18.02 Practice Exam 3 — S-2010 (120 pts. = 60 minutes)

Problem 1. (15 pts: 3 each part)

a) Write down in xy-coordinates the vector field $\mathbf{F}$ whose vector at (x, y) is obtained by rotating 90^o counterclockwise the radially-outward-pointing unit vector at (x, y).

b) Let $\mathbf{F}$ be the field in part (a). Let C_1 be the line segment running from (1,1) to (2,2), and C_2 the positively-oriented circle of radius a, center at origin. Using intuition, give the value of the following (short answer; no calculation required):

(i) $\int_{C_1} \mathbf{F} \cdot d\mathbf{r}$: (ii) $\oint_{C_2} \mathbf{F} \cdot d\mathbf{r}$: (iii) flux of $\mathbf{F}$ across C_1: (iv) flux of $\mathbf{F}$ across C_2:

Problem 2. (10 pts.) Let $\mathbf{F} = \nabla f = \text{grad} f$, where $f(x, y) = x^2 + 4y^2$.

a) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is a curve running from (1,1) to (2,2).

b) Find the locus of all points $P : (x, y)$ in the plane such that $\int_{(1,1)}^{(x,y)} \mathbf{F} \cdot d\mathbf{r} = 0$.

Problem 3. (20) Let $\mathbf{F} = y(ax + y)\ \mathbf{i} + (3x^2 + bxy + y^3)\ \mathbf{j}$, a, b constants.

a) Prove: if $\mathbf{F}$ is conservative, then $a = 6$, $b = 2$. (Use these values in part (b).)

b) Using a systematic method (show work), find a function $f(x, y)$ such that $\mathbf{F} = \nabla f$.

Problem 4. (20 pts: 8, 12) Let C be the portion of the parabola $y = 1 - x^2$ lying over the x-axis, oriented so that the positive direction is to the left, i.e., the direction in which x decreases. Taking

$$\mathbf{F} = (6xy^5)\ \mathbf{i} + (1 + x^2y - y^6)\ \mathbf{j}\ ,$$

a) Set up an integral in x alone which represents the flux of $\mathbf{F}$ over C. (Give integrand and limits, but do not evaluate.)

b) Calculate the flux of $\mathbf{F}$ over C by using Green's theorem in the normal form. (Note that C is not a closed curve.)

Problem 5. (10 pts.) Show that the value of $\oint_C (y^2 - 2y)\, dx + 2xy\, dy$ around a positively oriented circle C depends only on the size of the circle, and not upon its position.

Problem 6. (15 pts.: 5, 10) Let $\mathbf{F} = \dfrac{x\mathbf{i} + y\mathbf{j}}{x^2 + y^2}$.

a) Show that $\text{curl}\ \mathbf{F} = 0$ at every point except $(0, 0)$, where $\mathbf{F}$ is undefined.

b) Show that $\oint_C \mathbf{F} \cdot d\mathbf{r}$ around any positively-oriented simple closed curve C surrounding the origin has a value independent of C.

Problem 7. (30) Set up, but do not evaluate, triple integrals for (a) and (b):

a) Using rectangular coordinates: the mass of the tetrahedron in the first octant cut off by the plane $3x + 2y + z = 1$; this is one face, the other three faces lie in the three cooordinate planes and one vertex is at the origin; take the density function to be $\delta = z$.

b) Using cylindrical coordinates: the moment of inertia about the z-axis of the solid upper hemispherical ball of radius a, flat side formed by the disc of radius a centered at the origin in the xy-plane.

c) Using spherical coordinates: same as (b), but both set up and evaluate the integral.

18.02 SOLUTIONS: PRACTICE EXAM 3

1 a) $\dfrac{-y\hat{\imath}+x\hat{\jmath}}{\sqrt{x^2+y^2}}$

b) (i) 0 ($\vec{F}$ is $\perp$ to C_1)

(ii) $2\pi a$ ($|\vec{F}|\cdot$ length of C_2) (since $\vec{F}$ has same dir'n as C_2)

(iii) $-\sqrt{2}$ ($|\vec{F}|\cdot$ length of C) flux is in neg. direction: $R \to L$

(iv) 0 (since $\vec{F}$ has dir'n of C_3)

2 Using F.T.C. for line int's

a) $\displaystyle\int_C \vec{F}\cdot d\vec{r} = f(x,y)\Big]_{P_0}^{P_1} = x^2+4y^2\Big]_{(1,1)}^{(2,2)} = 15$, $\vec{F} = \nabla f$

b) Want all (x,y) such that $x^2+4y^2\Big]_{(1,1)}^{(x,y)} = 0$: $\boxed{x^2+4y^2=5}$

3 a) $\frac{\partial}{\partial y}(axy+y^2) = ax+2y$

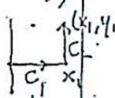
$\frac{\partial}{\partial x}(3x^2+bxy+y^3) = 6x+by$

$\vec{F}$ conservative $\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\Rightarrow a=6,\ b=2$

(converse is also true here).

b) Method 1:



$\displaystyle\int_{C_1+C_2} y(6x+y)\,dx + (3x^2+2xy+y^3)\,dy$

$C_1: \begin{cases} x=x \\ y=0 \end{cases}$ $\quad \displaystyle\int_{C_1} = 0$ since $y=0$, $dy=0$

$C_2: \begin{cases} x=x_1 \\ dx=0 \\ y=y \end{cases}$ $\quad \displaystyle\int_{C_2} = \int_0^{y_1}(3x_1^2+2x_1y+y^3)\,dy$

