
Syllabus: 14.01, Principles of Microeconomics:

<http://stellar.mit.edu/S/course/14/fa09/14.01/>

Course Description: 14.01 is an introductory undergraduate course that teaches the fundamentals of microeconomics. 14.01 is a HASS elective and is offered both terms.

Course Format: 14.01 is offered in two formats. Both use the same textbook, cover the same topics and have the same requirements, including exams.

1). Lecture-recitation. Students attend two one-hour lectures on Monday and Wednesday at 10:00am in E25-111. Students also attend a 1 hour recitation on Fridays – which are not optional. Recitations and Lectures are *complements* and not substitutes. Students may select from the following recitations.

- Friday 9 AM, 10 AM, 11 AM, 1 PM, 2 PM, 3 PM in 2-142.

2). Class Sections. Students attend three one-hour sections weekly on Monday, Wednesday and Friday. Sections are taught by graduate students but are organized to closely follow the lecture material so that there is little variation in content or teaching style between sections. The following sections are available (note that a section may be cancelled if insufficient enrollment).

- MWF 9 AM (5-233), 11 AM (5-233), 1 PM (5-233), 2 PM (13-5101).

Selecting a Format: During the first 2 weeks of the semester students may switch between formats. At the due date of the first homework (see below) students must submit the completed assignment at the recitation or section of their choice. This submission will automatically enroll them in that choice. Thereafter you can change only with special Faculty approval. If sessions are extremely unbalanced, the Faculty reserves the right to re-assign students.

Textbook: Microeconomics, by Jeffrey M. Perloff, Addison Wesley, May 31, 2008 (5th edition).

Prerequisites: some calculus at the level of 18.01 will be used. Otherwise there are no prerequisites. 14.01 is a 3-0-9 course.

Problem sets, exams, grading: there will be 9 mandatory problem sets which will be individually graded and returned to the students. At the end of the term students will have the best 8 homework grades cumulated up and this will count for 22% of the final course grade. Problem sets are due in sections on Fridays, as indicated. Late problem sets will not be accepted for any reason.

There will be two mid-term exams held throughout the term (see schedule below). These will each cover roughly 1/3 of the course material, will be 2 hours long, and will count as 22% (collectively 44%) of the grade.

There will also be a three hour final during finals period, which will be cumulative and cover all of the course materials. This will count as the remaining 34% of the grade.

Grading, exam policy: At the end of the semester, the faculty will report *internal* grades to the MIT registrar with + and – modifiers, where appropriate, and only for grades A, B, C. The faculty will be happy to *regrade* any problem set or exam – *in entirety*. Consequently it is possible for your revised grade to fall as well as rise. If you are an MIT freshman, the faculty will evaluate your performance on the first exam and first 3 problem sets and if it falls below passing, we will send out a warning to you and your advisor.

Conflicts/Makeup: Conflicts must be received by the head TA *at least one week prior* to the exam.

Illness on the day of the exam: If you should happen to be sick the day of the exam, contact one of the Deans in Student Support Services at <http://web.mit.edu/s3/www/> to verify your illness and let the head TA know.

A fixed Makeup exam time will be scheduled by the head TA.

Faculty:

Lecture: Professor Jon Gruber, gruberj@mit.edu.

Office hours by email appointment, E52-355

Head TA: Jessica Leight, jeleight@mit.edu

Office hours: Thurs 1-3pm and by appointment, E52-398

Graduate TAs: running sections and recitations.

Edward Cho, edcho@mit.edu , MWF 1 P.M., 2 P.M.

Office hours: W 3-4:30, Th 4:30-6, 2-142

Jennifer Peck, jpeck1@mit.edu, MWF 9 A.M., 11 A.M.

Office hours: M 1:15-2:15 and W 2-4, E53-394

Plamen Nenov, nenov@mit.edu. Recitations Friday 9, 10, 11, 1, 2, 3.

Office hours: Th 10-12 and 1-2 P.M, E52-369

Tutoring: Students are encouraged to seek help from the faculty and their assistants. An undergraduate TA will be in charge of offering tutorial help. There will be two tutorial sessions with different objectives. On Tuesdays from 5-7 pm, the tutorial will focus on doing 14.01 problem sets. On Thursdays from 7-9 pm there will be a general Q&A tutorial where students can bring questions from lecture, class or the text. The names of the tutors and locations of the tutorial sessions will be announced in lecture and on the class Stellar site.

Course Schedule, Fall 2010:

Date	Event	Material Covered
8-September	Class / Lecture	Chapter 1, Introduction
10-September	Class/ Recitation Problem Set 1: out	Chapter 2, Supply-demand analysis
13-September	Class / lecture	Chapter 2, Applying supply and demand
15-September	Class / Lecture	Chapter 3, Shape of supply and demand and empirical work
17-September	Class / Recitation Problem Set 1: due Problem Set 2: out	Chapter 3, Calculating elasticities. Review Problem set 1.
20-September	Class / Lecture	Chapter 4, Preferences and utility functions
22-September	Class / Lecture	Chapter 4, Budget constraints and constrained choice
24-September	Class / Recitation Problem Set 2: due Problem Set 3: out	Chapter 4, Mathematics of constrained maximization. Review Problem set 2.
27-September	Class / Lecture	Chapter 5, Food stamps, demand curves and income/substitution effects
29-September	Class / Lecture	Chapter 5, Applying consumer theory – labor supply
1-October [last add date]	Class / Recitation Problem Set 4: out Problem Set 3: due	Review problem set 3 and exam review
4-October	Class / Lecture	Chapter 5 and 6, Labor supply and introduction to production theory
5-October	<u>Exam #1 (7:30 pm)</u> Location TBA	
6-October	Class / Lecture	Chapter 6, Production theory and start Costs
8-October	Class / Recitation Problem Set 4: due Problem Set 5: out	Chapter 7, Costs
11-October	No class – Columbus Day	
13-October	Class / Lecture	Chapter 8, Competition
15-October	Class / Recitation Problem Set 5: out	Review Problem set 4 and Exam 1.

18-October	Class / Lecture	Chapter 8, Competition II
20-October	Class / Lecture	Chapter 8, Competition III
22-October	Class / Recitation Problem Set 5: due Problem Set 6: out	Review competition and Problem set 5.
25-October	Class / Lecture	Chapter 9, Welfare economics
27-October	Class / Lecture	Chapter 11, Monopoly I
29-October	Class / Recitation Problem Set 6: due Problem Set 7: out	Review examples of welfare economics and Problem set 6.
1- November	Class / Lecture	Chapter 11, Monopoly II
3- November	Class / Lecture	Chapter 12, Oligopoly
5- November	Class / Recitation Problem Set 7: due	Review Problem set 7 and prepare for exam.
8- November	Class / Lecture	Chapter 12, Oligopoly continued
9 - November	<u>Exam #2 (7:30 PM.)</u> Location TBD	
10 - November	Class / Lecture	Chapter 15, Factor markets
12 - November	Class / Recitation Problem Set 8: out	Chapter 12, Oligopoly continued
15 - November	Class / Lecture	International trade
17 - November	Class / Lecture	Equity vs. Efficiency
19 - November	Class / Recitation Problem Set 8: due Problem Set 9: out (DUE IN TWO WEEKS)	Review Problem set 8 and Exam 2.
22 - November	Class / Lecture	Equity vs. efficiency application: taxation & welfare
24 - November	Class / Lecture	Chapter 17, Uncertainty; Chapter 16, capital supply & capital markets I
26- November	No section – Thanksgiving break	
22 - November	Class / Lecture	Equity vs. efficiency application: taxation & welfare
24 - November	Class / Lecture	Chapter 17, Uncertainty
26- November	No section – Thanksgiving break	
29 - November	Class / Lecture	Chapter 16, capital supply & capital markets I

1 - December	Class / Lecture	Chapter 16, capital supply & capital markets II
3- December	Class / Recitation	Review Problem set 9 and review for final.
6- December	Class / Lecture	Psychology & economics
8- December	Class / Lecture	Review for final

①

14.01 Day 1

9/8

John Gruber!

← white house
health policy

gruberj@mit.edu

no office hrs

email to set up appointment

Come to sections Fri

- req

or all section format run by grad student
figure out by next Fri

First p-set posted Fri, next - Fri

need 18.01 - not req

9 p-sets - due in section
- no late P-set

drop lowest p-set

P-set 22% grade

2 Midterm 22%

Final 34%

Book just useful ref/backup

If you understand everything in class, no book needed
(he talks fast)

~~not~~ not everything written on board
Wants classroom participation

② What is micro econ?

- Scarcity *

- making decisions about that
 - constrained optimization
 - trade offs
 - like in engineering
- Consumers + producers
 - build models of how they behave
 - description of relationship
 - never precise
 - simplify to main tendencies
 - accurate vs. simple
 - limited wealth (budget constraint)
 - choose goods to make them well off
 - ~~but~~ maximize utility
 - firms maximize profits
 - subject to consumer demand + input costs

③ * fundamental questions

① What goods + services should be produced?

② How produce " ?

③ Who gets " ?

④ * all solved w/ prices

- interacting in a market

(3)

prices don't determine everything
- long lines to see Lady Gaga

(*) Theoretical vs. empirical economics

↑
building models
to explain
world

↑
test models
to see how good
model represents world

↑ 95% of
this class

(*) positive vs. normative economics

↑
way things
are

↑
way things
should be

← should ebay allow body part
auctions?

ebay = perfectly competitive market

(*) supply + demand

but is everything about money

- fairness?

- equity?

- several lectures at end

- behind much of what we discuss

(should do gov/health econ)
class

efficiency: how to get resources to right place

④ Every decision you make is made through this framework
- by 4th or 5th edition

"as if" principle - you could do the math + compute ~~any~~ ~~the~~ everything
- pro players just know
- behave as if they solved the math

get handouts from back of lecture

14.01 Principles of Microeconomics, Fall 2007

Chia-Hui Chen

September 5, 2007

match up fairly well

Lecture 1

Overview: Themes, Types of Markets, Economic Measurement, Economic Analysis

Microeconomics is a branch of economics that studies how individuals and firms make decisions to allocate limited resources, typically in markets where goods or services are being bought and sold.

Outline

1. Chap 1: *Optimization and Allocation*
2. Chap 1: *Definition and Various Type of Markets*
3. Chap 1: *Economic Measurement*
4. Chap 1: *Economic Analysis*

1 Optimization and Allocation

Consumer theory. Maximize preference (with limited income or time)

Producer theory. Maximize profit (with limited capital)

2 Definition and Various Type of Markets

Market. A place where buyers and sellers come together to exchange some product or good.

Product and Factor Markets

Market	Buyers	Sellers
Product Market	individuals	firms
Factor Market	firms	individuals

Table 1: Product and Factor Markets.

In a factor market, buyers are firms who need to hire workers and borrow money for capital expenditure, and sellers are individuals who provide labor and save money in banks.

Types of Markets Based on Influence on Price

Market Type	Products	Sellers	Buyers
Competitive	homogeneous	many	many
Monopolistic	heterogeneous	many	many
Oligopoly		a few	many
Monopoly		one	many
Monopsony		many	one
Oligopsony		many	a few

Table 2: Types of Markets Based on Influence on Price.

Table 2 shows different markets based on product differentiation and influence on price. Influence on price increases in moving from Competitive markets to Monopoly.

3 Economic Measurement

Flow and Stock Variables

Stock variables. Not measured with respect to time. e.g. price, wealth, inventories.

Flow variables. Measured per some unit of time. e.g. production, consumption, income.

Two additional flow variables:

Expenditure.

$$\text{EXPENDITURE} = \text{PRICE} \times \text{CONSUMPTION}.$$

Revenue.

$$\text{REVENUE} = \text{PRICE} \times \text{PRODUCTION}.$$

Prices

Nominal price. The absolute or current dollar price of a good or service when it is sold.

Real price. The price relative to an aggregate measure of prices or constant dollar price. It also measures prices relative to others. Price after adjustment for inflation.

CPI(Consumer Price Index). Total spending on a market basket of goods.

Formula of inflation rate:

$$(\text{Gross}) \text{ Inflation rate} = \frac{\text{CPI (current year)}}{\text{CPI (base year)}}.$$

Formula of real price:

$$\text{Real price} = \frac{\text{Nominal price (current year)}}{\text{Inflation rate (base year to current year)}},$$

or

$$\text{Real price} = \frac{\text{Nominal price (current year)}}{\text{CPI(current)/CPI(base)}}.$$

Example. For instance, the average tuition of college:

Year	Nominal Price	CPI	Real Price (base year 1970)
1970	2,530	38.8	2,530
1990	12,018	130.7	3,569
2002	18,273	181.0	3,917

Table 3: Average Tuition of College 1970 to 2002.

Notice that from 1970 to 2002 *nominal price* increases by 7 times but *real price* increases by 1.5 times.

4 Economic Analysis

Positive analysis. Study the relationship of cause and effect (Questions that deal with explanation and prediction).

Normative analysis. Analysis examining questions of what ought to be (Often supplemented by value judgments).

5 People showed up 9AM

- can go to any, decide by next week

P-Set due by noon

P-Set 1 is out

1. Competitive market

2. Demand

3. Supply

4. Market equilibrium

What determines the value of goods ??

1. Competitive Market

- each producer can not set price "price taker"

- each product is a substitute "non-differentiated"

- Symmetric info

- no transaction costs

ie agricultural, financial market

we know quantity producers will make at each price
" " " consumers " buy " " "

②

2. Demand

- quantity demanded

$$Q_0 = D(P)$$

↑ based on price

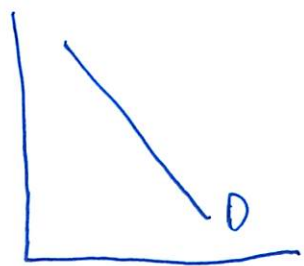
→

$$P = D^{-1}(Q_0)$$

↑ inverse welfare property

- downward sloping

$$\frac{\partial Q_0}{\partial P} < 0 = \text{Law of demand}$$



market for pork

$$Q_0 = 9 - 2P + 3 \cdot I + P^{\text{beef}}$$

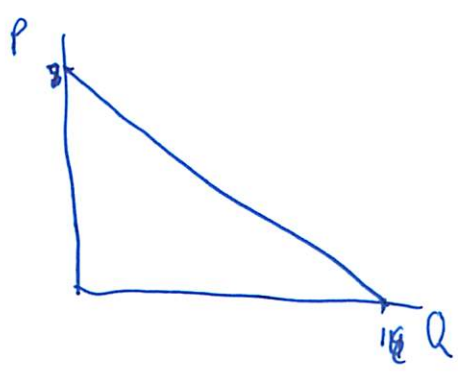
$I = \text{income} = 2$

$P^{\text{beef}} = \text{price of beef} = 1$

$$Q_0 = 16 - 2P$$

↑ inverse →

$$P = 8 - \frac{1}{2}Q$$



↳ no negative prices

↑ and demand should asymptote at 0

③

- change in demand curve shifts
- change in qtd. demanded
- movements along curve



income
general preference / fashion
price of other goods

- substitute \rightarrow $p_{\text{other}} \uparrow$ demand
complements $p_{\text{compl}} \uparrow = \downarrow$ demand

ie if income shifts

$$I = 4$$

$$p_{\text{Beet}} = 1$$

$$Q_D = 20 - 2P$$

$$P = 7 - \frac{1}{2}Q$$



poorly drawn is

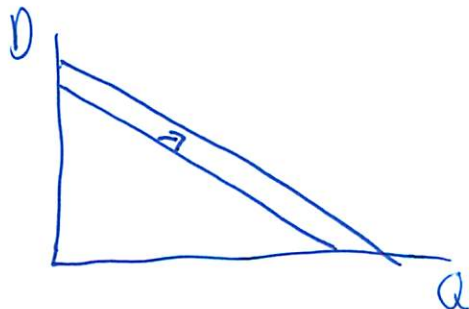
6 unit increase in demand

but people
really asked in
this section
- switch to
later

ie change in substitute price

$$I = 2 \quad p_{\text{Beet}} = 3$$

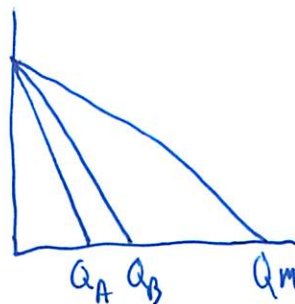
$$Q_D = \cancel{20 - 2P} \quad 18 - 2P$$



2 unit increase in demand

$$\left. \begin{aligned} Q(A) &= D_A(P) \\ Q(B) &= D_B(P) \end{aligned} \right\} \text{individual demand}$$

$$Q_M = Q_A + Q_B = \text{market demand}$$

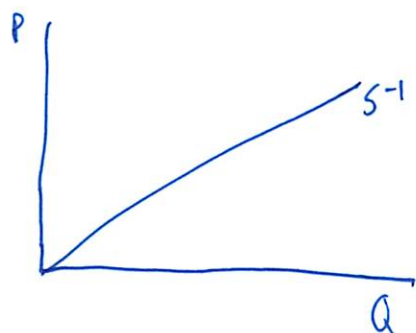


(4)

3. Supply

- similar to demand

- but look at $Q_s = S(P) \rightarrow P = S^{-1}(Q_s)$
 how much supply at price



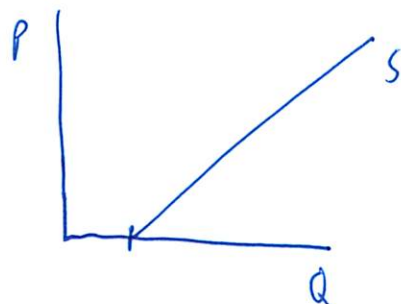
$$\frac{dQ_s}{dP} > 0 = \text{Law of Supply}$$

Market for Pork

$$Q_s = 10 + 4p - p^6$$

$$p^6 = 2 \text{ (grain)}$$

$$Q_s = 8 + 4P \rightarrow P = \frac{1}{4}Q - 2$$



Changes in qty. supplied

- move along curve

Changes in supply

- technology

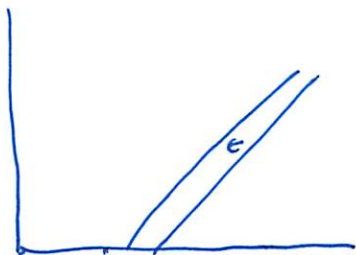
- prices of inputs

(5)

Price of input P
 $p_6 = 4$

$$Q_s = 6 + 4P$$

$$P = \frac{1}{4}Q - \frac{3}{2}$$

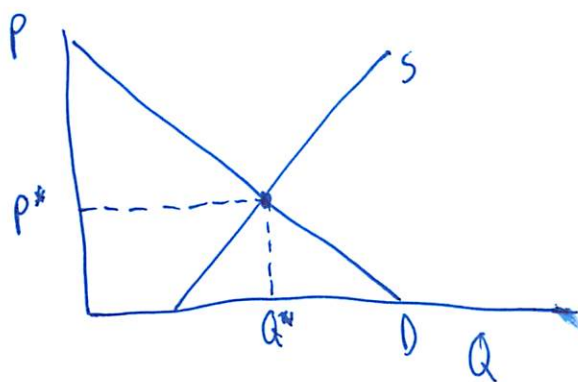


4. Market Equilibrium

- meeting of supply + demand

- equilibrium = everyone is satisfied, no change (P^*, Q^*)

$$\text{s.t. } Q^* = D(P^*) = S(P^*)$$



$$Q_D = 16 - 2P$$

$$Q_S = 8 + 4P$$

$$Q_D = Q_S$$

$$16 - 2P = 8 + 4P$$

$$8 = 6P$$

$$P = \frac{8}{6} = \frac{4}{3}$$

⑥

Market will want to return to \approx equilibrium

"market mechanism"

market clears

- no excess demand or supply

Elasticity of Demand

9/10

demand of 1 good based on consumer income, price of other goods
supply based on production cost

- how much will qtd. supplied + demanded change?

elasticity = sensitivity of 1 variable to another

- # that tells us % change in 1 variable based on a 1% change in other variable

Demand

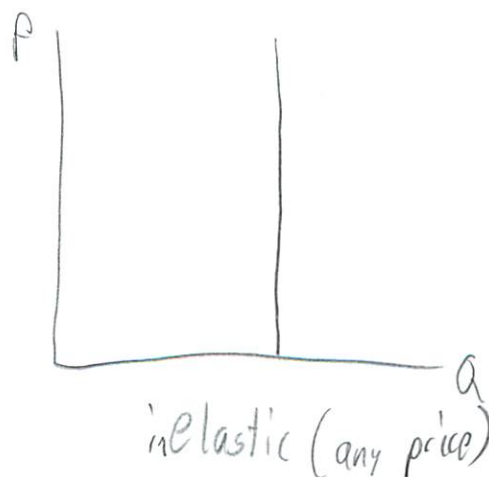
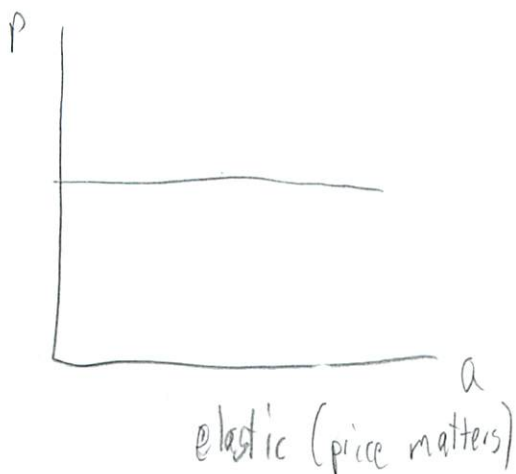
$$E_p = (\% \Delta Q) / (\% \Delta P) = \frac{\text{percent change quantity}}{\text{" " " price}} = \frac{\Delta Q / Q}{\Delta P / P} = \frac{P}{Q} \frac{\Delta Q}{\Delta P}$$

price elasticity of demand

- usually negative (as $P \uparrow, Q \downarrow$)
- so sometimes refers to magnitude

> 1 = price elastic

< 1 = price inelastic



(2)

Linear Demand Curve

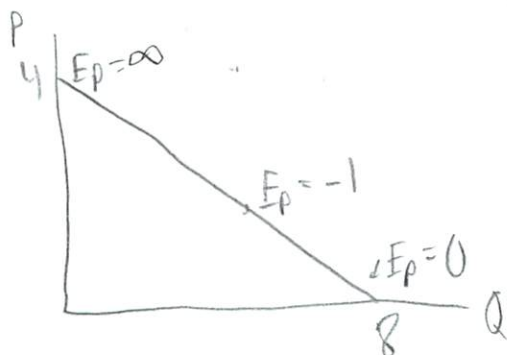
- measured at a certain point on the ^{supply demand} curve
- changes as you move along curve

$$Q = a - bP$$

↓ example

$$Q = 8 - 2P \quad \leftarrow \frac{\Delta Q}{\Delta P} \text{ is constant} = -2$$

but not constant elasticity



The steeper the curve, the less elastic the demand

Other Variables

- besides price
- demand ↑ as income ↑

$$E_I = \frac{\Delta Q/Q}{\Delta I/I} = \frac{I}{Q} \frac{\Delta Q}{\Delta I}$$

- substitutes + complements

- or substitutes (cross-price)

$$F_{Q_B P_M} = \frac{\Delta Q_B / Q_B}{\Delta P_M / P_M} = \frac{P_M}{Q_B} \frac{\Delta Q_B}{\Delta P_M}$$

③ Supply

- same except w/ supply
- Usually +
 - higher price = more incentive to produce
- interest, wages, raw materials

Point vs Arc Elasticities

Point elasticity - at a particular point

but sometimes want to calculate elasticity over portion of demand/supply curve

- do you use old or new price + quantity?

- So use average \bar{P}, \bar{Q}

$$E_p = \frac{\Delta Q}{\Delta P} \frac{\bar{P}}{\bar{Q}}$$

- lies somewhere in the middle

2.6 Effects of Changing Market Conditions

9/10

P^*, Q^* = equilibrium

find supply + demand curves that fit the #s

$$Q_D = a - bP$$

$$Q_S = c + dP$$

- we need to find the constants

$$E = \frac{P}{Q} \left(\frac{\Delta Q}{\Delta P} \right)$$

↑ constant on linear curves

$$\frac{\Delta Q}{\Delta P} = d$$

demand

$$\frac{\Delta Q}{\Delta P} = -b$$

supply

$$E_D = -b \left(\frac{P^*}{Q^*} \right) \quad E_S = d \left(\frac{P^*}{Q^*} \right)$$

substitute the # in + solve

$$a = Q^* + bP^*$$

①

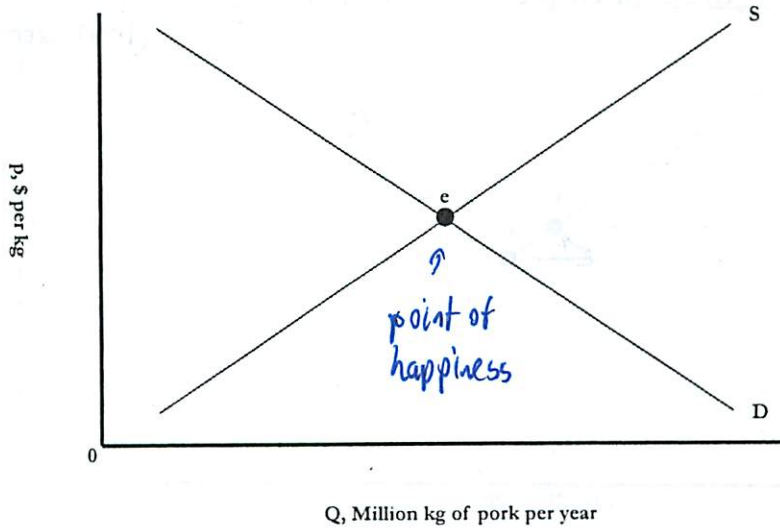
14.01 Lecture 2

9/13/2010

Supply + Demand interact to yield market price + quantity

Today: shocks / changes

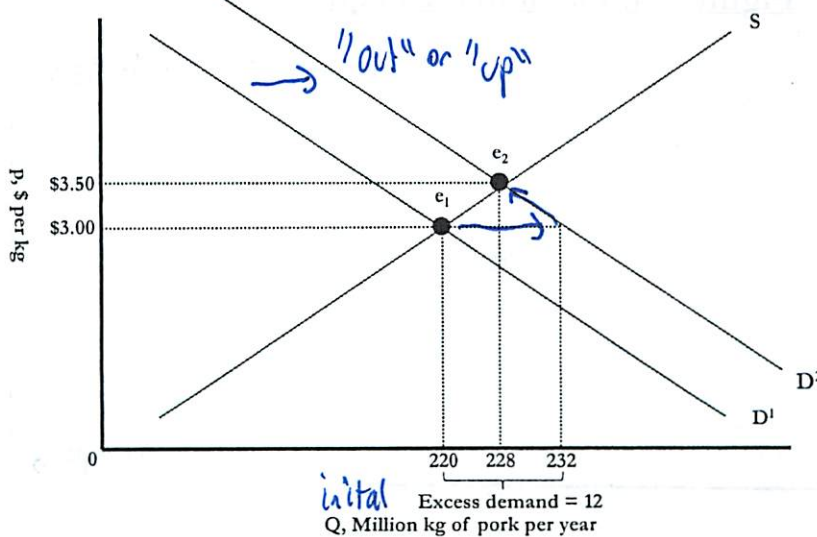
Figure 2-1: Pork market equilibrium



price people
willing to pay

intuitively
graphically
mathematically

Figure 2-2: Impact of a demand shift

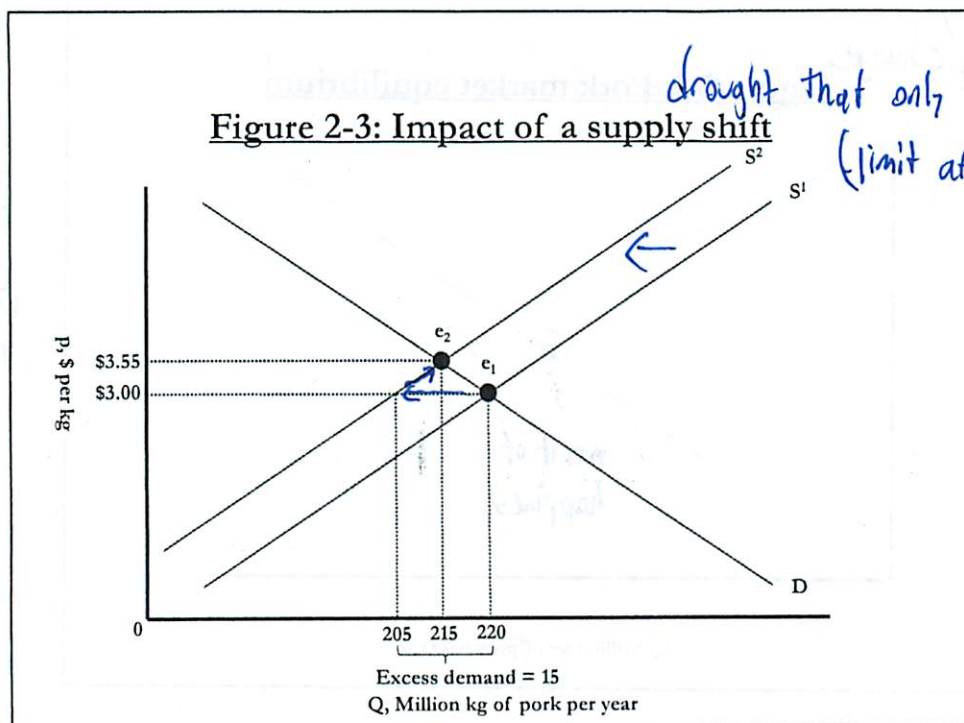


price of beef rises
substitute

We are considering 1 level

①

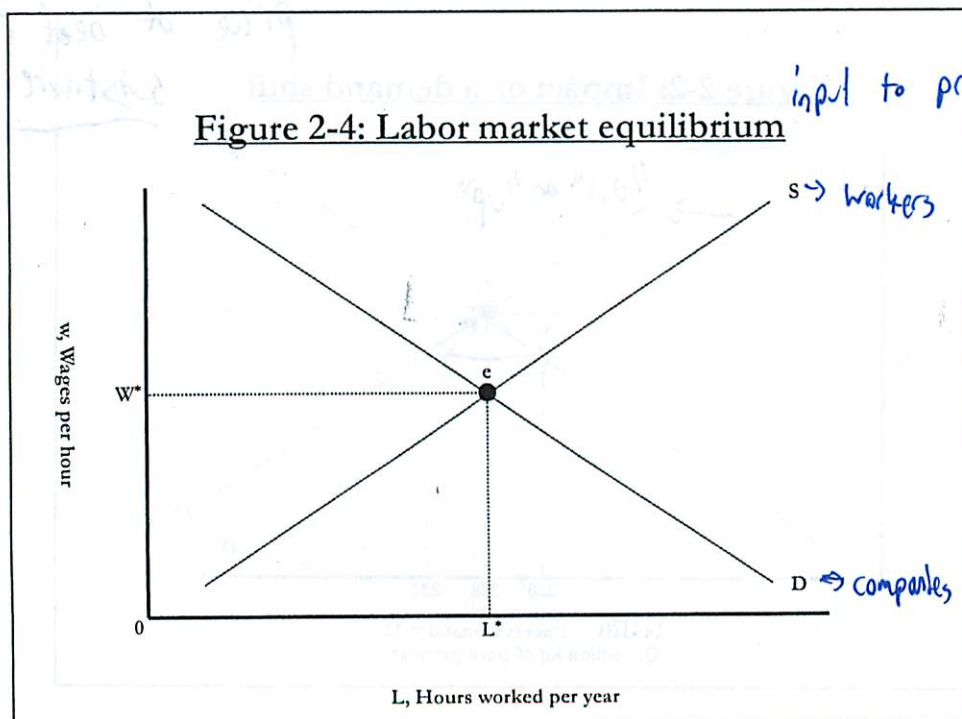
9/13/2010



drought that only affects pork
(limit affects to simplify)

in both cases, price increase

in this classic
simplified analysis:
→ gov always messes
things up

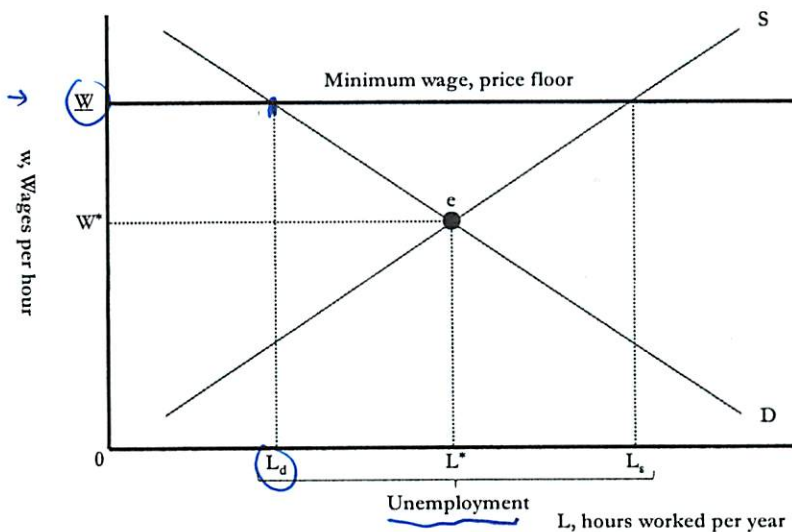


input to production

3

9/13/2010

Figure 2-5: Labor market with minimum wage

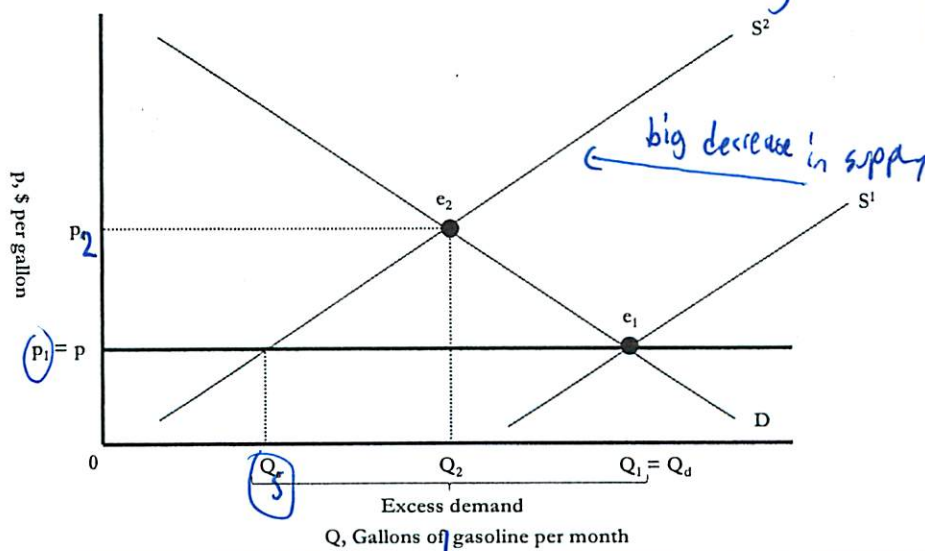


w lower bar

dis equilibrium

P + Q determined by constrained party → the firm

Figure 2-6: Gasoline price cap/ceiling



much less gas
being sold
than before
and before rig explosion

④

Cost + Benefits of Government Intervention

- to societies welfare (well being)
- cost: efficiency lost
 - if both parties willing to trade and they don't then that is an efficiency loss
- cost: allocation inefficiencies
 - prices: what, how produced, who gets it
 - the people who want the good get it
 - the people would be willing to work at x wage
 - not productive
- mechanical efficiency of idling waiting for gas
- must allocate gas somehow
 - price is most efficient mechanism
- Equity - thing economists don't like to think about makes things a lot more messy

Water Shortage

- in CA
- no watering lawns, x gallons per day
- not efficient
- tiered pricing

0-70 gallons	.10 / gallon
71-110 "	.15
111 +	.20

(5)

Or give each person 1000 coupons
and they can trade them freely

~~can set~~ In drought year issue 600 coupons
Markets set the price

14.01 Principles of Microeconomics, Fall 2007

Chia-Hui Chen

September 7, 2007

Lecture 2

The Basics of Supply and Demand

$$\text{MARKET} \left\{ \begin{array}{l} \text{BUYERS} \Rightarrow \text{DEMAND} \\ \text{SELLERS} \Rightarrow \text{SUPPLY} \end{array} \right\} \text{EQUILIBRIUM}$$

Outline

1. Chap 2: *Demand and Supply Curves*
2. Chap 2: *Equilibrium in the Market*
3. Chap 2: *Government Interventions*

1 Demand and Supply Curves

Quantity Demanded and Quantity Supplied

 Q_D (Quantity demanded). Depends on price.

$$Q_D = D(P). \quad (1.1)$$

 Q_S (Quantity supplied). Depends on price.

$$Q_S = S(P). \quad (1.2)$$

- Notes:**
1. Market demand/supply is the sum of individual demands/supplies.
 2. Assume individuals are price takers who cannot affect price.

