16	18	6
5	0	15	15	7
6	0	14	14	8
7	0	13	13	9
:		12	12	10
9		11	11	11
20	0	0	0	22



(b) (5 points) Suppose that the market supply for frozen yogurt in Smallville is given by $Q^S = 2 + P$. Find the equilibrium price and quantity. How much does each consumer buy at the equilibrium price? (Hint: Check the equilibrium price and quantity you get on a graph.)

$S = 2 + P$

$2 + P = 20 - P \qquad Q = 2 + P = 20 - P$

$2 - 20 = -P - P \qquad = 11 \quad = 11$

$-18 = -2P$

$P = 9$

Lovers buy 11 "crazy"

Likers buy none :(

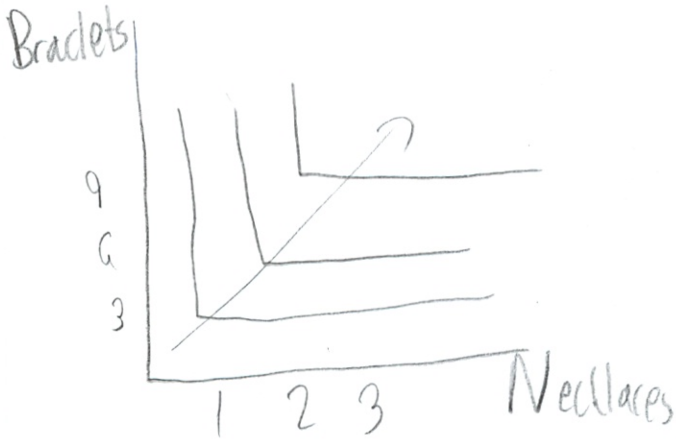
3. Consumer preferences and optimal allocations (23 points)

Mary is starting a jewelry collection. She wants to own matched sets of three bracelets and one necklace that can be worn together, and she doesn't want to own any bracelets or necklaces that are not in a matched set of this size.

(a) Draw Mary's indifference curves and write her utility function. Put bracelets on the y axis and necklaces on the x axis. Assume she receives utility of 3 utils from each matched jewelry set she owns.

$U(B,N) = \cancel{3}\min(B, 3N)$

Y



(b) (5 points) Currently, Mary has 32 dollars to spend. The price of necklaces is $p_n = 2$ and the price of bracelets is $p_b = 2$. What is the optimal allocation of necklaces and bracelets for Mary? How much utility does she achieve from this allocation?

can have non int B, N

$B \cdot P_B + N \cdot P_N = I$ $\qquad 3N = B$

$B \cdot 2 + N \cdot 2 = 32$

$3N \cdot 2 + N \cdot 2 = 32$ $\qquad B \cdot 2 + 4 \cdot 2 = 32$

$6N + 2N = 32$ $\qquad B \cdot 2 = 24$

$8N = 32$ $\qquad B = 12$

$N = 4$

$U = 3 \cdot 4 = 12$ "utility units"

(c) (4 points) Due to a shortage of gold, the price of necklaces increases to $p_n = 10$. What is the new allocation of necklaces and bracelets at this price level, and what utility does Mary obtain?

$B \cdot P_B + N \cdot P_N = I$ $\quad 3N = B$

$B \cdot 2 + N \cdot 10 = 32$ $\quad B \cdot 2 + 2 \cdot 10 = 32$

$3N \cdot 2 + 10N = 32$ $\quad B \cdot 2 = 12$

$6N + 10N = 32$ $\quad B = 6$

$16N = 32$

$N = 2$

$U = 2 \cdot 3 = 6$ "utility units"

(d) (4 points) Luckily, Mary's parents value their daughter's utility, and are willing to give her enough income to ensure that she has the same utility she did prior to the price change. How much extra money do they have to give her?

$B \cdot 2 + N \cdot 10 = I$ $\quad N \cdot 3 = 12$ $\quad 3N = B$

$N = 4$ $\quad 3 \cdot 4 = B$

$2B + 4 \cdot 10 = I$ $\quad B = 12$

$2 \cdot 12 + 4 \cdot 10 = I$

$24 + 40 = I$

$I = 64$

"no cross price elasticity since she requires $3N = B$"

extra \$ = $64 - 32$

= \$32

(e) (5 points) Mary has a sister Lily who doesn't like wearing matched sets of jewelry and has different preferences. Her utility function is nb^2. If she started with the same jewelry budget as Mary of 32 dollars and then faced the same price shock, what would be the decrease in her utility when the price of necklaces increases from \$2 to \$4?

10 correction

$U = nb^2$

$B \cdot P_B + N \cdot P_N = I$

$B \cdot 2 + N \cdot 2 = 32$

MRS = MRT

$$\frac{\frac{\partial U}{\partial B}}{\frac{\partial U}{\partial N}} = \frac{P_B}{P_N}$$

$$\frac{2nb}{b^2} = \frac{2}{2}$$

$$\frac{2n}{b} = 1$$

$B = 2N$

$2N \cdot 2 + N \cdot 2 = 32$

$4N + 2N = 32$

$6N = 32$

$N = 5\frac{1}{3}$

$B = 2N$

$B = 2 \cdot 5\frac{1}{3}$

$B = 10\frac{2}{3}$

$U = 5\frac{1}{3}(10\frac{2}{3})^2$

≈ 500

$\frac{2N}{B} = \frac{2}{10}$

$2B = 20N$

$B = 10N$

$10N \cdot 2 + N \cdot 10 = 32$

$20N + 10N = 32$

$N \approx 1$

$B = 10(1) = 10$

$U' = (1)(10)^2$

≈ 100

utility would fall from ≈ 500 to ~ 100

(f) (3 points) Mary and Lily's parents are going to give a gift of equal monetary value to both sisters. They are trying to decide whether to give cash or give jewelry. Which sister is more likely to prefer cash? Please explain intuitively and/or graphically; there is no need for algebra in this section.