$= 3x_1^2y_1 + x_1y_1^2 + \frac{1}{4}y_1^4$ (+c)

3 b) Method 2:

$f_x = 6xy+y^2$

$\therefore f = 3x^2y+xy^2+g(y)$ $\nearrow$ $f_y = 3x^2+2xy+g'(y)$

$= 3x^2+2xy+y^3$

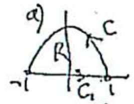
$\therefore g' = y^3,\ g = \frac{1}{4}y^4$ (+c)

$\therefore f = 3x^2y+xy^2+\frac{1}{4}y^4$ (+c)

4 $M = 6xy^5$, $N = 1+x^2y-y^6$

The curve C: $\begin{cases} x=x \\ y=1-x^2 \end{cases}$ x goes from 1 to -1

a)



$\therefore \displaystyle\int_C M\,dy - N\,dx$ (flux)

$= \displaystyle\int_1^{-1} 6x(1-x^2)^5(-2x\,dx)$ [Pretty awful!]

$- (1+x^2(1-x^2)-(1-x^2)^6)\,dx$

b) Make a closed curve by adding C_1: $x=x$, $y=0$

$\displaystyle\int_{C_1} 6xy^5\,dy - (1+x^2y-y^6)\,dx = \int_{-1}^{1} -dx = -2$ (*)

(with $y^5 dy = 0$, $x^2y = 0$, $y^6 = 0$)

$\therefore$ by Green's thm in normal form:

$\displaystyle\int_{C+C_1} M\,dy - N\,dx = \iint_R (M_x+N_y)\,dA$

$\displaystyle -2+\int_C M\,dy - N\,dx = \int_{-1}^{1}\int_0^{1-x^2} x^2\,dy\,dx$ (**)

(by *)

[since $M_x+N_y = 6y^5+(x^2-6y^5)$]

Inner integral: $x^2(1-x^2)$

Outer integral: $\frac{1}{3}x^3-\frac{1}{5}x^5\Big]_{-1}^{1} = \frac{4}{15}$

Therefore by (**):

$\displaystyle\int_C M\,dy - N\,dx = \frac{4}{15}+2 = \frac{34}{15}$

5 $\displaystyle\oint_C (y^2-2y)\,dx + 2xy\,dy$

$= \displaystyle\iint_R [\overset{N_x}{2y} - (\overset{M_y}{2y-2})]\,dA$

$= \displaystyle\iint_R 2\,dA = 2\cdot(\text{area of } R)$, area of $R = \pi a^2$

which is indep't of the position of R (a = radius of circle)

6 a) $\frac{\partial}{\partial x}\left(\frac{y}{x^2+y^2}\right) = \frac{-2xy}{(x^2+y^2)^2}$

$-\frac{\partial}{\partial y}\left(\frac{x}{x^2+y^2}\right) = -\frac{-2xy}{(x^2+y^2)^2}$

$\therefore$ the sum curl $\vec{F} = N_x - M_y = 0$.

b)

C_1 is a small circle inside C.

$\displaystyle\oint_{C_1} + \oint_C \vec{F}\cdot d\vec{r} = \iint_R \text{curl}\,\vec{F}\,dA = 0$

(by the extension of Green's thm) (Green's thm cannot be applied directly to C since the curl is undefined at $(0,0)$ inside C)

$\therefore \displaystyle\oint_C \vec{F}\cdot d\vec{r} = -\oint_{C_1}\vec{F}\cdot d\vec{r} = 0$. since $\vec{F}$ is outward radially, $\therefore$ perpendicular to C_1.

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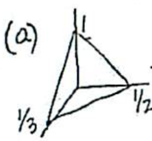
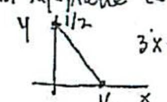
(a)



$3x+2y+z=1$

In xy-plane ($z=0$)



$3x+2y=1$

$y = \frac{1}{2}(1-3x)$

$\therefore \displaystyle\int_0^{1/3}\int_0^{\frac{1}{2}(1-3x)}\int_0^{1-3x-2y} z\,dz\,dy\,dx$

(b)

$\displaystyle\int_0^{2\pi}\int_0^a\int_0^{\sqrt{a^2-r^2}} r^2\,\underbrace{dz\,r\,dr\,d\theta}_{dV}$

$\underbrace{x^2+y^2}_{r^2}+z^2 = a^2$, $\therefore z = \sqrt{a^2-r^2}$

(c) $\displaystyle\int_0^{2\pi}\int_0^{\pi/2}\int_0^a \underbrace{(\rho\sin\varphi)^2}_{r^2}\cdot\underbrace{\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta}_{dV}$

$\displaystyle\iiint \rho^4\sin^3\varphi\,d\rho\,d\varphi\,d\theta$:

Inner: $\frac{\rho^5}{5}\sin^3\varphi\Big]_0^a = \frac{a^5}{5}\sin^3\varphi$

Middle: $\displaystyle\frac{a^5}{5}\int_0^{\pi/2}\sin^3\varphi\,d\varphi = \frac{a^5}{5}\cdot\frac{2}{3\cdot1} = \frac{2a^5}{15}$