Demand and Supply Curves

From Equations (1.1) and (1.2), draw demand curves and supply curves as follows:

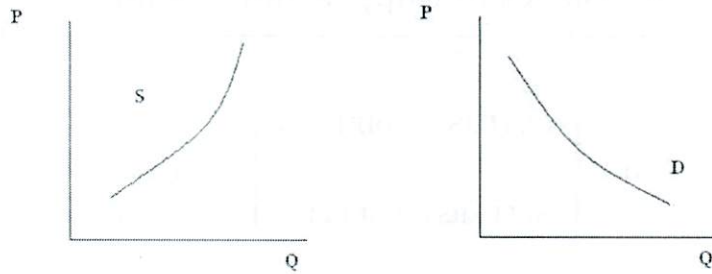


Figure 1: Supply curve. Price higher, quantity supplied more. Figure 2: Demand curve. Price higher, quantity demanded less.

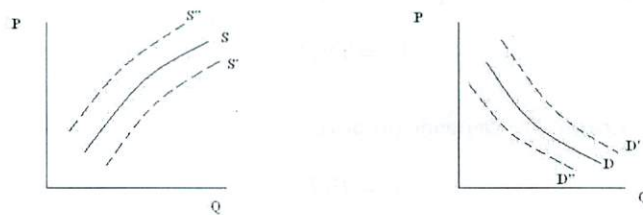


Figure 3: Shift in supply curve.

Figure 4: Shift in demand curve.

Supply curve

See Figure 1 and Figure 3:

1. Change in price causes change in quantity supplied, on the graph, there is movement along the curve accordingly.
2. Change in something other than price causes change in supply, on the graph, the supply curve shifts.

Example. Production cost falls \rightarrow supply curve S shifts to S' (See Figure 3).

Demand curve

See Figure 2 and Figure 4:

1. Change in price causes change in quantity demanded, on the graph, there is movement along the curve accordingly.
2. Change in something other than price causes change in demand, on the graph, the demand curve shifts.

Example. People's income increases \rightarrow demand curve D shifts to D' (Figure 4).

Substitutes and Complements

Substitutes. Increase in the price leads to an increase in the demand of the other.

Example (Italian and French bread). Price of Italian bread increases, demand of French bread increases.

Complements. Increase in the price leads to a decrease in the demand of the other.

Example (Pasta and pasta sauce). Price of pasta increases, demand of pasta sauce decreases.

2 Equilibrium in the Market

Equilibrium state:

- No shortage
- No surplus
- Equilibrium price clears the market.

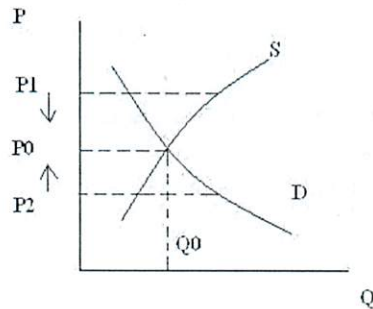


Figure 5: Demand and Supply curves. Equilibrium state.

Refer to Figure 5. (P_0, Q_0) is the equilibrium state, which is the intersection point of the demand and supply curves.

	Supply		Price
Change in	\Rightarrow	Change in equilibrium	
Demand		Quantity	

Surplus and Shortage

Surplus. Price P_1 is higher than P_0 and will fall down.

Shortage. Price P_2 is lower than P_0 and will raise up.

Comparative Static Analysis and Comparative Dynamics

Comparative static analysis. Compares the new and old equilibrium and not the actual path through time of the change.

Comparative dynamic analysis. Traces out the path over time.

This course will cover primarily Comparative Static analysis.

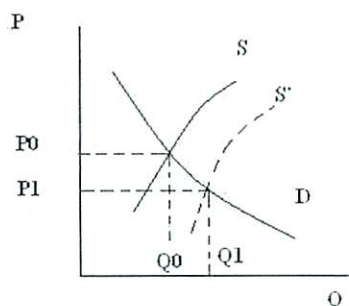


Figure 6: Decrease in raw material prices.

Examples

Example (Decrease in raw material prices). Raw material prices $\searrow \rightarrow$ Supply $\nearrow \rightarrow$ Price \searrow and Quantity \nearrow (Figure 6).

Example (Increase in income). Income $\nearrow \rightarrow$ Demand $\nearrow \rightarrow$ Price \nearrow and Quantity \nearrow (Figure 7).

Dual shifts in supply and demand

When supply and demand change simultaneously, the impact on the equilibrium price and quantity is determined by the size and direction of the changes and the slope of two curves.

3 Government Interventions

How can government help sellers? Discuss two methods.

Problem Description

Assume that

$$Q_D = 10 - P,$$

$$Q_S = -2 + P.$$

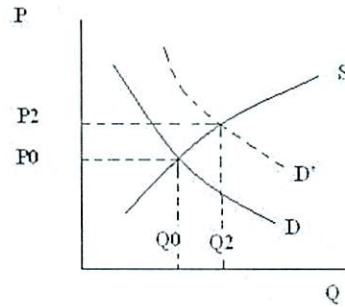


Figure 7: Increase in income.

The original equilibrium point is

$$P_0 = 6,$$

$$Q_{D0} = Q_{S0} = 4,$$

and the revenue before government intervention is:

$$REVENUE = P_0 \times Q_{D0} = 6 \times 4 = 24.$$

The government's goal: increase sellers' revenue.

Price Floor

The first method: set a price floor. Assume the lowest price is set to be 8, thus:

$$Q_D = 2,$$

$$Q_S = 6.$$

The revenue after using method 1 is:

$$REVENUE = P \times Q_D = 8 \times 2 = 16 < 24.$$

Subsidy

The second method: provide subsidy.

Customers get a 2 unit price refund per unit quantity bought, thus the quantity demanded changes:

$$Q_D = 10 - (P - 2) = 12 - P.$$

The new intersection point is

$$P = 7,$$

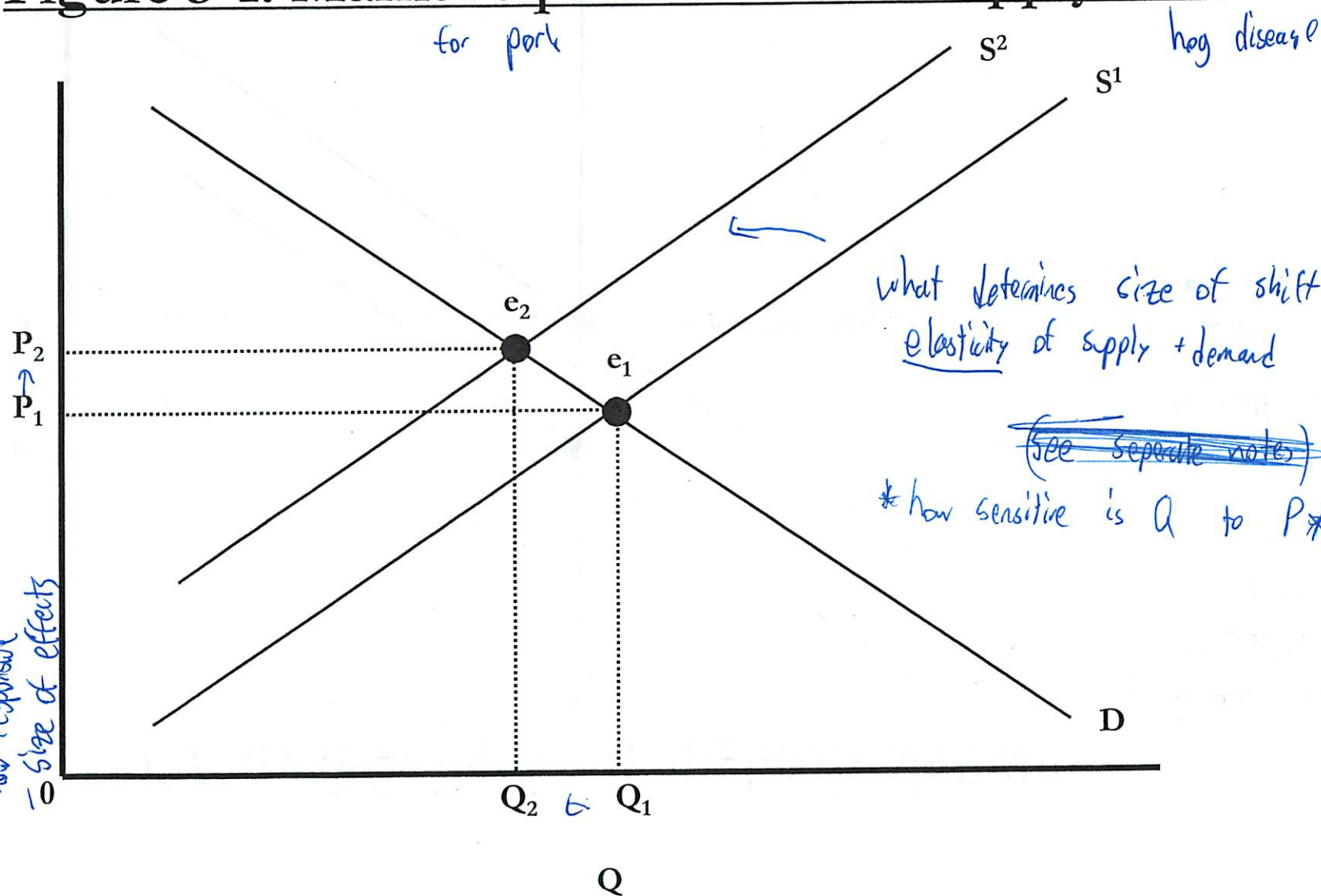
$$Q_D = Q_S = 5.$$

The revenue after using method 2 is:

$$\text{REVENUE} = P \times Q_D = 7 \times 5 = 35 > 24.$$

For this example, providing subsidies achieves the government's goal to increase seller's revenue, but setting price floor does not and even makes the revenue less.

Figure 3-1: Market equilibrium with supply shift



Lecture 3
More Supply + Demand

- shapes of curves
- how responsive
- size of effects

9/15

Figure 3-2: Perfectly inelastic demand

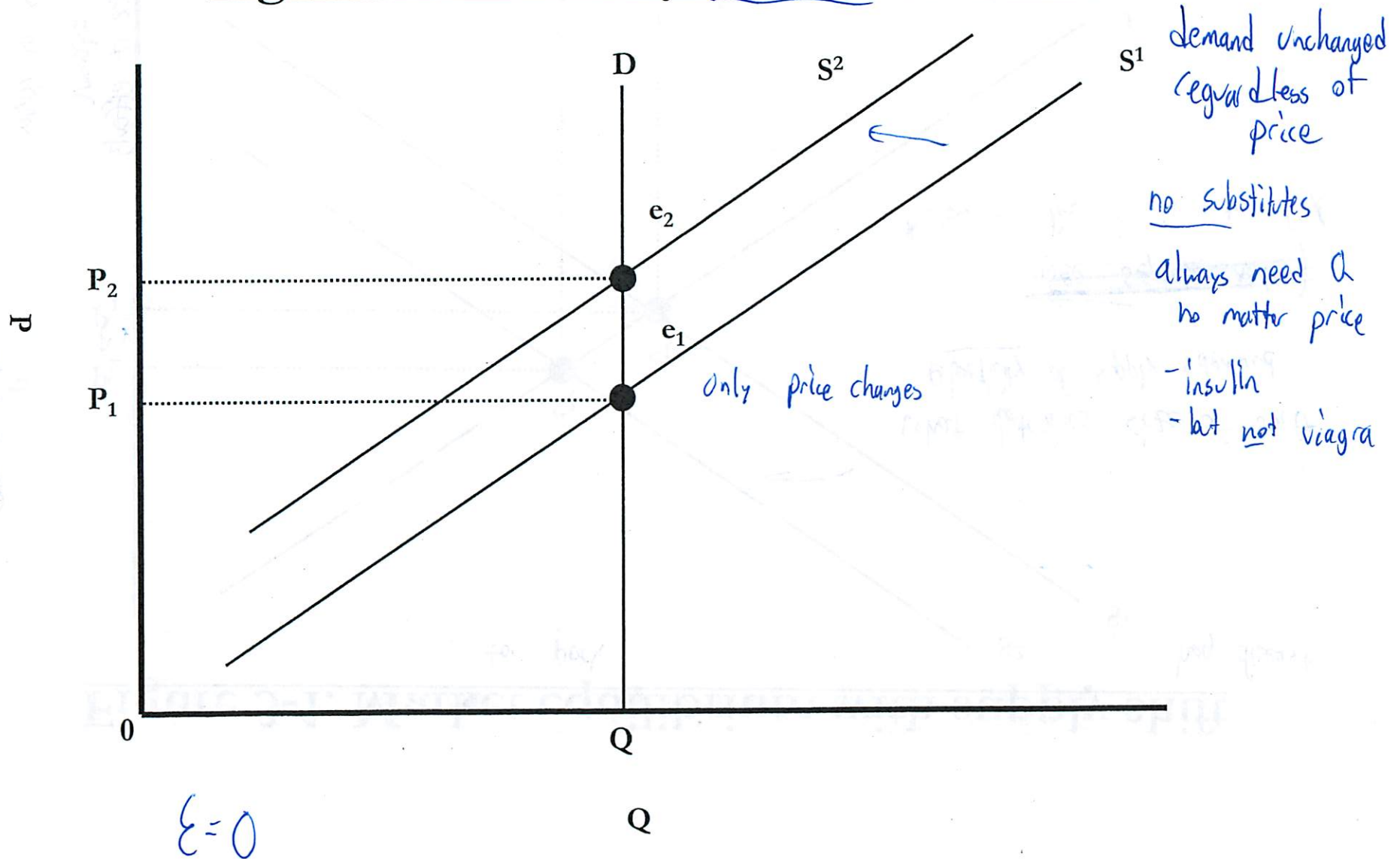
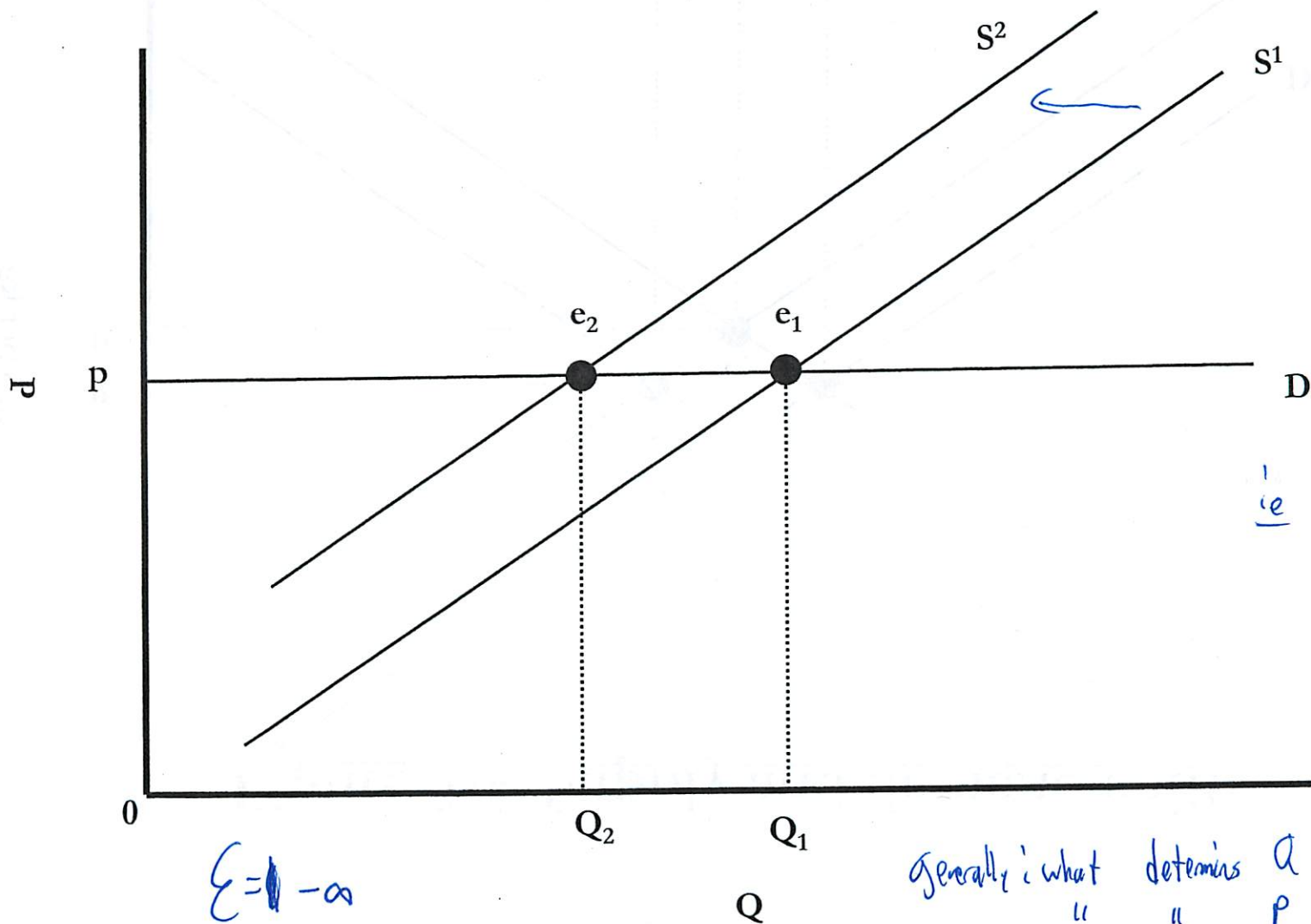


Figure 3-3: Perfectly elastic demand



perfectly ∞
~~lots~~ of substitutes

Consumers just care
 about price

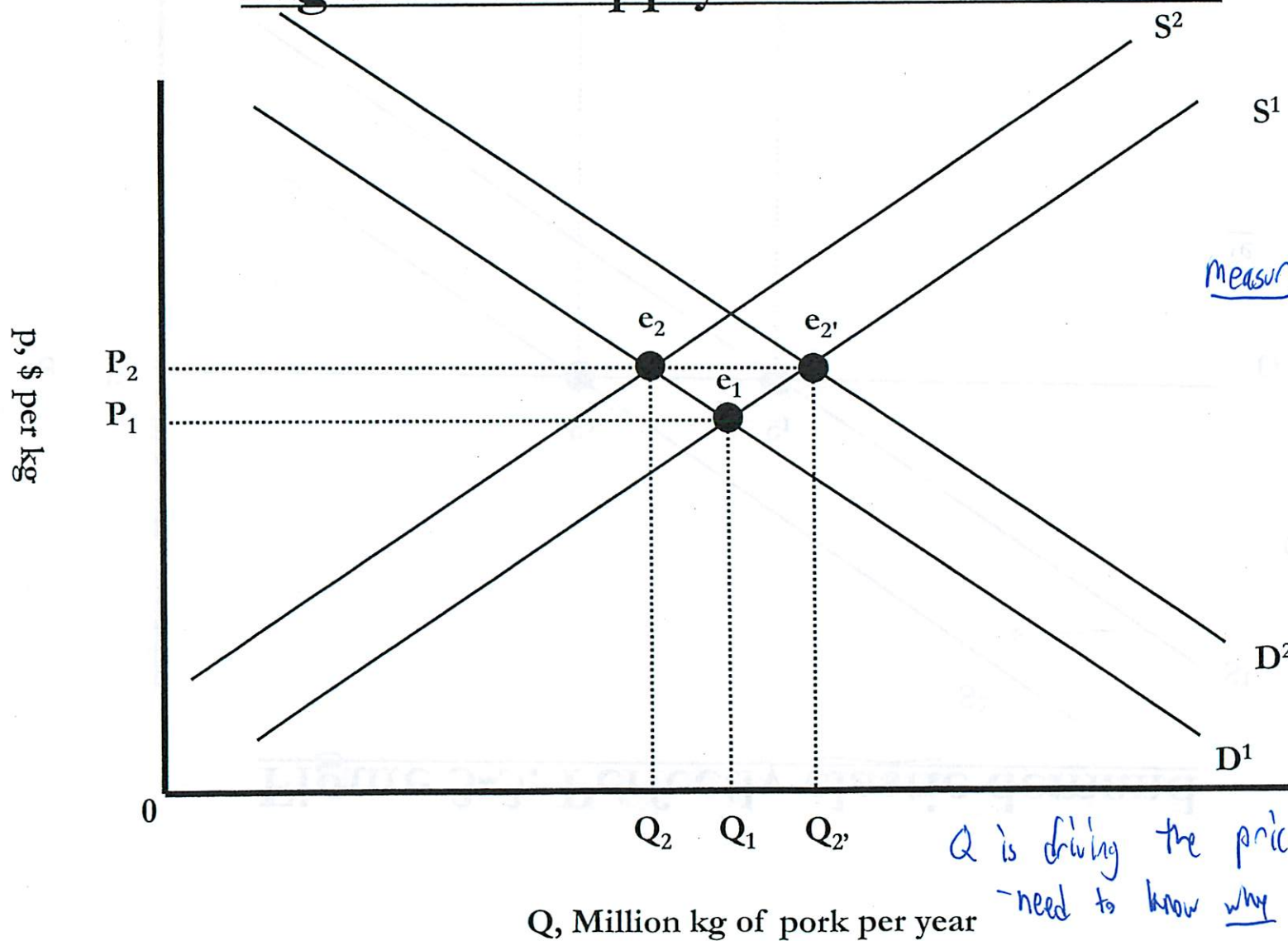
if prices change at
 all, immediately change

ie types of candy

$$\epsilon = -\infty$$

generally, what determines $\Delta Q/Q$ change = elasticity
 " " " " $\epsilon = \frac{\Delta Q/Q}{\Delta P/P}$
 price elasticity of demand

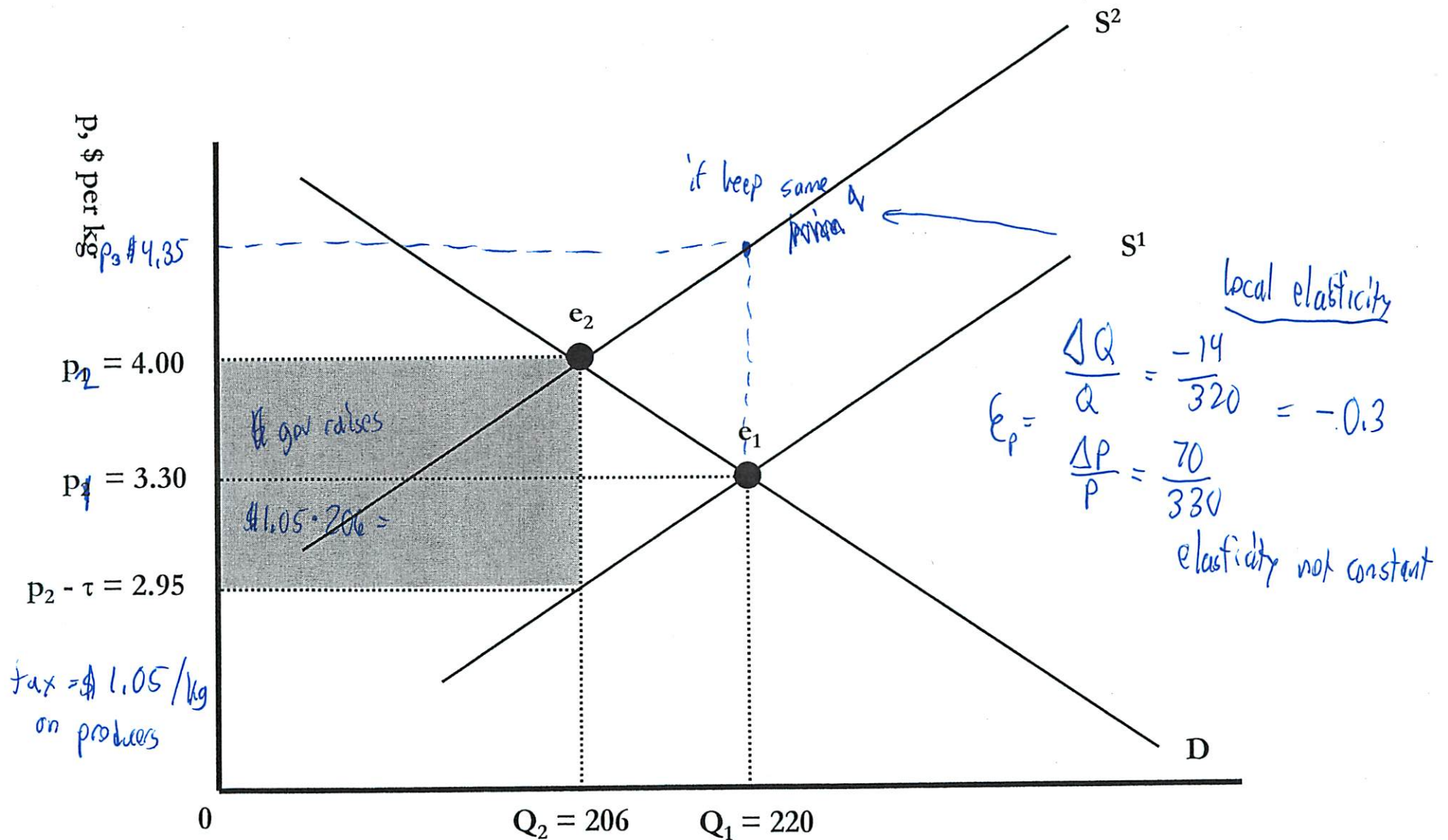
Figure 3-4: Supply and demand shifts



measure slope of demand curve
-not shift of curve

Q is driving the price
-need to know why price increased

Figure 3-5: Impact of a producer tax



\$\text{gov raises directly depends on elasticity}\$

- elastic goods \rightarrow raises little $\$$
- inelastic \rightarrow raises more $\$$

Q, Million kg of pork per year

hard for gov to figure out how much $\$$ gov raise

⑥

Producers'

Revenue = price \times quantity

$$\frac{\Delta R}{\Delta p} = Q + p \frac{\Delta q}{\Delta p} = Q(1 + \epsilon)$$

\uparrow if producer deciding if \uparrow price will \uparrow revenue

$0 \leq \epsilon \leq -1$ = revenue \uparrow

$\epsilon \geq -1$ = revenue \downarrow

Cost of production matters too

Where get it \rightarrow Empirical economics

- estimating these elasticities

\rightarrow pro issue causation vs correlation

- Very common mistake

- fundamental conundrum

ϵ_s = same but of suppliers

ϵ_d = of demand

$\epsilon_s \rightarrow$ changed by gov policy

ie tax on pork (Fig 8-5)

⑦

Another issue in real world

Consumers face choice which price they pay health care

- Co pay - you pay compay at doctor's visit
- rapid rise in comp payments b/c \uparrow h/c cost
- today 17% of GDP
- 2075 40% of GDP

if consumers pay more \Rightarrow understand the consequence

- will it cause them to change how much care they get?
- must look at data

People w/ \uparrow co payments + deductibles use a lot less care

- but those are healthy people
- causation vs correlations
- ^{doctors} run a randomized trial
- hard to do this in social science
- but did Rand Health Insurance

Medical care demand is elastic - .2

- not that much less

- if people had to pay all demand \downarrow 45%

is it a good or bad thing?
look at their health?

- not at all

We waste a huge amt of h/c in US

- could cut back a lot

how much pay?

efficiency vs equity

14.01 Principles of Microeconomics, Fall 2007

Chia-Hui Chen

September 10, 2007

Lecture 3

Elasticities of Demand

Elasticity. Elasticity measures how one variable responds to a change in another variable, namely the percentage change in one variable resulting a one percentage change in another variable. (The percentage change is independent of units.)

Outline

1. Chap 2: *Price Elasticity of Demand*
2. Chap 2: *Income Elasticity of Demand*
3. Chap 2: *Cross Price Elasticity of Demand*
4. Chap 2: *Comparison of Elasticity Over Short Run and Long Run*

1 Price Elasticity of Demand

Price elasticity of demand. Price elasticity of demand measures the percentage change in quantity demanded resulting from one percentage change in price.

$$E_E^D = \frac{\% \Delta Q_P}{\% \Delta P} = \frac{\frac{\Delta Q}{Q}}{\frac{\Delta P}{P}}.$$

Example Calculation

Figure 1 shows a demand curve:

$$Q(P) = 8 - 2P.$$

When the price changes from 2 to 1, the price elasticity of demand is:

$$E_P^D|_{p=2 \rightarrow 1} = \frac{\frac{\Delta Q}{Q}}{\frac{\Delta P}{P}} = \frac{\frac{2}{4}}{\frac{-1}{2}} = -1.$$

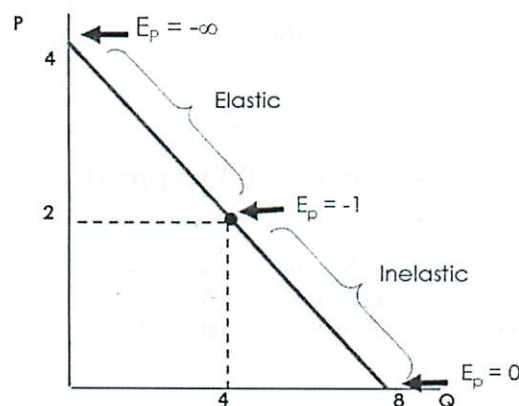


Figure 1: Price Elasticity of Demand.

If the direction of change is opposite, from 1 to 2, then the price elasticity of demand is:

$$E_P^D|_{P=1 \rightarrow 2} = \frac{\frac{\Delta Q}{Q}}{\frac{\Delta P}{P}} = \frac{\frac{-2}{6}}{\frac{1}{1}} = -\frac{1}{3}.$$

The two quantities are different. To solve this conflict, consider small changes in P and Q , and define:

$$E_P^D = \frac{\frac{dQ}{Q}}{\frac{dP}{P}} = \frac{P}{Q} \frac{dQ}{dP}.$$

Thus, at the point $P = 2$, the price elasticity of demand is:

$$E_P^D|_{P=2} = \frac{P}{Q} \frac{dQ}{dP} = \frac{2}{4} \times (-2) = -1.$$

Properties of Price Elasticity of Demand

1. Price elasticity of demand is usually a negative number.
2. $|E_P| > 1$ indicates that the good is price elastic, perhaps because the good has many substitutes; $|E_P| < 1$ indicates that the good is price inelastic, perhaps because the good has few substitutes.
3. Given a linear demand curve, E_P is not a constant along the curve. For example, for curve in Figure 1, $E_P = -\infty$ at top portion, but zero at bottom portion.
4. Discuss two extreme situations: $|E_P| = 0$, quantity independent of price Figure 2 and $|E_P| = \infty$, quantity very sensitive to price. See Figure 3.

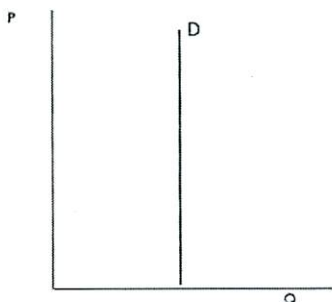


Figure 2: Extreme demand elasticity. $|E_P| = 0$, quantity independent of price.

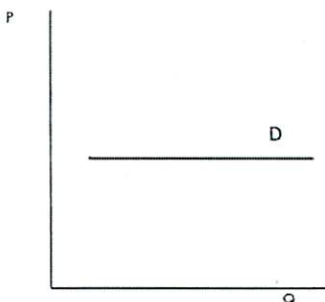


Figure 3: Extreme demand elasticity. $|E_P| = -\infty$, quantity very sensitive to price.

5. The constant elasticity demand function is

$$Q = aP^b,$$

since

$$E_P = \frac{dQ}{dP} \frac{P}{Q} = abP^{b-1} \frac{P}{Q} = b \frac{aP^b}{Q} = b.$$

Refer to Figure 4.

6. How do total consumer expenditure change when the price of a good changes?

$$\frac{dExp}{dP} = \frac{d(PQ_D(P))}{dP} = Q + P \frac{dQ}{dP} = Q(1 + E_P) = Q(1 - |E_P|).$$

- If $|E_P| > 1$, total expenditure decreases when price increases;
- If $|E_P| < 1$, total expenditure increases when price increases.

Example (Cell phone). People need to do business in the morning, so E_P is low, so cell phone companies increase the rate while customers will expend more; but E_P is high in the evening since people do not have to talk, so cell phone companies lower the rate to encourage customer expenditure.

2 Income Elasticity of Demand

Income elasticity of demand. Income elasticity of demand measures the percentage change in quantity demanded resulting from one percentage change in income. Similarly,

$$E_I = \frac{\frac{dQ}{Q}}{\frac{dI}{I}} = \frac{I}{Q} \frac{dQ}{dI}.$$

The income elasticity of demand is usually positive.

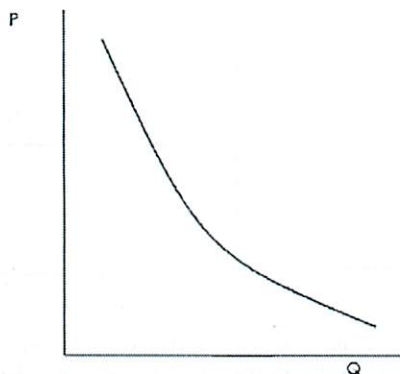


Figure 4: Constant Demand Elasticity.

3 Cross Price Elasticity of Demand

Cross price elasticity of demand. Cross price elasticity of demand measures the percentage change in quantity demanded of a good (x) resulting from one percentage change in price of another good (y).

$$E_{Q_x P_y} = \frac{\frac{dQ_x}{Q_x}}{\frac{dP_y}{P_y}} = \frac{P_y}{Q_x} \frac{dQ_x}{dP_y}.$$

- If y is a substitute of x , the cross price elasticity of demand is positive.
- If y is a complement of x , the cross price elasticity of demand is negative.

4 Comparison Between Elasticity Over Short Run and Long Run

Is demand more elastic in the long run or short run?

Consumption goods. For consumption goods, the demand is more elastic in the long run. Because people need goods for daily life and buy them constantly, the short run demand is inelastic. Faced with high prices in the long run, they may change habits or find more substitutes.

4 Comparison Between Elasticity Over Short Run and Long Run 5

Durable goods. For durable goods, the demand is more elastic in the short run. Consider cars. If price of cars increase, in the short run people might use their current cars longer. In the long run, though, people have to replace their cars.

Michael Plasmeier

theplaz@mit.edu

F 10 AM

14.01 Fall 2010

Problem Set 1

Due in class on September 17th

91

9/18
Due 9/17

Red Review post grade

1. (25 points) For each of the following scenarios, use a supply and demand diagram to illustrate the effect of the given shock on the equilibrium price and quantity in the specified competitive market. Explain whether there is a shift in the demand curve, the supply curve or neither.

(a) (5 points) An unexpected temporary heat wave hits the East Coast. Show the effect in the ice-cream market in New England.

(b) (5 points) The government introduces a tax on ice-cream which is paid by producers. What is the effect in the ice-cream market.

(c) (5 points) China and Mexico are major producers of textiles. Workers in Mexico decide to go on strike. Show the effect on the market for Mexican textiles.

(d) (5 points) Show the effect of the situation described in (c) on the market for Chinese textiles.

(e) (5 points) Suppose the government imposes a price cap on bottled water. Show the effect in the bottled water market.

2. (20 points) For each of the following pairs of goods, identify which one you would expect to have more own-price elastic demand. Please explain your reasoning.

(a) (5 points) Computers (generally) vs. Apple MacBook Pro laptops.

(b) (5 points) Stereo headphones (generally) vs. hearing aids.

For each of the following goods, identify whether you would expect demand to be more (own-price) elastic in the short run or the long run. As above, please briefly explain your reasoning.

(c) (5 points) Retail gasoline in the suburbs of Chicago.

(d) (5 points) Air conditioning units in Miami Beach, Florida.

3. (30 points) Consider the market for apple-juice. In this market the supply curve is given by $Q_S = 10P_J - 5P_A$ and the demand curve is given by $Q_D = 100 - 15P_J + 10P_T$, where J denotes apple juice, A denotes apples and T denotes tea.

(a) (7 points) Assume that P_A is fixed at \$1 and $P_T = 5$. Calculate the equilibrium price and quantity in the apple juice market.

(b) (7 points) Suppose that a poor harvest season raises the price of apples to $P_A = 2$. Find the new equilibrium price and quantity of apple juice. Draw a graph to illustrate your answer.