Lilly has a much higher for Jewelry because it is exponential where as Marry's utility function has a minimum, If Marry got 2 bracelets, she would not care (no change in utility). Lilly would, depending on how many bracelets she has, perhaps double or triple her utility, Perhaps there is something else which Marry would like better with the cash.

Correct answer ~~reason~~ but reason not quite right

-2

TA: pretty hard

cash more substitutable

Lilly can substitute away easier

20/25

4. Labor markets and labor supply shocks (25 points)

Consider the labor market in the country of Widgetland. The demand for labor is given by:

$$L^D = 34 - 4w$$

where w is the wage rate.
Labor supply is:

$$L^S = \overline{L} + 2w$$

where $\overline{L}$ is the number of people in the country willing to work at a wage of zero.

5/5

(a) (5 points) Suppose that $\overline{L} = 10$. Find the equilibrium wage and equilibrium demand for labor. Is demand for labor elastic or inelastic at the equilibrium wage?

Equallibrium $L^D = L^S$

$34 - 4w = 10 + 2w$

$34 - 10 = 2w + 4w$

$24 = 6w$

$w = 4$ ✓

$L = 10 + 2(4)$

$L = 18$ ✓

$L = 34 - 4(4)$

$L = 18$

$\frac{\partial L}{\partial w} \cdot \frac{w}{L} = -4 \cdot \frac{4}{18}$

$= -\frac{16}{18} = -\frac{8}{9}$ ✓

<1 so it is slightly inelastic at this wage - about what you would expect

Suppose that there is a sudden influx of migrant labor, which increases the number of people willing to work at a wage of zero to $\overline{L} = 16$. For the remainder of the problem set $\overline{L} = 16$.

5/5

(b) (5 points) Compute the new market equilibrium. What happens to the equilibrium wage rate?

$34 - 4w = 16 + 2w$

$34 - 16 = 2w + 4w$

$18 = 6w$

w went down → $w = 3$

$L = 16 + 2(3)$

work done went up → $L = 22$

$L = 34 - 4(3)$

$L = 22$

$\frac{\partial L}{\partial w} \cdot \frac{w}{L} = -4 \cdot \frac{3}{22}$

$= -\frac{12}{22}$

more inelastic

1/5

(c) (5 points) In reality an increase in population should affect the demand for labor as well as the supply. Explain how the equilibrium wage and labor demanded will change compared to the market equilibrium in part (a) if demand for labor were to increase as well.

- So the qtd. of labor demanded did change from 18->22

Are you asking about a shift in the demand curve, due to a stronger economy or other factors, If so, then the equation would be different and there would be a new equilibrium

See solutions

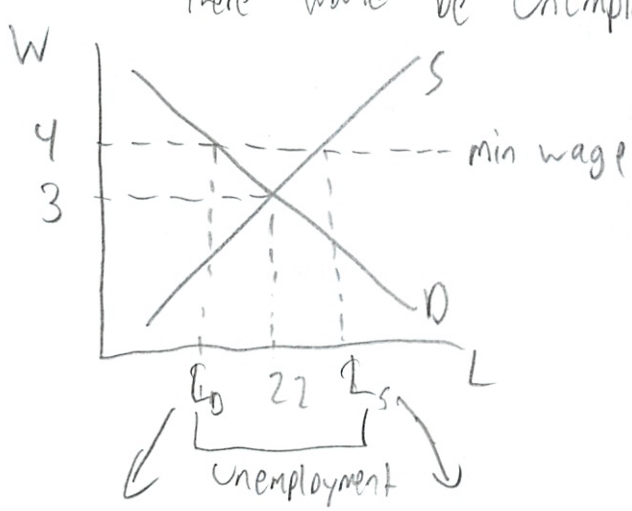
ie $64-4w = 16+2w$
$48 = 6w$
$w = 7$ ← wage rate shot up

$L = 16+2(7)$
$L = 30$ ← amt of labor demanded increased as well

5/5

(d) (5 points) The government in Widgetland becomes worried about the upcoming election and decides to appease voters by imposing a minimum wage of $\underline{w} = 4$. What happens in the labor market as a result? What is the demand and supply for labor now? Include a graph in your explanation.

There would be unemployment



$L_D = 34-4(4)$
18

$L_S = 16+2(4)$
$L_S = 24$

$24-18$

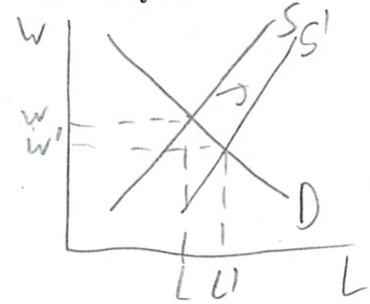
6 laborers would want to work, but can't so they are unemployed.

-think of if gov make apples $1 cheaper
- supply shifts out
- or is it something fair more?