Outer: $\frac{2a^5}{15}\cdot 2\pi = 4\pi a^5/15$

<u>18.02 Test 3</u>

4/21

Go through notes

Starting 3/30

Vector fields + line integrals

force fields

differ at each pt

$\vec{F}$ = magnitude $\cdot$ direction

directional vector tangent

Line integral = work done $\boxed{\text{by}}$ a force along curve

$F \cdot \overrightarrow{P_1 P_2}$

F non constant as move along curve

$F = \langle M(x,y), N(x,y) \rangle \cdot \vec{r}$

$\vec{r}$ arrow vector

Sum up all those vector fields

$\sum M \Delta x + N \Delta y$

$\int_C \vec{F} \cdot dr$

paramatrize curve in terms of t

$\int M dx + N dy$

(2)

$$\int_C \vec{F} \cdot d\vec{r}$$

$$=$$

$$\int_C M\,dx + N\,dy$$

$$\int_{t_1}^{t_2} \left[F(x(t), y(t)) \cdot \frac{dr}{dt} \right] dt$$

or 3rd way: intrinsic calculation

$$\int \vec{F} \cdot \vec{T}\, ds$$

↑ $\frac{dr}{ds}$ unit tangent vector

(This all makes more sense now!)

in a <u>constant</u> field / $\boxed{\text{"gradiant"}}$

path does not matter

Rember $\perp$ dot product $= 0$

Remember the

$dx = 0$ $x = 5$

etc

Learn to see it in all 4 forms

- think I did
- but not the work vs flux

(3) traveling the other way is $\ominus$ neg

Paramitrarization

$x = t^2$

$\vec{F} = \langle y^2, 2xy \rangle$

$y = t^3$

$\int y^2\,dx + 2xy\,dy$

take differential

replace w/ t

and integrate

~~$x = t^2$~~

Gradiant field

$\vec{F} = \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \rangle$ has $\nabla f(x,y)$

$\uparrow \qquad \uparrow$

$M \qquad N$

$\int \vec{F} \cdot dr = f(x,y)$ yeah gradiant is the ans

(know how this works - clarify)

nothing to add since its constant

$\vec{F} = \nabla \vec{f}$

So F is the derivitive of f (the function)

So ~~$\int f = F$~~

$\vec{F} = \langle \frac{\partial x}{\partial f}, \frac{\partial y}{\partial f} \rangle$

f

$\downarrow$ derivitive

F

integrate

4

$$\int \left(\frac{\partial f}{\partial x} \frac{dx}{dt} \quad \frac{\partial f}{\partial y} \frac{dy}{dt} \right) dt$$

? when do you split it like this

$F = \langle M, N \rangle$

$f = \frac{\partial M}{\partial x} \quad \frac{\partial N}{\partial y}$

vector

- path inndependent
- Conservative

So given F

- paramatize w/ curve

$M\,dx + N\,dy$

?   ?

$F = \langle x+y, x \rangle$

do pratice test of course

Think I am clearer on paramitizing now than before

Can do whatever parameter

Wat is this FTC thing

- w/ gradiant fields

$$\int_{P_0}^{P_1} \quad f(P_1) - f(P_0)$$

(5) How to know is parameter field?

$M_y = N_x$

$\langle M, N \rangle = \vec{\nabla} f = \vec{F}$

Proof to memorize

Hyp: $F = \vec{\nabla} f$

$\int \vec{F} \cdot d\vec{r} = \int_{t_0}^{t_1} \left(\frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt} \right) dt$ ← long calculating form

$= \int_{t_0}^{t_1} \left[\frac{df}{dt} (x(t), y(t)) \right] dt$

$= f(x(t), y(t)) \Big|_{t_0}^{t_1}$

$= f(x_1, y_1) - f(x_0, y_0)$

wait - how is that notable?

In the past practice test just like real exam

- expect that, but be preped w/ common context

$M_y = f_{xy}$

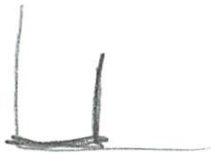
$N_x = f_{yx}$

) and they usually = each other

Can prove w/ approximation thing

(forget name)

⑥ Actually Finding $f(x,y)$
if gradient test



Now greens theorm (4/8/10)
-seems so recent

- smooth curve, no over lap
- divided ~~...~~ plane into 2 parts

∞ exterior
Finite R

$\vec{F} = M\hat{i} + N\hat{j}$ as usual - differential etc

(-didn't do work vs flux - think later...)

* interior always on left *

$\oint$ ← closed path integral

$$\oint M\,dx + N\,dy = \iint_R (N_x - M_y)\,dA = \iint \text{curl } F \cdot dA$$

note
not usual

and this is ~~flux~~ work I think

* curl = work

(7) (goes so slow 45 min - 7th ~~pg~~ pg of notes 9:30)

Area under hypocycloid

divide curves into parts

-could perhaps do as on example

$\oint M\,dy = -\iint_R \frac{\partial M}{\partial y}\,dA$

(~~th~~ was 2x integrals on last test)

~~y~~yeah, main focus

then proof not writing

~~Flux~~

Remember $F = M\hat{\imath} + N\hat{\jmath}$ = Force Field

$\oint \vec{F} \cdot \hat{T}\,ds$

Curl $= N_x - M_y$ = ~~...~~ work

---

flux = net flow across C

$\vec{F}$ constant

only perpendicular counts

(I guess w/ work on parallel counts)

divide curve up into little paths

8

remember $\hat{t}$ = unit tangent vector $\langle \frac{dx}{ds}, \frac{dy}{ds} \rangle$

$\hat{n} = \langle -\frac{dy}{ds}, \frac{dx}{ds} \rangle$

$\int F \cdot \hat{n}$

as opposed to $\int F \cdot t$ for work

~~div~~ divergence $= M_x + N_y$ = flux

notice back to normal

and $F = \langle M, N \rangle = \Delta f$

?