(c) (8 points) Suppose $P_A = 1$ but the price of tea drops to $P_T = 3$. Find the new equilibrium price and quantity of apple juice.

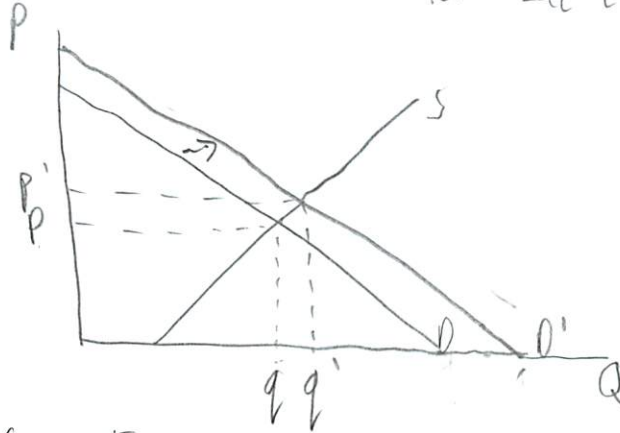
(d) (8 points) Suppose $P_A = 1$, $P_T = 5$, and there is a price ceiling on apple juice of $P_J^* = 5$. What is the excess demand for apple juice as a result? Draw a graph to illustrate your answer.

4. (25 points) You have been asked to analyze the market for steel. From public sources, you are able to find that last year's price for steel was \$20 per ton. At this price, 100 million tons were sold on the world market. From trade association data you are able to obtain estimates for the own price elasticities of demand and supply on the world markets as -0.25 for demand and 0.5 for supply. **Assume that steel has linear demand and supply curves throughout.**

a) (10 points) Solve for the equations of demand and supply in this market and sketch the demand and supply curves.

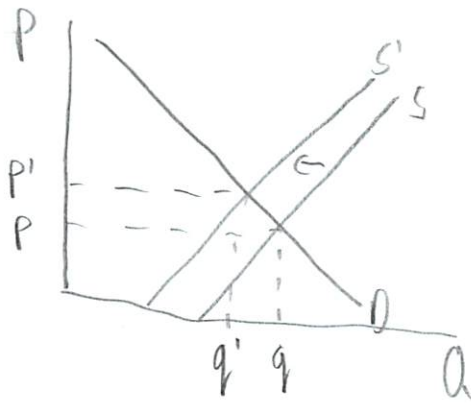
b) (15 points) Suppose that you discover that the current price of steel is \$15 per ton and the current level of worldwide sales of steel is 150 million tons. The most recent elasticity estimates from the trade association this year are -0.125 for demand and 0.25 for supply. Describe the change in the supply and demand curves over the past year using your diagram from part a). What sort of event(s) might explain the change?

1. a. Heat wave on Market for Ice Cream



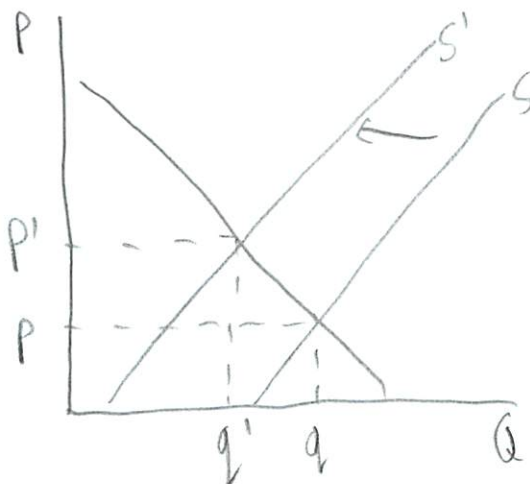
Increase in demand
- curve shifts

b. New Tax on Producers



Increase in cost of supplies
Decrease in supply

c. Workers go on strike

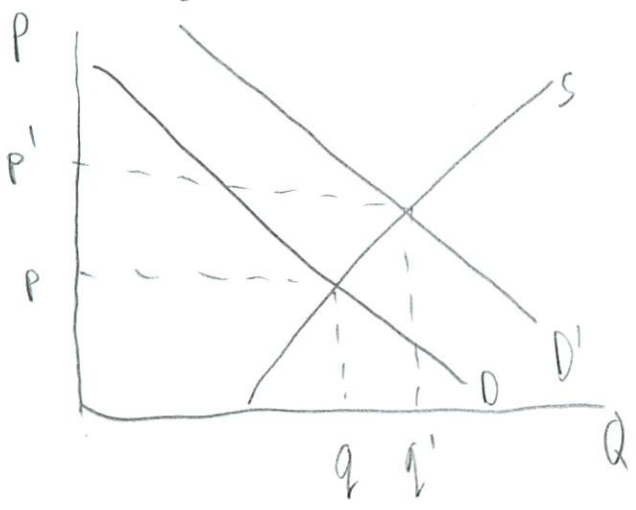


Huge \uparrow in cost of supplies
(Assuming producers can hire
high-cost scabs)

②

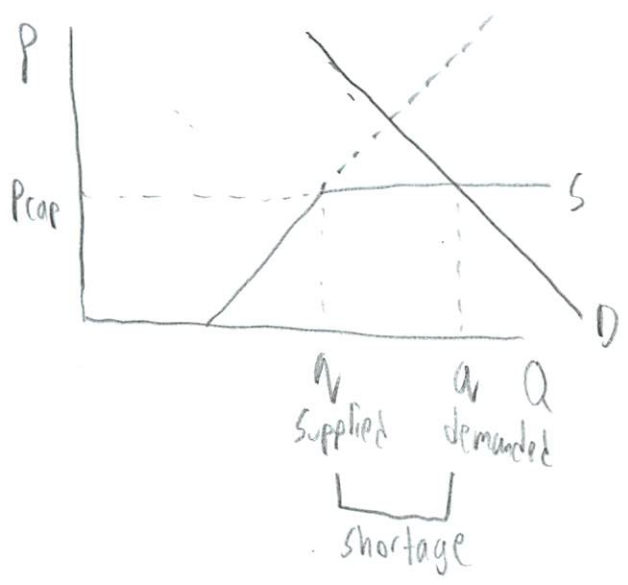
do fr, insist
on graphs formatted
a special way

d. Workers go on strike in other countries



Increase in demand for this
countries' output

e. Government imposes price cap



What about if
cap $>$ equil price?
-2

Oh the boring case

- no one cares "no effect"
- duh
- that is why I did not mention it

3)

2. For each of the following pair of goods, identify which one would have more own-price elastic demand

a) Computers vs MacBook Pro

- Computers very competitive market, substitutes
 - Apple = luxury, status symbols, no substitutes
 - more elastic \rightarrow small change in P = large Δ in Q
- but no subs for computers in general

Computers would be more elastic *they said MacBook*

b) Headphones vs Hearing Aids

Headphones = competitive, lots of substitutes

Hearing aids = paid by insurance, more variability

Headphones more elastic

c) *Retail* gas in Chicago

- as the book explains this is long term. In the short term people still need to get to work etc, but in long run can buy smaller cars, and in super long run build transit, electric cars

d) AC units in FL

- short term - *Can sub for fans in short term* the poor souls who do not yet have an AC (or central air) rush to buy them whatever the cost \rightarrow

When it gets hot. They are generally cheap ($\approx \$100$) that people will not shop around. On the other hand they are durable and need to be replaced. But many are not because they do not know operating cost (rent) or it is included in rent.

④

3. Market for apple juice

$$Q_S = 10P_J - 5P_A$$

$$Q_D = 100 - 15P_J + 10P_T$$

J = apple juice

A = apple

T = tea

a) $P_A = \$1$

$P_T = \$5$

$$10P_J - 5P_A = 100 - 15P_J + 10P_T$$

$$10P_J - 5 \cdot 1 = 100 - 15P_J + 10 \cdot 5$$

$$10P_J - 5 = 150 - 15P_J$$

$$25P_J = 155$$

$$P_J = \$6.20$$

$$Q_E = 10 \cdot 6.20 - 5$$

$$Q_E = 57$$

b) $P_A = 2$ new

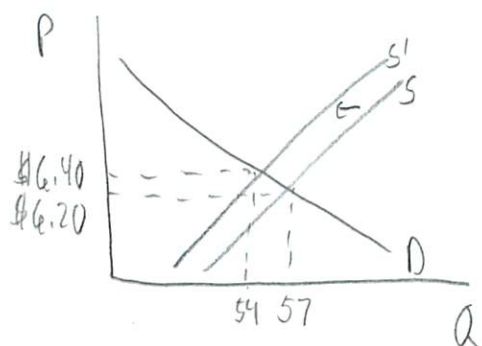
$$10P_J - 10 = 100 - 15P_J + 50$$

$$25P_J = 160$$

$$P_J = \$6.40$$

$$Q_E = 10 \cdot 6.40 - 10$$

$$Q_E = 54$$



5

c. Suppose $P_A = 1$

$$P_T = 3$$

$$10P_J - 5 = 100 - 15P_J + 30$$

$$25P_J = 135$$

$$P_J = \$5.40$$

$$Q_E = 10 \cdot 5.40 - 5$$

$$Q_E = 49$$



d. $P_A = 1$ $P_J^* \text{ ceiling} = 5$
 $P_T = 5$

Equilibrium

$$10P_J - 5 = 100 - 15P_J + 50$$

$$25P_J = 155$$

$$P_{JE} = \$6.20$$

$$Q_E = 10 \cdot 6.2 - 5$$

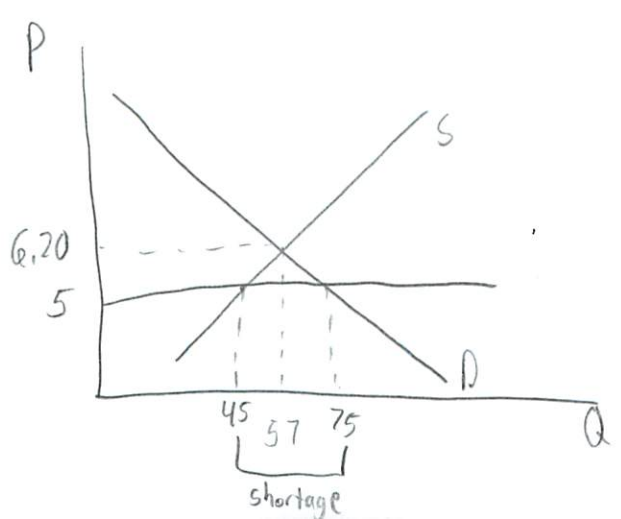
$$Q_E = 57$$

Price Ceiling

$$P_J^* = \$5$$

$$Q_S^* = 10 \cdot 5 - 5$$
$$45$$

$$Q_D^* = 100 - 15 \cdot 5 + 10 \cdot 5$$
$$100 - 75 + 50$$
$$75$$



6

4. Steel

P is \$ / ton
Q in million tons

Last year $P = 20$ $Q = 100$

$E_D = -1.25$ $E_S = 1.5$

$\epsilon \frac{P_D}{Q_D} \frac{\Delta Q}{\Delta P}$ $\epsilon \frac{P_S}{Q_S} \frac{\Delta Q}{\Delta P}$

~~$Q_D = a - bP + b$ $Q_S = c + dP$~~

~~$-1.25 = \frac{P_D}{a - bP + b} \cdot \frac{\Delta Q_D}{\Delta P_D}$~~

~~$1.5 = \frac{P_S}{c + dP} \cdot \frac{\Delta Q_S}{\Delta P_S}$~~

~~?? not enough data ??~~

~~try plugging in #~~

~~at $P = 25$~~

~~$-1.25 = \frac{25}{Q_D} \cdot \frac{\Delta Q_D}{5}$~~

~~$1.5 = \frac{25}{Q_S} \cdot \frac{\Delta Q_S}{5}$~~

~~So what?~~

read 2.6

$Q_D = a - bP$

$Q_S = c + dP$

$\frac{\Delta Q}{\Delta P} = \text{constant on linear graph} = -b = d$

$E_D = -b \left(\frac{P^*}{Q^*} \right)$ $E_S = d \left(\frac{P^*}{Q^*} \right)$

$$-1.25 = -b \frac{20}{100}$$

$$1.5 = d \frac{20}{100}$$

$$-1.25 = -b$$

$$b = 1.25$$

$$d = 2.5$$

$$Q_D = a - 1.25P$$

$$100 = a - 1.25(20)$$

$$a = 125$$

$$Q_S = c + 2.5P$$

$$100 = c + 2.5(20)$$

$$(c=60) \quad (50)$$

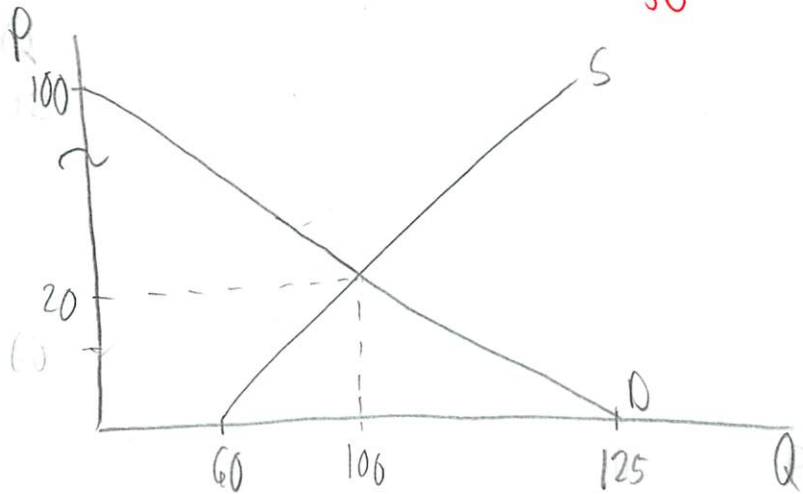
-1

stupid math error

$$Q_D = 125 - 1.25P$$

$$Q_S = \cancel{60} + 2.5P$$

50



or

$$Q_D - 125 = -1.25P$$

$$Q_S - 60 = 2.5P$$

$$\frac{Q_D}{-1.25} + 100 = P_D$$

$$\frac{Q_S}{2.5} - 24 = P_S$$

(8)

b. Current price = \$15
 $Q = 150$

$$E_D = -1.25$$

$$E_S = 1.25$$

$$-1.25 = -b \frac{15}{150}$$

$$1.25 = d \frac{15}{150}$$

$$-1.25b = 1.25$$

$$d = 2.5$$

$$150 = a - 1.25(15)$$

$$150 = c + 2.5(15)$$

$$150 = a - 18.75$$

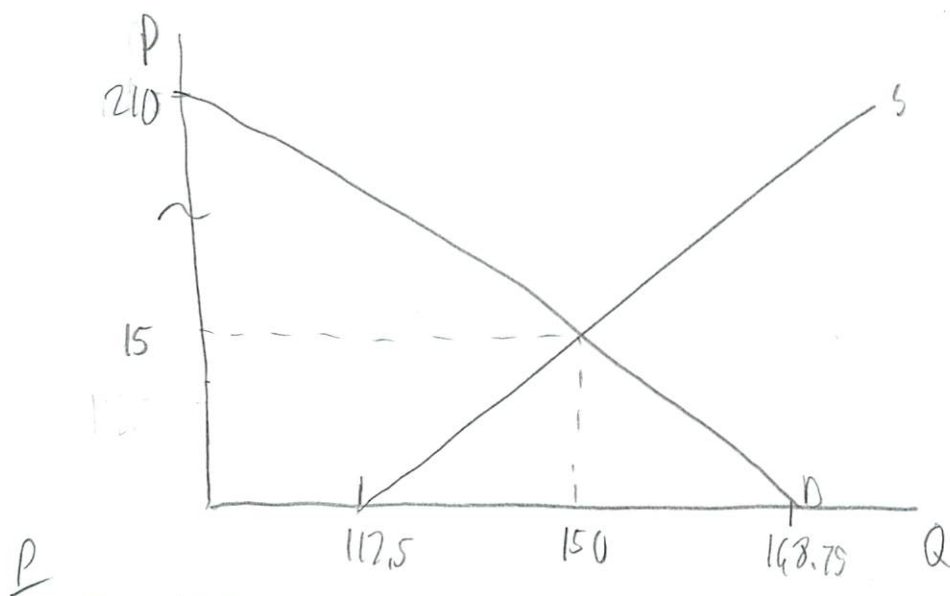
$$150 = c + 37.5$$

$$a = 168.75$$

$$c = 112.5$$

$$Q_D = 168.75 - 1.25P$$

$$Q_S = 112.5 + 2.5P$$



$$Q_D - 168.75 = -1.25P$$

$$Q_S - 112.5 = 2.5P$$

$$\frac{Q_D}{-1.25} + 210 = P$$

$$\frac{Q_S}{2.5} - 45 = P$$

9

The demand shifted to the right (increased) by a moderate amt.
The supply also shifted to the right (increased) by a large amt.

This increased the production sharply, while dropping prices.

A possible scenario is that production forecasters forecast a large increase in demand, and build new factories. Some of that new demand materializes, but not enough to save the jobs of the forecasters.

See solutions

-6

~~W~~ - curves have same slopes as last year
but intercepts shifted outwards for both (what I said)

demand shift reasons

- ↑ WTP → ↑ income
- ↑ substitute price
- ↑ demand for buildings (built w/ steel)

Supply

- ↑ # of firms ← what I said, but demand also shifts
- ↓ input prices ↑
↑ said that too, but no reason given (kinda explanatory)

14.01 Fall 2010

Problem Set 1

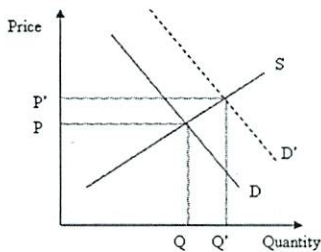
Due in class on September 17th

Answers

1. (25 points) For each of the following scenarios, use a supply and demand diagram to illustrate the effect of the given shock on the equilibrium price and quantity in the specified competitive market. Explain whether there is a shift in the demand curve, the supply curve or neither.

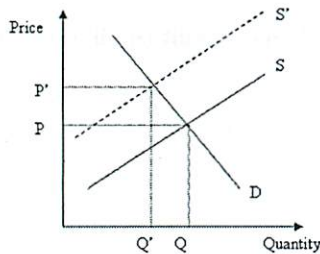
(a) (5 points) An unexpected temporary heat wave hits the East Coast. Show the effect in the ice-cream market in New England.

The temporary heat wave shifts the demand curve to the right from D to D' . As a result equilibrium price and quantity both go up.



(b) (5 points) The government introduces a tax on ice-cream which is paid by producers. What is the effect in the ice-cream market.

The supply curve shifts up from S to S' by the amount of the tax. As a result the equilibrium price increases and the equilibrium quantity decreases. However, the rise in the equilibrium price from P to P' is smaller than the tax.



(c) (5 points) China and Mexico are major producers of textiles. Workers in Mexico decide to go on strike. Show the effect on the market for Mexican textiles.

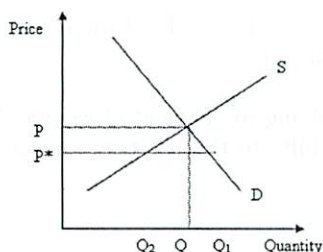
The supply curve for Mexican textiles shifts to the left. This results in a higher equilibrium price and lower equilibrium quantity in the market for Mexican textiles.

(d) (5 points) Show the effect of the situation described in (c) on the market for Chinese textiles.

The demand curve for Chinese textiles shifts to the right. This results in a higher equilibrium price and higher equilibrium quantity in the market for Chinese textiles.

(e) (5 points) Suppose the government imposes a price cap on bottled water. Show the effect in the bottled water market.

If the price ceiling P^* is set below the equilibrium price P then there will be a shortage of ice-cream in the amount of $Q_1 - Q_2$ and hence ice-cream will be rationed. If the price ceiling is above the equilibrium price then there is no effect.



2. (20 points) For each of the following pairs of goods, identify which one you would expect to have more own-price elastic demand. Please explain your reasoning.

(a) (5 points) Computers (generally) vs. Apple MacBook Pro laptops.

Apple MacBook because it is a specific brand and has more substitutability.

(b) (5 points) Stereo headphones (generally) vs. hearing aids.

Stereo headphones since they tend to be less of a necessity good than hearing aids.

For each of the following goods, identify whether you would expect demand to be more (own-price) elastic in the short run or the long run. As above, please briefly explain your reasoning.

(c) (5 points) Retail gasoline in the suburbs of Chicago.

More price elastic in the long-run because people cannot effectively adjust to necessity goods like gasoline in the short-run, while in the long-run, people can substitute it with electricity (hybrid cars), diesel, public transportation etc.

(d) (5 points) Air conditioning units in Miami Beach, Florida.

More price elastic in short-run because there are other good substitutes in the short-run such as fans. If there were a sudden increase in the price of A/C units, people could delay their purchase of a unit for a few days or weeks. But in the long run, there is no good substitute for A/C.

3. (30 points) Consider the market for apple-juice. In this market the supply curve is given by $Q_S = 10P_J - 5P_A$ and the demand curve is given by $Q_D = 100 - 15P_J + 10P_T$, where J denotes apple juice, A denotes apples and T denotes tea.

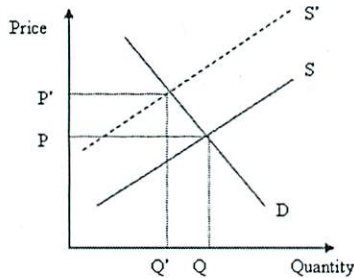
(a) (7 points) Assume that P_A is fixed at \$1 and $P_T = 5$. Calculate the equilibrium price and quantity in

the apple juice market.

We have the system of equations $Q = 10P_J - 5 \cdot 1$ and $Q = 100 - 15P_J + 10 \cdot 5$. Solving for P_J and Q we get that $P_J = 6.2$ and $Q = 57$.

(b) (7 points) Suppose that a poor harvest season raises the price of apples to $P_A = 2$. Find the new equilibrium price and quantity of apple juice. Draw a graph to illustrate your answer.

We now have to solve the system: $Q = 10P_J - 10$, $Q = 150 - 15P_J$. Solving for P_J and Q we get that $P_J = 6.4$ and $Q = 54$. In a supply and demand graph, the supply curve shifts to the left resulting in the higher equilibrium price and lower equilibrium quantity.



(c) (8 points) Suppose $P_A = 1$ but the price of tea drops to $P_T = 3$. Find the new equilibrium price and quantity of apple juice.

$$Q = 10P_J - 5, Q = 130 - 15P_J. P_J = 5.4, Q = 49.$$

(d) (8 points) Suppose $P_A = 1$, $P_T = 5$, and there is a price ceiling on apple juice of $P_J^* = 5$. What is the excess demand for apple juice as a result? Draw a graph to illustrate your answer.

Note that the price ceiling will be binding since the equilibrium price from (a) is $P_I = 6.2$. Plugging the price ceiling level into the supply and demand equations we get that $Q_S = 45$ and $Q_D = 75$. Hence, there will be excess demand for ice-cream of $Q_E = 30$. The graph in Question 1(e) shows the identical case as the one here.

4. (25 points) You have been asked to analyze the market for steel. From public sources, you are able to find that last year's price for steel was \$20 per ton. At this price, 100 million tons were sold on the world market. From trade association data you are able to obtain estimates for the own price elasticities of demand and supply on the world markets as -0.25 for demand and 0.5 for supply. Assume that steel has linear demand and supply curves throughout.

a) (10 points) Solve for the equations of demand and supply in this market and sketch the demand and supply curves. Assume that this is a competitive market and assume that demand and supply are linear.

Thus, $X^d = a - bP$ and $X^s = c + dP$. We know from the equation for own price elasticity of demand that

$$E_{Q_X P_X} = \frac{dX}{dP_X} \frac{P_X}{X} = -b \frac{P_X}{X} = -b \frac{20}{100} = -0.25.$$

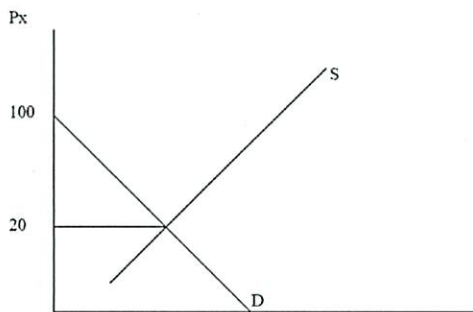
Solving for b , then, we have $b = 1.25$. Substituting back into the equation for demand, $X^d = a - 1.25P$

or $100 = a - 1.25(20)$. Solving for "a" we have $a = 125$. Hence, the equation for last year's demand is $X^d = 125 - 1.25P$.

We know that the price elasticity of supply is

$$E_{Q_X P_X} = \frac{dX^s}{dP_X} \frac{P_X}{X^s} = d \frac{20}{100} = 0.5.$$

Solving for d , then, we have $d = 2.5$. Substituting back into the equation for supply, $X_s = c + 2.5P$ or $100 = c + 2.5(20)$. Solving for c , we have $c = 50$. Hence, the equation for last year's supply is $X_s = 50 + 2.5P$.



b) (15 points) Suppose that you discover that the current price of steel is \$15 per ton and the current level of worldwide sales of steel is 150 million tons. The most recent elasticity estimates from the trade association this year are -0.125 for demand and 0.25 for supply. Describe the change in the supply and demand curves over the past year using your diagram from part a). What sort of event(s) might explain the change?

Using the same functional forms as in the first part of the answer, with the new data we have that $X_d = a - bP$ becomes $150 = a - b(15)$. Our equation for elasticity of demand yields

$$E_{Q_X P_X} = \frac{dX}{dP_X} \frac{P_X}{X} = -b \frac{P_X}{X} = -b \frac{15}{150} = -0.125.$$

Solving for b yields $b = -1.25$. Substituting this value for b into the equation for (linear) demand, we have $150 = a - 1.25(15)$ or $a = 168.75$. Hence, $X^d = 168.75 - 1.25P$.

For supply, we have $X^s = c + dP$ or $150 = c + d(15)$. The equation for elasticity yields

$$E_{Q_X P_X} = \frac{dX^s}{dP_X} \frac{P_X}{X^s} = d \frac{15}{150} = 0.25.$$

Solving for d yields $d = 2.5$. Substituting this value for d into the equation for linear supply, we have $150 = c + 2.5(15)$ or $c = 112.5$. Thus, $X^s = 112.5 + 2.5P$.

The demand and supply have kept the same slope as last year, but the intercepts have changed for both curves: demand and supply have shifted out. The demand shift could occur with any number of factors that

increase the willingness to pay for steel at any given price, such as an increase in income, an increase in the price of other (substitute) materials, or the increase in demand for a good that requires steel as an input, like cars. The supply shift could occur with any of a number of factors that increase the willingness to produce steel for the markets at a given price, such as an increase in the number of firms that sell steel, or a decrease in the prices of inputs required to produce steel (such as steel workers wages).

Revision 2

9/17

1. Price elasticity of demand
 2. elasticity of supply
 3. Effect of taxes on equilibrium $P+Q$
 4. PS 1
-

How Q changes w/ respect to price

$$\epsilon_D = \frac{\% \Delta \text{Quantity demanded}}{\% \Delta \text{Price}} = \frac{\frac{\Delta Q}{Q}}{\frac{\Delta P}{P}} = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q} = \frac{dQ}{dP} \cdot \frac{P}{Q}$$

$$Q_D = 24 - 2P$$

What is ϵ_D at $P=3$

First calc $Q \rightarrow 18$
at $P=3$

$$\epsilon_D = \frac{dQ}{dP} \cdot \frac{P}{Q} = -2 \cdot \frac{3}{18} = \boxed{-\frac{1}{3}}$$

$0 \leq \epsilon_D \leq -1$ "inelastic"

$\epsilon_D > -1$ "elastic"

$$\epsilon = 0$$

$$\epsilon = -\infty$$

perfectly inelastic

perfectly elastic

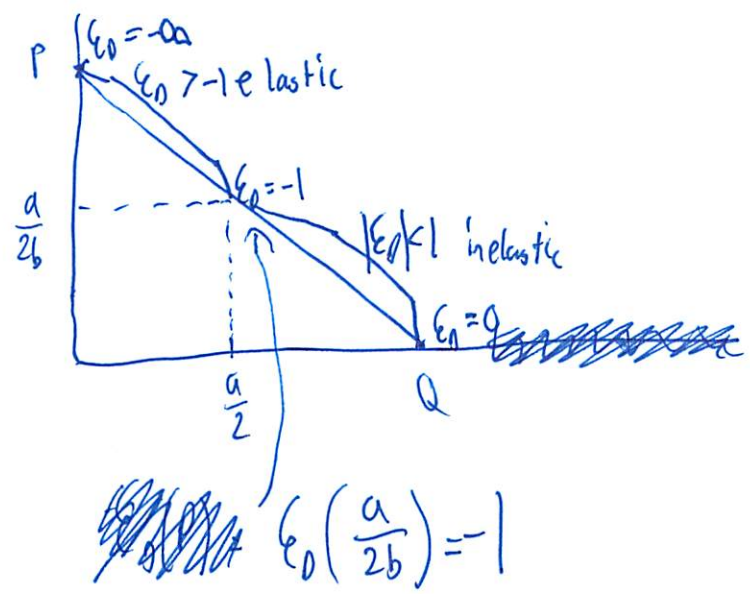
⌋⌋

⌋⌋

(2)

On a linear demand curve

$$Q = a - bP \quad a, b > 0$$



$$\begin{aligned} \epsilon_D(P) &= -b \cdot \frac{P}{a} \\ &= -b \cdot \frac{P}{a - bP} \end{aligned}$$

What about non linear demand curves?

- Potential to have constant elasticity

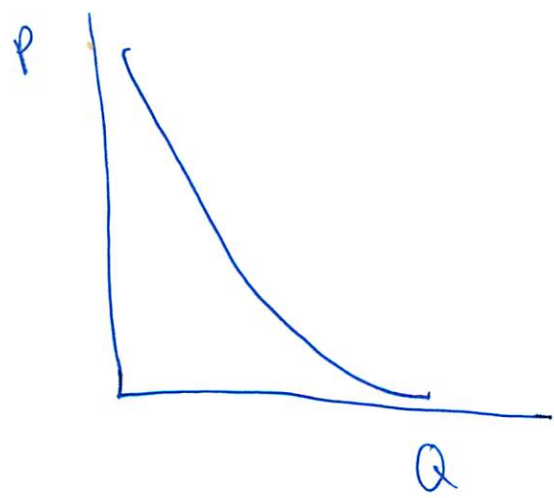
$$\epsilon_D = \frac{dQ}{dP} \cdot \frac{P}{Q} = C \rightarrow \frac{dQ}{dP}, P = C \cdot Q \quad C < 0$$

↓ the curves that follow that

$$Q = A \cdot P^C$$

$$\frac{dQ}{dP} = C \cdot A P^{C-1}$$

$$\frac{dQ}{dP} \cdot P = C \cdot A P^C = C \cdot Q$$



3

Can also calculate elasticity vs income
other good (cross price elasticity)

$$\frac{\Delta \% \text{ quantity}}{\Delta \% (\text{price of another good})}$$

" < 0 Complements

" > 0 substitutes

Economists work a lot w/ elasticity

2. Price Elasticity of Supply

$$\eta = \frac{\Delta \% \text{ qtd. supplied}}{\Delta \% \text{ price}} = \frac{\frac{\Delta Q_s}{Q_s}}{\frac{\Delta P}{P}} = \frac{dQ_s}{dP} \cdot \frac{P}{Q_s}$$

$$Q_s = 12 + 2P \quad P = 3 \quad Q \rightarrow 18$$

$$\eta = 2 \cdot \frac{3}{18} = \frac{1}{3}$$

④

Effect of a tax on equilibrium $P+Q$

- sales tax in US = ad valorem tax
 - proportional to price
 - complicated to calculate
- tax on gas in US = unit tax
 - each gallon $\approx .20 \$USD$
 - tax added to price

$$Q_D = 24 - 2P$$

$$Q_S = 12 + 2P$$

$$24 - 2P = 12 + 2P$$

$$P = 3$$

$$\rightarrow Q = 18$$

$$\epsilon_D = \frac{1}{3}$$

$$\epsilon_S = \eta = \frac{1}{3}$$

$T = 1$ - collected from producers

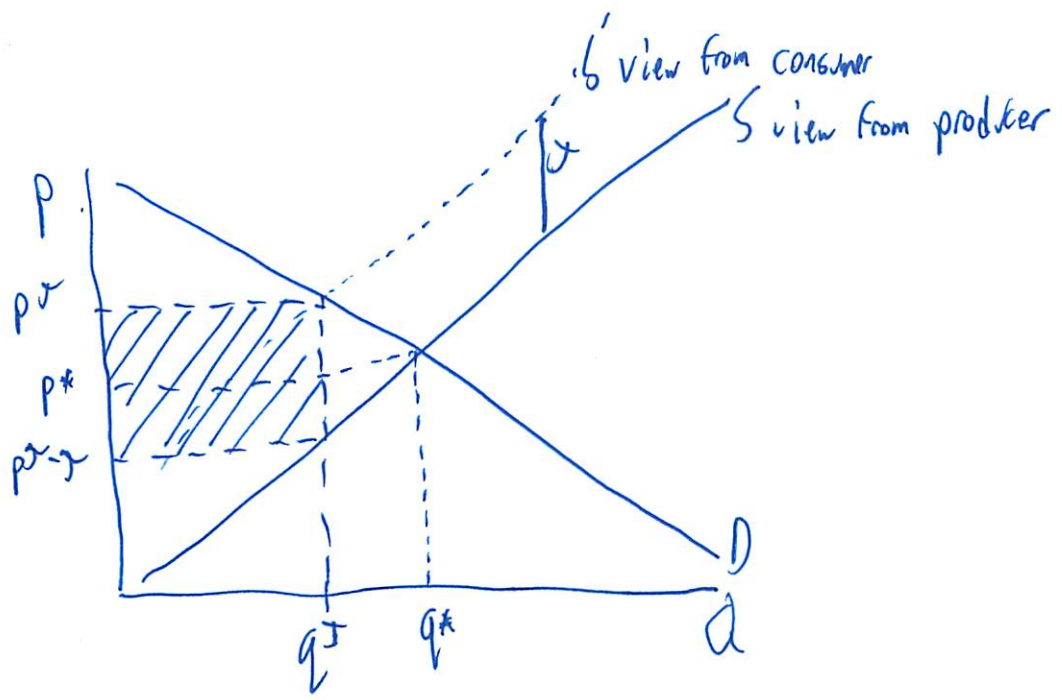
producer only keeps $p - T$

- so he behaves as though this is the price

$$Q_S = 12 + 2(p - T)$$

but consumer needs to pay $p + T$ to get same Q
as before the tax

5)



 = Revenue from tax

$$Q_D = 24 - 2P$$

$$Q_S = 12 + 2(P - T)$$

$$24 - 2P = 12 + 2(P - T)$$

$$\bar{P} = \frac{14}{4} = 3.5 \quad Q^T = 17$$

~~Gov Revenue~~

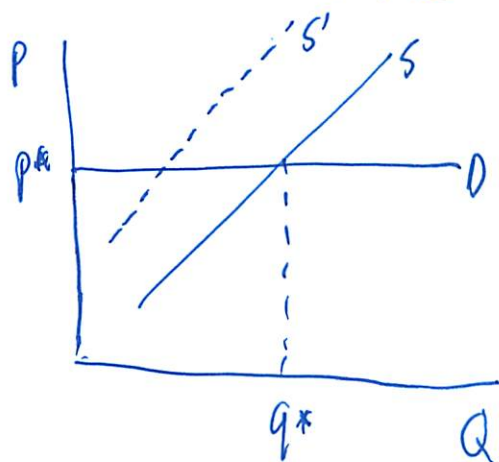
$$\text{Gov Revenue} = T \cdot Q^T = 17$$

Price only increases by half dollar

Producer's income falls by half dollar

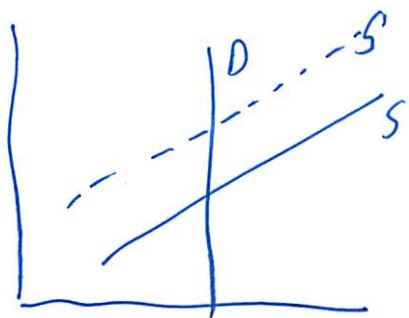
⑥ Extrem Example

perfectly elastic



Price stays the same
Producers can't pass along price increase
Must swallow it

perfectly inelastic



Producers can pass along the entire tax

$Q_D(P)$ = demand curve

$Q_S(P)$ = supply curve

τ = unit tax

$$Q_D(P) = Q_S(P - \tau)$$

$$Q_D(P) - Q_S(P - \tau) = 0$$

$$\left(\frac{dQ_D}{dP} - \frac{dQ_S}{dP} \right) dP + \frac{dQ_S}{dP} d\tau = 0$$

$$\frac{dP}{d\tau} = \frac{\frac{dQ_S}{dP}}{\frac{dQ_S}{dP} - \frac{dQ_D}{dP}}$$

⑦

$$\frac{dP}{d\tau} = \frac{\frac{dQ_s}{dP} \cdot \frac{P}{Q}}{\frac{dQ_s}{dP} \cdot \frac{P}{Q} - \frac{dQ_d}{dP} \cdot \frac{P}{Q}} = \frac{\eta}{\eta - \epsilon_D}$$

*

With linear supply & demand - this holds everywhere

$$\frac{\Delta P}{\Delta \tau} = \frac{\eta}{\eta - \epsilon_D}$$

in our Example: $\frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$

$\frac{\Delta P}{\Delta \tau} = \frac{1}{2}$ w/ the price we had, the price should
 $\uparrow \frac{1}{2}$ of the tax

"Incidence of a tax" -

- consumers \Rightarrow amt. that the price rises w/ respect to tax

$$-\Delta P = \frac{\eta}{\eta - \epsilon_D} \Delta \tau$$

Chap 3 Consumer Behavior

9/19

How much are consumers willing to pay?