? like a tax

4/5 (e) (5 points) The government is unhappy with the results of the minimum wage law and repeals it. Instead it introduces a subsidy of $\tau = 1$ dollar on labor that is paid to workers. What happens to the equilibrium wage and labor used as a result of this subsidy? How much do workers get in equilibrium?

Suppliers



firms → $D = 34 - 4w$

$S = 16 + 2(w+1) = 18 + 2w$

?

$34 - 4w = 18 + 2W$

$34 - 18 = 2w + 4w$

$16 = 6w$

$2\frac{2}{3} w = \frac{8}{3}$

wages fall slightly

$L = 34 - 4(2\frac{2}{3})$

$23\frac{1}{3}$

Firms pay wages $2\frac{2}{3}$

Gov pays 1

Workers receive $3\frac{2}{3}$

$L = 18 + 2(2\frac{2}{3})$

$18 + \frac{16}{3}$

$23\frac{1}{3}$

slightly more people working

Same as graph

5. Income and substitution effects (26 points)

(6) Glenn's utility function for goods X and Y is represented as $U(X,Y) = X^{0.2}Y^{0.8}$. Assume his income is \$100 and the prices of X and Y are \$10 and \$20, respectively.

(-1) (a) Express his marginal rate of substitution (MRS) of good Y for good X. As the amount of X increases relative to the amount of Y along the same indifference curve, does the absolute value of the MRS increase or decrease? Explain (4 points)

See sol

MRS

$$-\frac{\frac{\partial U}{\partial Y}}{\frac{\partial U}{\partial X}} \quad \frac{.8Y^{-.2}X^{.2}}{.2X^{-.8}Y^{.8}} \quad - \frac{.8\frac{X^{.2}}{Y^{.2}}}{.2\frac{Y^{.8}}{X^{.8}}} \quad -.8\frac{X^{.2}}{Y^{.2}} \cdot \frac{1}{.2}\frac{X^{.8}}{Y^{.8}} \quad -4\frac{X}{Y}$$

true

Yes? if we were to hold Y fixed increasing X would increase the absolute value of the MRS

(b) What is his optimal consumption bundle (X^*, Y^*), given income and prices of the two goods? (5 points)

$P_x \cdot X + P_y \cdot Y = I$

$10 \cdot x + 20 \cdot y = 100$

MRS = MRT

$-4\frac{x}{y} = -\frac{20}{10}$

$4x \cdot 10 = 20y$

$2x = y$

$10 \cdot x + 20 \cdot 2x = 100$

$10x + 40x = 100$

$x = 2$

$x = \frac{20y}{40}$

$x = \frac{1}{2}y$

$10 \cdot \frac{1}{2}y + 20 \cdot y = 100$

$5y + 20y = 100$

$y = 4$

$(2, 4)$

(c) How will this bundle change when all prices double and income is held constant? When all prices double AND income doubles? (5 points)

$-4\frac{x}{y} = -\frac{40}{20}$

$4x \cdot 20 = 40y$

$20x = 10y$

$2x = y$

$20 \cdot x + 40 \cdot 2x = 100$

$x = 1$

$20 \cdot \frac{1}{2}y + 40 \cdot y = 100$

$10y + 40y = 100$

$y = 2$

So just half of everything

$20x + 40 \cdot 2x = 200$

$x = 2$

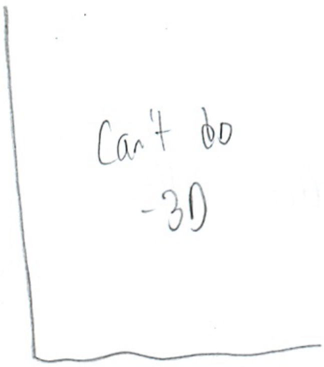
$20 \cdot \frac{1}{2}y + 40y = 200$

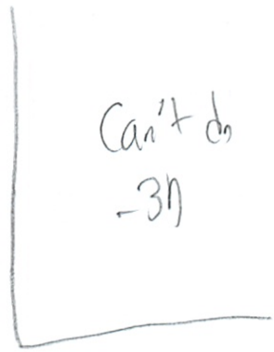
$y = 4$

So in this case nothing changes

(d) Derive the demand curve for good X and the demand curve for good Y as a function of prices assuming income is \$100. (4 points)

-3





$P_X \cdot X + P_Y \cdot 2x = 100$

$X \cdot (P_X + 2P_Y) = 100$

$X = \dfrac{100}{P_X + 2P_Y}$

See Sol

$P_X \cdot \frac{1}{2}Y + P_Y \cdot Y = 100$

$Y(\frac{1}{2}P_X + P_Y) = 100$

$Y = \dfrac{100}{\frac{1}{2}P_X + P_Y}$

Now a government subsidy program lowers the price of Y from \$20 per unit to \$10 per unit.

here we go again - well just do lower price

(e) Calculate the change in good Y consumption resulting from the program. In a clearly labeled diagram with Y on the y-axis and X on the x-axis, graphically show the change in consumption of good Y resulting from the program. (4 points)

