Calc Exam 2 Review

Lin approx $f(x) \approx f(x_0) + f'(x_0)(x - x_0)$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad$ Ta

quad $+\frac{f''(x_0)}{2}(x - x_0)^2$

don't forget!

<u>Standard approx</u>

e^x	$\boxed{1+x}$	$1 + x + \frac{1}{2}x^2$
$\sin x$	x	x
$\cos x$	1	$1 - \frac{1}{2}x^2$
$\ln(1+x)$	x	$x - \frac{1}{2}x^2$
$(1+x)^r$	$\boxed{1+rx}$	$1 + rx + \frac{r(r-1)}{2}x^2$

Graphing

- on this as well

1. deriv
2. critical pt $= 0$
3. 2nd deriv $= 0 \to$ inflection pts
4. Vert asy

$\frac{2x^2}{x}$ = slant asy $= 2x$

$\frac{2x^2}{x^2}$ = asy horiz $x = 2$

$\frac{2x}{x^2}$ = asy $\overset{x}{y} = 0$

symmetries periodicity
-even odd

- zeros

Min-Max

explicit - on closed interval [ ] $a \le x \le b$

- no asymptotes in the middle

- open ( )

- could be asymptote

implicit - oh just means need to find int from word problems

pick variable

domain - means find limits

x cant > height

$x \in [0, \sqrt{2}/2]$

find crit pts & endpoints

<u>or</u> look at 2nd deriv   min   max

Related Rates

again all in setup

think got this

Newton's Method

find approx solution $f(x) = 0$
differentiable

iterative $x_n - \dfrac{f(x_n)}{f'(x_n)}$ pick x_1
goes till $x_n \approx x_{n+1}$

does not work always

MVT

$f(x)$ is differentiable $a < b$ then c $a < c < b$

$$f'(c) = \frac{f(b) - f(a)}{b - a} \text{ exact}$$

$$f(b) = f(a) + f'(c)(b-a)$$

$$f(x) = f(a) + f'(c)(x-a)$$ 2 same except letter

$\sin < x$ for all $x > 0$

$\sin x - x < 0$
swap

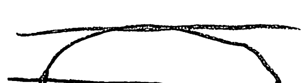
$f(x) = \sin x - x$

$f'(x) = \cos x - 1$
$-$ for $x > 0$

So $\ominus$

$$\frac{f\,x - f\,k}{x-1} = -$$

Rolle's Thoerm



differentiable Contineous
point somewhere deriv $= 0$

proves the MVT
Special case of MVT

① Pratice ~~Question~~ Questions

$3x^5 - 5x^3 + 1$ sketch

f' $15x^4 - 15x^2 = 0$

? how to solve

yeah $x = 0, \pm 1$

? where from ?

$x^4 - x^2 = 0$

oh yeach can plug in to $= 0$ guess just have to think about

$f'' = 60x^3 - 30x = 0$

0 $\frac{\sqrt{2}}{2}$

~~$2x^3 = x$~~

* factor out $30x$ * don't forget the f-ing steps *

$2x^2 - 1 = 0$

$2x^2 = 1$

$x^2 = \frac{1}{2}$

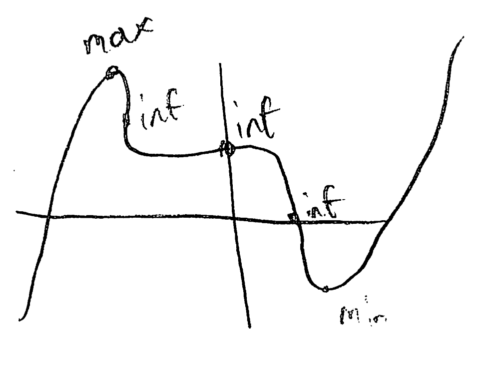
$x = \sqrt{\frac{1}{2}} \to \frac{\sqrt{2}}{2}$