What will consumers do?

1. Consumer Preferences

- prefer one good to another

2. Budget Constraints

- consumers have limited \$

3. Consumer Choice

- consumers want to maximize utility w/ limited \$
- may not always be rational
 - impulse, ignoring, or going into debt (ignoring budget)
- hard to know all of the choices
 - ↑ called behavioral economics
 - psychology
 - here we simplify it away

Consumer Preferences

market basket - a specific group (and qtd) of specific items
ie: consumers decide how ~~much~~ ~~to~~ ~~get~~ many food + clothes
units to buy each month
to maximize well being

②

3 basic assumptions

1. Completeness

- Consumers can compare + rank all baskets

2. Transitivity

if $A < B$ and $B < C$ then $A < C$

3. More is better than less

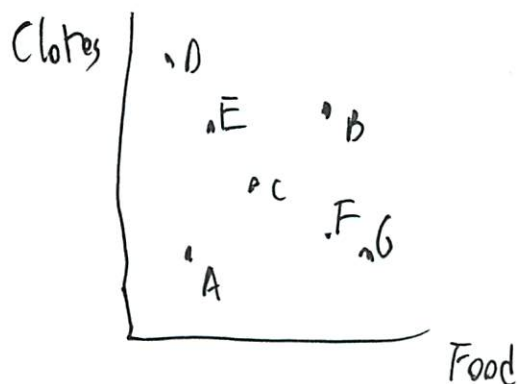
- ignoring bad goods like air pollution

- ie non satiation

Indifference Curve

All ~~poss~~ combos of market baskets that provide same satisfaction

- person indifferent among the baskets



$C > A$

$B > C$

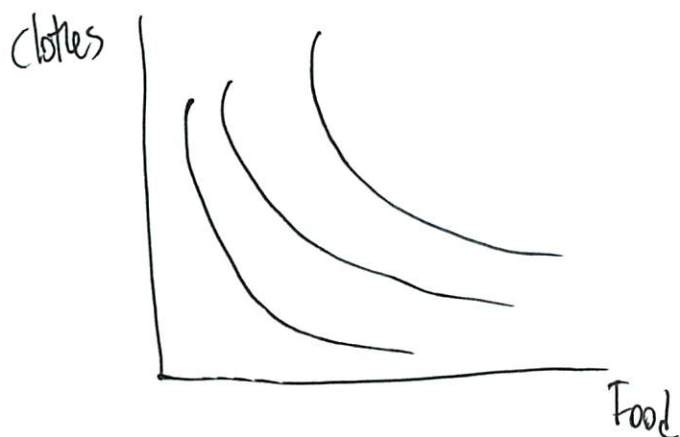
but don't know
about D, E, F, G
w/o add. info



anything above + to the
right is preferred

③ Indifference Maps

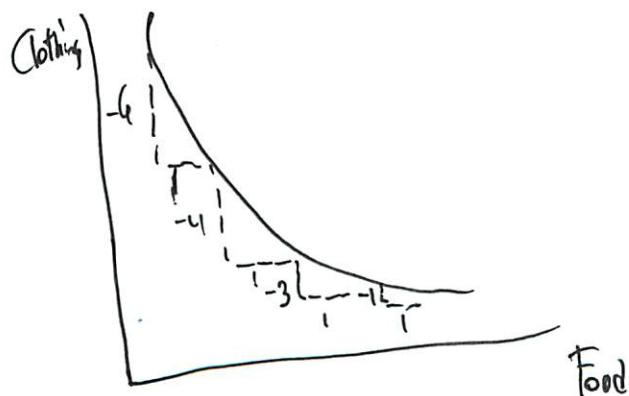
- set of 'indifference' curves
- along each curve the person is 'indifferent'
- can not 'intersect'



downward sloping - more is better (fairly obvious)

Marginal Rate of Substitution

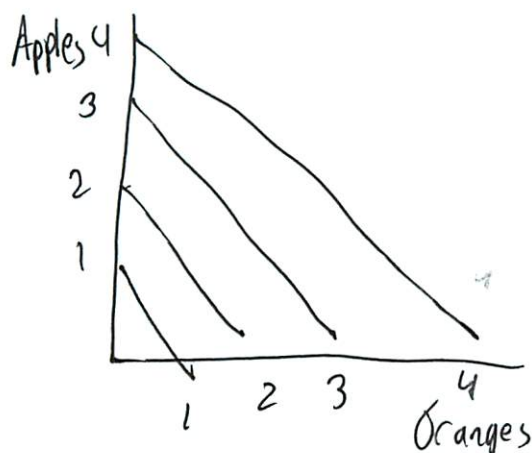
- max amt of clothing someone will give up for 1 unit food



- vertical = amt to give up
- horiz = to get 1 unit
- convex if diminishing marginal rate of substitution

(4)

Perfect Substitutes



marginal rate of substitution
= constant

Perfect Complements



only good together
if have 2 right shoes
~~can have any amt left shoes~~
but only want 2 left shoes
will give up up to 2 left shoes
for more right shoes

Bads less is more

- so taste is less is more
- or talk about the reverse \rightarrow clean air, asbestos removal

Utility

- attach a # to people's desire, satisfaction, need, etc
- utility function assigns a level of utility to each market basket
- i.e. $u(F, C) = F + 2C$
- indifferent to everything having same u
- numbers only for ranking

⑤

Ordinal vs Cardinal Utility

- ordinal = generates a ranking of baskets
 - does not indicate how much one is preferred to another (#)
 - numbers are somewhat arbitrary
(↑ ??? in general?, person to person?)
- Cardinal - can describe how much one basket is preferred to other
 - ?? what → the numbers matter now?
- but - numbers don't really help you compare
- so will only care about the rank (ordinal)

Consumers want utility maximization

- preferences
 - budget constraints
 - constrained maximization
-) what bundle of goods is optimal?

2 goods easiest to graph

- but can be many, many dimensions

3 steps to model consumer behavior

- ① Preference assumptions
- ② Utility function \Rightarrow math model of consumer preferences
- ③ Budget constraints
 \uparrow not today

Preference Assumptions

- ① Completeness - you can always tell which is better
 - must be some slight preference
 - can't be "not sure"
- ② Transitivity $\Rightarrow A \prec B$ and $B \prec C$ then $A \prec C$
- ③ Non-satiation \Rightarrow more is always better
 - you never would turn down more
 - might not like next unit as much, but always > 0

② Indifference Curves

- ie preference maps
- graphical representation of preferences
- decision: Pizza or movies w/ allowance

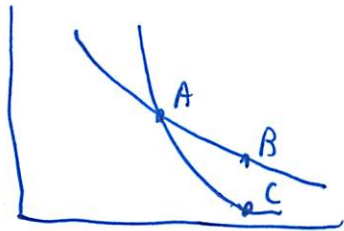
See 4.1 graph

4 key properties

① Prefer higher indifference curves
↳ due to non-satiation

② Always downward sloping
↳ (4.2) violates more is better

③ Can not cross



$$A=B \quad A=C$$

~~A=B~~ but $B \neq C \rightarrow$ more is better

④ ... (did not hear)

③ Utility

- math representation of preferences
- ie $U = \sqrt{P \cdot M}$
- Does not mean anything 'inherently'
- but it tells us your preferences
- Utility does not mean anything
 - only ordinal - just rank
 - not cardinal
 - can't say absolutely how happy you will be

Marginal Utility

- how utility changes w/ each add. unit
- derivative of utility
- ie you have 2 pizzas
- graph 4-4, 4-5

Shape of Preference Maps

- linkage \rightarrow marginal rate of substitution

$$MRS = \frac{\Delta P}{\Delta M} = \text{slope indifference curve}$$

- rate you are willing to trade of Y axis for 1 unit of X axis
- trade off by allocating budget

4

Graph 4-6

falls since marginal utility ↑ the fewer you have

- When I have 4 pizzas, I am willing to give one up for an extra movie

- when I have 2 pizzas, 2 movies I'd rather have my pizza and ~~I don't want~~ then an extra movie

$$MRS = \frac{\Delta P}{\Delta M} = \frac{-MU_M \text{ (x-axis)}}{MU_P \text{ (y-axis)}}$$

↑ marginal utility is a \ominus function for quantity

As you get more movies + less pizza

$$\frac{-MU_M \text{ (↓)}}{MU_P \text{ (↑)}} \rightarrow \text{so } MRS \downarrow$$

As you move down the curve

- does not count addition

Where do S+D curves come from
D: consumer + consumer preferences

Figure 4-1a: Pizza/movie preferences

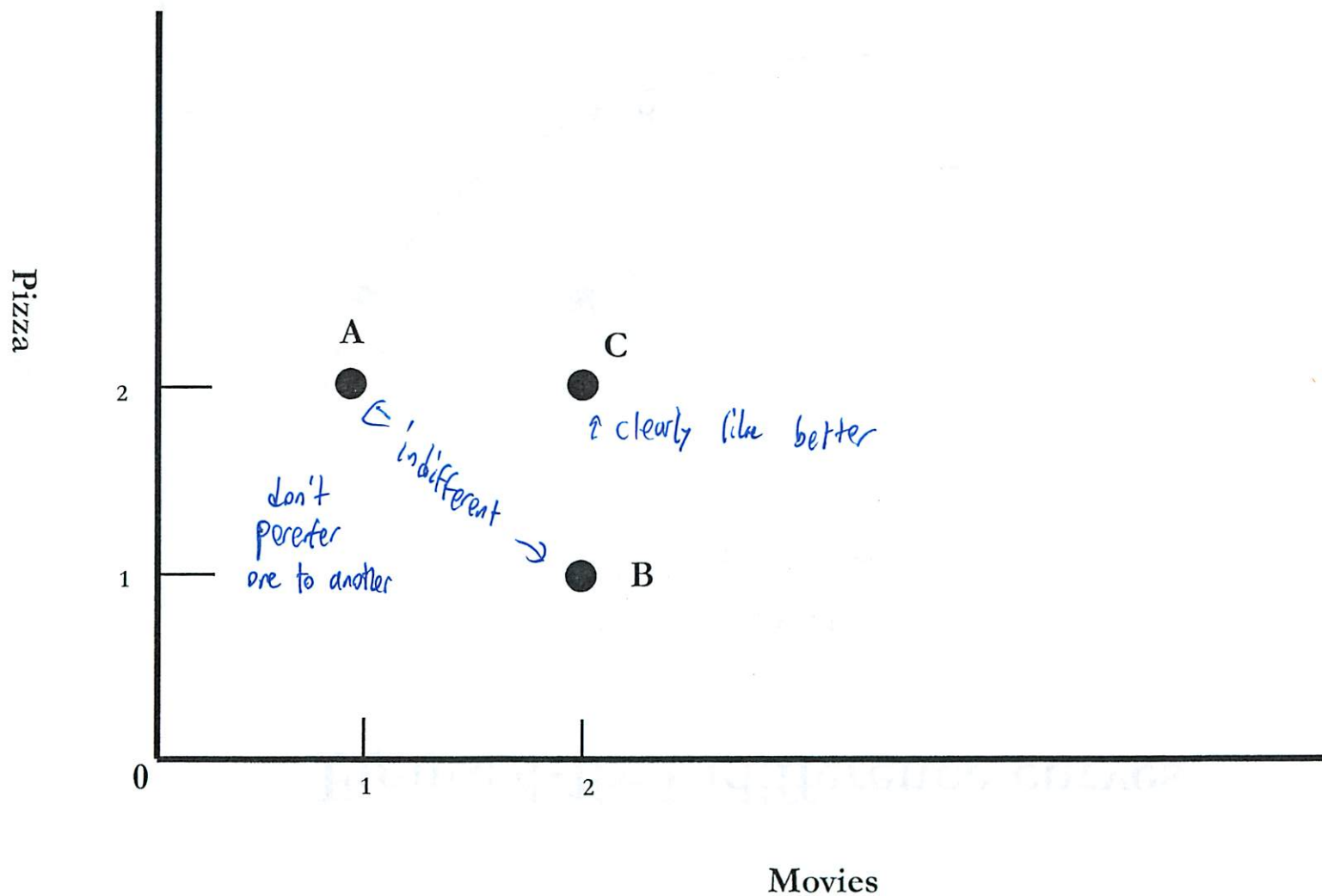


Figure 4-1b: Indifference curves

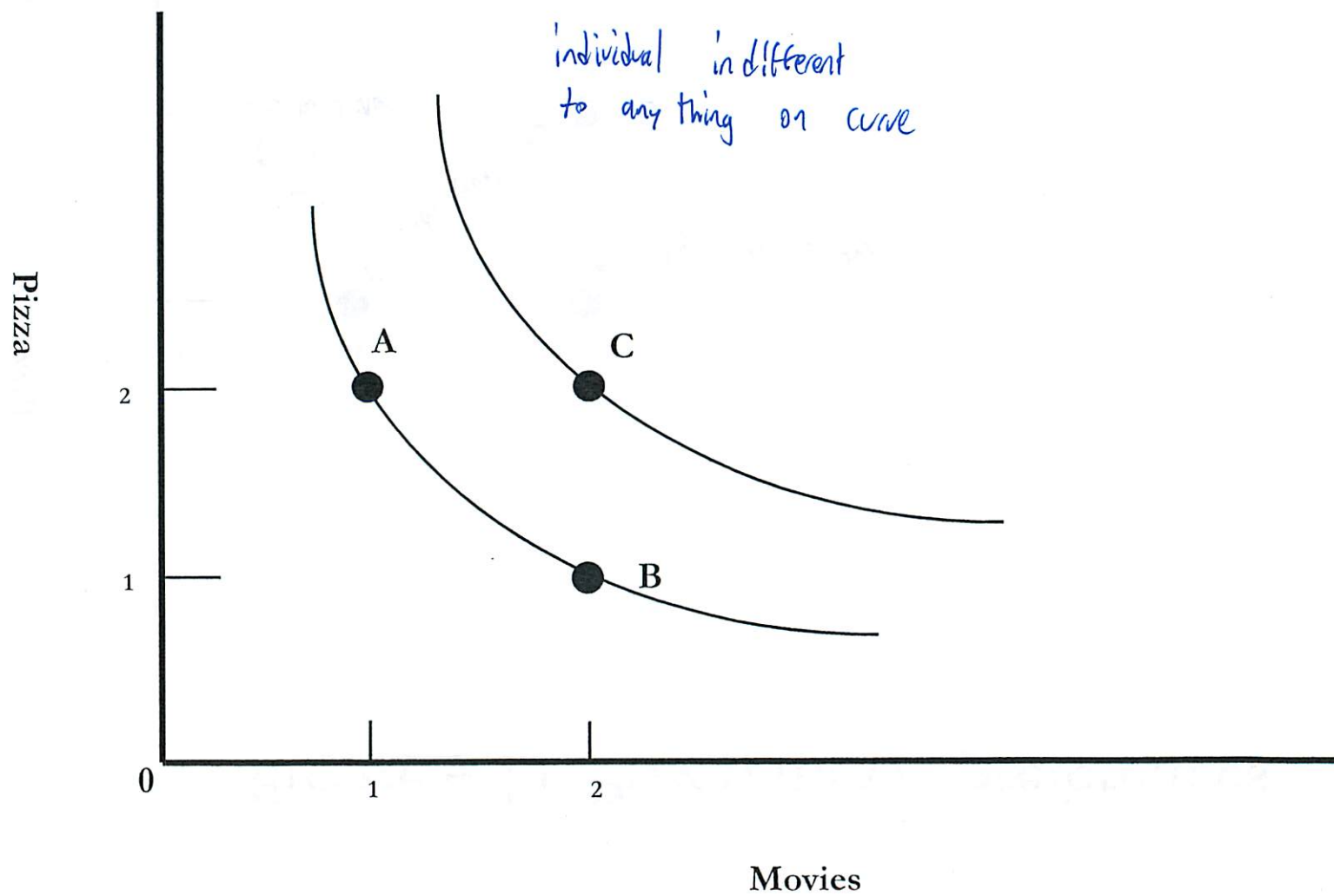


Figure 4-2: Upward-sloping indifference curve

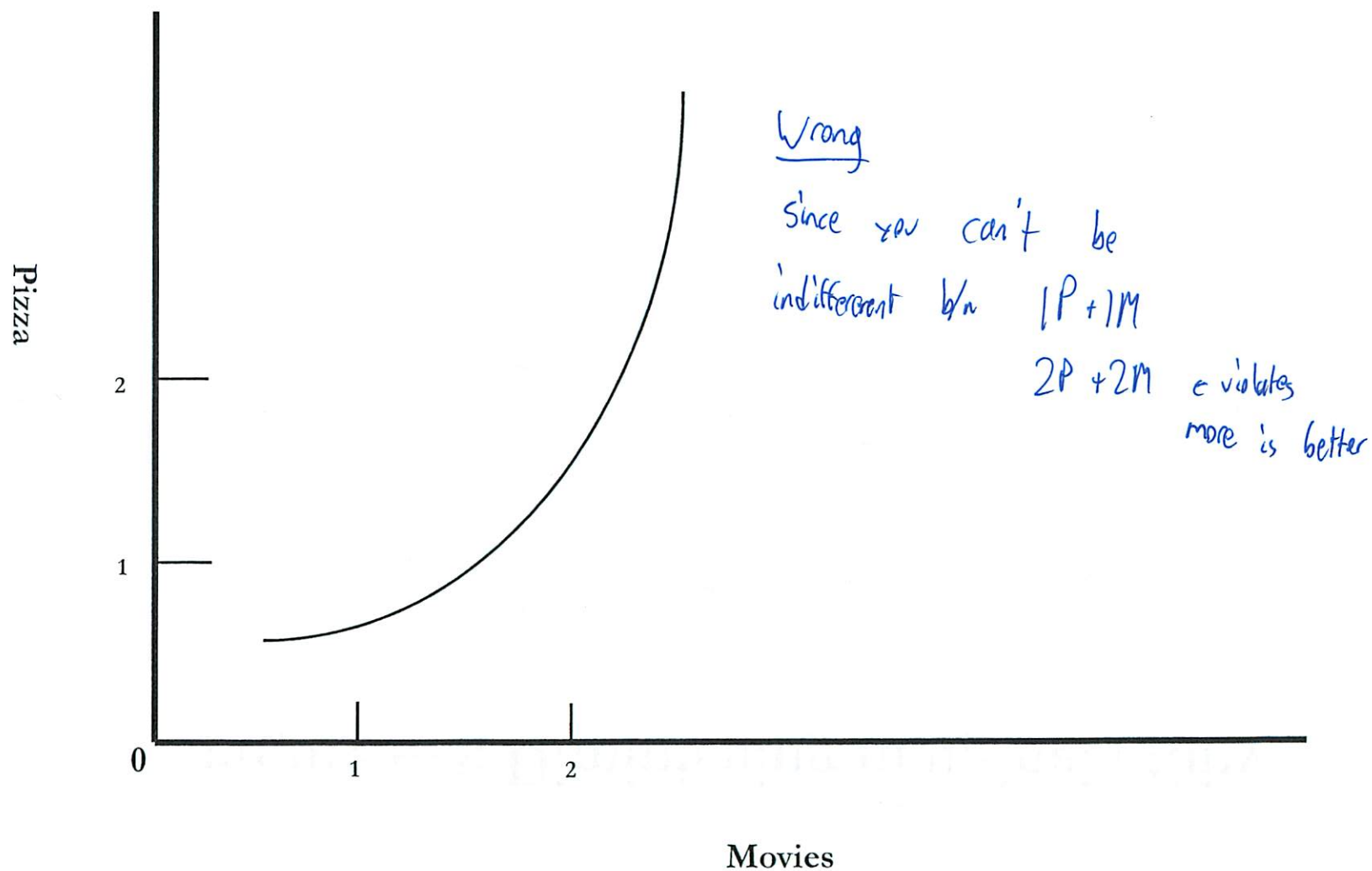


Figure 4-4: Diminishing marginal utility

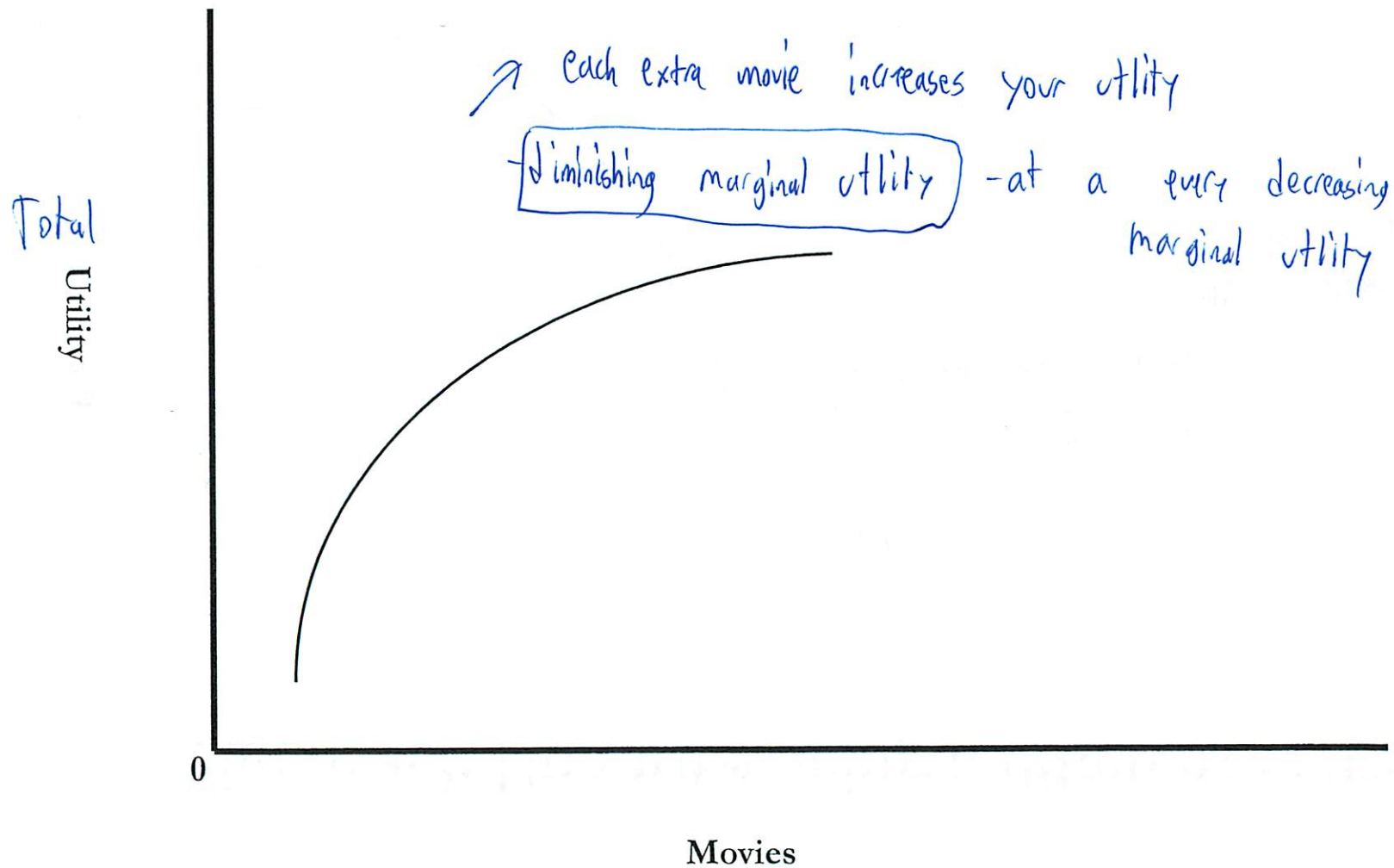


Figure 4-5: Diminishing marginal utility for $U = \sqrt{P \cdot M}$

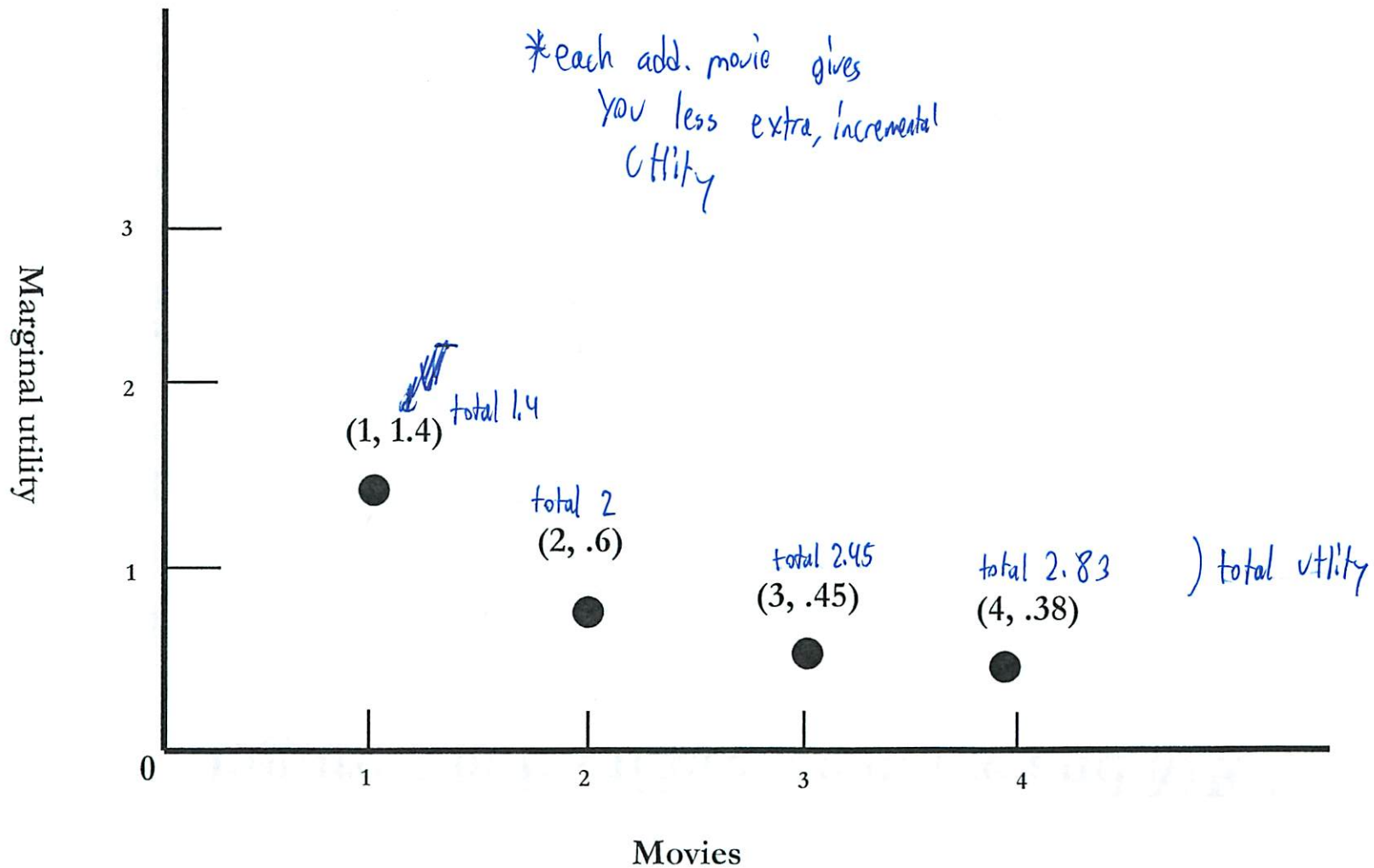
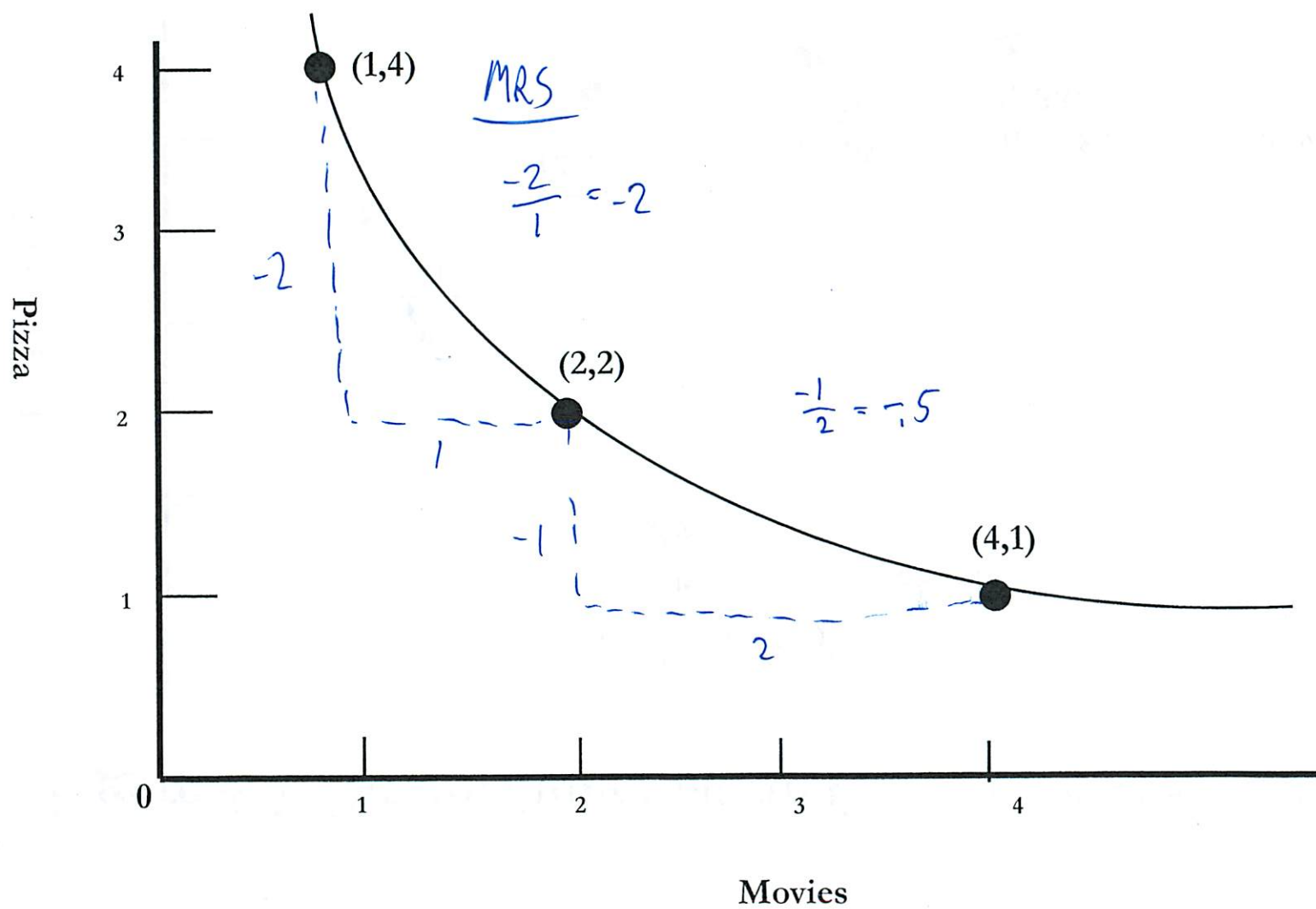


Figure 4-6: Indifference curve and MRS



14.01 Principles of Microeconomics, Fall 2007

Chia-Hui Chen

September 12, 2007

Lecture 4

Price Elasticity of Supply; Consumer Preferences

Outline

1. Chap 2: *Elasticity - Price Elasticity of Supply*
2. Chap 3: *Consumer Behavior - Consumer Preferences*

1 Price Elasticity of Supply

Price elasticity of supply. The percentage change in quantity supplied resulting from one percentage change in price.

$$E_P^S = \frac{\frac{dQ_S}{Q_S}}{\frac{dP}{P}} = \frac{P}{Q_S} \frac{dQ_S}{dP}.$$

In the short run, if price increases, firms will want to produce more but cannot hire workers and buy machines immediately, thus the supply is less elastic. In contrast, supply is more elastic in the long run.

Example (Example in Elasticities of Demand). Assume the quantity demanded is

$$Q^D = 14 - 3P + I + 2P_S - P_C.$$

- P - Price
- I - Income
- P_S - Price of substitute
- P_C - Price of complement

Calculate E_P^D , E_I , E_{QP_S} and E_{QP_C} when $P = 1$, $I = 10$, $P_S = 2$ and $P_C = 1$.

Solution:

Given the values of variables, the quantity demanded is:

$$Q^D = 14 - 3 \times 1 + 10 + 2 \times 2 - 1 = 24.$$

The elasticities are

$$E_P^D = \frac{P}{Q_D} \frac{dQ_D}{dP} = \frac{1}{24} \times (-3) = -\frac{1}{8},$$

$$E_I = \frac{I}{Q} \frac{dQ}{dI} = \frac{10}{24} \times 1 = \frac{5}{12},$$

$$E_{QP_S} = \frac{P_S}{Q} \frac{dQ}{dP_S} = \frac{2}{24} \times 2 = \frac{1}{6},$$

$$E_{QP_C} = \frac{P_C}{Q} \frac{dQ}{dP_C} = \frac{1}{24} \times (-1) = -\frac{1}{24}.$$

2 Consumer Preferences

$$\text{Consumer behavior} \left\{ \begin{array}{l} \text{Consumer preferences} \\ \text{Budget constraints} \end{array} \right\} \Rightarrow$$

- What amount and types of goods will be purchased.
- Origin of demand, how to decide demand.

Topics

1. Preference
2. Indifference Curve, Marginal Rate of Substitution (MRS)
3. Utility Functions

Preference

Notation

- $A \succ B$: A is preferred to B.
- $A \sim B$: A is indifferent to B.

Basic assumptions for preferences

- Completeness - can rank any basket of goods.
(always possible to decide preference or indifference)
- Transitivity - $A \succ B$ and $B \succ C$ implies $A \succ C$.
This assumption seems obvious, but can have contradiction (see example below).

	Property I	Property II	Property III
Good A	3	1	2
Good B	2	3	1
Good C	1	2	3

Table 1: Example of contradiction of transitivity.

Example (A contradiction of transitivity). Chart below lists 3 goods and 3 properties, assume that people will prefer one to another if 2 properties are better. Table 1. Actually $A \succ B$, $B \succ C$ and $C \succ A$ - this loop contradicts the assumption.

- Non-satiation - more is better. (Monotonicity) Assume we discuss goods, since in general, more is not always better.
- Convexity - given two indifferent bundles, always prefer the average to each of them. In Figure 1, the average point C is more preferred to A or B.

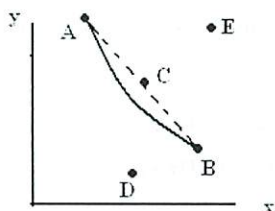


Figure 1: Convexity of indifference curve.

Indifference Curve, Marginal Rate of Substitution (MRS)

Properties of indifference curves

- Downward sloping: if not, non-satiation violated. Refer to Figure 1.

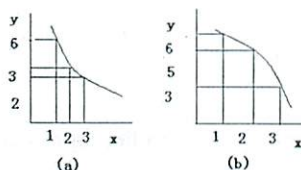


Figure 2: Compare the Shapes of Indifference Curve.

- Cannot cross: if not, non-satiation and transitivity cannot be satisfied simultaneously. In Figure 1, assume there is another indifference curve through A and D.

$$A \sim B, A \sim D \implies B \sim D.$$

However,

$$B \succ D$$

in this figure. Contradiction exists.

- Shape: describes how willing one is to substitute one good for another. See Figure 2.