∇ oops

or $\frac{\text{net flux}}{\text{surface area}}$

Applications + extensions

$F = \frac{-y\,\hat{i} + x\,\hat{j}}{\sqrt{x^2+y^2}}$ means ~~pa~~ tangent to circle

- calc curl = 0

Oh right - not always applicable

like when $\vec{F}$ not defined at origin

think I got holes theory

- but have not done 1 qu on it

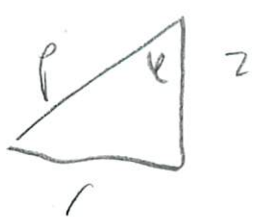
(9)

w/ holes = extended Green's Theorm

think I get this stuff much better now

this triple $\int$ was from this week

- did P-set last night ....
  - don't know the formal conversions



$r = \rho \sin \varphi$

$z = \rho \sin \varphi$ - if I can picture it can I make it

- still good to know conversions
  - like review today

$\rho^2 \sin \varphi \, d\rho \, d\varphi \, d\theta$

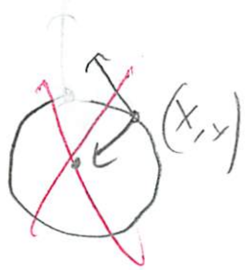
- also know how those fields convert like recitation today
  - think I just need to do pratice test

Pratice Test

10

Practice Test

1. Write down in x-y coord the vector Field $\tilde{F}$ whose vector at (x,y) is obtained by 90° CCW radial out vector at (x,y)



$t = \langle \frac{dx}{dt}, \frac{dy}{dt} \rangle$

$\hat{n} = \langle -\frac{d\cancel{x}}{dt}, \frac{d\cancel{x}}{dt} \rangle$

This is − that

$\langle \frac{dy}{dt}, \frac{dx}{dt} \rangle$

$$\tilde{F} = \frac{-y\hat{i} + x\hat{j}}{\sqrt{x^2+y^2}}$$

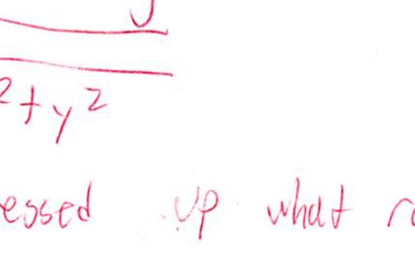
- So messed up what radially meant

$$\frac{-y\hat{i} + x\hat{j}}{x^2+y^2} = \text{tangent to circle}$$

is no $\sqrt{\ }$ a mistake?

~~and what~~ is

Oh and



just remember $\frac{-y\hat{i} + x\hat{j}}{\sqrt{x^2+y^2}}$ = tangent to circle

(12)

b) Let $\vec{F}$ be field in part a

two seperate - read closer - cure always screws one up

⊕ oriented → region to left ✓

$\int_{C_1} F \cdot dr$ ← will be X

$$\frac{-y}{\sqrt{x^2+y^2}}\,dx + \frac{+x}{\sqrt{x^2+y^2}}\,dy$$

$$\boxed{\cancel{\frac{-y+x}{\sqrt{x^2+y^2}}}} \quad 0$$

$\vec{F}$ is $\perp$ to C_1 →

The next morning I know

Or is it that param itrizing thing I never d.

$y = x \quad x[1,2]$

$x = x$

$dx = 1 \quad dy = 1 \quad \int_1^2$

- ~~So what did I do wrong here?~~
- ~~draw field always help~~
- ~~but it was process for~~ only doing $\perp$??

and it cancles

ii) $\oint_{C_2} \vec{F} \cdot dr$ is it 0 because closed loop?

- well is it conserative field — no

but in a circle?

(13)

No

$|F| \cdot$ lenght of C_2 $\cdot ds$

$F \cdot 2\pi a$

circumerence

$2\pi a$

F has same dir C

i why did this get so screwed up?

iii) Flux across C_1

$F = \langle M, N \rangle$

$\langle \frac{-y}{\sqrt{x^2+y^2}}, \frac{x}{\sqrt{x^2+y^2}} \rangle$

$M_x =$

$-y\,(x^2+y^2)^{-1/2}$

$-y \cdot -\frac{1}{2} \cdot (x^2+y^2)^{-3/2} \cdot 2x$

$\frac{+2xy}{2(x^2+y^2)^{3/2}} = \frac{xy}{(x^2+y^2)^{3/2}}$

did differenintation correct

(14)

~~flux across C_2~~

$$\int_0^{2\pi} \frac{xy}{\sqrt{x^2+y^2}} - \frac{xy}{\sqrt{x^2+y^2}}$$

forgot dr

dr

$= 2\pi\sqrt{x^2+y^2}$

$\cancel{0}$

$-\sqrt{2}$

$|F|$ · lenght of C

flux in - dir

next day still dont get

~~could $\oint M\,dx + N\,dy$~~

$\cancel{\oint\left(M\frac{dy}{ds} - N\frac{dx}{ds}\right)ds}$

$\cancel{\oint M\,dy - N\,dx}$

$\boxed{\oint x\,dy - y\,dx}$

d) 0 ✓

Was I mixing things

F · dr

not including curve component

- why getting this wrong - should be slam dunk

do these again later

(15)

2. $F = \nabla f = \text{grad } f$

$f(x,y) = x^2 + 4y^2$

$\nabla f = \langle 2x, 8y \rangle$

a) Eval $\int_C \vec{F} \cdot d\vec{r}$ C from (1,1) to (2,2)

- 2nd chanch

well this might have been what thinking of above

$\cancel{dx = 1 \quad dy = 1}$

$\cancel{\int 2x \cdot 1 + 8y \cdot 1}$

but what is x and y

Wrong again

$\int F \cdot dr = f(x,y)\Big|_{P_0}^{P_1}$ ← they already took gradient

And in constant field

just gradiant

- special rules

Remember. $F = \nabla f$

f

derive ↓ integrate F to get f

F

; so I ~~should~~ have done it

- all ~~confused~~

(a)

$x^2 + 4y^2 \Big|_{1,1}^{2,2}$

$2^2 + 4(2)^2 - [1^2 + 4(1)^2]$

$4 + 16 - 1 - 4$

15 ✓

b) Find the locus of all pts $P(x, y)$
↑?

in the plane such that $F \cdot dr = 0$

→? so find what $x + y$ would make it 0?