-1 $\frac{\sqrt{2}}{2}$ 0 $\frac{\sqrt{2}}{2}$ 1

test where f' ⊕ ⊖

f'' ↑ ↓

find f



2. Same w/ $4x^2 - \frac{1}{x}$

$f' = 8x + \frac{1}{x^2}$ derive right

$f'' = 8 + \frac{2}{x^3}$

$8x + \frac{1}{x^2} = 0$

$\cancel{8x = -\frac{1}{x^2}}$ $8x + x^{-2} = 0$ $-\frac{1}{2}$

$\cancel{x(8}$

$\cancel{x}$ ~~by~~ x^2 $\cancel{\pm 1}$ ~~again just test I guess~~

$\cancel{8x^3 + 1 = 0}$

$x\left(8 + 1 - \frac{8}{x^3}\right)$

$x\left(8 - \frac{1}{x^3}\right) =$

$\neq 0$

$x^3 = \frac{1}{8}$

$x = \sqrt[3]{\frac{1}{8}}$ $-\frac{1}{2}$

find asy $x = 0$

Sphere

$SA = 4\pi r^2$

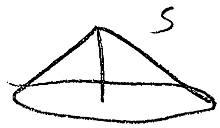
$V = \frac{4}{3}\pi r^3$

← coincidanse? $V = SA$

Cone

$SA = \pi r s + \pi r^2$

$V = \frac{1}{3}\pi r^2 h$



Cylinder

$SA = 2\pi r^2 + 2\pi r h$

$V = \pi r^2 h$

Circcle

$A = \pi r^2$

$Circ = 2\pi r$

"It's just actually doing it"

3. How many solutions to $\sqrt{x} = x$

$\sqrt{1} = 1$ $\sqrt{-1} = i$

$\sqrt{0} = 0$

↑2

they say consider $\sqrt{x} - x$

$f(0) = 0$
$f(1) = 0$ } yeah what I said

continuous $[0, \infty)$

differentiable $(0, \infty)$

↑always

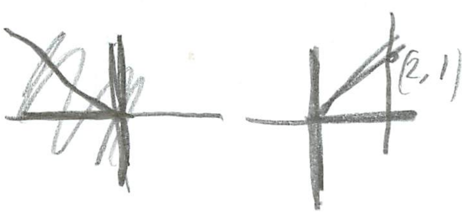
} prove those are the only

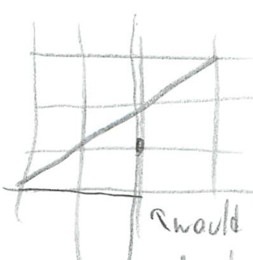
Rolle's theorm

$f'(x) = \frac{1}{2\sqrt{x}} - 1$ $= 0$ only at $x = \frac{1}{4}$

- Justifys that are only 2 roots

4.





↑would have to be that

- what is there to optimize

- no could be

$y = 0$

$y = x$

$y - 1 = m(x - 2)$ ← find int

↑horizontal, or <u>not</u>

$A = \frac{1}{2} b h$

↑ x highest

↑at $(0, y)$

★ but how put it all together???

· Identify

* have control over slope of line
  - area depends on this

(I realized this but did not act on it)

$y-1=m(x-2)$

can write $x = 2 - \frac{1}{m}$ at intersection

$y = \frac{1-2m}{1-m}$

$y=x$ - can sub in (I thought of that)