Marginal rate of substitution (MRS)

Marginal rate of substitution (MRS). How many units of Y one is willing to give up in order to get one more unit of X.

$$\frac{-\Delta y}{\Delta x} = \frac{-dy}{dx}$$

People prefer a balanced basket of goods.

- MRS decreasing.
- Preferred set is convex.
- The left one in Figure 2 makes more sense in the real world.

Perfect substitution. MRS is constant.

Perfect complements. Indifference curves are shaped as right angles.

Example (Perfect complements). Buying shoes. People need both the left one and the right one.

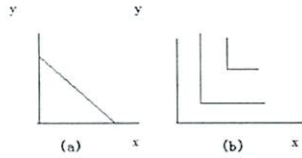
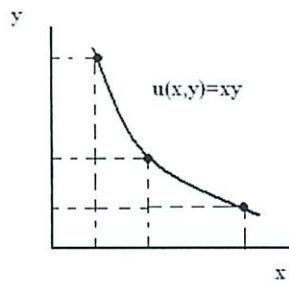


Figure 3: Perfect Substitution and Perfect Complements.

Figure 4: Indifference Curve with Utility Function $u(x, y) = xy$.

Utility Functions

Utility function. Assigns a level of utility to each basket of consumption.

Example (A sample utility function).

$$u(x, y) = xy.$$

For example, (5,5) is indifferent to (25,1) and (1,25).

Ordinal utility function. Ranks the preferences, but does not indicate how much one is preferred to another.

Cardinal utility function. Describes the extent to which one of the bundles is preferred to another. Only the ordinal utility function is required in this course.

- Budget constraint stops consumers from buying what you want
- for now income = budget
 - ignore savings for now
- $Y = \text{income} / \text{budget}$

$$X = P \cdot p_p + M \cdot p_m$$

\uparrow \uparrow price of pizza
 $\#$ pizzas

- Graph 5.1 -

Opportunity cost = value of forgone alternative

- if you chose to see a movie, you are forgoing half a pizza
- not really in \$
- ~~even~~ because you have a budget constraint

- Graph 5.2 -

Opportunity ~~set~~ set - just \downarrow , restricted

- you are effectively poorer
- even though income did not change

- Graph 5.3 -

- income fell
- opportunity set contracted

area under graph is potential utility

- but don't know preferences
- do you hate movies?

(2)

So now add indifference curve

- What is the furthest out point you can achieve??

- figure 5.4 -

$$\begin{aligned}\text{Utility} &= \sqrt{p \cdot m} \\ &= \sqrt{6 \cdot 3} \\ &= \sqrt{18}\end{aligned}$$

marginal \rightarrow next unit

What is that next \$ of expenditure get you in terms of utility

equilibrium point \rightarrow the marginal benefit of the pizza + movie should be =

$$\begin{aligned}A \quad P &= 2 \\ M &= 5 \quad U = \sqrt{10}\end{aligned}$$

at this point

$$MU_p = \frac{dU}{dp} = \frac{0.5 \cdot M}{\sqrt{p \cdot m}} \quad \text{b/c does this come from}$$

$$MU_m = \frac{dU}{dm} = \frac{0.5 \cdot P}{\sqrt{p \cdot m}} = \frac{2.5}{\sqrt{10}}$$

$$\} \text{MRS} = 2.5$$

You would give up 2.5 pizzas to see one movie

You would really like to see more movies and lighten up on the pizza

Indifference curve is very steep here

③

Willing to give up 2.5 pizza for movie

But market says .5 pizza for movie

So no where near equilibrium

↳ inefficiency happens when people make trades they don't value

C A

MRS does = MRT

but you also need to spend all your money

E

MRS < MRT again

but does not meet budget constraint

Your desired trade ~~not~~ should be same as market for most efficiency

- Figure 5.5 —

- Corner solution

- Your prefs are very different

- constant rate willing to trade off pizza + movies

- unrealistic

- have to check for corner solution

- if ~~it~~ don't seem to work

- since have — q

(4)

In practice need multidimensional graphs

But can break it up into categories (food, entertainment) or pairs

- mental accounting

Government + Taxes

- What if government taxed pizza?

- just like a price P in pizza

- so like graph 5.2

- will lower consumption of pizza

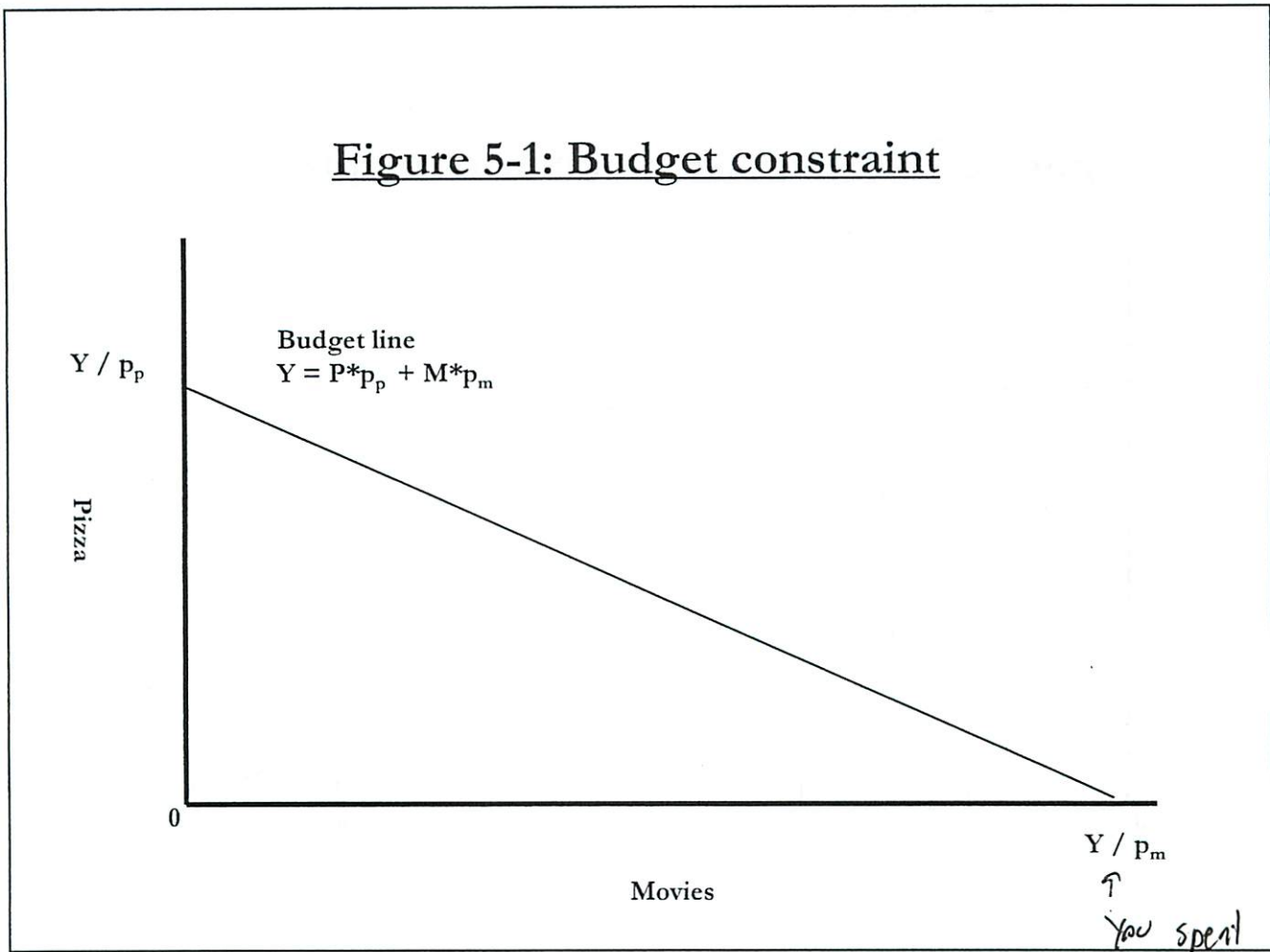
but w/ mental accounting

- if you shift pizza's budget category

- to one where it has less $\$$ in there

- called "Nudge" - in behavioral economics

not in 14.01

Figure 5-1: Budget constraint

Slope = ~~$\frac{P_p}{P_m}$~~ ~~$\frac{P_m}{P_p}$~~ $-\frac{P_m}{P_p}$ = Marginal Rate of Transformation

Example 196

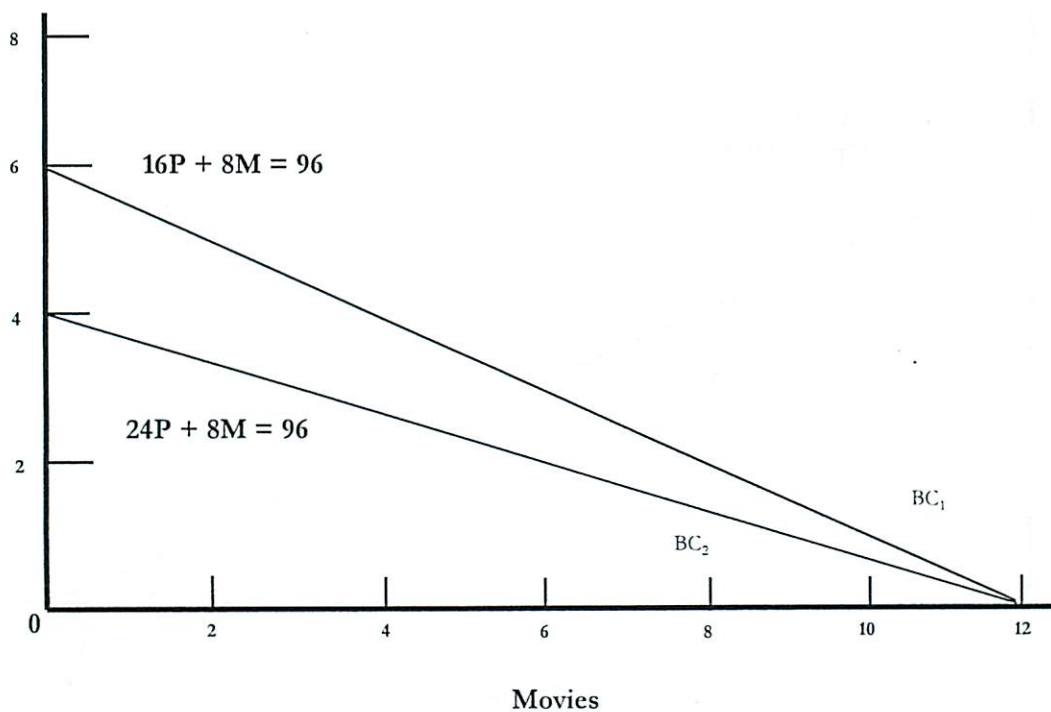
$P_m = \$8$
 $P_p = \$16$

You can buy 8 Pizzas
 or 12 movies

Can trade 1 pizza
 for 2 movies

Slope = $-\frac{1}{2} = MRT$

Figure 5-2: Increase in the price of pizza



$MRT = -\frac{1}{3}$

Figure 5-3: Decrease in income

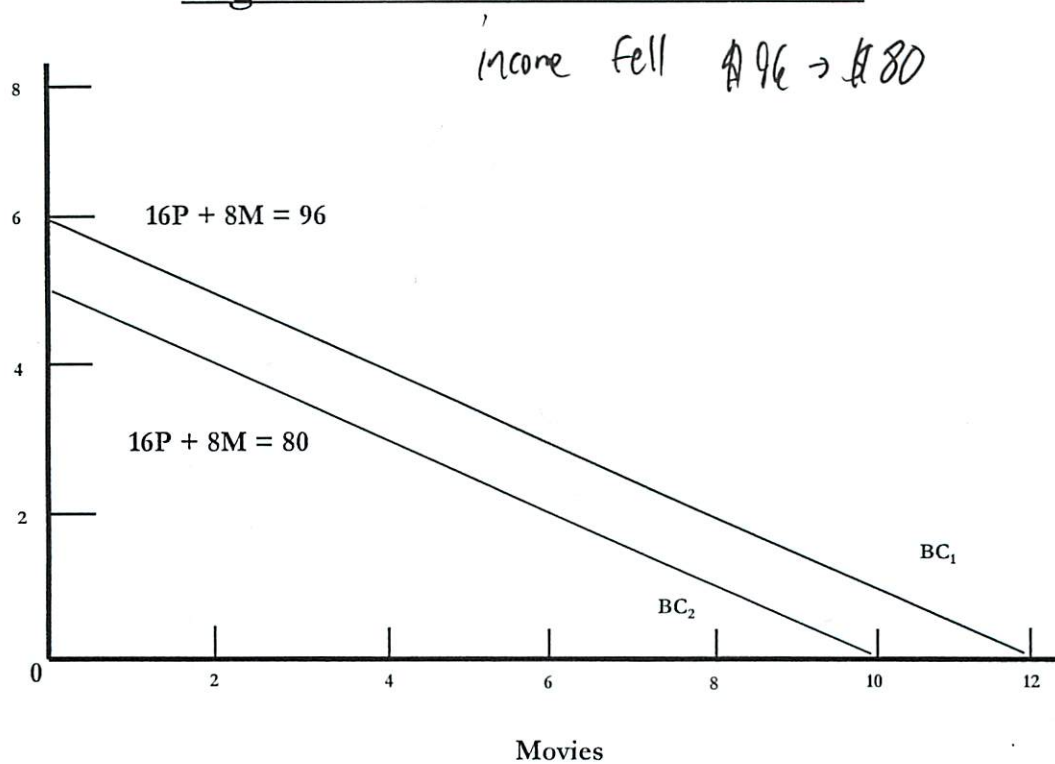
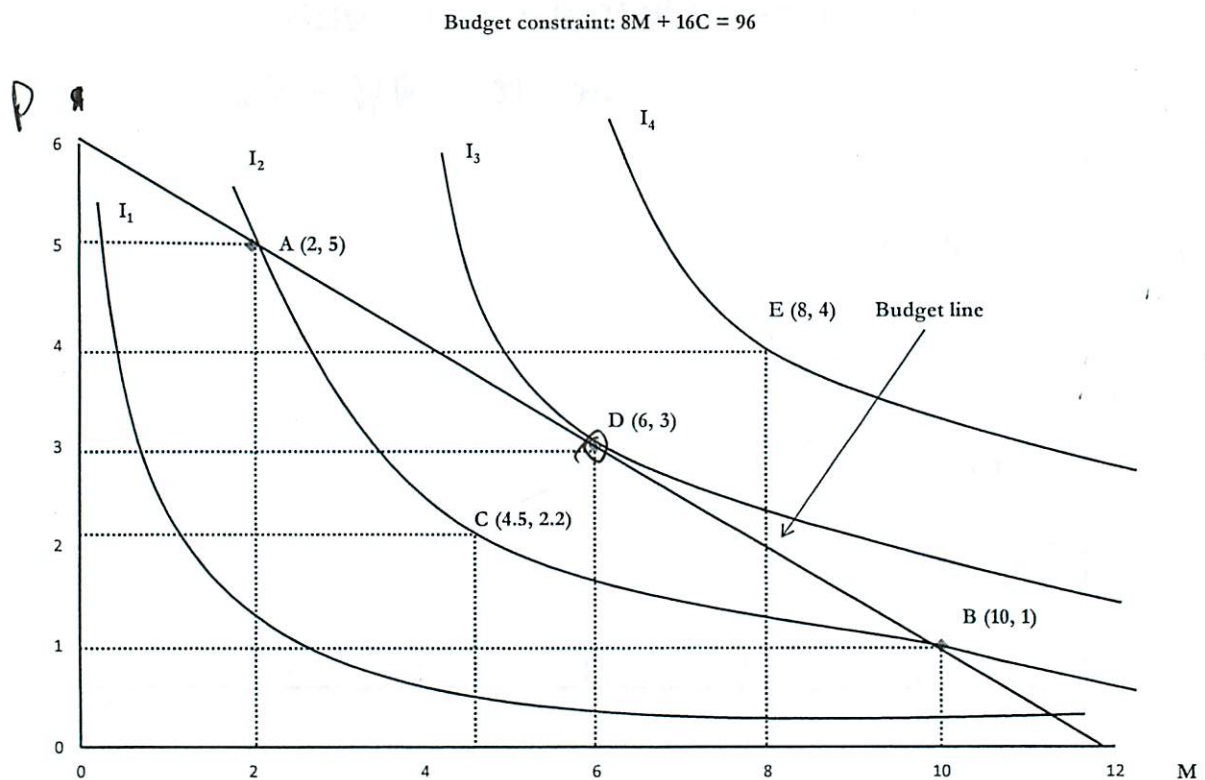


Figure 5-4: Budget constraint and indifference curves



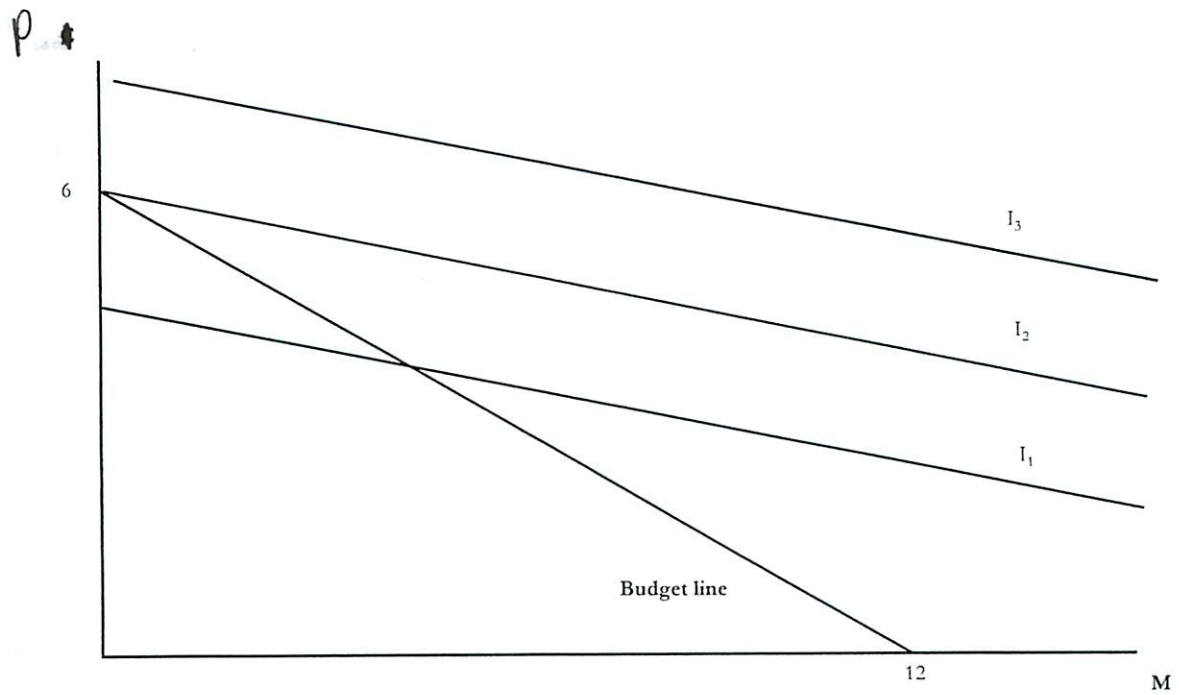
D = best choice

$\text{preference} \quad \text{market}$
 $MRS = MRT$
 $\text{slope indifference} = \text{slope budget constraint}$
 $\hookrightarrow \text{place where they are} =$

* Marginal benefit = marginal cost

$$\frac{-MU_M}{MU_C} = \frac{-P_M}{P_C}$$

Figure 5-5: Corner solution



14.01 Principles of Microeconomics, Fall 2007

Chia-Hui Chen

September 14, 2007

Lecture 5

Deriving MRS from Utility Function, Budget Constraints, and Interior Solution of Optimization

Outline

1. Chap 3: *Utility Function, Deriving MRS*
2. Chap 3: *Budget Constraint*
3. Chap 3: *Optimization: Interior Solution*

1 Utility Function, Deriving MRS

Examples of utility:

Example (Perfect substitutes).

$$U(x, y) = ax + by.$$

Example (Perfect complements).

$$U(x, y) = \min\{ax, by\}.$$

Example (Cobb-Douglas Function).

$$U(x, y) = Ax^b y^c.$$

Example (One good is bad).

$$U(x, y) = -ax + by.$$

An important thing is to derive MRS.

$$MRS = -\frac{dy}{dx} = |\text{Slope of Indifference Curve}|.$$

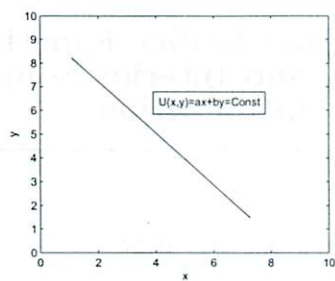


Figure 1: Utility Function of Perfect Substitutes

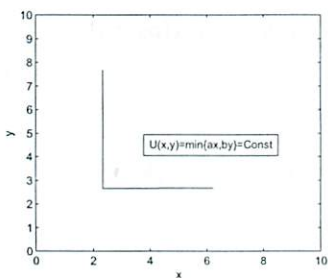


Figure 2: Utility Function of Perfect Complements

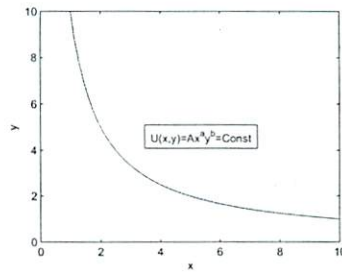


Figure 3: Cobb-Douglas Utility Function

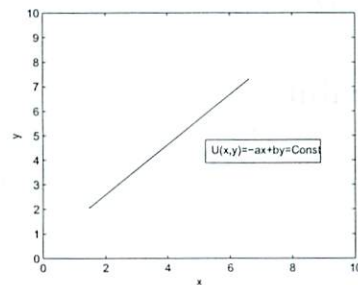


Figure 4: Utility Function of the Situation That One Good Is Bad

Because utility is constant along the indifference curve,

$$u = (x, y(x)) = C, \implies$$

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \frac{dy}{dx} = 0, \implies$$

$$-\frac{dy}{dx} = \frac{\frac{\partial u}{\partial x}}{\frac{\partial u}{\partial y}}.$$

Thus,

$$MRS = \frac{\frac{\partial u}{\partial x}}{\frac{\partial u}{\partial y}}.$$

Example (Sample utility function).

$$u(x, y) = xy^2.$$

Two ways to derive MRS:

- Along the indifference curve

$$xy^2 = C.$$

$$y = \sqrt{\frac{C}{x}}.$$

Thus,

$$MRSd = -\frac{dy}{dx} = \frac{\sqrt{C}}{2x^{3/2}} = \frac{y}{2x}.$$

- Using the conclusion above

$$MRS = \frac{\frac{\partial u}{\partial x}}{\frac{\partial u}{\partial y}} = \frac{y^2}{2xy} = \frac{y}{2x}.$$

2 Budget Constraint

The problem is about how much goods a person can buy with limited income.

Assume: no saving, with income I , only spend money on goods x and y with the price P_x and P_y .

Thus the budget constraint is

$$P_x \cdot x + P_y \cdot y \leq I.$$

Suppose $P_x = 2$, $P_y = 1$, $I = 8$, then

$$2x + y \leq 8.$$

The slope of budget line is

$$-\frac{dy}{dx} = \frac{P_x}{P_y}.$$

Bundles below the line are affordable.

Budget line can shift:

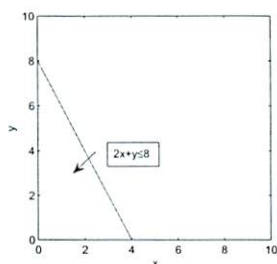


Figure 5: Budget Constraint

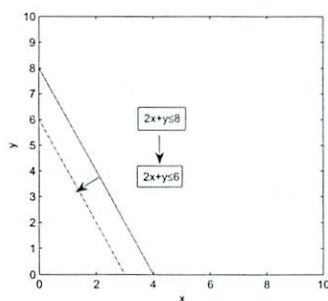


Figure 6: Budget Line Shifts Because of Change in Income

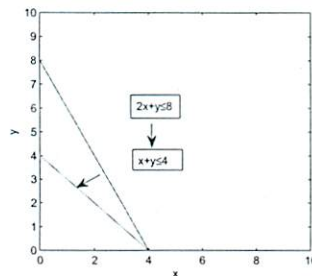


Figure 7: Budget Line Rotates Because of Change in Price

- Change in Income Assume $I' = 6$, then $2x + y = 6$. The budget line shifts right which means more income makes the affordable region larger.
- Change in Price Assume $P'_x = 2$, then $2x + 2y = 8$. The budget line changes which means lower price makes the affordable region larger.

3 Optimization: Interior Solution

Now the consumer's problem is: how to be as happy as possible with limited income. We can simplify the problem into language of mathematics:

$$\max_{x,y} U(x,y) \text{ subject to } \begin{cases} xP_x + yP_y \leq I \\ x \geq 0 \\ y \geq 0 \end{cases}.$$

Since the preference has non-satiation property, only (x, y) on the budget line can be the solution. Therefore, we can simplify the inequality to an equality:

$$xP_x + yP_y = I.$$

First, consider the case where the solution is interior, that is, $x > 0$ and $y > 0$. Example solutions:

- Method 1

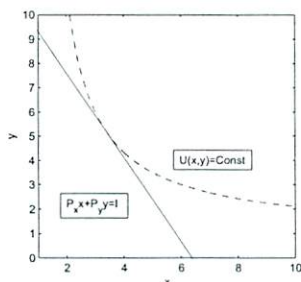


Figure 8: Interior Solution to Consumer's Problem

From Figure 8, the utility function reaches its maximum when the indifference curve and constraint line are tangent, namely:

$$\frac{P_x}{P_y} = MRS = \frac{\partial u / \partial x}{\partial u / \partial y} = \frac{u_x}{u_y}.$$

– If

$$\frac{P_x}{P_y} > \frac{u_x}{u_y},$$

then one should consume more y , less x .

– If

$$\frac{P_x}{P_y} < \frac{u_x}{u_y},$$

then one should consume more x , less y . Intuition behind $\frac{P_x}{P_y} = MRS$: $\frac{P_x}{P_y}$ is the market price of x in terms of y , and MRS is the price of x in terms of y valued by the individual. If $P_x/P_y > MRS$, x is relatively expensive for the individual, and hence he should consume more y . On the other hand, if $P_x/P_y < MRS$, x is relatively cheap for the individual, and hence he should consume more x .

- Method 2: Use Lagrange Multipliers

$$L(x, y, \lambda) = u(x, y) - \lambda(xP_x + yP_y - I).$$

In order to maximize u , the following first order conditions must be satisfied:

$$\begin{aligned}\frac{\partial L}{\partial x} = 0 &\implies \frac{u_x}{P_x} = \lambda, \\ \frac{\partial L}{\partial y} = 0 &\implies \frac{u_y}{P_y} = \lambda, \\ \frac{\partial L}{\partial \lambda} = 0 &\implies xP_x + yP_y - I = 0.\end{aligned}$$

Thus we have

$$\frac{P_x}{P_y} = \frac{u_x}{u_y}.$$

- Method 3

Since $xP_x + yP_y - I = 0$,

$$y = \frac{I - xP_x}{P_y}.$$

Then the problem can be written as

$$\max_{x,y} u(x,y) = u\left(x, \frac{I - xP_x}{P_y}\right).$$

At the maximum, the following first order condition must be satisfied:

$$\begin{aligned}u_x + u_y\left(\frac{\partial y}{\partial x}\right) &= u_x + u_y\left(-\frac{P_x}{P_y}\right) = 0. \\ &\implies \\ \frac{P_x}{P_y} &= \frac{u_x}{u_y}.\end{aligned}$$

3.2 Budget Constraints

- Consumers face limits in income so must make choices

$$- I = P_F \cdot F + P_C \cdot C$$

\uparrow \uparrow
 price amt
 food units
 food

- budget line = where total \$ spent = income

↳ buys on the budget line (more is better)

- linear usually (not qtd. discounts normally)

- to see how much C needs to be given up for F solve for C

$$C = \frac{I}{P_C} - \frac{P_F}{P_C} F$$

\uparrow Slope of budget line

Income change - line shifts out (\uparrow) or in (\downarrow)

Price change - line pivots

- inc tax

- (this was all well covered in lecture)

- if both prices change (ratio unchanged) slope will change
+ line shifts, but still parallel

- purchasing power - ability to generate utility

- depends on income + prices

(2)

3.3 Consumer Choice

Consumers want to maximize their satisfaction given their limited budget
So it's on the budget line & must be most preferred combo

$$MRS = \frac{P_F}{P_C}$$

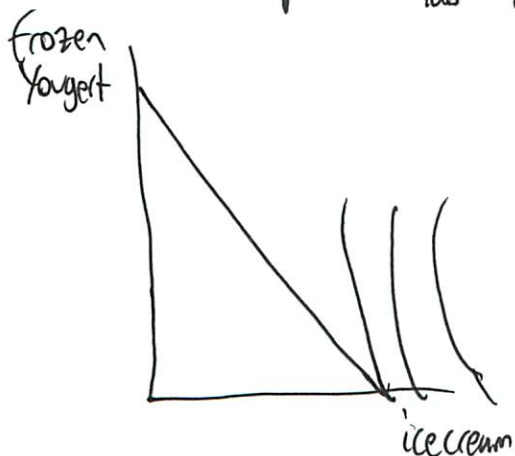
$$MB = MC$$

Slopes =

When no longer want to make trade offs

Corner Solutions

- if consumers don't want any of one good
- slopes will not = because graph does not extend
- ie consumer would want to give up frozen yogurts but at this price has all ice cream and 0 yogurt



- if price FY were to drop a lot, consumer would start buying

$$MRS \geq \frac{P_{IC}}{P_{FY}}$$

MRS does not necessarily = price ratio

③

3.4 Revealed Preferences

If we know choices a consumer made, can we determine his/her preferences?

If a consumer picks a more expensive budget, he must prefer this.



(The more budget lines you can test, the more you reveal)



$$MRT = -\frac{dB}{dA}$$

minus the slope of the production transformation curve

ie if $B = (25 - A^2)^{1/2}$

$$\frac{dB}{dA} = \frac{1}{2} (25 - A^2)^{-1/2} (-2A) \quad \text{take derivative}$$

$$= \frac{-A}{(25 - A^2)^{1/2}}$$

$$\text{So } MRT = \frac{A}{(25 - A^2)^{1/2}} = \frac{A}{B}$$

measure of opportunity cost

(ate) give up one good to get another

Law of increasing Opportunity Costs

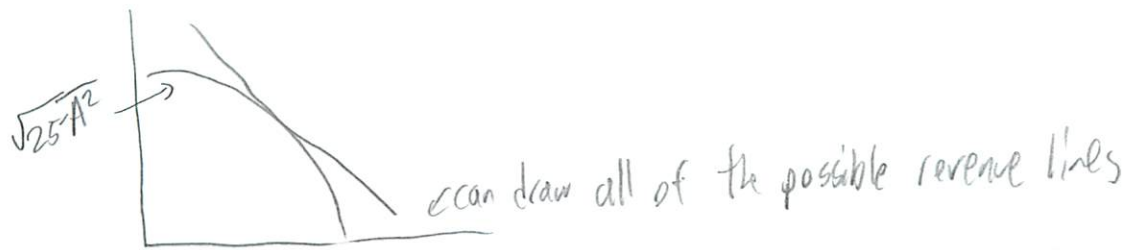
$$R(A, B) = P_A A + P_B B$$

or put in prices

$$= 2A + B$$

subject to constraint $B = \sqrt{25 - A^2}$ $0 \leq A \leq 5$

- ② So maximize a function of 2 variables subject to constraint
 - ok I get how to do graphically



$$R = 2A + B$$

iso revenue lines

$$\text{So } B = R - 2A$$

put in graphable form

So how combine again?
 - put one in the other

$$B = \frac{R}{P_b} + \left(\frac{P_a}{P_b} \right) A$$

↑
intercept

↑ slope

- well just graph both and see intersection?

- but R is not fixed

$$\text{- well } R = a \cdot p_a + b \cdot p_b$$

$$B = a \cdot p_a + b \cdot p_b - 2A$$

$$B - B \cdot P_b = a \cdot p_a - 2A$$

$$P_b = P_a \cdot A - 2A$$

But need a B as y

③

But that 'intercept / slope makes sense
- but how to get that?

Book:

Sub to get

$$R(A) = 2A + \sqrt{25 - A^2}$$

- duh - just put in for B

- but graph is a vs b

now take derivative

$$\frac{dR}{dA} = 2 + \frac{1}{2} (25 - A^2)^{-1/2} - 2A$$

Set deriv to 0 to maximize

- why that is —

- critical point

- yeah min or max

- but



* Oh I see they made new revenue only graph *

- duh!

- unshown temporary graph!

④

$$(25-A^2)^{-1/2} = 2A^{-1}$$

now solve for A

$$A = \sqrt{20}$$

and can find B w/ one of our constraints

now let me try p-set again.

- much clearer

- long summer ☺

77/100

= 57.75/75 I got such a poor grade even after going to OH for help!

14.01 Fall 2010

Problem Set 2

Due in class on September 24th

1. (10 points) Annie and David are painting their apartment. At the paint store, David says he prefers Canary Yellow to Bumblebee Yellow, Lime Yellow and Crayola Yellow. Annie finds new paint samples and asks David to compare Canary Yellow to School Bus Yellow and to Sunrise Yellow. David prefers Sunrise Yellow to Canary Yellow and prefers School Bus Yellow to Canary Yellow. He also prefers Sunrise Yellow to School Bus Yellow. The store is out of Sunrise Yellow, so they buy School Bus Yellow and paint their apartment with it. David then insists that they go back, buy Lime Yellow and repaint the apartment.

True/False/Uncertain: David has rational preferences (as we define them).

2. (20 points) In each of the following examples, a consumer purchases just two goods: x and y . Based on the information in each of the following parts, sketch a plausible set of indifference curves (that is, draw at least two curves on a set of labeled axes, and indicate the direction of higher utility). Also, write down a utility function $u(x, y)$ consistent with your graph. Note that although all these preferences should be assumed to be complete and transitive (as required for utility representation), not all will be monotone.

(a) (4 points) Jessica enjoys bagels x and coffee y , and consuming more of one makes consuming the other more enjoyable.

(b) (4 points) Plamen loves Mocha Swirl ice cream x , but he hates mushrooms y .

(c) (4 points) Jennifer likes Cheerios x , and neither likes nor dislikes Frosted Flakes y .

(d) (4 points) Edward always buys three white tank tops x for every pair of jeans y .

(e) (4 points) Nancy likes both peanut butter x and jelly y , and always gets the same additional satisfaction from an ounce of peanut butter as she does from two ounces of jelly.

3. (20 points) A consumer's preferences are representable by the following utility function:

$$u(x, y) = x^{\frac{1}{2}} + y.$$

(a) (10 points) Obtain the MRS of the consumer at an arbitrary point (\bar{x}, \bar{y}) , where $\bar{x} > 0$ and $\bar{y} > 0$.

(b) (10 points) Suppose the price of the second good is 1, and the price of the first good is denoted by $p > 0$. If the consumer's income is $m > 0$, obtain the optimal consumption bundle of the consumer (in terms of m and p). [Caution: make sure you cover cases in which m is relatively low as well as cases in which m is relatively high.]

4. (25 points) It is exactly 24 hours before Lauren's physics final, she has an economics final directly after the physics final and has no time to study in between. Lauren wants to be a physicist, so she places more weight on her physics test score. Her utility function is given by

$$u(p, e) = 0.6 \ln(p) + 0.4 \ln(e),$$

where p is the score on the physics final and e is the score on the economics final. Although she cares more about physics, she is better at economics; for each hour spent studying economics she will increase her score by 3 points but her physics score will only increase by 2 points for every hour spent studying physics. Studying zero hours results in a score of zero on both subjects (although $\ln(0)$ is not defined, assume her utility for a score of zero is negative infinity).

(a) (5 Points) What constraints does Lauren face in her test score maximization problem?

(b) (5 Points) How many hours should Lauren optimally spend studying physics? How many hours studying economics? (hours are divisible)

(c) (5 Points) What economics and physics test scores will she achieve (i.e. what are e^* and p^*)?

(d) (5 Points) What utility level will she achieve?

(e) (5 Points) Suppose Lauren can get an economics tutor. If she goes to the tutor she will increase her economics test score by 5 points for every hour spent studying instead of 3 points, but will lose 4 hours of study time by going to the tutor. She can not study while at the tutor, and going to the tutor does not directly improve her test score. Should Lauren go to the tutor?