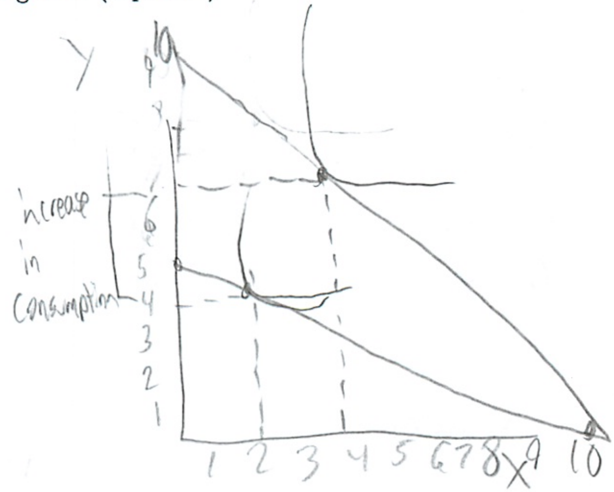
-2

$X = \dfrac{100}{10 + 2(10)} = \dfrac{100}{30} = \dfrac{10}{3} = 3\frac{1}{3}$

$= 3\frac{1}{3}$

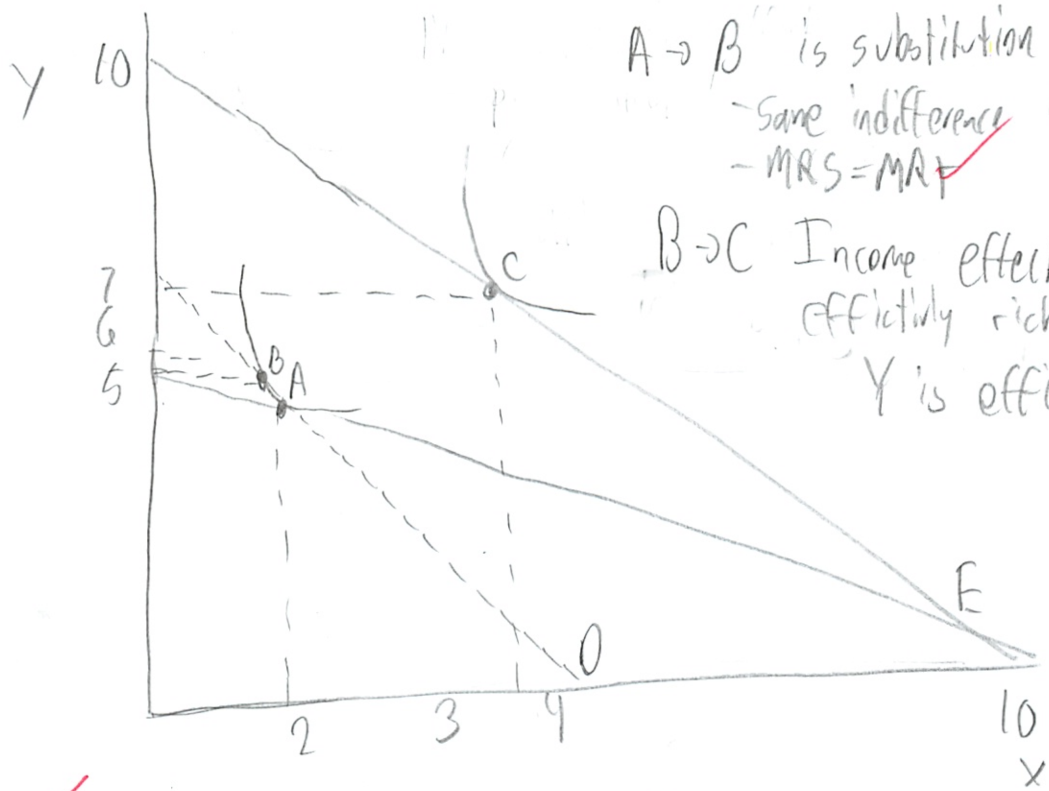
$Y = \dfrac{100}{\frac{1}{2}(10) + 10} = \dfrac{100}{15} = \dfrac{10}{1.5} = \dfrac{20}{3}$

$= 6\frac{2}{3}$



(f) In a clearly labeled diagram with Y on the y-axis and X on the x-axis, graphically show the change in consumption attributable to the separate income and substitution effects for good Y only. Explain the intuition of the income and substitution effects. No calculations are required for this part. (4 points)

effect of them having different ↓ prices



A → B is substitution effect
- Same indifference curve
- MRS = MRT

- new budget line D

B → C Income effect due to being effictivly richer because Y is effictivly cheaper

D ∥ E
∥ is parallel to

(g) How much does the program cost the government?

He purchases 8 Y goods → So it cost them $80.

END OF EXAM

done w/ 15 min to ✓

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14.01 Fall 2010: Midterm 1 Solution Set

October 13, 2010

1. True/False/Uncertain Questions

Write whether each statement is True, False or Uncertain. You should fully explain your answer, including diagrams where appropriate. Points will be given based on your explanation.

1. A consumer finds two goods to be perfectly substitutable. Claim: The optimal bundle for this consumer will always be a corner solution.

   False. The optimal bundle will be a corner solution if the prices of the two goods are different; if the prices are the same, then any allocation is optimal.

2. Innovations in the production of batteries lead to a rightward shift in the market supply for hybrid cars while demand stays the same. Since this leads to a decrease in the equilibrium price and an increase in the equilibrium quantity demand is more inelastic at the new equilibrium.

   Uncertain. This is true in the case of linear demand since elasticity of demand is increasing in price along a linear demand curve. However, this is false in the case of a constant elasticity demand curve.

3. A consumer has selected an optimal bundle of two goods that includes some of each good. The price of one good increases. Claim: her utility is lower after the price increase compared to before it.