(1, 1) would

$x^2 + 4y^2 - 5 = 0$

$\boxed{x^2 + 4y^2 = 5}$

$x = \sqrt{5 - 4y^2}$ $\qquad y = \sqrt{\frac{5 - x^2}{4}}$

so everything that satisfies this

they answer this

locus = set of paths that satisfy a condition

(17)

3. Let $\vec{F} = y(ax+y)\vec{\imath} + (3x^2 + bxy + y^3)\vec{\jmath}$

a, b constants

a) prove that if F is conservative

$a = 6, b = 2$

So $N_x = M_y$

$N = 3x^2 + 2xy + y^3$

$N_x = 3 \cdot 2x + 2y$

$M = 6xy + y^2$ ← did not distribute y.

$M_y = \cancel{1} \quad 6x + 2y$

~~$6x + 2y = 1$~~

~~what more to prove~~

~~$x = \frac{1}{12}$ $x = \frac{1}{6}$~~

~~$y = \frac{1}{4}$ $y = -\frac{1}{2}$~~

$6x + 2y = 6x + 2y$

✓

Cool, knew how to do

got it done

(except for the oversight)

(18)

b) Using a systematic method find $f(x,y)$ $\bar{F} = \nabla f$

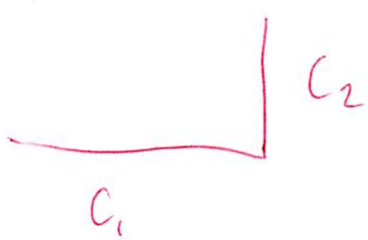
$$\begin{matrix} f \\ \downarrow \text{gradient} \\ F \end{matrix}$$
integrate back

$\int 6xy + y^2 \, dx$ $\qquad$ $\int 3x^2 + 2xy + y^3 \, dy$

$\frac{6x^2y}{2} + xy^2$ $\qquad$ $3x^2y + \frac{2xy^2}{2} + \frac{y^4}{4}$

$3x^2y + xy^2$ $\qquad$ f $\qquad$ $3x^2y + xy^2 + \frac{y^4}{4}$

but they want line integral - not regular integral - which was not in this unit



Must integrate over ~~the field~~ - over the dr along curve

$\int C_1 = 0$ since $y, dy = 0$

$\int_{C_2} = \int_0^4 \ldots$

blah blah

Same what I got for y only

$3x_1^2 y_1 + x_1 y_1^2 + \frac{1}{4} y_1^4 + C$

Remember line integral ~~- but I don't understand why~~

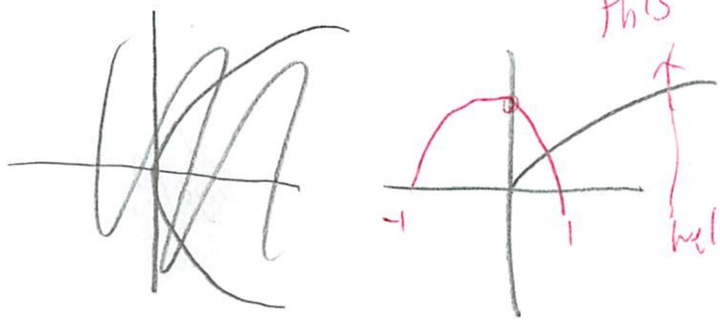
Must integrate along curve

(19)

4. Let C be a portion of porapola

$Y = 1 - x^2$

I know how to draw a porapola!! this is 8th grade



← i like this

the Cs are always cryptic

well I froze up, did not recognize

$\vec{F} = (6xy^5)\hat{\imath} + (1 + x^2y - y^6)\hat{\jmath}$

a) Set up an integral in x & curve : w/ Frelletes
flux

$\iint_A M_x - N_y \, dA$

$M = 6xy^5 \qquad N = 1 + x^2y - y^6$

$M_x = 6y^5 \qquad N_y = x^2 - 6y^5$

$\int \cancel{6y^5} - [x^2 - \cancel{6y^5}] \, dA$ ← cancle!

i then what for limits

how does curve play into

why don't I know this??

i no parameterizing or curve

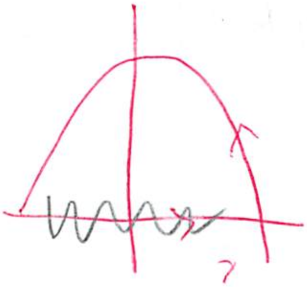
(next pg)

forgot this on every problem!

and do it w/o greens like #1

20

Write what C is



what I had was not even Close!

$$\begin{cases} x = x \\ y = 1-x^2 \end{cases}$$ ← poramatize

$x \in [-1,1]$ ↑ don't forget!