OH = office hrs

~ = After grade

-10

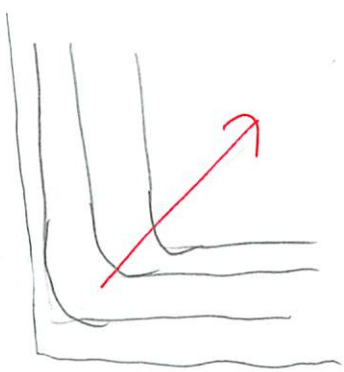
1. (WTF?)

Yes, David has a preferred paint color,

David is a little weird because he tries to suggest a different color after the apartment is painted. It is rational to disagree with David + Annie, but he should have done so before they started painting. It would be rational for Annie + David to kick this troublemaker out of the apartment.

OH they did not like this answer → they were looking for transfer

2. a, Coffee



$U = X \cdot Y$

Consuming more of one, makes consuming the other more enjoyable

~~perfect complements~~ OH: not

Bagles

b,

Mushrooms

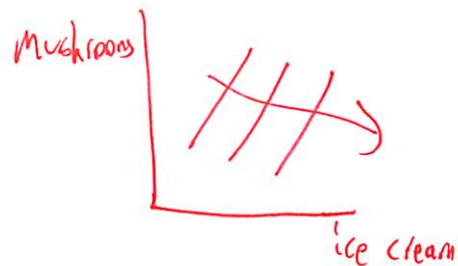


$U = \sqrt{X}$

OH: he pays to get rid of them

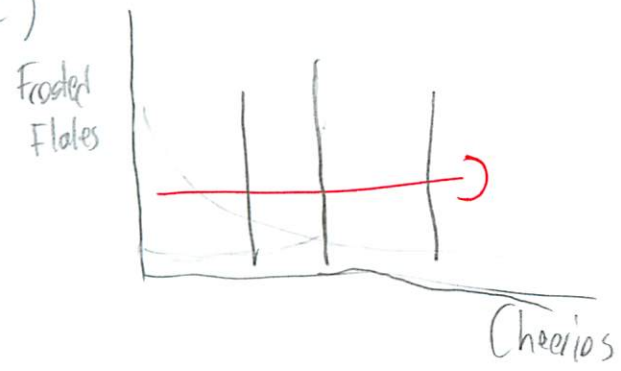
-4

ice cream Plamon told me wrong thing!



2

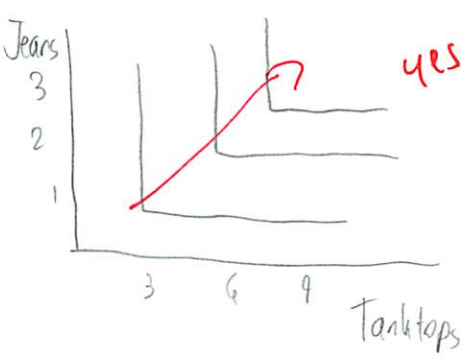
c)



$$U = \sqrt{x + \frac{1}{2}y}$$

Oh! throw out frosted flakes

d)



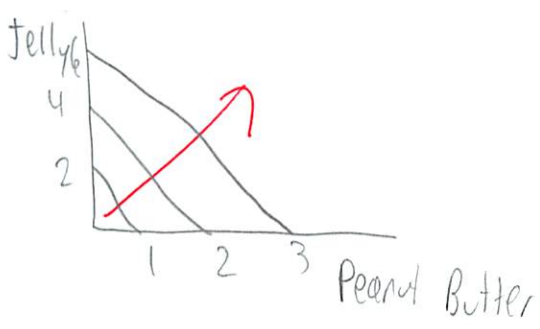
Complement

$$U = 3x + y \quad \text{X} \quad (-1)$$

$$\min(x, 3y)$$

↑ don't forget min function

e)



perfect substitutes, quantity adjusted

$$\text{slope}^{\text{indifference}} = -2 = -\frac{MU_x}{MU_y}$$

$$U = 2x + y$$

need 2 of y to x

↑ think carefully about slope



③

3. $U(x, y) = x^{1/2} + y$

a) Obtain MRS at an arbitrary point

$$-\frac{\Delta y}{\Delta x} = -1$$

$$(4, 7)$$

$$\downarrow$$

$$U = \sqrt{4} + 7 = 9$$

$$U(5, _) = 9$$

$$U = \sqrt{5} + _ = 9$$

$$_ = 6.76$$

$$7 - 6.76 = .23$$

or take derivative

$$\frac{dU}{dx} = \frac{1}{2}x^{-1/2} + y'$$

$$= \frac{1}{2\sqrt{x}} + y'$$

Changes at each point

$$\frac{dy}{dx} = \sqrt{x} + y(x)$$

$$\frac{y + y'}{x + x'}$$

$$MRS = -1.23 \text{ at } (4, 7)$$

b) Suppose price of $y = 1$, price of $x = p > 0$

Income = $m > 0$

What is optimal bundle

$$MRS = MRT$$

$$MRS =$$

$$-\frac{\Delta y}{\Delta x} = -\frac{p}{1}$$

did in office hrs \rightarrow

4)

are we supposed to guess values of m , p ?

- no ans in terms of $m + p$

- which is only the budget constraint side, so should we use previous value for \bar{x} \bar{y}

- budget $m = \bar{x} \cdot p + \bar{y} \cdot 1$

- $MU_x = \frac{du}{dx} = \frac{??}{\sqrt{x+y}} = \frac{1}{9}$

$MU_y = \frac{du}{dy} = \frac{??}{\sqrt{x+y}} = \frac{1}{9}$

MRS = -123 last time

I should work out too

$MRT = -\frac{p}{1}$

So $MRS = MRT$

-123 = -p

so p should = 123

and then income would be

$4 \cdot 123 + 7 \cdot 1 = 28$

but income is determined 1st

- but we just look at slope

I don't think I can assign $p = 123$

5) Retry

$$M = \bar{X} p + \bar{Y} \cdot 1$$

Solve for p

$$M - \bar{Y} = \bar{X} p$$

$$p = \frac{M - \bar{Y}}{\bar{X}}$$

$$\frac{\bar{Y} + \bar{Y}'}{\bar{X} + \bar{X}'} = \frac{M - \bar{Y}}{\bar{X}}$$

;

no don't we
want in terms of p

(6)

$u(x, y)$

$$\{ \bar{u} = u(x, y) \}$$

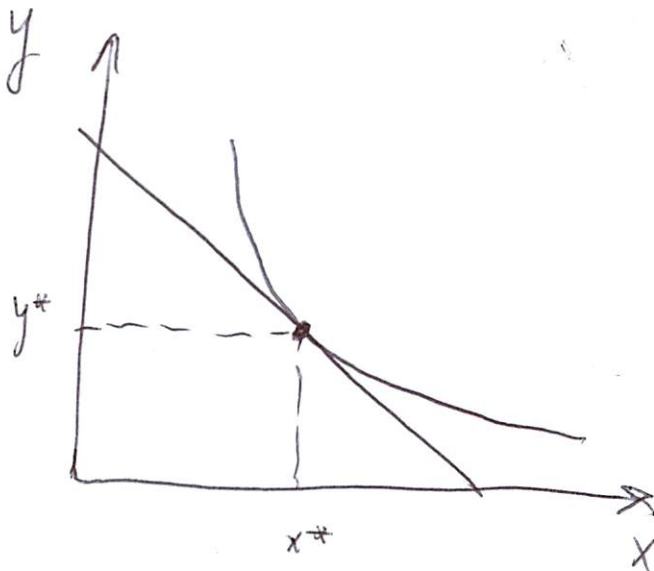
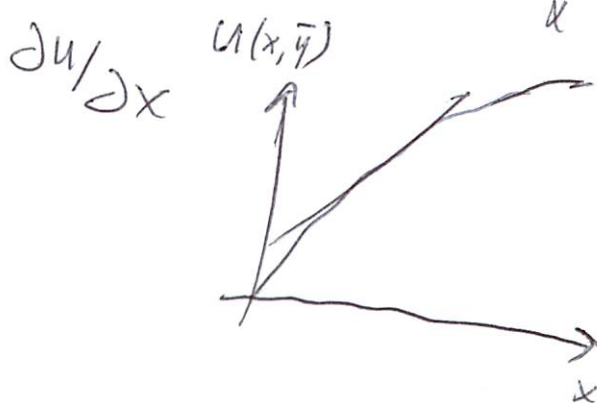
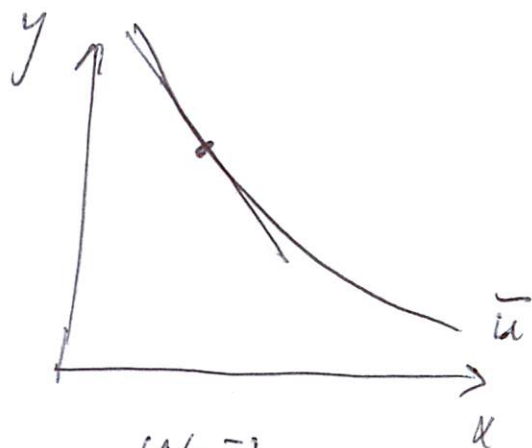
~~$\bar{u} = u(x, y)$~~

$$\frac{dy}{dx}$$

$$\frac{dy}{dx} = - \frac{\partial u / \partial x}{\partial u / \partial y}$$

$$MRS = - \frac{\partial u / \partial x}{\partial u / \partial y}$$

from Oth



7) OH

a Slope indifference = $-\frac{MU_x}{MU_y}$



$\frac{dy}{dx}$ implicit differentiate

$$\frac{dy}{dx} = - \frac{du/dx}{du/dy}$$

now plug in

$$\frac{du}{dx} = \frac{1}{2} x^{-1/2} + 0^{\text{constant}}$$

$$\frac{du}{dy} = 0 + 2$$

$$\frac{-\frac{1}{2\sqrt{x}}}{1} = \boxed{-\frac{1}{2\sqrt{x}}}$$

8

3b O.H.:

~~MAT~~ $M = x \cdot p_x + y \cdot p_y$

$$M = p_x x + p_y y$$

tangent = slopes =

slopes $MRS = MAT$

$$-\frac{1}{2\sqrt{x}} = \text{slope budget line} = -p$$

- lies on budget line

Solve for x in terms of p

$$\frac{1}{2\sqrt{x}} = p$$

$$\frac{1}{\sqrt{x}} = 2p$$

$$\sqrt{x} = \frac{1}{2p} \quad \downarrow x^{-1}$$

$$x = \frac{1}{4p^2}$$

optimal amt y ✓

(91)

Non optimal y

$$m = p \left(\frac{1}{4p^2} \right) + y$$

$$m = \frac{1}{4p} + y$$

$$y = m - \frac{1}{4p} \quad \checkmark$$

Now the caution

- Can not consume ~~#~~ $y < 0$

- Corner case

- So m and p must be such that $m - \frac{1}{4p} \geq 0$

- if $< 0 \rightarrow$ corner case

then y just $= 0 \quad \checkmark$

buy all of y ~~x~~ (use all income) \checkmark

because ~~budget~~/income too low

- like a poor person who spends all $\$$ on food

(10)

4. 24 hrs before physics final

1 hr study econ = +3 pts

1 hr " physics = +2 "

$$U(p, e) = .6 \ln(p) + .4 \ln(e)$$

p = score on physics

e = econ

a) What are constraints

24 hrs max ✓

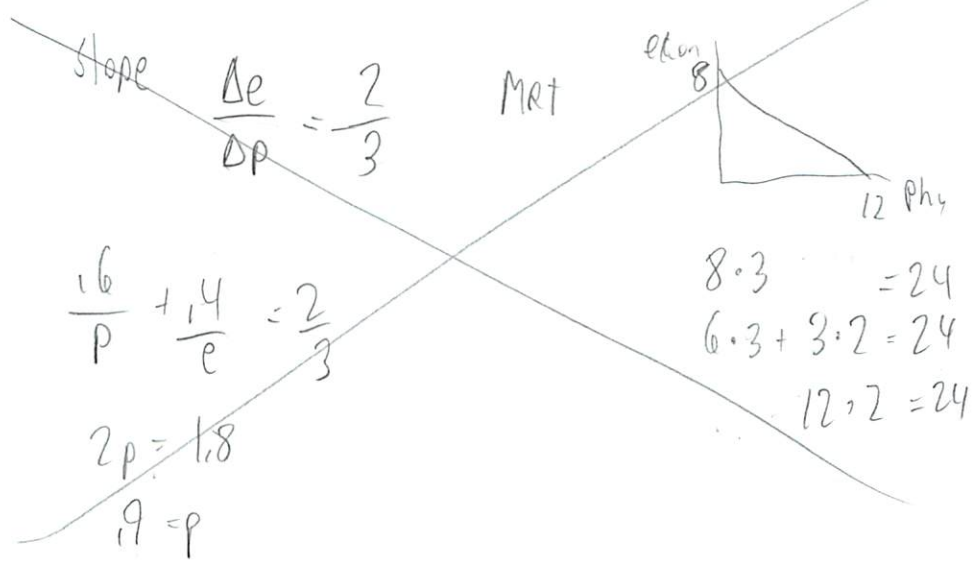
(assuming sleep does help hort)

b) What is max utility?

take derivative

$$\frac{dU}{dp} = \frac{.6}{p} + \frac{.4}{e} \quad \text{MRS}$$

~~$$24 = 3e + 2p$$~~



also need nonnegativity constraint -1

well duh!
(but need to put on test)

lets see if I can get this
I think I was not derivative

Why can't I do a simple optimization problem

(11)

try graphically



Where are slopes = ?

(Read Econ w/ Calculus Book) → (been long summer)

So want to maximize Utility

w/ constraint $3e + 2p = 24$

$$3e = 24 - 2p$$

$$e = 8 - \frac{2}{3}p$$

$$U = .6 \ln(p) + .4 \left(\ln \left(8 - \frac{2}{3}p \right) \right)$$

take deriv

$$\frac{.6}{p} + \frac{.4 \cdot 2}{3 \left(8 - \frac{2}{3}p \right) p^2}$$

← put in wolfram as $2p/3$ not $2/3p$!

Set = 0 + solve for p

calculator

$$p = 7.2$$

(12)

~~$$\text{So } e = 8 - \frac{2}{3}p = 8 - \frac{2}{3}(7.2)$$

$$= 3.2$$~~

- finally, took a hr to review optimization
- I feel so stupid)

C. What will the score

no $e + p = 24$

don't care that 3 and 2 points yet, just maximize utility

$$U = .6 \ln(p) + .4 \ln(24 - p)$$

$$\frac{dU}{dp} = \frac{.6}{p} - \frac{.4}{24 - p}$$

$$p = 14.4 \quad \checkmark$$

$$e = 24 - 14.4 = 9.6 \quad \checkmark$$

c) Scores

$$p = 2 \cdot 14.4 = 28.8 \quad \checkmark$$

$$e = 3 \cdot 9.6 = 28.8 \quad \checkmark$$

d) She has achieved maximum utility level for the function under the constraint.

In this case she gets the same score on each exam

what is U ?

(-5)

$$U(p, e) = .6 \ln(28.8) + .4 \ln(28.8) = 3.36$$

(13)

e) Tutor

5 points for every hr w/ tutor
but lose 4 hrs

$$\text{So } U = .6 \ln(p) + .4 \ln(20-p)$$

$$\frac{dU}{dp} = \frac{.6}{p} - \frac{.4}{20-p}$$

$$p = 12$$

$$e = 8$$

$$\text{Score } p^* = 12 \cdot 2 = 24 \checkmark$$

$$e^* = 8 \cdot 5 = 40 \checkmark$$

Her econ score will improve, but at the expense of her physics score. If she values her physics score more it's a bad plan. She may also want to update her utility function.

(2)

→ no. Given this utility function, is this a good choice?

$$U = .6 \ln(24) + .4 \ln(40) = 3.38$$

↑ higher, so
should go

I really should redo this + p set 3 # 5 before test

14.01 Fall 2010

Problem Set 2

Due in class on September 24th

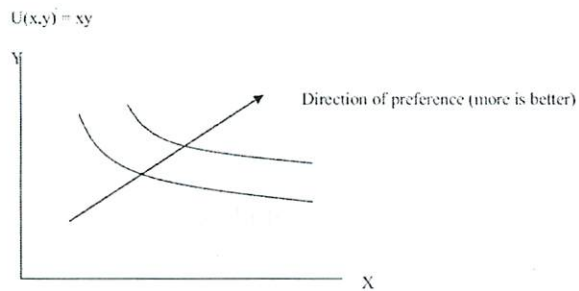
1. (10 points) Annie and David are painting their apartment. At the paint store, David says he prefers Canary Yellow to Bumblebee Yellow, Lime Yellow and Crayola Yellow. Annie finds new paint samples and asks David to compare Canary Yellow to School Bus Yellow and to Sunrise Yellow. David prefers Sunrise Yellow to Canary Yellow and prefers School Bus Yellow to Canary Yellow. He also prefers Sunrise Yellow to School Bus Yellow. The store is out of Sunrise Yellow, so they buy School Bus Yellow and paint their apartment with it. David then insists that they go back, buy Lime Yellow and repaint the apartment.

True/False/Uncertain: David has rational preferences (as we define them).

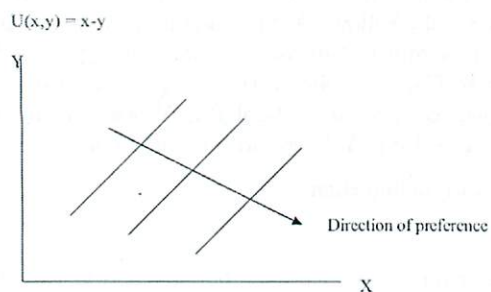
False. He violates transitivity.

2. (20 points) In each of the following examples, a consumer purchases just two goods: x and y . Based on the information in each of the following parts, sketch a plausible set of indifference curves (that is, draw at least two curves on a set of labeled axes, and indicate the direction of higher utility). Also, write down a utility function $u(x, y)$ consistent with your graph. Note that although all these preferences should be assumed to be complete and transitive (as required for utility representation), not all will be monotone.

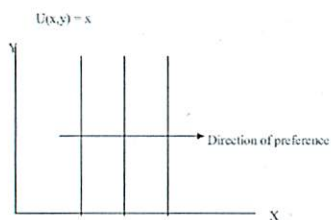
(a) (4 points) Jessica enjoys bagels x and coffee y , and consuming more of one makes consuming the other more enjoyable.



(b) (4 points) Plamen loves Mocha Swirl ice cream x , but he hates mushrooms y .

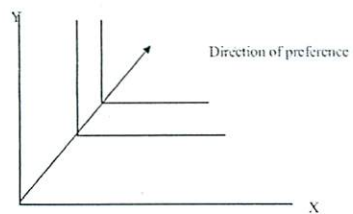


(c) (4 points) Jennifer likes Cheerios x , and neither likes nor dislikes Frosted Flakes y .



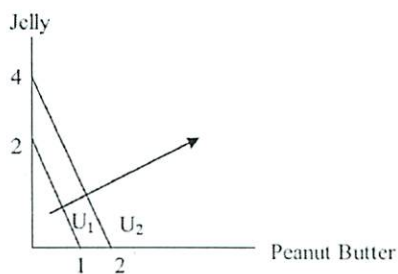
(d) (4 points) Edward always buys three white tank tops x for every pair of jeans y .

$$U(x,y) = \min(x, 3y)$$



- (e) (4 points) Nancy likes both peanut butter x and jelly y , and always gets the same additional satisfaction from an ounce of peanut butter as she does from two ounces of jelly.

$$U(x,y) = 2x + y \text{ where } U_1 < U_2$$



3. (20 points) A consumer's preferences are representable by the following utility function:

$$u(x, y) = x^{\frac{1}{2}} + y.$$

- (a) (10 points) Obtain the MRS of the consumer at an arbitrary point (\bar{x}, \bar{y}) , where $\bar{x} > 0$ and $\bar{y} > 0$.

$$MRS = -\frac{\frac{1}{2}x^{-\frac{1}{2}}}{1} = -\frac{1}{2}x^{-\frac{1}{2}}$$

(b) (10 points) Suppose the price of the second good (y) is 1, and the price of the first good (x) is denoted by $p > 0$. If the consumer's income is $m > 0$, obtain the optimal consumption bundle of the consumer (in terms of m and p). [Caution: make sure you cover cases in which m is relatively low as well as cases in which m is relatively high.]

The consumer solves $\max x^{\frac{1}{2}} + y$ s.t. $px + y = m$. We look for stationary values of the lagrangian $L = x^{\frac{1}{2}} + y + \lambda(m - px - y)$. The first-order conditions for stationarity are

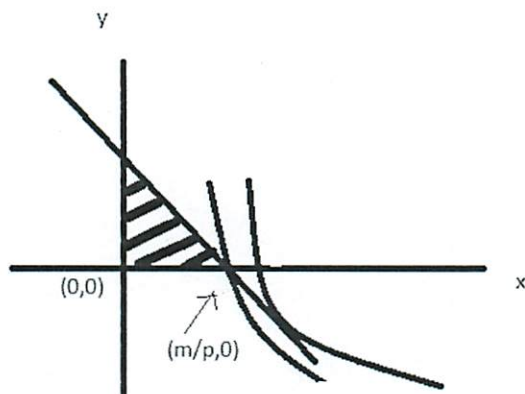
$$\begin{aligned}\frac{\partial L}{\partial x} &= \frac{1}{2}x^{-\frac{1}{2}} - \lambda p = 0 \\ \frac{\partial L}{\partial y} &= 1 - \lambda = 0 \\ \frac{\partial L}{\partial \lambda} &= m - px - y = 0\end{aligned}$$

Combining the first two equations above gives $\frac{1}{2x^{\frac{1}{2}}} = p$, or $x^* = \frac{1}{4p^2}$. Substituting x^* into the budget constraint gives $y = m - px^* = m - \frac{1}{4p}$.

Case 1): $m \geq \frac{1}{4p} \Rightarrow x^* = \frac{1}{4p^2}$ and $y = m - \frac{1}{4p} \geq 0$.

Case 2): $m \leq \frac{1}{4p} \Rightarrow x^* = \frac{m}{p}$ and $y = 0$.

Graphically,



4. (25 points) It is exactly 24 hours before Lauren's physics final, she has an economics final directly after the physics final and has no time to study in between. Lauren wants to be a physicist, so she places more

weight on her physics test score. Her utility function is given by

$$u(p, e) = 0.6\ln(p) + 0.4\ln(e),$$

where p is the score on the physics final and e is the score on the economics final. Although she cares more about physics, she is better at economics; for each hour spent studying economics she will increase her score by 3 points but her physics score will only increase by 2 points for every hour spent studying physics. Studying zero hours results in a score of zero on both subjects (although $\ln(0)$ is not defined, assume her utility for a score of zero is negative infinity).

(a) (5 Points) What constraints does Lauren face in her test score maximization problem?

$$\begin{aligned} H_P + H_e &\leq 24 \\ H_P &\geq 0, H_e \geq 0. \end{aligned}$$

One might also consider production constraints:

$$p = 2H_p, e = 3H_e.$$

(b) (5 Points) How many hours should Lauren optimally spend studying physics? How many hours studying economics? (hours are divisible)

$$\begin{aligned} &\max_{H_e} 0.6\ln(2 \times (24 - H_e)) + 0.4\ln(3H_e) \\ \implies F.O.C. &0.6 \times \frac{2}{48 - 2H_e} = 0.3 \times \frac{1}{H_e} \\ \implies &H_e = 9.6, H_p = 14.4 \end{aligned}$$

(c) (5 Points) What economics and physics test scores will she achieve (i.e. what are e^* and p^*)?

$$\begin{aligned} e^* &= 3H_e = 28.8 \\ p^* &= 2H_p = 28.8 \end{aligned}$$

(d) (5 Points) What utility level will she achieve?

$$u(p, e) = 0.6\ln(p) + 0.4\ln(e) = 3.36$$

(e) (5 Points) Suppose Lauren can get an economics tutor. If she goes to the tutor she will increase her economics test score by 5 points for every hour spent studying instead of 3 points, but will lose 4 hours of study time by going to the tutor. She can not study while at the tutor, and going to the tutor does not directly improve her test score. Should Lauren go to the tutor?

$$\begin{aligned} &\max_{H_e} 0.6\ln(2 \times (20 - H_e)) + 0.4\ln(5 \times H_e) \\ F.O.C. &0.6 \times \frac{2}{40 - 2H_e} = \frac{0.4}{H_e} \\ \implies &H_e = 8, H_p = 20 - H_e = 12 \\ &e = 40, p = 24, u(24, 40) = 3.38 \end{aligned}$$

Lauren has higher utility when she goes to the tutor, so she should go.

14.01 Recitation

9/24

Tutor Tu 3:30-5 and 7-9

Exam in week
and half

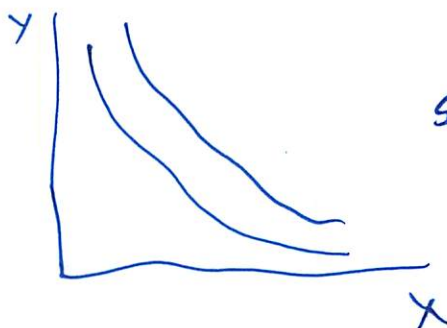
1. Review intuition behind utility maximization
2. Math. of Consumer Choice
3. Corner Solutions

Going over P-set², did in Otl

P-set 1 got a lower grade than most

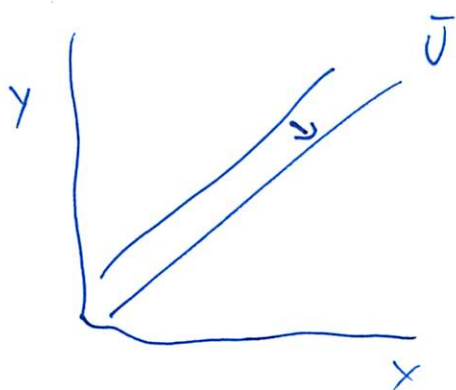
2a.

$$MRS = - \frac{MU_x}{MU_y}$$



$$U(x, y) = x \cdot y \\ \text{or } \sqrt{x \cdot y}$$

2b.

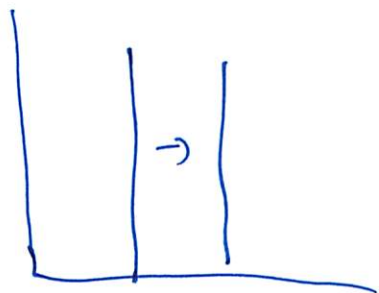


Need ice cream to wash out
mushrooms

to keep utility constant

$$U(x, y) = x - y$$

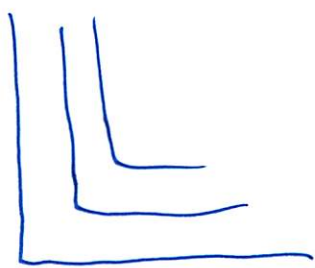
2c.



Chereos vs Frosted flakes

②

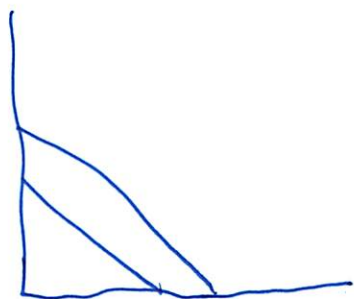
d)



indifference kicks in after a point

$$U(x, y) = \min\{x, 3y\} = \begin{cases} x & x \leq 3y \\ 3y & x > 3y \end{cases}$$

e)



add. satisfaction always the same
marginal utility does not change
MRS = Constant

3. a) $U(x, y) = \sqrt{x} + y$

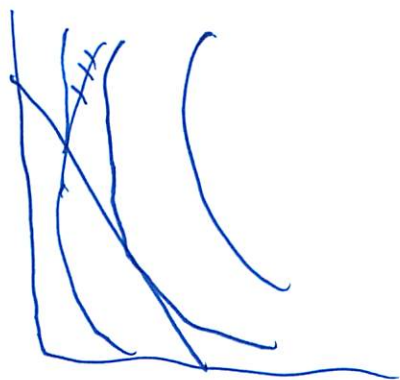
$$MRS = -\frac{MU_x}{MU_y} = \frac{-\frac{1}{2}x^{-1/2}}{1} = -\frac{1}{2}x^{-1/2}$$

b) $P_y = 1, P_x = p, I = m$

MRT = $-p$

Now optimal point

$$MRS = MRT$$



For interior solution
 $x > 0, y > 0$

③

$$1) \text{ MRS} = \text{MRT}$$

$$-\frac{1}{2}x^{-1/2} = -p$$

$$x = \frac{1}{4p^2}$$

2.) Budget constraint

$$p \cdot \frac{1}{4p^2} + y$$

$$x = m - \frac{1}{4p}$$

) interior solution

edge cases / corner solutions

$m \geq \frac{1}{4p}$ then interior solution works

$$x^* = \frac{1}{4p^2}$$

$$y^* = m - \frac{1}{4p}$$

$m < \frac{1}{4p}$ then $x^* = \frac{m}{p}$ $y^* = 0$

$$\text{MRS} = \frac{-MU_x}{MU_y} < -\frac{p_x}{p_y} = \text{MRT}$$

$$\frac{MU_x}{p_x} < \frac{MU_y}{p_y}$$

$$\frac{MU_x}{p_x} = \frac{\frac{1}{2} \left(\frac{m}{p}\right)^{-1/2}}{p}$$

$$\frac{MU_y}{p_y} = 1$$

4)

if utility is linear in ^{at least} one of the goods \Rightarrow good chance of corner solution

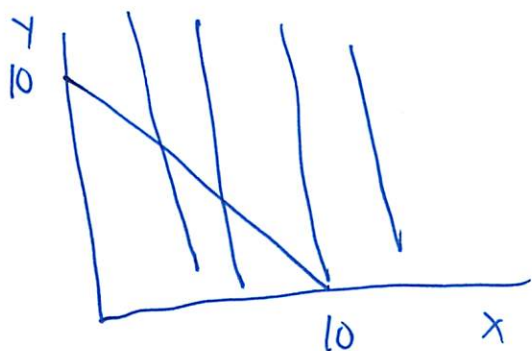
$$U(x, y) = 2x + y$$

$$P_x = 1$$

$$P_y = 1$$

$$I = 10$$

so budget $x + y = 10$



$$\left. \begin{array}{l} MRS = -2 \\ MRT = -1 \end{array} \right\} \text{can't ever be } =$$

$$\frac{MU_x}{P_x} = \frac{MU_y}{P_y}$$

$$\frac{2}{1} \neq \frac{1}{1}$$

⑤

4. Last problem \rightarrow straight forward

$$U(p, e) = .6 \ln(p) + .4 \ln(e)$$

$$\text{budget: } \frac{1}{2}p + \frac{1}{3}e = 24$$

or

$$U(p, e) = .6 \ln(2h_p) + .4 \ln(3h_e)$$

$$h_e + h_p = 24$$

Topps perhaps I did it wrong
assumed that utility function
already controls for 2/3 pts
this p-set was not well written

$$U = U(x, y)$$

$$dU = 0 = \frac{\partial U(x, y)}{\partial x} \cdot dx + \frac{\partial U(x, y)}{\partial y} \cdot dy$$

Solve for

$$\frac{dy}{dx} = - \frac{\partial U(x, y) / \partial x}{\partial U(x, y) / \partial y} = - \frac{MU_x}{MU_y}$$

$$MRS = MRT$$

- must use constraint optimization

$$\max_{x, y} U(x, y)$$

$$\text{such that } p_x x + p_y y = I$$

⑥ Lagrange function

$$L(x, y, \lambda) = U(x, y) + \lambda \cdot (I - p_x \cdot x - p_y \cdot y)$$

↑
the add. utility you get
if your income ↑ 1 unit

Example: $U(x, y) = \sqrt{x \cdot y}$

First order differentiation

$$\begin{aligned} \frac{\partial L}{\partial x}: \quad & \frac{1}{2} x^{-1/2} y^{1/2} - \lambda \cdot p_x = 0 \\ y: \quad & \frac{1}{2} y^{-1/2} x^{1/2} - \lambda \cdot p_y = 0 \\ \lambda: \quad & I - p_x(x) - p_y(y) = 0 \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} \frac{1}{2} x^{-1/2} y^{1/2} = \lambda \cdot p_x \\ \frac{1}{2} x^{1/2} y^{-1/2} = \lambda \cdot p_y \end{array}$$

$$\lambda = \frac{\frac{1}{2} x^{-1/2} y^{1/2}}{p_x} = \frac{\frac{1}{2} x^{1/2} y^{-1/2}}{p_y}$$

$$MRS = MRT$$

3.5 Marginal Utility + Consumer Choice

Want highest indifference curve given consumer income w/ max utility

Marginal utility - extra utility from 1 more unit of the good

diminishing marginal utility - fries less good after 1st one
sick by 3rd or 4th box

MU_F = marginal utility of 1 unit of food

ΔF = # units of food
add.

$$0 = MU_F(\Delta F) + MU_C(\Delta C)$$

$$\boxed{-\frac{\Delta C}{\Delta F} = \frac{MU_F}{MU_C} = MRS} = \frac{P_F}{P_C}$$

$$\frac{MU_F}{P_F} = \frac{MU_C}{P_C}$$

? budget allocated when ~~each~~ marginal \$
for ~~it~~ brings same utility for each good

"equal marginal principle"

Food Stamps

- \$ for food for poor people
 - ↳ below US poverty line
- today ~~debit~~ debit card not coupons

Figure A-1

X → cares a lot about other stuff

Y → cares a lot about food

gov could send check for \$500

- budget line then just out parallel

X → would spend almost all of it on other goods

Y → would spend a lot of it on food

So gov could only give \$500 food coupon

- line in blue



Y - spending \$900 on food

- does not spend any more on food
- just funds \$500 w/ stamps
- and spends the \$500 "savings" as if it was cash

(2)

X \rightarrow could choose that point but didn't

- revealed preference: shows that they would be less well off here
- made worse off by stamps rather than cash

Paternalism

- don't trust them with cash
- what is the cost of stamps rather than cash

Y \rightarrow no change

X \rightarrow worse off: welfare loss

Empirical

~~15% of the~~

- ~~the~~ people spend 15% more on food
w/ stamps rather than cash

depends how paternalistic we are

Where do demand curves come from?

- figure 6-2
-

Demand curves can change

- based on income
- figure 6-3
- can make Engle curve: income vs demand for a good
- income elasticity of demand

③

- ~~change of 2%~~

- $\gamma = \frac{\Delta Q/Q}{\Delta Y/Y} \leftarrow \frac{\Delta P/P}{\text{income}}$

- for most goods: upward sloping $\gamma > 0$
"normal goods"

- Some goods "inferior ~~good~~ goods" $\gamma < 0$

- goods substitute away from as you get richer

- potatoes

- canned food

- more is always better, but you substitute away from

- necessities $\gamma < 1$

- as income doubles don't buy double

- food

- luxuries $\gamma > 1$

- increase more proportional w/ income

- cars

- can have constant or changing curve
↑ usual

④

What determines slope of demand curve?