   True. If the consumer was indifferent to one of the goods, she would have selected a bundle of only the other good. Clearly, she obtains utility from both goods; thus when the price of either increases and her budget constraint shifts in, she will be on a lower indifference curve.

4. When market demand and supply shift in opposite directions, we can unambiguously say how the equilibrium price and quantity change.

   False. When demand and supply shift in the same direction we can unambiguously say how the equilibrium price changes (equilibrium price increases if supply shifts to the left and demand shifts to the right and vice versa). However, the effect on equilibrium quantity is ambiguous; it could either increase or decrease in both cases.

2. Market demand for frozen yogurt

Market surveys show that there are two types of consumers for frozen yogurt. The first type like frozen yogurt and have an inverse demand curve of $P = 5 - \frac{1}{2}Q$. The second type are crazy about frozen yogurt and have an inverse demand curve of $P = 20 - Q$. In the town of Smallville there are only 2 consumers: one of them likes frozen yogurt and the other is crazy about frozen yogurt.

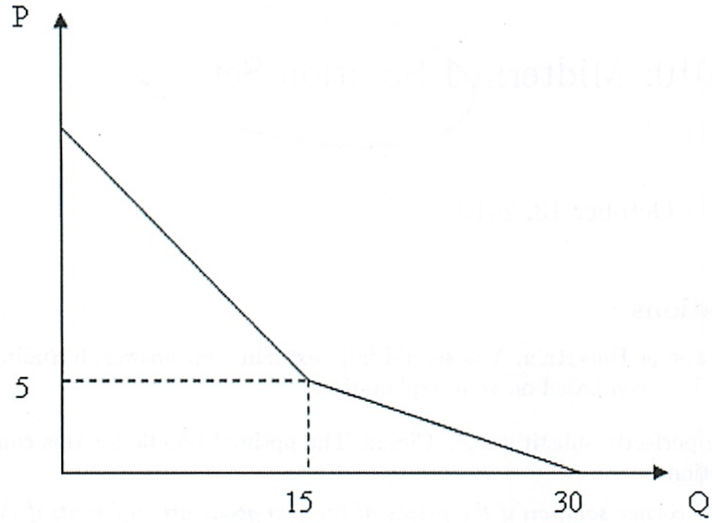
1. Using the individual demand curves above derive the market demand for frozen yogurt in Smallville. Plot the market demand curves.

   The demand of the first group is

$$Q_1^D = \begin{cases} 10 - 2P & , P \leq 5 \\ 0 & , P > 5 \end{cases}$$

   and the demand of the second group is

$$Q_2^D = 20 - P$$



Summing the two demand curves we get a market demand curve of

$$Q_M^D = Q_1^D + Q_2^D = \begin{cases} 30 - 3P & , P \leq 5 \\ 20 - P & , P > 5 \end{cases}$$

The inverse market demand curve is then:

$$P = \begin{cases} 20 - Q & Q \leq 15 \\ 10 - \frac{1}{3}Q & Q > 15 \end{cases}$$

2. Suppose that the market supply for frozen yogurt in Smallville is given by $Q^S = 2 + P$. Find the equilibrium price and quantity. How much does each consumer buy at the equilibrium price? (Hint: Check the equilibrium price and quantity you get on a graph)

   The market demand curve is piecewise linear. However, since at $p = 5$ $Q^S = 7 < 15 = Q^D$ it follows that supply would intersect demand to the left of the kink. Hence, $Q^S = Q^D$ or $2 + P = 20 - P$ and hence $P^* = 9$. This means that $Q^* = 11$. Only the crazy consumer buys at the equilibrium price.

3. Consumer preferences and optimal allocations

Mary is starting a jewelry collection. She wants to own matched sets of three bracelets and one necklace that can be worn together, and she doesn't want to own any bracelets or necklaces that are not in a matched set of this size.

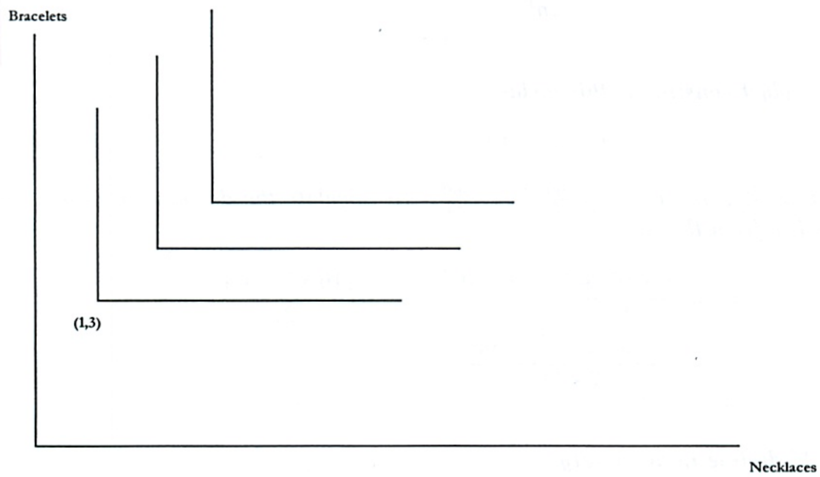
1. Draw Mary's indifference curves and write her utility function. Put bracelets on the y axis and necklaces on the x axis. Assume she receives utility of 3 utils from each matched jewelry set she owns.