? ✱ Really know how to do this

$$\int M\,dy - N\,dx \quad \leftarrow$$

– Said pretty awful math

– they did not want greens in (a) ← So they subbed in parameters?

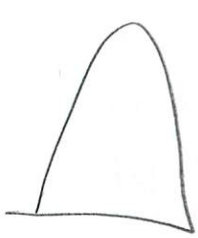
$$\int_{1}^{-1} 6x(1-x^2)^5(-2x\,dx) - \left(1 + x^2(1-x^2) - (1-x^2)^6\right)dx$$

b) Now Green's theorm normal form

↑ flux

✱ Have to add that vertical line ✱

– So do we do each part seperat'y



(21)

~~By Green's theorm~~ in normal

Make it a closed curve

$x = x$ ← don't forget paramatrize curve

$y = 0$

$\int_{C_1} 6xy^5\, dy - (1 + x^2 y - y^6)\, dx$ } have to do this line seperatly

(to 0; $y \to 0$, $y \to 0$)

$\int_{-1}^{1} -dx = -2$ ← find

By Green's theorm in normal form

now normal like

$\int_{C_1 + C} M\,dy - N\,dx$

$= \iint_R (M_x + N_y)\, dA$ So many different forms

? oh it is over R in this form

$-2 + \int_{-1}^{1} \int_{0}^{1-x^2} x^2\, dy\, dx$

From above — or this

not this, forgot over region

got this part

then solve . . .

$\oint M\,dy - N\,dx = \iint_R (M_x - N_y)\, dA$

(22)

$2+\int_0^{1-x^2} x^2 \, dy$

$x^2 y \Big|_0^{1-x^2}$

$x^2(1-x^2)$

$x^2 - x^4$

$\int_{-1}^{1} x^2 - x^4 \, dx$

$\frac{x^3}{3} - \frac{x^5}{5} \Big|_{-1}^{1}$

$\frac{1}{3} - \frac{1}{5} - \left[\frac{-1}{3} + \frac{1}{5}\right]$

$\frac{1}{3} - \frac{1}{5} + \frac{1}{3} - \frac{1}{5}$

$2+\ \frac{2}{3} - \frac{2}{5}$

$2+\ \frac{10}{15} - \frac{6}{15}$

$2+\frac{4}{15}$

$\frac{34}{15}$ ✓ solved right

but would have prob forgot the +2 — dont do it

(23) Still troubled how baddly I do on basic math
- understand it better next day

5. $\oint (y^2-2y)\,dx + 2xy\,dy$

depends only on size of circle - not position

- this was on p-set and I ~~forgot how to do~~ did not get it then

Looked ?

$\cancel{\iint M_x - N_y}$ Not flux but work

$\cancel{M = y^2 - 2y}$ $N_x = 2y$

$\cancel{M_x = 0}$ $M_y = 2y - 2$

$\cancel{N = 2xy}$

$\cancel{N_y = 2x}$

$\cancel{\iint -2x\,dA}$

$\iint 2y - [2y - 2]$

$\iint +2$ ← so depends only on size?
- but <u>not</u> position
(no x or y)

know this part

$= 2 \cdot \text{area of } R = 2 \cdot 2\pi a^2$

this type of problem seems easy

(24)

6.

$$\vec{F} = \frac{x\,\vec{i} + y\,\vec{j}}{x^2+y^2}$$

? so this is not w/ $\sqrt{\ }$ — does that mean anything special ??

- something about radially out
  - not tangent

a) Show curl $\vec{F} = 0$ except $(0,0)$ where undefined

- so I know radially outward
- from recognizing pattern
- but how prove

- curl is work
- which is $\perp$ and 0

Oh actually do it

=

$M = \dfrac{x}{x^2+y^2}$ $\qquad$ $N = \dfrac{y}{x^2+y^2} = y(x^2+y^2)^{-1}$

$N_x = 0$ $\qquad$ $M_y = 0$

$S\ 0+0 = 0$

actually do differentiation don't just guess

(25) Well

$$N_x = \frac{-2xy}{(x^2+y^2)^2}$$

$$- M_y = -\frac{2xy}{(x^2+y^2)^2}$$

$$N_x - M_y = 0$$ curl is addintonal normally

- somewhat confused on signs
- and how did deriv

$$N_x = y \cdot -1 (x^2+y^2)^{-2} \cdot 2x$$

$$\frac{-2xy}{(x^2+y^2)^{-2}}$$ it works out if I would actually do it

b) Show $\oint F\,dr$ around any curve has value independent of C

- or conservative

$N_x = M_y$

✓ which it does

gradient field

oh for this need to show it has hole

(26)

$\oint_{C_1} + \oint_C = 0$

I think I pretty much got it except the hole.

7. Set up, but don't eval tripple Integral

Mass of tetrahedron cut by plane

$3x + 2y + z = 1$ 1st quad

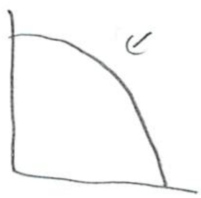
3 cord planes

$\delta = 2$

M

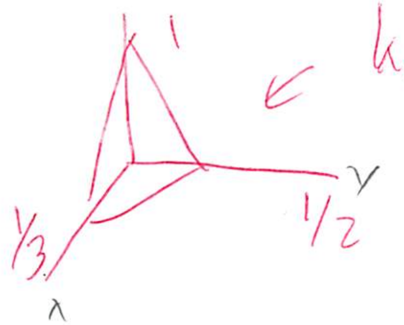
$$\int_{x=0}^{\cancel{1}} \int_{y=0}^{3/2x} \int_{z=0}^{z = 1-3x-2y} 2 \, dz \, dy \, dx$$

No!