Price Elasticity

① Substitution effect

- change in demand as $P \uparrow$, holding utility constant

$$\left| \frac{\Delta P/P}{\Delta Q/Q} \right|_{\bar{U}}$$

- as good gets expensive, ~~will~~ how do you shift away

② Income effect

- change in q demanded as income \uparrow , holding prices constant

$$\left| \frac{\Delta Q/Q}{\Delta Y/Y} \right|_{\bar{P}}$$

figure 6.4

- decompose income + substitution effect

Substitution effect is always $\ominus (< 0)$

- has to be because graphically

$$\frac{MU_M}{MU_P} = \frac{P_M}{P_P}$$

$$MRS = MRT$$

- utility constant

- so if have less, you are less happy

⑤

2 reasons

- relative prices change
- you are relatively poorer

Giffen good

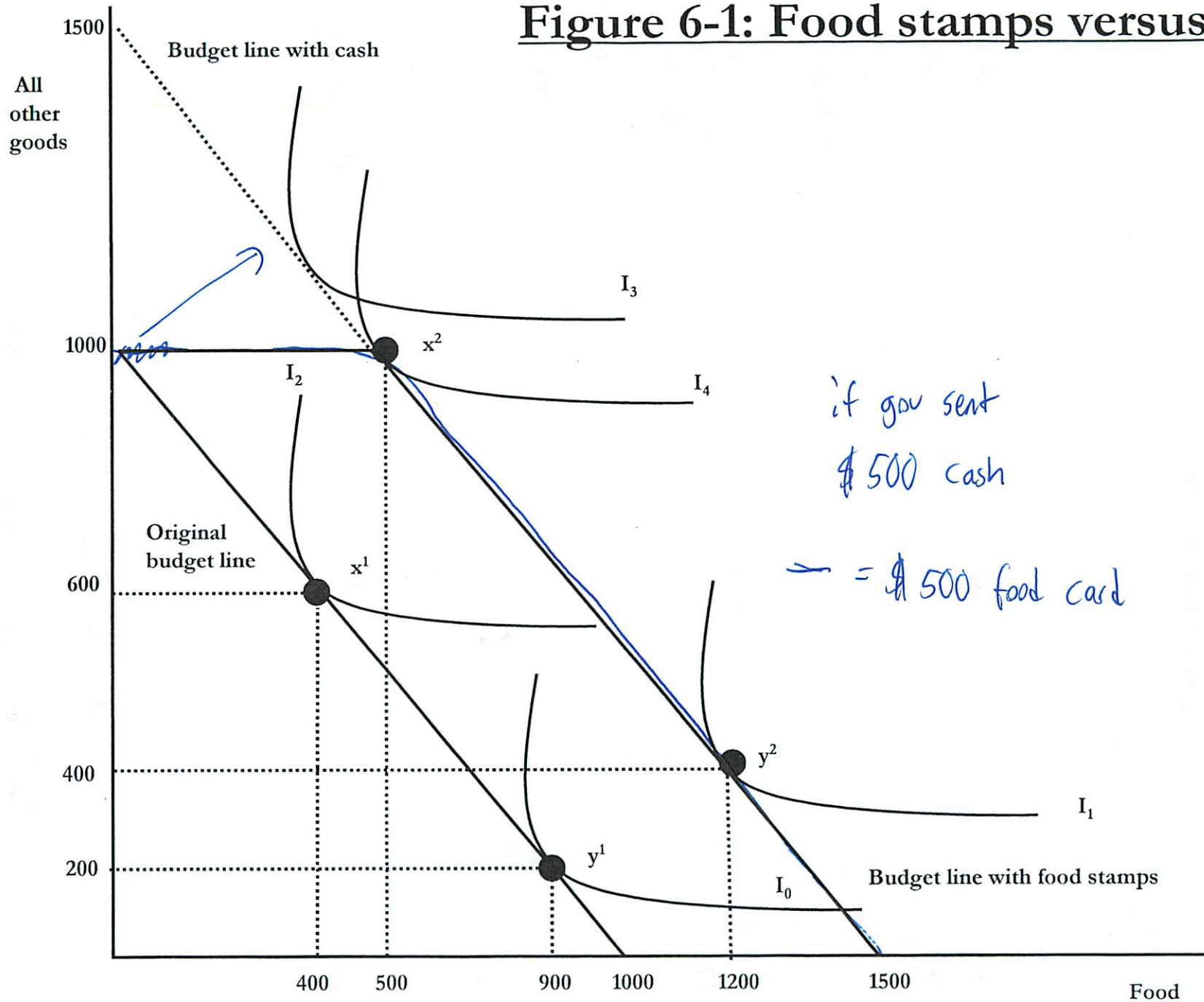
- inferior
- price goes \uparrow , you want more
- luxury goods, status symbols
- he thinks that none of these exist

~~x 20 giffen, I think~~

upward sloping demand curve

- income effect $>$ sub effect
+

Figure 6-1: Food stamps versus cash



if you sent
\$500 cash

→ = \$500 food card

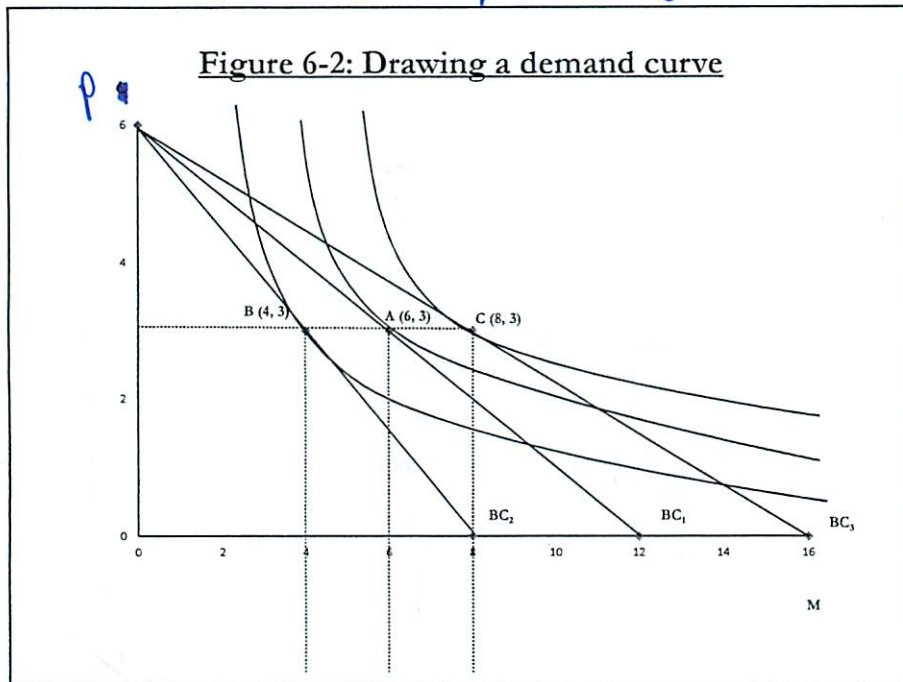
Lecture 6

9/20/17

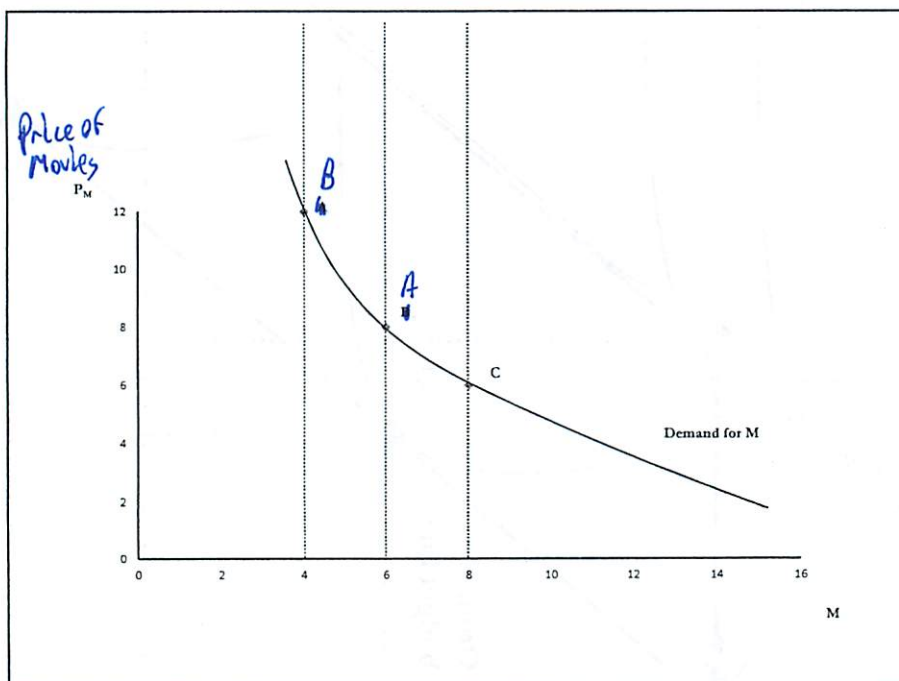
Choices you make as prices change

Given $U = \sqrt{p \cdot m}$

~~where~~
Special case \rightarrow
that demand for
pizza does not
change =
Special case :
no cross price
elasticity



Start at pt A
Say price of Movies
\$8 \rightarrow \$12
- steepens budget
constraint &
moves it inward
 $-\frac{1}{2} \rightarrow -\frac{3}{4}$
- would choose B



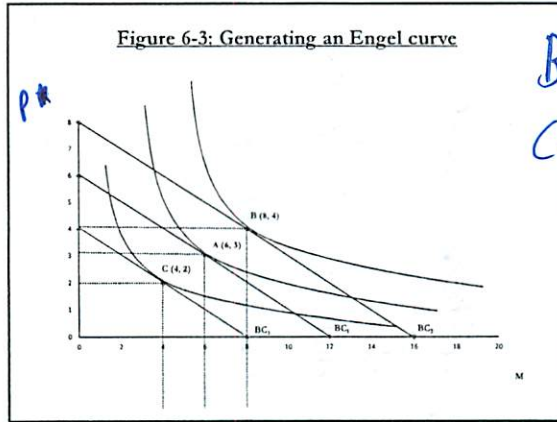
if price movies \downarrow
than budget curve
flattens

- would choose C

MRS always = MRT

\uparrow where demand curves come from
should be able to draw for pizzas

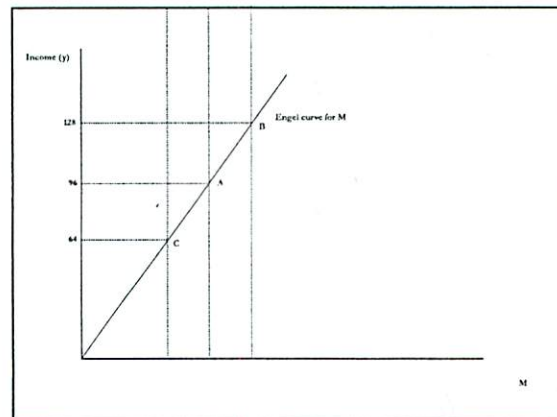
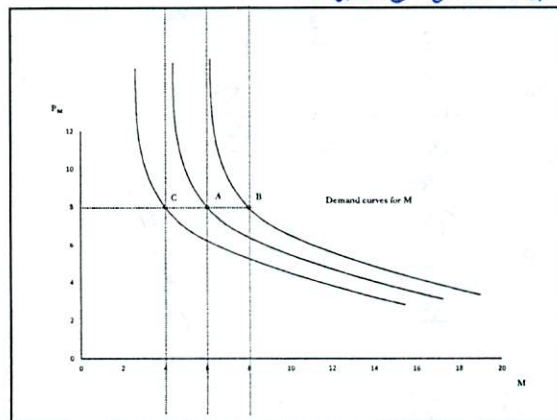
Figure 6-3: Generating an Engel curve



B = income rises
 C = income falls

↓ translate

Demand curve for movies

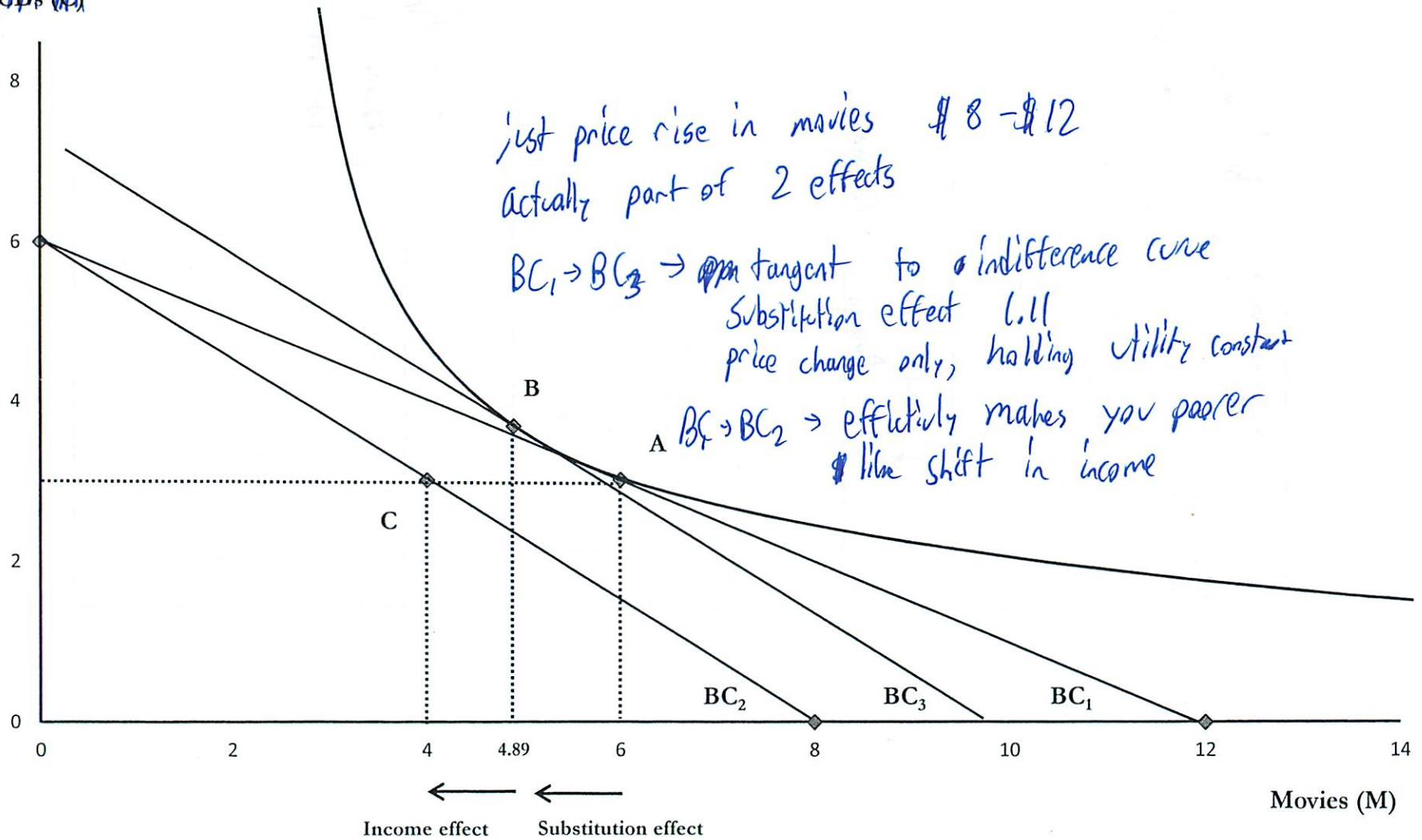


relationship b/w
 income + demand
 for movies

Figure 6-4 and 7-1: Income and substitution effect

Pizzas (P)
\$ (M)

normal good



real world is just shift $BC_1 \rightarrow BC_2$
- BC_3 is imaginary middle step

14.01 Principles of Microeconomics, Fall 2007

Chia-Hui Chen

September 17, 2007

Lecture 6

Optimization, Revealed Preference, and Deriving Individual Demand

Outline

1. Chap 3: *Corner Solution of Optimization*
2. Chap 3: *Revealed Preference*
3. Chap 4: *Deriving Individual Demand, Engle Curve*

1 Corner Solution of Optimization

When we have an interior solution,

$$\frac{P_x}{P_y} = \frac{U_x}{U_y}$$

must be satisfied. However, sometimes a consumer gets highest utility level when $x = 0$ or $y = 0$. If that's the case, we have corner solutions, and

$$\frac{P_x}{P_y} \neq \frac{U_x}{U_y},$$

as shown in Figure 1.

In Figure 1, because people cannot consume negative amounts of goods (bundle A), their best choice is to consume bundle B, so the quantity of y consumed is zero. Conditions for corner solutions:

•

$$MRS = \frac{U_x}{U_y} > \frac{P_x}{P_y} \text{ when } y = 0.$$

•

$$MRS = \frac{U_x}{U_y} < \frac{P_x}{P_y} \text{ when } x = 0.$$

Example (An example of consumer's problem). The parameters are

$$P_x = 1,$$

$$P_y = 1,$$

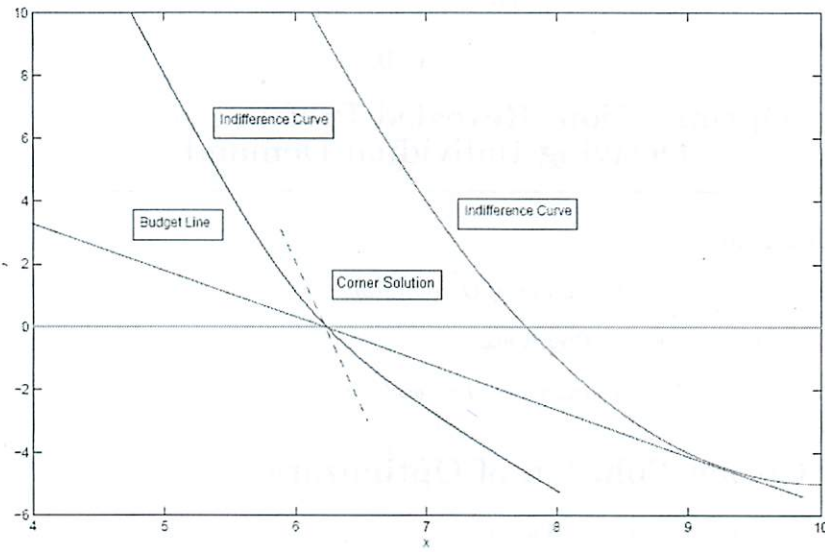


Figure 1: Corner Solution to Consumer's Problem.

$$I = 2.$$

The utility function is

$$U(x, y) = x + 2\sqrt{y}.$$

The budget constraint is

$$x + y = 2.$$

According to the condition for an interior solution:

$$\frac{P_x}{P_y} = \frac{U_x}{U_y}.$$

\Rightarrow

$$\frac{1}{1} = \frac{1}{\frac{1}{\sqrt{y}}}.$$

\Rightarrow

$$y = 1 \Rightarrow x = 1.$$

If the price y changes to 1:

$$P_y = 1,$$

then the solution is

$$y = 4 \Rightarrow x = -3 < 0,$$

which is impossible.

Then we have the corner solution:

$$x = 0, y = 2.$$

$x = 0$ since consumer wants to consume as little as possible.

2 Revealed Preference

In the former chapters, we discussed how to decide optimal consumption from utility function and budget constraint:

Utility Function

\Rightarrow Optimal Consumption

Budget Constraint

And now we discuss how to know consumer's preference from budget constraint and consumption:

Budget Constraint

\Rightarrow Preference

Consumption

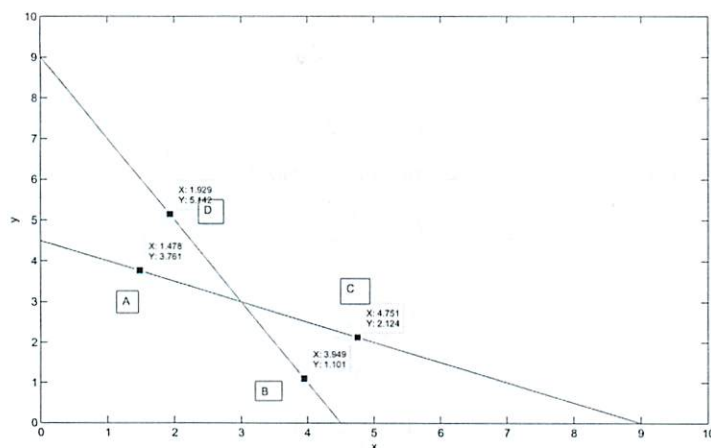


Figure 2: A Contradiction of Preference. A and B are the Choices.

Example (Revealed preference). In Figure 2, two budget constraint lines intersect. Assume one person's choices are A and B respectively. Then we have

$$A \succsim C,$$

$$B \succsim D.$$

And Figure 2 obviously shows that

$$C \succ B,$$

$$D \succ A.$$

Thus,

$$A \succsim C \succ B \succ D \succ A,$$

which is a contradiction, which means utility does not optimized and the choice is not rational.

3 Deriving Individual Demand, Engle Curve

Use the following utility function again:

$$U(x, y) = x + 2\sqrt{y},$$

with a budget constraint:

$$P_x x + P_y y = I.$$

When

$$I \geq \frac{P_x^2}{P_y},$$

we have an interior solution. $MRS = P_x/P_y$. Thus,

$$x = \frac{I}{P_x} - \frac{P_x}{P_y},$$

$$y = \left(\frac{P_x}{P_y} \right)^2.$$

When

$$I \leq \frac{P_x^2}{P_y},$$

we have a corner solution.

$$x = 0,$$

$$y = \frac{I}{P_y}.$$

- Figure 3 shows a demand function of y and P_y as an example. (Assume that I , x and P_x are held constant.)
- Engle Curve describes the relation between quantity and income. Figure 4 shows the relation between x and income, and Figure 5 shows that between y and income.

Normal good. Quantity demanded of good increases with income.

Inferior good. Quantity demanded of good decreases with income.

Substitutes. Increase in price of one leads to an increase in quantity demanded of the other.

Complements. Increase in price of one leads to a decrease in quantity demanded of the other.

For this problem,

- if $I < \frac{P_x^2}{P_y}$, x and y are neither substitutes nor complements, and x is a normal good.
- if $I \geq \frac{P_x^2}{P_y}$, x and y are substitutes, and y is a normal good.

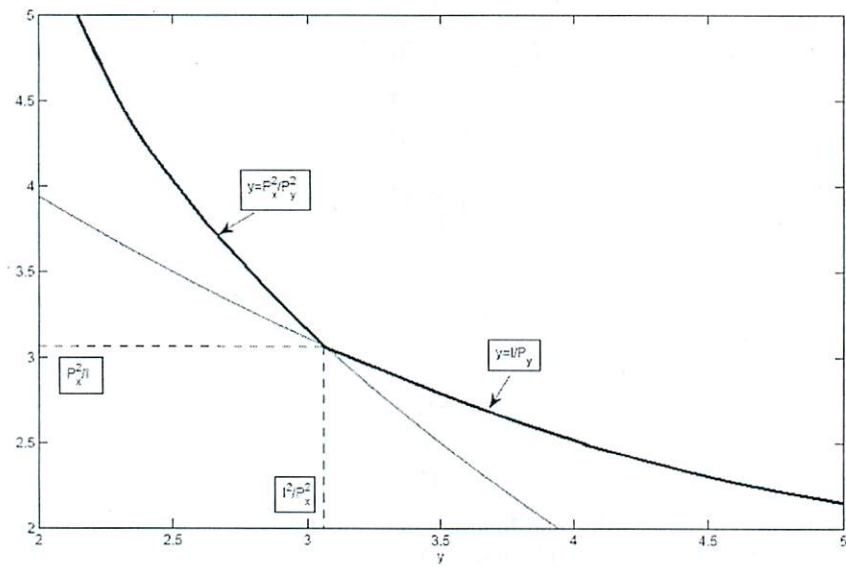


Figure 3: Demand Function for Goods 'y'.

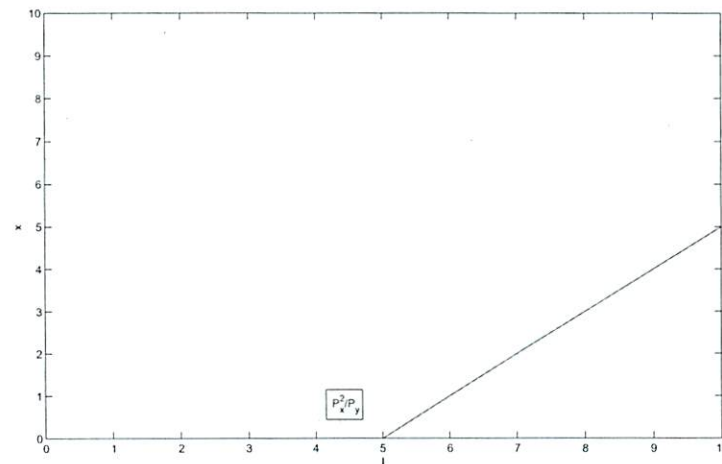


Figure 4: The Relation between Income and Quantity Demanded of 'x'. Engle curve of x .

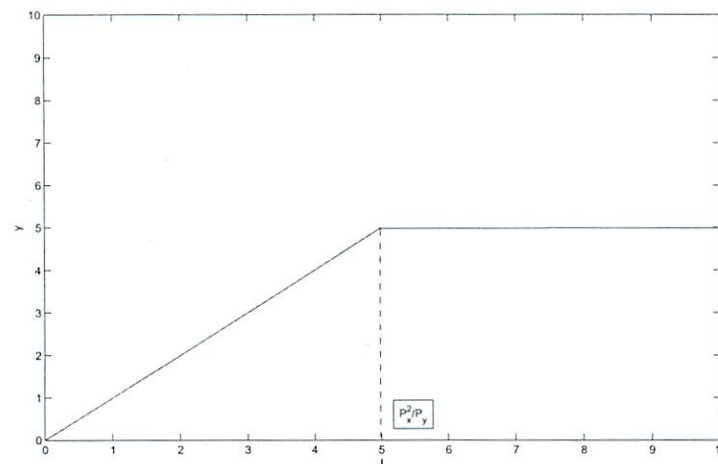
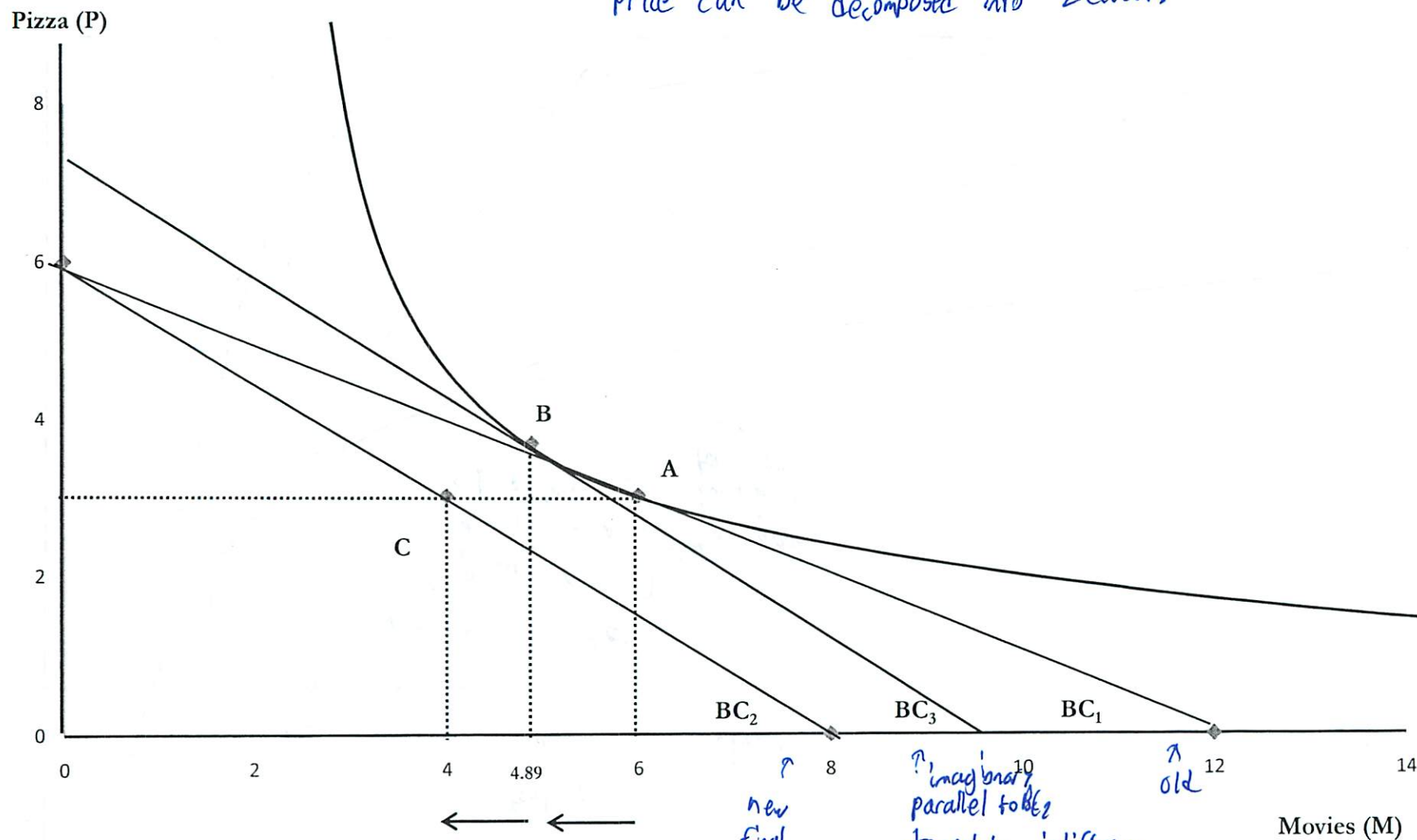


Figure 5: The Relation between Income and Quantity Demanded of 'y'. Engle curve of y .

Figure 6-4 and 7-1: Income and substitution effect

Price can be decomposed into 2 effects



Income effect
effectively poorer
for normal goods

Substitution effect
price ratios change
always \ominus

new final

? imaginary parallel to BC_2 tangent to indifference curve

old

Lecture 7

9/29

Figure 7-2: Price change with an inferior good

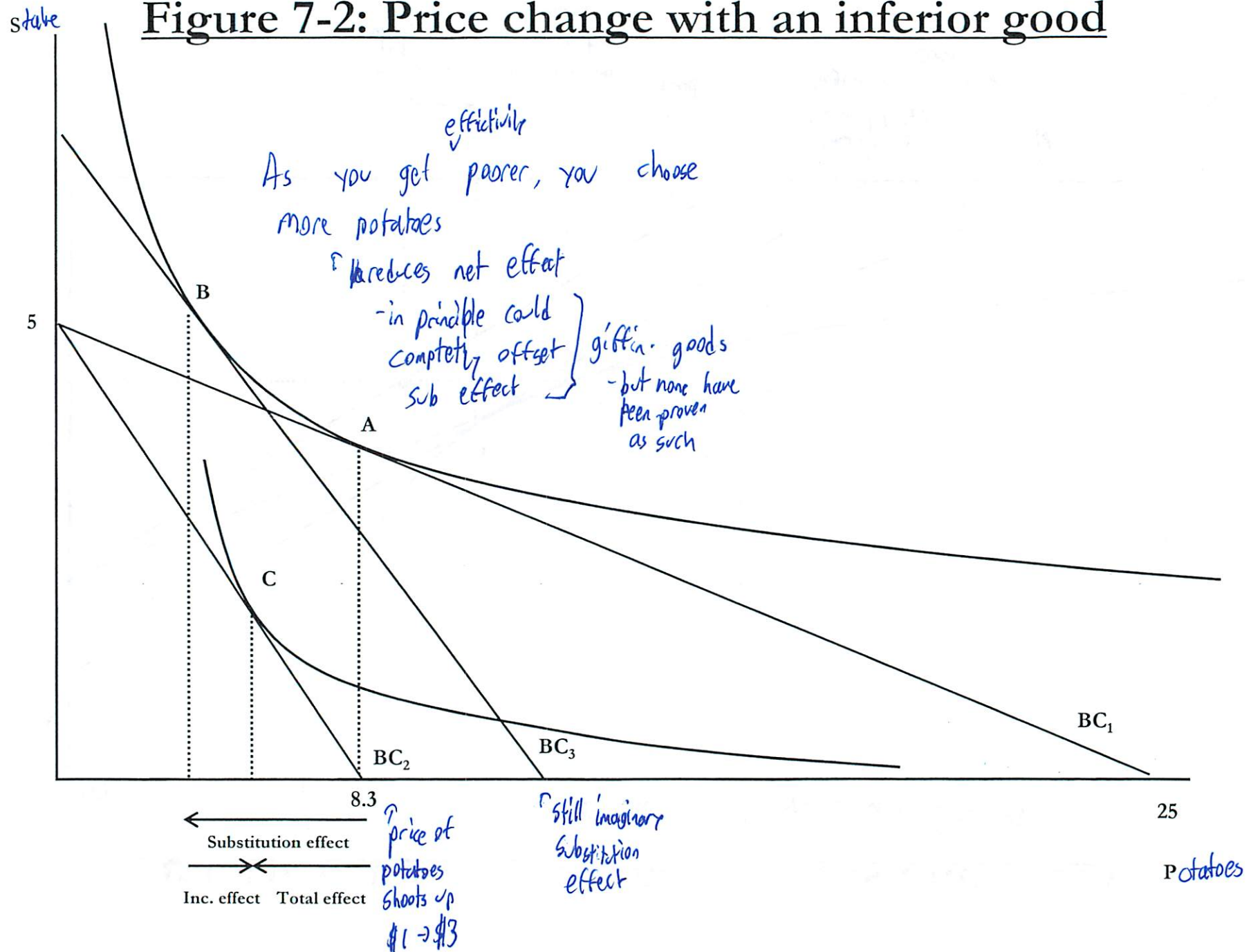


Figure 7-3: Labor – leisure trade-off

Goods

Labor Supply

-inferior good

price of leisure =
forgiving the wage

How hard do people work?
typical person does not want to work

$$H = 24 - N_{\text{leisure}}$$

Econ not good at modeling bads

- so measure the good
- that is the complement

Slope of budget constraint = $-w$

working 24 hrs
↓

24w

W = amt of
goods you can
buy w/ salary

BC

24

hrs of
leisure

Leisure →

← Work

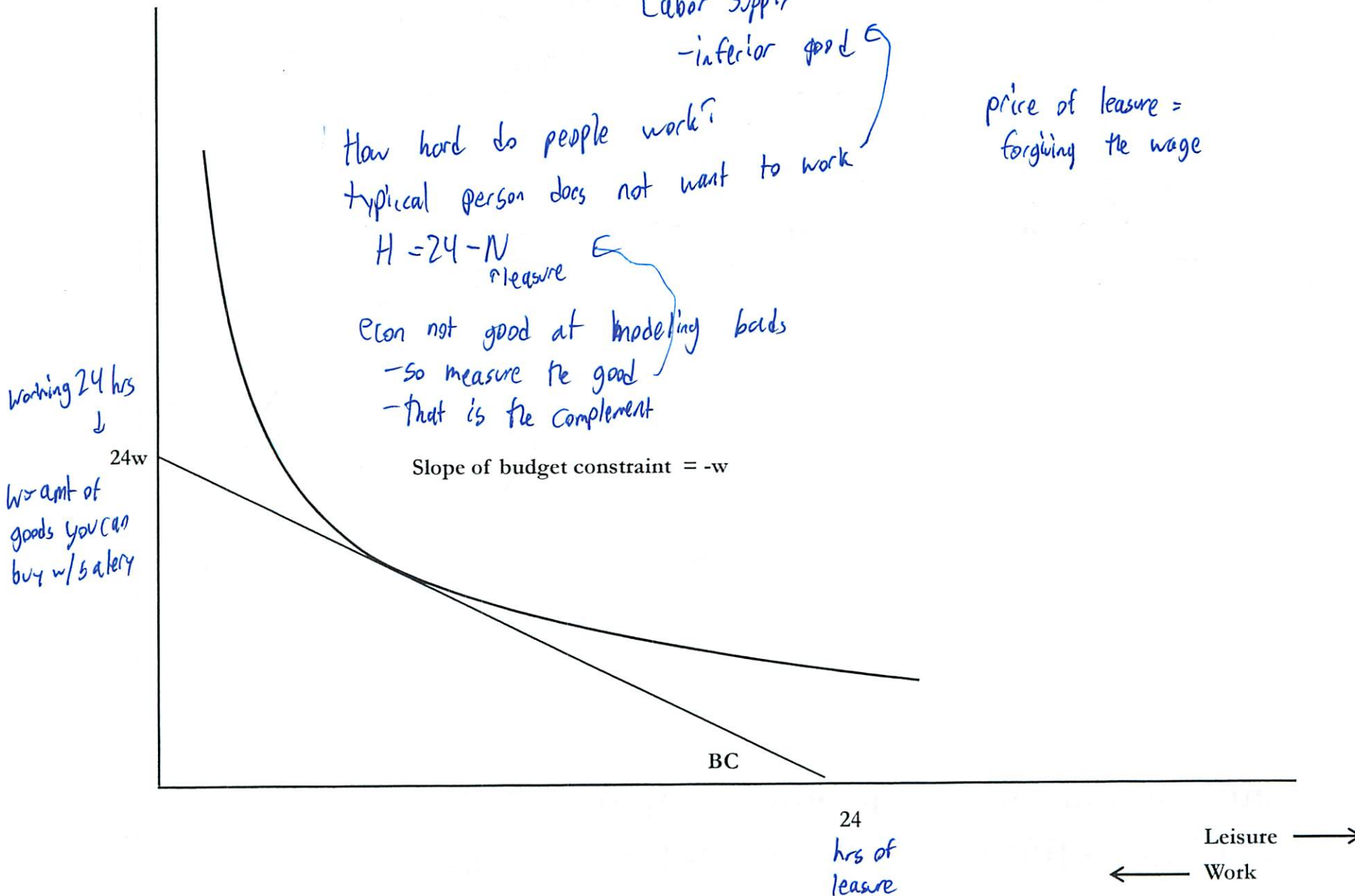


Figure 7-4: Income and substitution effects for labor supply: income effect does not dominate