   U(b,n)=3 min (n, $\frac{1}{3}$b) or U(b,n)=3min (3n, b)

2. Currently, Mary has 32 dollars to spend. The price of necklaces is $p_n = 2$ and the price of bracelets is $p_b = 2$. What is the optimal allocation of necklaces and bracelets for Mary? How much utility does she achieve from this allocation?

   We can find the optimal allocation by finding the intersection of the lines $b = 3n$ and $2n + 2b = 32$. Substituting in to the budget constraint, this yields the following.

$$\begin{aligned} 2n + 6n &= 32 \\ n &= 3 \end{aligned}$$



We have $n = 4$, $b = 12$, $u = 12$.

3. Due to a shortage of gold, the price of necklaces increases to $p_n = 10$. What is the new allocation of necklaces and bracelets at this income level, and what utility does Mary obtain? What is the proportional decrease in her utility?

   Re-solve the problem with a new budget constraint. This yields

$$10n + 6n = 32$$

   We have $n = 2$, $b = 6$, $u = 6$.

4. Luckily, Mary's parents value their daughter's utility, and are willing to give her enough income to ensure that she has the same utility she did prior to the price change. How much extra money do they have to give her?

   Mary's parents have to give her sufficient income to buy two more matched sets at the new prices. This requires the purchase of two necklaces and six bracelets, at a cost of $2 * 10 + 6 * 2 = 32$. They have to double her income, increasing it by \$32, in order to maintain her at the same utility level.

5. Mary has a sister Lily who doesn't like wearing matched sets of jewelry and has different preferences. Her utility function is nb^2. If she started with the same jewelry budget as Mary of 32 dollars and then faced the same price shock, what would be the decrease in her utility when the price of necklaces increases from \$2 to \$10?

   First, solve for Lily's original optimal allocation. We set the ratio of marginal utilities equal to the ratio of prices.

$$\begin{aligned} \frac{b^2}{2nb} &= \frac{2}{2} \\ b &= 2n \end{aligned}$$

   Substituting into the budget constraint, this yields

$$2(2n) + 2n = 32$$

This yields $n = \frac{16}{3}$, $b = \frac{32}{3}$, and $u = \frac{16}{3}(\frac{32}{3})^2 = \frac{512}{27}$
When prices increase, we now re-solve the problem.

$$\begin{aligned}\frac{b^2}{2nb} &= \frac{10}{2}\\ b &= 10n\end{aligned}$$

Substituting into the budget constraint, this yields

$$2(10n) + 10n = 32$$

This yields $n = \frac{16}{15}$, $b = \frac{160}{15}$, and $u = \frac{16}{15}(\frac{160}{15})^2 = \frac{2560}{225}$. To calculate the decrease in utility, subtract the second level of utility from the first.

$$\begin{aligned}&= (\frac{16*16^2*4}{27} - \frac{16*16^2*100}{27*5^3})/(\frac{16*16^2*4}{27})\\ &= \frac{16*16^2(4*5^3-100)}{27*5^3}\\ &= 485.4\end{aligned}$$

Lily experiences a 80% decline in her utility.

6. Mary and Lily's parents are going to give a gift of equal monetary value to both sisters. They are trying to decide whether to give cash or give jewelry. Which sister is more likely to prefer cash? Please explain intuitively and/or graphically; there is no need for algebra in this section.

   Mary is more likely to prefer cash, because she only receives utility from a gift that is given as a matched set and no utility from any other type of gift. You can portray this graphically by showing a graph of the two utility functions and indicating that for Mary, any increase in the quantity of one good or the other keeps her on the same indifference curve, while for Lily, any increase will move her to a new indifference curve. The below graph shows the result of a gift of necklaces only to both girls. Mary is on the same utility curve, but Lily has increased utility. In this case and others like it, Mary is more likely to prefer cash.

4. Labor markets and labor supply shocks

Consider the labor market in the country of Widgetland. The demand for labor is given by:

$$L^D = 34 - 4w$$

where w is the wage rate.
Labor supply is:

$$L^S = \overline{L} + 2w$$

where $\overline{L}$ is the number of people in the country willing to work at a wage of zero.

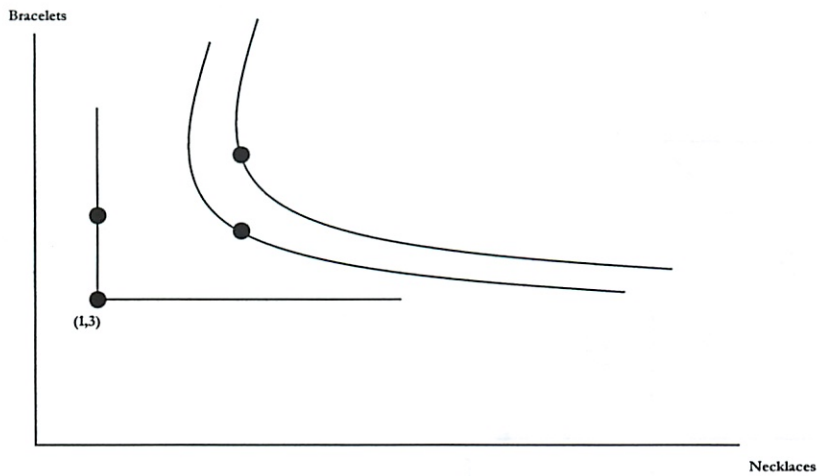
1. Suppose that $\overline{L} = 10$. Find the equilibrium wage and equilibrium demand for labor. Is demand for labor elastic or inelastic at the equilibrium wage?