$y = \frac{3}{2}x$

(27)



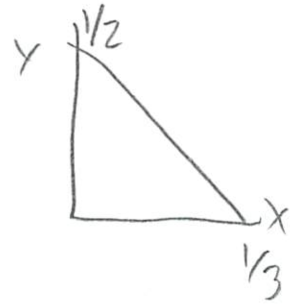
know this

keep z fixed
Sove fo y

$$\int_0^{1/3}\int_{\cancel{\frac{3}{2}x}\;0}^{\frac{1}{2}(1-3x)}$$

$\frac{3}{2}\cdot\frac{1}{3} = \frac{3}{6} = \frac{1}{2}$

Its not that hard!



max value   function   solve for z

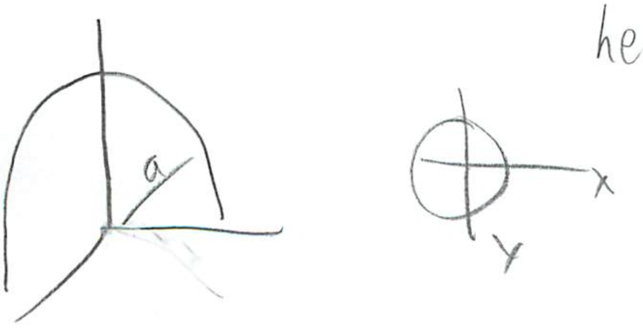
$$\int_0^{1/3}\int_0^{\frac{1}{2}(1-3x)}\int^{1-3x-2y} z\,dz\,dy\,dx$$

close - need to better on limits

b) Using cylindrical coords

Moment of inertia about z-axis

hemisphere



(28)

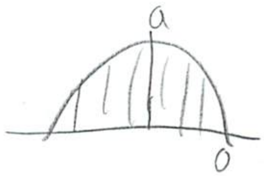
And what is moment of inertia?

$$\frac{\iiint -}{\text{Mass}}$$

~~remember from~~

$\boxed{r^2}$ ← ? how fand

$$\underset{\theta}{\int_0^{2\pi}} \underset{r}{\int_0^{a}} \underset{z}{\int^{\cancel{r\cos\theta}\ \sqrt{a^2-r^2}}} r^2 \quad r\,dz\,dr\,d\theta$$



$r\cos\theta$

$\underbrace{x^2+y^2}_{r^2}+z^2=a^2 \quad z=\sqrt{a^2-r^2}$

↑ do it like that

knew like $r^2+z^2=a^2$

↑

r is just across, right?

(2a)

c) Using spherical coords do b + evaluate

- eaiser

~~correct~~

$$\int_0^{2\pi}\int_0^{\pi/2}\int_0^{a} \rho^2 \; \rho^2 \sin\varphi \, d\rho \, d\phi \, d\theta$$

$\theta \quad \cancel{\phi}\,\varphi \quad \rho$ ✓

φ as letter

did not find right

$$r^2 = (\rho \sin\varphi)^2$$

not just ρ^2

$$\int_0^{2\pi}\int_0^{\pi/2}\int_0^{a} (\rho\sin\varphi)^2 \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$$

$$\rho^4 \sin^3\varphi \, d\rho \, d\varphi \, d\theta$$

$$\int_0^{a} \rho^4 \sin^3\varphi \, d\rho$$

$$\rho^5 \Big|_0^{a}$$

$$\int_0^{\pi/2} a^5 \cancel{d\varphi} \sin^3\varphi \, d\varphi$$

calc error

$$a^5 \frac{\sin^4\varphi}{4} \Big|_0^{\pi/2}$$

$$a^5 \cancel{\frac{\sin^4 \frac{\pi}{2}}{4}} \; \frac{2}{3\cdot 1}$$ ← what math here

30

~~th~~ I got something really long on calc undef w/ limits I got ~~$\frac{32}{16}$~~ $\frac{2}{3}$ ✓

-think he said this may be given

Could do it perhaps

$$-\frac{\sin^2 x}{3} - \frac{2}{3} \cdot \cos x$$

No -no way

and oliver dropped that qu at oh

$$\int_0^{2\pi} \frac{2a^3}{15}\, d\theta$$

$$= \boxed{\frac{4\pi a^3}{15}}$$ ✓

<u>Test Reflection</u> 4/25

Was able to do toward origin

1b ii) Lost one point for not identfing $r=1$

- gave stupid mistake
- not using all info in problem

2) forgot parenthsis
   - but no effect on problem!

and they don't do limit
   - but area

~~did on test~~ See test

Green's Theorm → must be closed

18.02 Exam 3 Thurs. Apr. 21, 2010 11:05-11:55

Directions:

1. There are 3 sheets, printed on both sides: eight problems in all.
2. Do all the work on these sheets; use the blank part below if truly necessary. Write down enough to show you are not guessing.
3. No books, notes, calculators, use of cell-phones, etc.
4. Please don't start until the signal is given; stop at the end when asked to; don't talk until your paper is handed in.
5. When the exam starts, read through the exam and start with what you are surest of.
6. Fill out the information below now.