$y-1=m(y-2)$

~~$m = \frac{y-1}{y-2}$~~

$y = m(y-2)+1$

~~$\frac{y}{y-2} = m + \frac{1}{y-2}$~~

goal $y = \frac{1-2m}{1-m}$

distribute $y-1 = my-2m$

$y = my - 2m + 1$

$y = \frac{my}{m} + \frac{1-2m}{m}$

$y = y + \frac{1-2m}{m}$

no clue here

find area

derivitive set $=0$

find $m = \frac{1}{2}$

then find stuff

but $a=0$ - check endpts

$0 < m < 1$ area $\ominus$

$m > 1$ area $\downarrow$ → want area as $m \to \infty$

turns out to be a right triangle

just think it through
- identified 80% of what to solve

∴ linear approx instead / ~~of~~ $\sqrt[3]{8}$

5 Calc $\sqrt[3]{10}$ to 6 decimal places

∴ Newton's method or approx ∴

on $x^3 - 10$

$r \omega t f$ ∴ so $= 0$

$x_1 = 1$

$\frac{f}{f'}$ $3x^2$

$x_2 = 1 - \frac{-9}{3} = -4$

$x_3 = -4 - \frac{-16 \cdot 4}{3 \cdot 16}$

$\frac{4 \cdot 4 \cdot 4 - 16 \cdot 4}{3 \cdot 16}$

first guess near 2

$2^3 = 8$

$3^3 = 27$

$0 = x^3 - 10$

$10 = x^3$

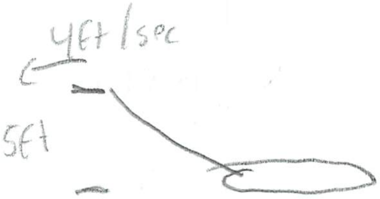
$x = \sqrt[3]{10}$ ← what we want

~ 2

$x^3 = 10$

$x^3 - 10 = 0$

6. Boat pulled into dock



13 ft rope

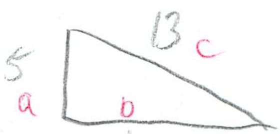
R = rope lenght

$R = 13$

$\frac{dR}{dt} = 4 \text{ ft/sec}$

$\frac{dB}{dt} = ??$

$B = ?$ can find 12



$13^2 = 5^2 + x^2$

$\sqrt{13^2 - 5^2}$

12

Say $a^2 + b^2 = c^2$

← all missing was this which I did think about - just did not write

$\frac{d}{dt}$ $2a\,a' + 2b\,b' = 2c\,c'$

$2 \cdot 5 \cdot 0 + 2 \cdot 12 \cdot b' = 2 \cdot 13 \cdot 4$

$\frac{dB}{dt}$ ✓

$24b' = 104$

$b' = \frac{104}{24} = 4\frac{1}{3}$ ft/sec

$24\overline{)104}$

13
-8
104

$\frac{4}{12} \frac{1}{3}$

48
72
96
$\frac{8}{24}$

7.

so must know formulas

$V = \pi r^2 h + \frac{1}{2}\frac{4}{3}\pi r^3$

$SA = 2\pi r h + \frac{1}{2}\frac{4}{3}\pi r^2$

Oh they see bottom as

which tanks don't usually have

Cost

$\overset{\swarrow c \text{ for cost} \downarrow}{2\pi r h c + 3c\pi r^2}$ ← sub in ← want in terms of v

$v = \pi r^2 h$ ← solve for $h = \frac{v}{\pi r^2}$

$C = \frac{2\cancel{\pi} r v}{\cancel{\pi} r^2} c + 3c\pi r^2$

cross out

$\frac{2v}{r}c + 3c\pi r^2$

* cost of whole = C
Cost unit = c

$C = 2vr^{-1}c + 3\pi r^2$

↑ constant ← find product rule

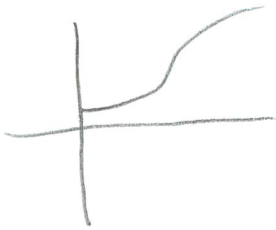
$\frac{dc}{dv}$

$2c \cdot -1 r^{-2} 2vc + 3\pi 2 r^3$

$-2\frac{v}{r^2} c + 6\pi r c$

8. $f(x) = \sqrt[3]{x}$ leads to $x_2 = -2x_1$

(what does that even look like??



Newton's method

$x - \frac{f(x)}{f'(x)}$

MVT

$\frac{f(b) - f(a)}{b-a} = f'(c)$

bounces back + forth

$-$

$+$

$-$

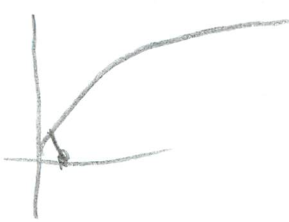
$+$

So useless

gets worse

9.

opt



distance formula

$d^2 = (y-0)^2 + (x - 1/2)^2$

eleven

really should be able to do this

Simp

derivitive

Find where $=$ to 0

$x=0$ pt $= (0,0)$

10. $x^2 + y^2 = a^2$

$\frac{dx}{dt} = -y$??

$2xx' + 2yy' = 2aa'$

$x' = \frac{2aa' - 2yy'}{2x}$

$\frac{aa' - xy'}{2x}$

??

a^2 = constant ← was thinking something like that

$2xx' + 2yy' = 0$

$x' = \frac{2yy'}{2x} = \frac{yy'}{x}$ first do

$y' = \frac{2xx'}{2y} = \frac{-x(x')}{y}$ ← $-y$ given

the given

$y' = \frac{-x \, -y}{y}$ $+x$

at $(1,0)$

$+$

clockwise

$(0,1)$

0 so $y' = 0$ ←

$y - 1 = m(y - 2)$

figure at

thanks

$$\boxed{y = \frac{1-2m}{1-m}}$$

$y - 1 = my - 2m$

$y = my - 2m + 1$

$y - my = -2m + 1$

$y(1-m) =$

factor $\quad y = \frac{1-2m}{1-m}$

$x - \frac{f}{f'}$

$f'c = \frac{fb - fa}{b-a}$

$0 = 0$

$f(a) - f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2$

$\frac{\text{1 or more}}{\text{where deriv} = 0}$