   To find equilibrium wage we set $L^S = L^D$ and hence $34 - 4w = 10 + 2w$. Hence, $w^* = 4$. $L^* = 18$. The elasticity of demand at the equilibrium wage is

$$\epsilon_D = \frac{dL^D}{dw}\frac{w^*}{L^*} = -4\frac{4}{18} = -\frac{16}{18} = -\frac{8}{9}$$

   Hence, demand for labor is inelastic at w^*.

   Suppose that there is a sudden influx of migrant labor, which increases the number of people willing to work at a wage of zero to $\overline{L} = 16$. For the remainder of the problem set $\overline{L} = 16$.



2. Compute the new market equilibrium. What happens to the equilibrium wage rate?

   In the new equilibrium $34 - 4w = 16 + 2w$. *Hence,* $w^* = 3$. $L^* = 22$. *Hence the equilibrium wage rate decreases.*

3. In reality an increase in population should affect the demand for labor as well as the supply. Explain how the equilibrium wage and labor demanded will change compared to the market equilibrium in part (a) if demand for labor were to increase as well.

   A shift in demand in the same direction as the shift in supply increases the equilibrium demand for labor unambiguously. However, the change in the wage rate is ambiguous as the wage rate could either increase or decrease.

4. The government in Widgetland becomes worried about the upcoming election and decides to appease voters by imposing a minimum wage of $\underline{w} = 4$. What happens in the labor market as a result? What is the demand and supply for labor now? Include a graph in your explanation.

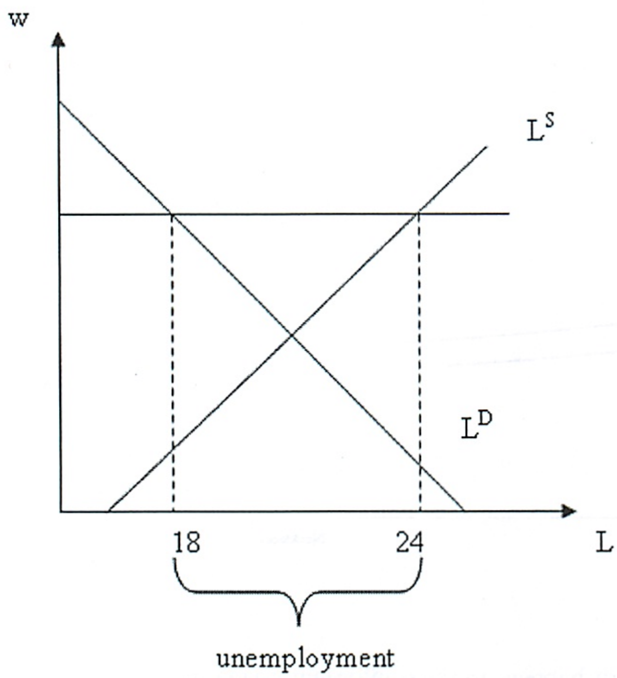
   Since $\underline{w} > w^*$ *it follows that the minimum wage will be binding. At the minimum wage demand for labor is* $L^D = 18$ *whereas supply for labor is* $L^S = 24$. *Hence, as a result of the minimum wage there is unemployment of* $L^S - L^D = 6$.

5. The government is unhappy with the results of the minimum wage law and repeals it. Instead it introduces a subsidy of $\tau = 1$ dollar on labor that is paid to workers. What happens to the equilibrium wage and labor used as a result of this subsidy? How much do workers get in equilibrium?

   A subsidy behaves like a negative tax. Hence, it effectively shifts supply down by τ. *The new equilibrium wage is determined where labor supply with the subsidy equals labor demand, i.e.* $L^S(p+\tau) = L^D$ *or* $16 + 2(w+\tau) = 34 - 4w$. *Hence,* $w^* = \frac{16}{6} = \frac{8}{3}$. *Hence workers get paid* $w^* + \tau = \frac{8}{3} + 1 = \frac{11}{3}$.

Income and substitution effects

Glenn's utility function for goods X and Y is represented as $U(X,Y) = X^{0.2}Y^{0.8}$. Assume his income is \$100 and the prices of X and Y are \$10 and \$20, respectively.



1. Express his marginal rate of substitution (MRS) between goods X and Y. As the amount of X increases relative to the amount of Y along the same indifference curve, does the MRS increase or decrease? Explain.

   $MU_X = 0.2(Y/X)^{0.8} and MU_Y = 0.8(X/Y)^{0.2}$
   $MRS_{Y forX} = -MU_X/MU_Y = -Y/4X$
   The MRS (in absolute value) gets smaller as the amount of X increases relative to Y. In other words, the more X (and less Y) one has, the less of Y one is willing to give up in order to obtain an additional unit of X.

2. What is his optimal consumption bundle (X^*, Y^*), given income and prices of the two goods?

   At the optimal consumption bundle, the MRS is equal to the ratio of prices. That is, $MRS_{Y forX} = -P_X/P_Y$. Plugging in our prices and the results from (a), $-Y/4X = -10/20 \Longrightarrow -Y/4X = -1/2 \Longrightarrow 2Y = 4X \Longrightarrow Y = 2X$
   The budget constraint must hold as well: $I = P_X X + P_Y Y \Longrightarrow 100 = 10X + 20Y$
   We now have two equations and two unknowns. Solve for (X^*, Y^*): $100 = 10X + 20(2X) \Longrightarrow 100 = 50X \Longrightarrow X^* = 2$
   $Y = 2X = 2(2) \Longrightarrow Y^* = 4$

3. How will this bundle change when all prices double and income is held constant? When all prices double AND income doubles?

