(0) # Review Exam 4 12/2

$\sin^2 x + \cos^2 x = 1$ — can derive others from this one

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ — if forget plug in 0 + check

$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

$\sin x \cos x = \frac{1}{2}\sin 2x$

- normal u substitution
- chip then away all powers but 1 → trig identity
- $\sin^3 x \to \sin^2 x \cdot \sin x$

if even split + half angle

$(\sin^2 x)^2$

$\left(\frac{1}{2}(1 - \cos^2 x)\right)^2 \to \frac{1}{4}(1 - 2\cos 2x + \cos^2 2x)$ split — still annoying

each stage loses half of powers before

$24 \to 12 \to 6 \to 3$ — now odd

$\frac{d}{dx}\tan x = \sec^2 x$

$\frac{d}{dx}\sec x = \sec x \tan x$

$\sec^2 x = 1 + \tan^2 x$

double angle

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

Derivatives

Basic Properties/Formulas/Rules

$\frac{d}{dx}\left(cf(x)\right) = cf'(x)$, c is any constant. $\quad \left(f(x) \pm g(x)\right)' = f'(x) \pm g'(x)$

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$, n is any number. $\quad \frac{d}{dx}(c) = 0$, c is any constant.

$(f\,g)' = f'\,g + f\,g'$ – **(Product Rule)** $\quad \left(\frac{f}{g}\right)' = \frac{f'\,g - f\,g'}{g^2}$ – **(Quotient Rule)**

$\frac{d}{dx}\left(f\left(g(x)\right)\right) = f'\left(g(x)\right)g'(x)$ **(Chain Rule)**

$\frac{d}{dx}\left(\mathbf{e}^{g(x)}\right) = g'(x)\mathbf{e}^{g(x)}$ $\quad \frac{d}{dx}\left(\ln g(x)\right) = \frac{g'(x)}{g(x)}$

$U = \ln x$

$du = \frac{1}{x}\,dx$

Common Derivatives

Polynomials

$\frac{d}{dx}(c) = 0 \quad \frac{d}{dx}(x) = 1 \quad \frac{d}{dx}(cx) = c \quad \frac{d}{dx}\left(x^n\right) = nx^{n-1} \quad \frac{d}{dx}\left(cx^n\right) = ncx^{n-1}$

Trig Functions

$\frac{d}{dx}(\sin x) = \cos x \quad \frac{d}{dx}(\cos x) = -\sin x \quad \frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{d}{dx}(\sec x) = \sec x \tan x \quad \frac{d}{dx}(\csc x) = -\csc x \cot x \quad \frac{d}{dx}(\cot x) = -\csc^2 x$

Inverse Trig Functions

$\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}} \quad \frac{d}{dx}\left(\cos^{-1} x\right) = -\frac{1}{\sqrt{1-x^2}} \quad \frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}$

$\frac{d}{dx}\left(\sec^{-1} x\right) = \frac{1}{|x|\sqrt{x^2-1}} \quad \frac{d}{dx}\left(\csc^{-1} x\right) = -\frac{1}{|x|\sqrt{x^2-1}} \quad \frac{d}{dx}\left(\cot^{-1} x\right) = -\frac{1}{1+x^2}$

Exponential/Logarithm Functions

$\frac{d}{dx}\left(a^x\right) = a^x \ln(a) \quad \frac{d}{dx}\left(\mathbf{e}^x\right) = \mathbf{e}^x$

$\frac{d}{dx}\left(\ln(x)\right) = \frac{1}{x},\ x > 0 \quad \frac{d}{dx}\left(\ln|x|\right) = \frac{1}{x},\ x \neq 0 \quad \frac{d}{dx}\left(\log_a(x)\right) = \frac{1}{x\ln a},\ x > 0$

Hyperbolic Trig Functions

$\frac{d}{dx}(\sinh x) = \cosh x \quad \frac{d}{dx}(\cosh x) = \sinh x \quad \frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$

$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x \quad \frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x \quad \frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$

Integrals

Basic Properties/Formulas/Rules

$\int cf(x)\,dx = c\int f(x)\,dx$, c is a constant. $\quad \int f(x) \pm g(x)\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$ where $F(x) = \int f(x)\,dx$

$\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$, c is a constant. $\quad \int_a^b f(x) \pm g(x)\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

$\int_a^a f(x)\,dx = 0 \quad \int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx \quad \int_a^b c\,dx = c(b-a)$

If $f(x) \geq 0$ on $a \leq x \leq b$ then $\int_a^b f(x)\,dx \geq 0$

If $f(x) \geq g(x)$ on $a \leq x \leq b$ then $\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$

Common Integrals

Polynomials

$\int dx = x + c \quad \int k\,dx = k\,x + c \quad \int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$

$\int \frac{1}{x}\,dx = \ln|x| + c \quad \int x^{-1}\,dx = \ln|x| + c \quad \int x^{-n} dx = \frac{1}{-n+1}x^{-n+1} + c,\ n \neq 1$

$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln|ax+b| + c \quad \int x^{\frac{p}{q}} dx = \frac{1}{\frac{p}{q}+1}x^{\frac{p}{q}+1} + c = \frac{q}{p+q}x^{\frac{p+q}{q}} + c$

Trig Functions

$\int \cos u\,du = \sin u + c \quad \int \sin u\,du = -\cos u + c \quad \int \sec^2 u\,du = \tan u + c$

$\int \sec u \tan u\,du = \sec u + c \quad \int \csc u \cot u\,du = -\csc u + c \quad \int \csc^2 u\,du = -\cot u + c$

$\int \tan u\,du = \ln|\sec u| + c \quad \int \cot u\,du = \ln|\sin u| + c$

$\int \sec u\,du = \ln|\sec u + \tan u| + c \quad \int \sec^3 u\,du = \frac{1}{2}\left(\sec u \tan u + \ln|\sec u + \tan u|\right) + c$

$\int \csc u\,du = \ln|\csc u - \cot u| + c \quad \int \csc^3 u\,du = \frac{1}{2}\left(-\csc u \cot u + \ln|\csc u - \cot u|\right) + c$

$\int \tan^2 x = \int \sec^2 x = \tan x + C$ [illegible]

Exponential/Logarithm Functions

$\int \mathbf{e}^u\,du = \mathbf{e}^u + c \quad \int a^u\,du = \frac{a^u}{\ln a} + c \quad \int \ln u\,du = u\ln(u) - u + c$

$\int \mathbf{e}^{au}\sin(bu)\,du = \frac{\mathbf{e}^{au}}{a^2+b^2}\left(a\sin(bu) - b\cos(bu)\right) + c \quad \int u\mathbf{e}^u\,du = (u-1)\mathbf{e}^u + c$

$\int \mathbf{e}^{au}\cos(bu)\,du = \frac{\mathbf{e}^{au}}{a^2+b^2}\left(a\cos(bu) + b\sin(bu)\right) + c \quad \int \frac{1}{u\ln u}\,du = \ln|\ln u| + c$