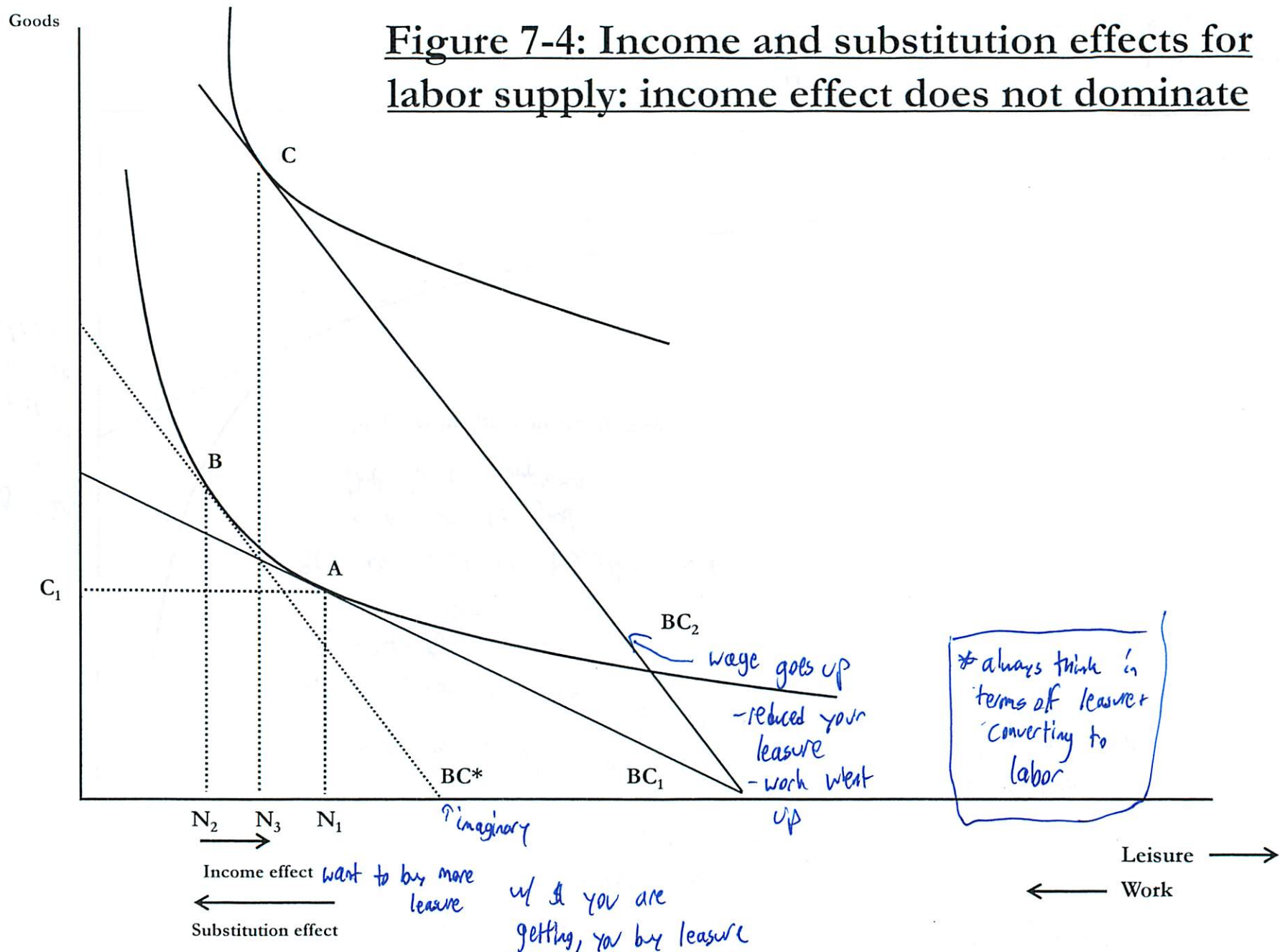
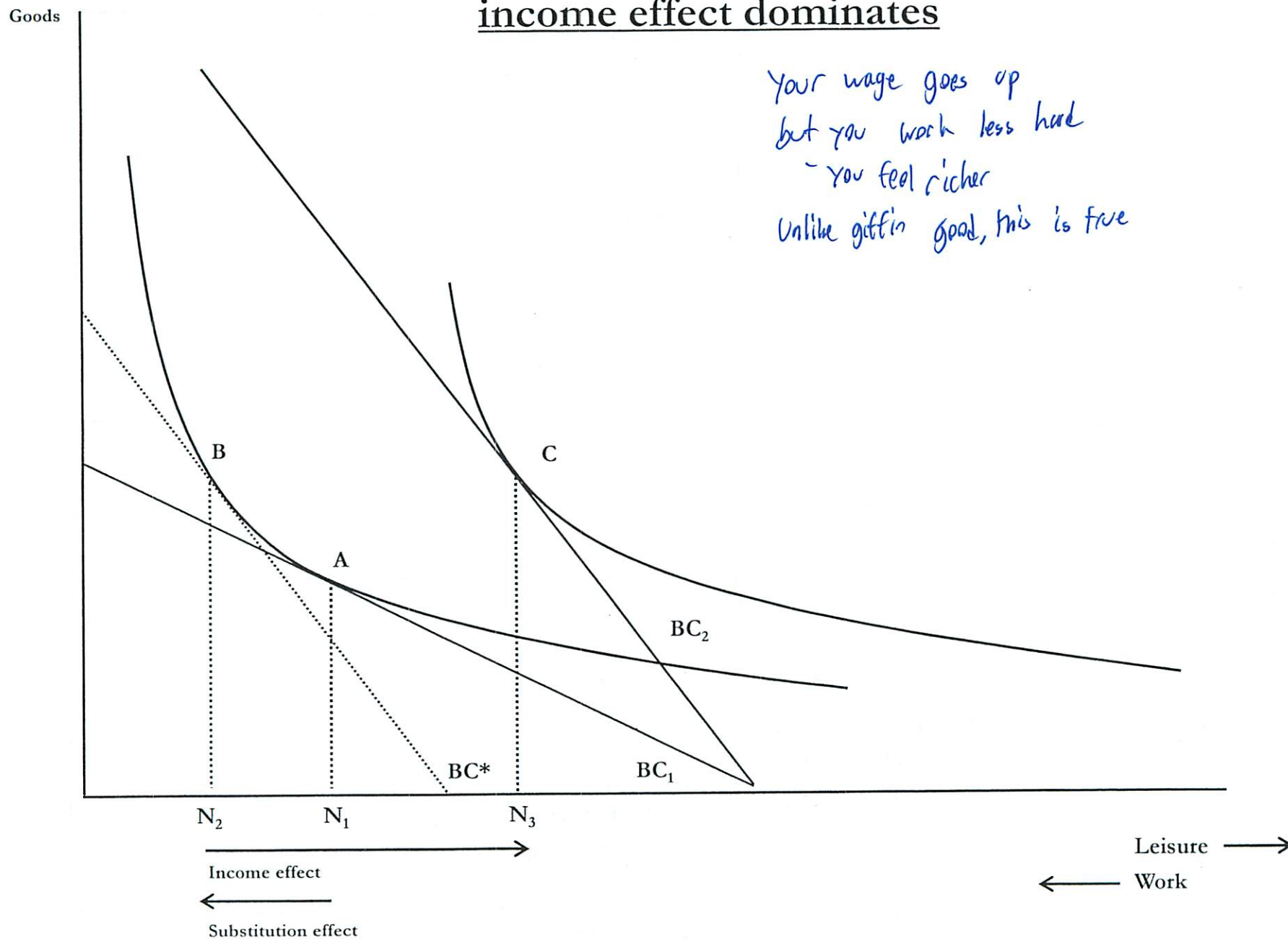
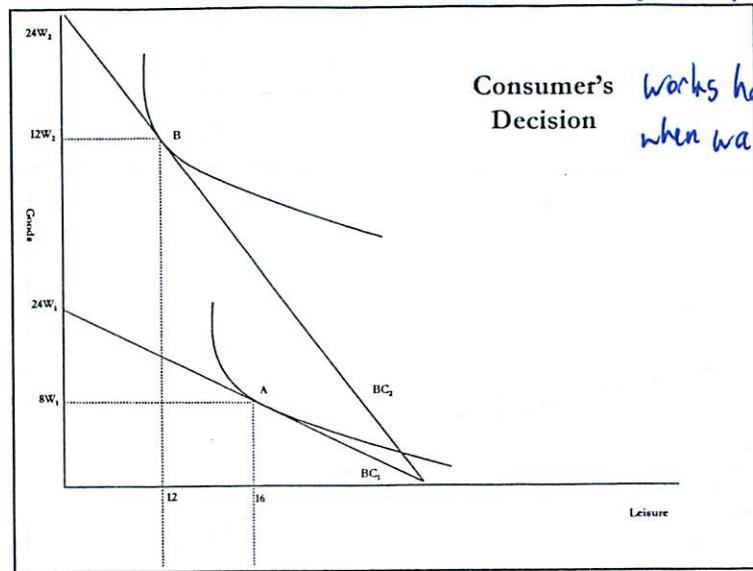


Figure 7-5: Income and substitution effects for labor supply:
income effect dominates

Your wage goes up
 but you work less hard
 ~ You feel richer
 Unlike giffen's good, this is true

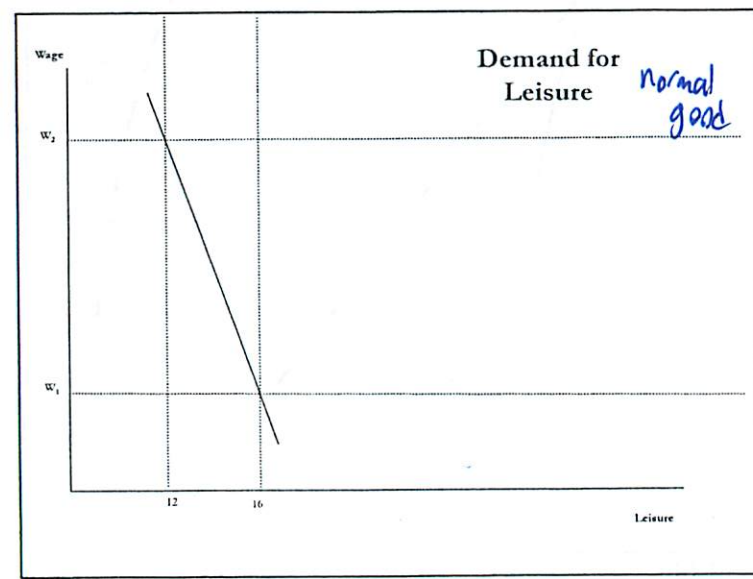


Wage goes up $w_1 \rightarrow w_2$

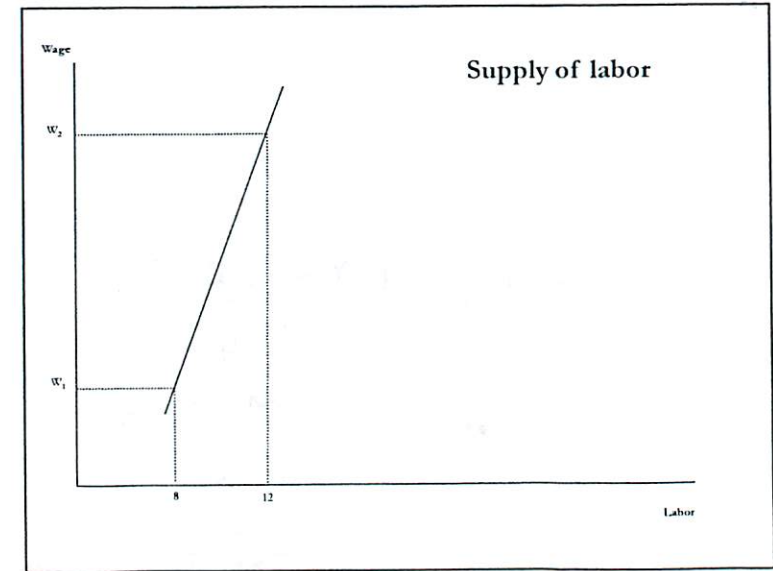


works harder when wage goes up

Figure 7.6:
Deriving leisure demand and labor supply



→
Convert



Graph people actually care about
- does it go up or down?

Lecture 7

Income and substitution effects for goods

Price Change	Substitution Effect	Income Effect	Total Effect
Normal Good			
Price Rises	≤ 0	≤ 0	≤ 0
Price Falls	≥ 0	≥ 0	≥ 0
Inferior Good			
Price Rises	≤ 0	≥ 0	????
Price Falls	≥ 0	≤ 0	????

law of demand
like richer

which is larger?

Income and substitution effects for labor supply

Price Change	Substitution Effect	Income Effect	Total Effect
Leisure			
Wage Rises	≤ 0	≥ 0	????
Wage Falls	≥ 0	≤ 0	????
Labor			
Wage Rises	≥ 0	≤ 0	????
Wage Falls	≤ 0	≥ 0	????

Handwritten bracket on the left side of the Labor section, grouping the Wage Rises and Wage Falls rows.

14.01 Principles of Microeconomics, Fall 2007

Chia-Hui Chen

September 19, 2007

Lecture 7

Substitution and Income Effect, Individual and Market Demand, Consumer Surplus

Outline

1. Chap 4: *Substitution Effect, Income Effect, Giffen Goods*
2. Chap 4: *From Individual Demand to Market Demand*
3. Chap 4: *Consumer Surplus*

1 Substitution Effect, Income Effect, Giffen Goods

Substitution and Income Effects

The impact of price change on quantity demanded are divided into two effects:

Substitution effect. Substitution effect is the change in an item's consumption associated with a change in the item's price with the utility level held constant.

Income effect. Income effect is a change in an item's consumption associated with a change in purchasing power with the price held constant.

Figure 1 shows the two effects: L is the old budget line. P_x decreases, and hence the new budget line is L' . A is the optimal consumption before price change, and C is the optimal consumption after price change. L'' is a line that has the same slope as L' and is tangent with the green indifference curve that passes through A , and B is the tangent point.

- The change from A to B is because of the substitution effect;
- The change from B to C is because of the income effect.

So the total effect is point A moving to C .

Inferior Good and Giffen Good

Now consider different positions of C (Figure 1):

- The income effect is B changing to C . In this case, an increase in income causes an increase in the demand of x . x is a normal good.

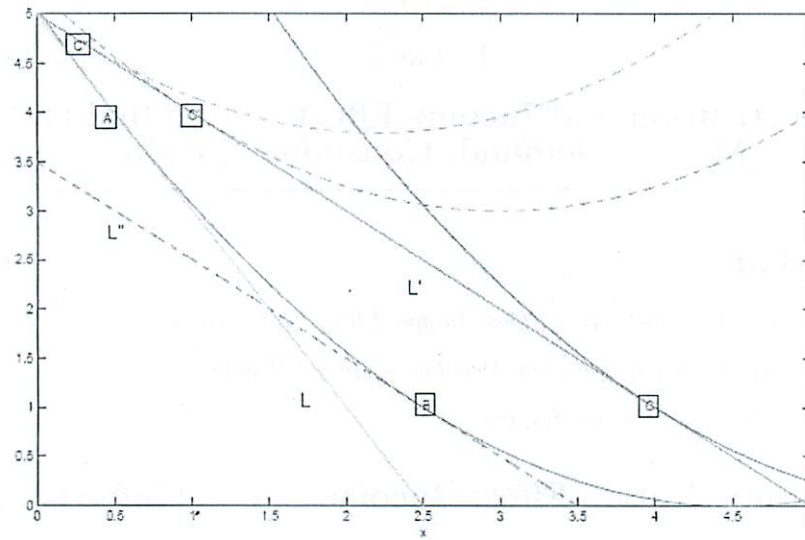


Figure 1: Substitution Effect and Income Effect.

Lecture 7

9/29

Is demand for labor up or down?

- 40 years ago women could work, but chose not to

	<u>Substitution</u>	<u>Income</u>	<u>Total</u>	<u>Figure</u>
married men	smaller	bigger	more like	7.5
married women	bigger	smaller	" "	7.4

↑
no good alternative
for men, lots of
outside options for
women
must be < 0

↑
 $h \cdot \frac{dh}{dy}$ complex algebra
if hrs you work important

found

~~upward~~
that wage Δ did not matter much for men

- inelastic ~ 0

- backward bending (?)

women: very elastic change in labor supply
 ~ 1

data matches theory

today: converging men ~ 1

women $\sim .5$

Is this good?

- in econ people do what is best for them ← simple ans
- but women less happy in surveys
- people can buy more goods, but leisure time ↓

More labor supply not necessarily more unemployment

Gov ran this experiment in the 1970s

- today look at changes in state tax rates

- The income effect is B changing to C' or C'' . In these cases, an increase in income causes a decrease in the demand of x . x is an inferior good;
- If the total effect is A changing to C'' , such that a decrease in price causes a decrease in the demand, we call x is a Giffen good.

	Price increases	
Normal good	substitution effect	quantity increases
	income effect	quantity increases
Inferior good	substitution effect	quantity increases
	income effect	quantity decreases

Table 1: Normal Good and Inferior Good

In Table 1, if x is a normal good, both substitution and income effects increase its quantity; if x is an inferior good, discuss as follows:

1. substitution effect $>$ income effect
→ quantity increases
2. substitution effect $<$ income effect
→ quantity decreases. This unusual good is called a Giffen good. A Giffen good must be an inferior good, but an inferior good is not necessarily a Giffen good.

Giffen good. Good with an upward demand curve. (Figure 2)

Example (Giffen Good Example: Irish Potato Famine). People consumed lots of potato but little meat (and other food) since meat was more expensive. Price of potato rose. People had less money to consume meat, so they ate more potatoes instead of meat.

An Example of Substitution Effects and Income Effects

Utility function Figure 3:

$$U(x, y) = x + 2\sqrt{y}.$$

Parameters:

$$P_x = 1,$$

$$P_y = 1,$$

$$I = 5.$$

The optimal solution is:

$$x = 4,$$

$$y = 1.$$

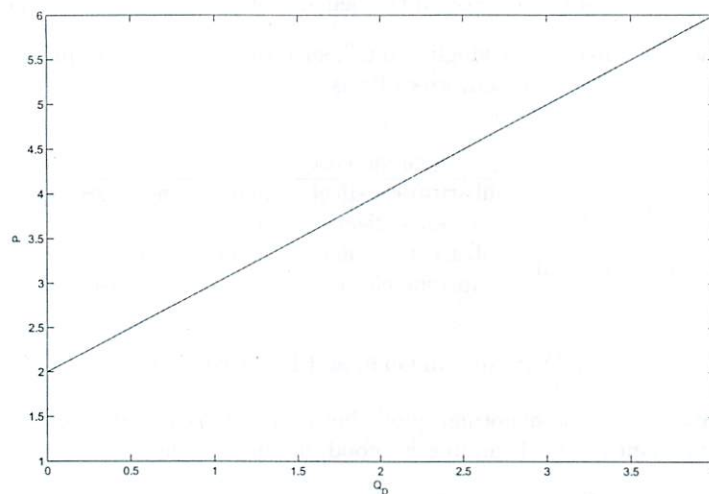


Figure 2: Demand Curve of Giffen Good.

i.e. the solution is at point A: $(4, 1)$.

If price of x changes to 2, $P'_x = 2$, then the new optimal solution is:

$$x = \frac{1}{2},$$

$$y = 4.$$

i.e. the solution is at point C: $(\frac{1}{2}, 4)$. Try to find out the substitution effect, i.e. the change from A to B.

At B, the slope of the indifference curve equals the slope of the new budget constraint.

Thus,

$$MRS = \frac{1}{\frac{1}{\sqrt{y}}} = \frac{P'_x}{P'_y} = \frac{2}{1}.$$

$$\Rightarrow y = 4.$$

On the other hand,

$$U(x, y) = x + 2 \times \sqrt{4} = 4 + 2 \times \sqrt{1}.$$

$$\Rightarrow x = 2.$$

Thus, point B is at $(2, 4)$.

Decomposition of the two effects:

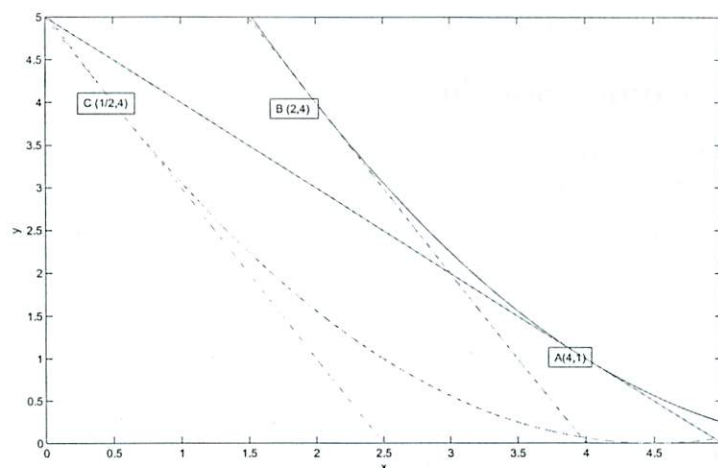


Figure 3: Showing the Substitution effect and Income Effect.

- Substitution effect (A to B)
 $(4,1) \Rightarrow (2,4)$.
- Income effect (B to C)
 $(2,4) \Rightarrow (\frac{1}{2},4)$.

2 From Individual Demand to Market Demand

Assume in a market there are two individuals A and B. And their demand functions are:

$$Q_A = 1 - P,$$

$$Q_B = 1 - \frac{1}{2}P.$$

When $P < 1$, both individuals consume, and the market demand is the sum of the individual demands:

$$Q = Q_A + Q_B = 2 - \frac{2}{3}P.$$

However, if P is larger than 1, only B consumes, so the market demand equals the demand of B. Thus, the market demand function is

$$Q = \begin{cases} 2 - \frac{2}{3}P & \text{if } P \leq 1 \\ 1 - \frac{1}{2}P & \text{if } P > 1 \end{cases}.$$

This is shown in Figure 4.

3 Consumer Surplus

Willingness to Pay. The sum of the 'values' of each of the units that consumers consume.

Consumer Surplus. The difference between Willingness to Pay and the actual Expenditure.

Example. Figure 5 shows the demand curve of a good. Assume now the price is 15, then only the highest 6 individuals consume:

$$WILLINGNESS\ TO\ PAY = 20 + 19 + 18 + 17 + 16 + 15 = 105.$$

On the other hand, the expenditure is

$$EXPENDITURE = 6 \times 15 = 90.$$

Therefore,

$$CONSUMER\ SURPLUS = 105 - 90 = 15.$$

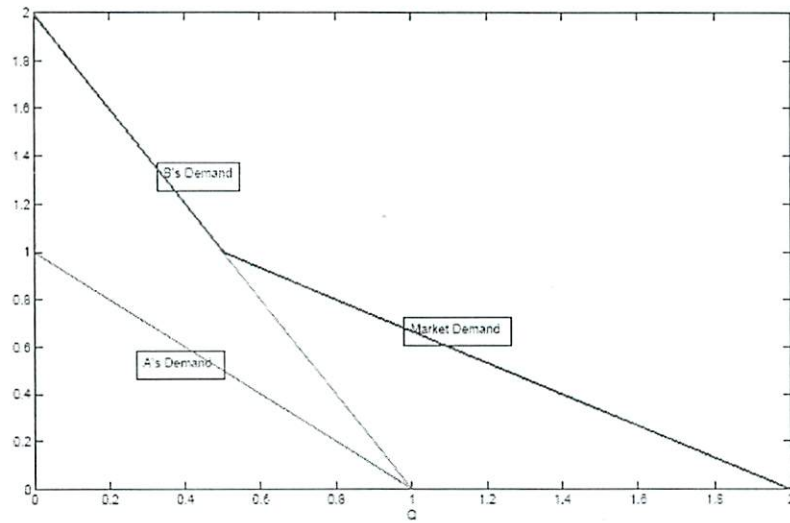


Figure 4: Derived Market Demand from Individual Demands.

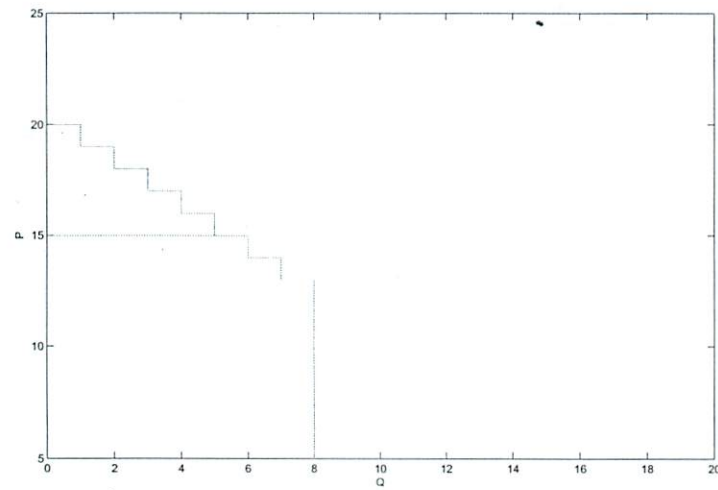
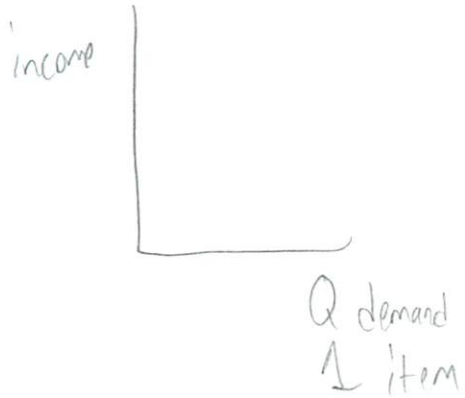


Figure 5: Demand Curve for a Good. Used in consumer surplus calculation.

Engel Curve Notes

- purchases of goods vs income



Exam 1 10/5 Walker 7:30 - 9:30pm

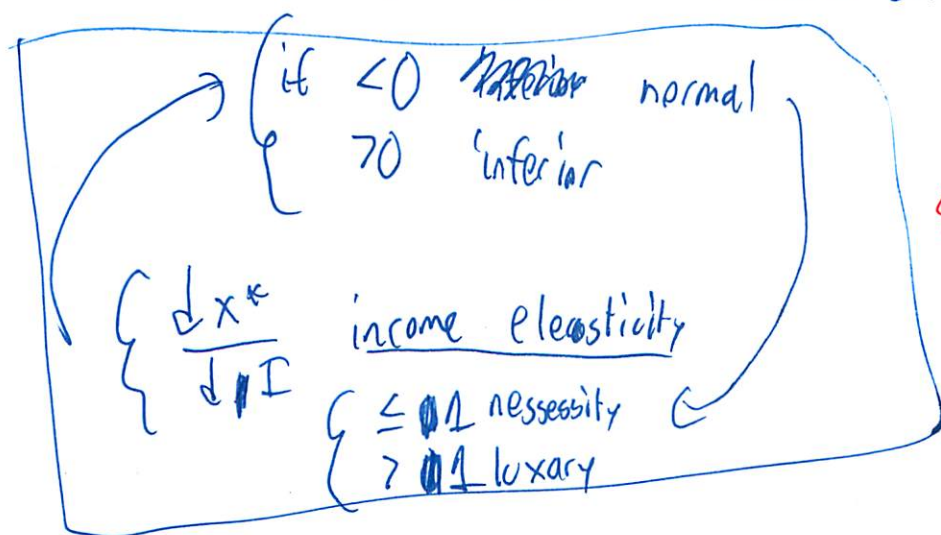
1. PS 5
2. Demand functions
3. Income/sub effect
4. Ind. vs market demand
5. Price floors/price ceilings

Post 3

- 2a. Compute price elasticity of demand
- 2b. Giffen good \rightarrow \oplus price elasticity of demand

$$X^*(P_x, P_y, I)$$

$\frac{dX^*}{dP_x}$ how price of good affects demand for it
- decompose \rightarrow substitution + income effect



See next
page for cleaner chart

(2)

Restated

For price ↑

⊖ income effect

$$\frac{dx^*}{dI} > 0$$

normal

$$\frac{dx^*}{dI} > 1 \text{ luxury}$$

⊕ " "

$$\frac{dx^*}{dI} < 0$$

inferior

could be
normal
or giffin

$$\frac{dx^*}{dI} \leq 1 \text{ necessity}$$

Substitution effect always ⊖
- need utility function

$$\frac{dx}{dp} > 0 \text{ giffin}$$

$$\frac{dx}{dp} < 0 \text{ not giffin}$$

Giffin: ⊕ income effect > ⊖ substitution effect

So

$$\frac{dx^*}{dp^*} > 0$$

- always an inferior good

Not Giffin

$$\frac{dx^*}{dp^*} < 0$$

(think about definition)

- could be normal or inferior

c) If are giffin → always inferior A
not → no info B

5. S C

$$U(S, C) = 4 \ln(S) + 6 \ln(C)$$

a) Compute MRS of clothes

- how many units of clothes Δ when 1 software change

$$\frac{-MU_S}{MU_C} = \frac{-4 \frac{1}{S}}{6 \frac{1}{C}} = -\frac{2}{3} \frac{C}{S} \quad \checkmark \text{ had that on p-set}$$

$$|MRS| = \frac{2}{3} \frac{C}{S}$$

③

Was \ominus , so decreasing

- as she buys more software, harder to sub software for clothes

b) ^{Find demand curve} $P_s \cdot S + P_c \cdot C = I$ \leftarrow had

- from last time, find optimum consumption

$MRS = MRT$
tangent pt \leftarrow part I missed

also on budget constraint

$$MRT = -\frac{P_s}{P_c} \Rightarrow MRS = -\frac{2}{3} \frac{C^*}{S^*}$$

\uparrow (part I missed)

Solve for C^*

$$C^* = \frac{3}{2} \frac{P_s}{P_c} \cdot S^*$$

Plug in to constraint \rightarrow income/budget

$$P_s \cdot S^* + P_c \cdot \frac{3}{2} \frac{P_s}{P_c} \cdot S^* = I$$

$$S^* \cdot P_s \left(1 + \frac{3}{2}\right) = I$$

$$\text{So } S^*(P_s, P_c, I) = \frac{2}{5} \frac{I}{P_s} \leftarrow \text{demand function for software}$$

Solve for C^* by plugging in

$$\begin{aligned} C^*(P_s, P_c, I) &= \frac{3}{2} \cdot \frac{P_s}{P_c} \cdot \frac{2}{5} \frac{I}{P_s} \\ &= \frac{3}{5} \frac{I}{P_c} \end{aligned}$$

$C^* = q^d_c$
demanded $= Q_c$

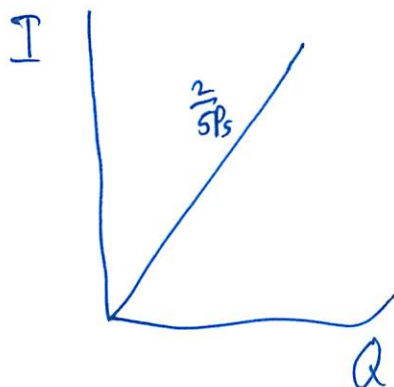
1. Get MRS
2. Set $=$ to MRT
3. Plug into income constraint
4. Solve for Q

demand
optimal consumptions
- generalized

Why did I not get this?

④

c) Engal Curve



$$S^* = \frac{2}{5} P_S \cdot I$$

? linear slope

d) Optimal Bundle

Plug in #

* if did b right \rightarrow can just plug in

$$S^* = \frac{2}{5} \cdot \frac{10}{2} = 2$$

$$C^* = \frac{3}{5} \cdot \frac{10}{3} = 2$$

are they complements or substitutes? \rightarrow independent

Price elasticity of demand for SW

constant elasticity of demand

$$S^* = \frac{2}{5} I \cdot P_S^{-1}$$

$$Q_D = A \cdot p^\alpha \quad \epsilon_D = \alpha$$

So linear demand curve

elastic $\rightarrow \alpha < -1$ elastic
 $\alpha > -1$ inelastic

tangent /
 I did
 not really
 understand

(5)

e) Suppose $P_3 = 4$

$$S^* = \frac{2}{5} \cdot \frac{10}{4} = 1$$

$$C^* = \frac{3}{5} \cdot \frac{10}{3} = 2 \quad \text{demand did not change - independent}$$

Substitution + income effect = , so net ~~do~~ does not change
- independent

f) Find substitution effect

$$U(S^*, C^*) = \text{initial bundle} = 4 \ln(2) + 6 \ln(2) = 10 \ln(2)$$

- before price change

What must income (i) be so utility new = old?

$$S^* = \frac{2}{5} \cdot \frac{i}{4} = \frac{i}{10}$$

$$C^* = \frac{3}{5} \cdot \frac{i}{3} = \frac{i}{5}$$

Get utility as a function of income

$$U(i) = \cancel{4 \ln \frac{i}{10} + 6 \ln \frac{i}{5}} \quad 4 \ln \frac{i}{10} + 6 \ln \frac{i}{5}$$

$$10 \ln(2) = 4(\ln i - \ln 10) + 6(\ln i - \ln 5)$$

Must solve for i - w/ calculator

$$i = 13.2 \quad \leftarrow \begin{array}{l} \text{new income to get same utility} \\ \text{old utility} \end{array}$$

Can sub into demand function to find quantities

$$S^* = 1.32 \quad C^* = 2.64$$

Price increased, what income was needed for same utility?

actually, calculating
utility did not
do on last p-set!

⑥
9) Decompose to income + substitution effects

What we have so far	$P_S = 2$ $P_C = 3$ $I = 10$	$P_S = 4$ $P_C = 3$ $I = 10$	$P_S = 4$ $P_C = 3$ $i = 1.32$
---------------------------	------------------------------------	------------------------------------	--------------------------------------

substitution effect keeping utility constant \rightarrow what does change of price do to Q_S demanded
 $2 \rightarrow 1.32 = .68$ well $1.32 - 2 = -.68$

"change in demand in response to a price change, holding utility constant"

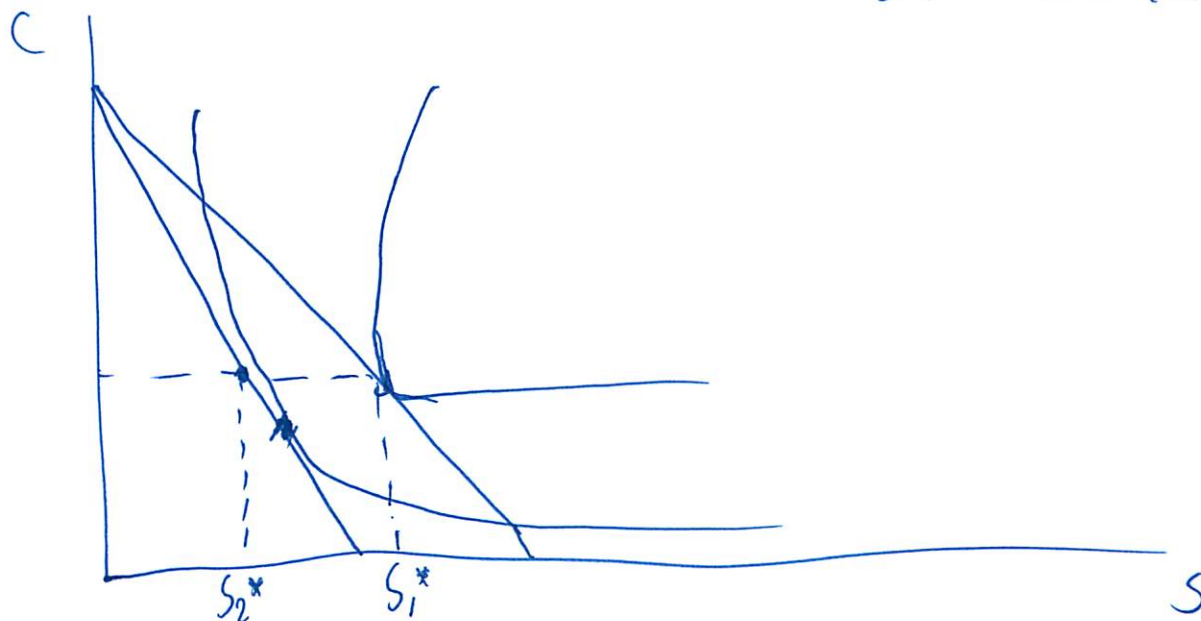
~~SE~~

'income effect' $TE_S - SE_E = -.32$
 can't calc income effect directly

$$SE_C = 2.64 - 2 = .64$$

$$TE_C = -.64$$

Graphs hard to draw \rightarrow big



⑦ Sub effect hold utility constant

- go to new price ratio

- where it touches indifference curve

IE \downarrow PSE



do this problem again