   The utility function does not change and therefore the formula for MRS does not change. Prices change, but the ratio $-P_X/P_Y$ does not. Therefore, from $MRS_{Y\ for\ X} = -P_X/P_Y$ we still get $Y = 2X$. The budget constraint changes to $100 = 20X + 40Y$. Solving as before (with two equations, two unknowns), we get $(X^*, Y^*) = (1, 2)$. If all prices and income change proportionally, the optimal bundle does not change.

4. Derive the demand curve for good X and demand curve for good Y.

Solve for X^* and Y^* as before, except this time do not plug in explicit values for P_X, P_Y and I.
$MRS_{Y \text{ for } X} = -P_X/P_Y \Longrightarrow -Y/4X = -P_X/P_Y \Longrightarrow P_Y Y = 4P_X X$
Budget constraint: $I = P_X X + P_Y Y$
Now let's solve for X^* by substituting out Y. $I = P_X X + 4P_X X = 5P_X X \Longrightarrow X^*(P_X; I) = I/(5P_X)$.
Holding income constant at \$100 *gives us a demand curve of* $X^*(P_X; I = 100) = 20/P_X$

We can solve for Y^* by substituting out X: $P_Y Y = 4P_X X \Longrightarrow Y = (4P_X X)/P_Y \Longrightarrow Y = (4P_X[I/(5P_X)])/PY$ $Y^*(P_y; I) = 4I/(5P_y)$
Holding income constant at \$100 *gives us a demand curve of* $Y^*(P_y; I = 100) = 80/P_y$

Now a government subsidy program lowers the price of Y from \$20 per unit to \$10 per unit.

5. Calculate and graphically show the change in good Y consumption resulting from the program.

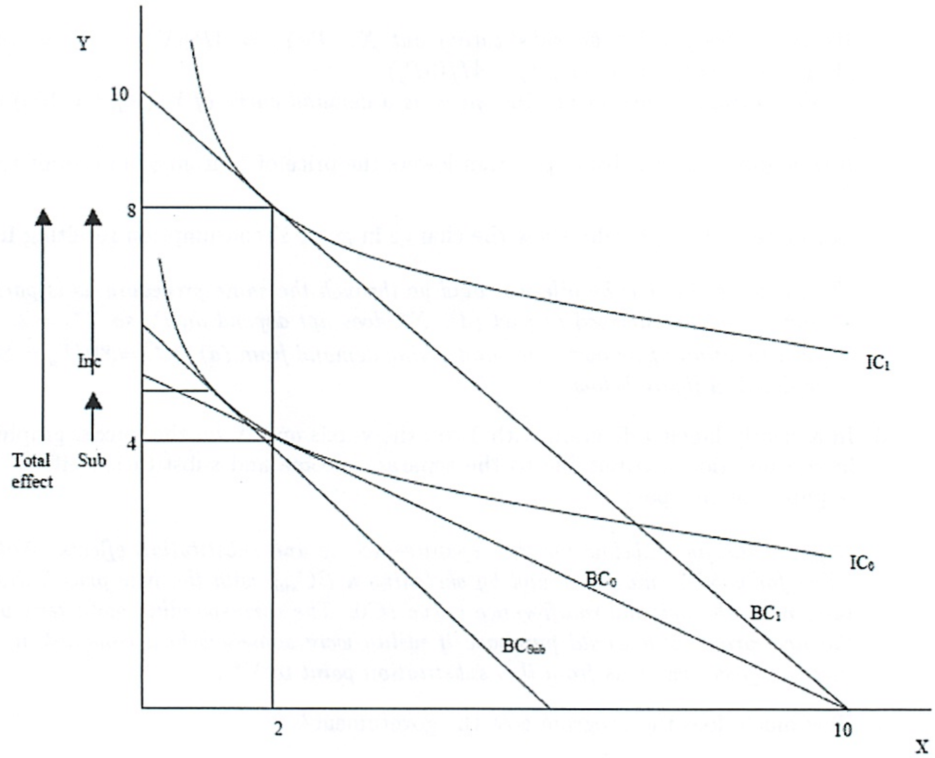
To calculate the new bundle we could go through the same procedure as is part (b) or simply use the demand equations derived in part (d). X^* does not depend on P_y so $X^{*'} = 2$.
Y^* is a function of its own price and, using demand from (d), $Y^{*'} = 80/P_y = 80/10 = 8$. *This change is depicted in figure below.*

6. In a clearly labeled diagram with Y on the y-axis and X on the x-axis, graphically show the change in consumption attributable to the separate income and substitution effects. No calculations were required for this part.

Refer to the figure below for the separate income and substitution effects. Note that the substitution effect for good Y may be found by sketching a BC_{sub} with the new price ratio (parallel to BC_1) but tangent to the original indifference curve $IC0$. The corresponding point tells us what a person facing the new price ratio would purchase if utility were somehow held constant at its original level. The income effect brings us from this substitution point to $Y^{*'}$.

7. How much does the program cost the government?

For each unit of Y purchased, the government pays out \$10. *Since* 8 *units of Y are currently purchased, the cost will be* 8(\$10) = \$80.



Y
10
8
4
Total effect
Sub
Inc
IC1
IC0
BC0
BC1
BCSub
2
10
X