Name Michael Plasmeier e-mail@mit.edu theplaz

Recitation teacher Oliver Rec. hour 12

$$\int_0^{\pi/2} \sin^n\theta\,d\theta = \int_0^{\pi/2} \cos^n\theta\,d\theta = \frac{(n-1)!!}{n!!}A, \quad \text{where } A = \begin{cases} \pi/2, & n \text{ even} \\ 1, & n \text{ odd} \end{cases} \quad \text{and } n!! = n(n-2)(n-4)\ldots$$

mean 76

passing 60

pg.1 16

pg.2 11

pg.3 6

pg.4 8

pg.5 11

Total 52

$$\frac{x\hat{\imath}+y\hat{\jmath}}{\sqrt{x^2+y^2}} \text{ (out)} \qquad \frac{-x\hat{\imath}-y\hat{\jmath}}{\sqrt{x^2+y^2}} \text{ (in)}$$

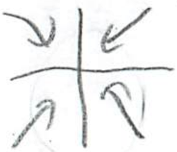
16

Problem 1. (10 pts: 4; 3,3)

a) Write down in xy-coordinates the vector field $\mathbf{F}$ whose vector at (x, y) points *towards* the origin and has unit length.

did not memorize

4



F = dir ∘ lenght

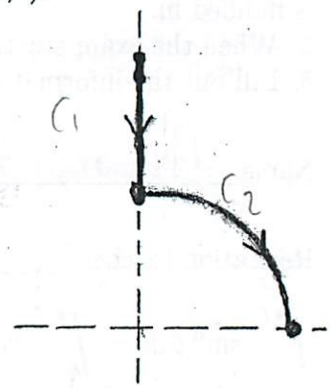
$$\frac{-x\hat{\imath}-y}{\sqrt{x^2+y^2}} \checkmark$$

$\cancel{\frac{-x}{x}} \circ 1$

b) (No calculation required or expected): For the oriented path C consisting of the line segment from (0,2) to (0,1), followed by the quarter-circle (center (0,0), radius 1) from (0,1) to (1,0):

(i) find the value of $\int_C \mathbf{F} \cdot d\mathbf{r}$ on the path C

(ii) find the flux of $\mathbf{F}$ across C.



3 i) Only C_1 matters, because $C_2 \perp$ so it $= 0$

$\int M\,dx + N\,dy$ with $dx = 0$, $x = 0$: $\int \frac{-y}{\sqrt{0^2+y^2}}\,dy$ ✓

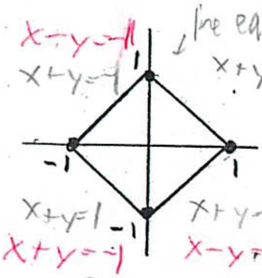
$\int_1^2 1\,dy = 1$

2

ii) Flux $= \int M\,dy - N\,dx$ $\qquad \int_0^{\pi/2} \vec{F} \cdot d\vec{r}$ $\qquad r = 1$

Only C_2 matters because $\perp$ $\qquad \frac{1}{4} \cdot 2\pi r = \frac{\pi r}{2}$ $\qquad$ Stupid

Problem 2. (10) Use Green's theorem to evaluate $\oint_C (y^3 + 2y)\,dx + x(3y^2 - 1)\,dy$ over the positively oriented square C having vertices at the four points ± 1 on the x- and y-axes.



$x - y = -1$, $x + y = 1$ $\qquad$ line equ: $x + y = 1$

$x + y = 1$, $x + y = -1$ $\qquad$ $x + y = -1$, $x - y = +1$

← think got C wrong

7

$M = y^3 + 2y \qquad N = 3xy^2 - x$

$N_x = 3y^2 - 1 \qquad M_y = 3y^2 + 2$

$\iint_R 3y^2 - 1 \,\cancel{+}\!-\, (3y^2 + 2)\,dA$

oh it was $N_x - M_y$
- and flux is $M_x + N_y$
- confused them

$\int_{-1}^{1} \int_{-1+x}^{1-x} 6y^2 - 1\,dy\,dx$

← they just multiplied
$-3 \circ$ area
$-3 \circ (\sqrt{2})^2$
← know lenght of 1 side
-6

$y = -1 - x \qquad \left. \frac{6y^3}{3} - y \right|_{-1-x}^{1-x}$

$(1-x)^2(1-x)$

$(1^2 - 2x + x^2)(1-x)$

$1 - 2x + x^2 + x + 2x^2 - x^3$

$-x^3 + 3x^2 - 3x + 1$

$2(1-x)^3 - (1-x) - \left[2(-1-x)^3 - (-1-x)\right]$

$2(-x^3 + 3x^2 - 3x + 1) - 1 + x - 2(-1-x)^3 - 1 - x$

Problem 3. (20 pts: 5,10,5) Let $\mathbf{F} = ay(x-y)\,\mathbf{i} + (x^2 - bxy + 3y^2)\,\mathbf{j}$, where a and b are constants.

a) Prove that if $\mathbf{F}$ is conservative, then $a = 2,\ b = 4$.

$N_x = M_y$

$M = 2xy - 2y^2 \qquad N = x^2 - 4xy + 3y^2$

$M_y = 2x - 4y \qquad N_x = 2x - 4y$

$2x - 4y = 2x - 4y$

$N_x = M_y$ so conservative

b) Let $a = 2,\ b = 4$.
Using a systematic method (show work), find a function $f(x, y)$ such that $\mathbf{F} = \nabla f$.

integrate along path

← stupid thought mistake

$\int_{C_1} M\,d\cancel{x} + N\,d\cancel{y} + \int_{C_2} M\,d\cancel{x} + N\,d\cancel{y}$ ← flipped

work $\int M\,dx + N\,dy$

flux $\int M\,dy - N\,dx$

$\to 0 \quad \cancel{x=0} \quad y=0 \qquad x = X \quad \to 0$

$\int_0^x \cancel{x^2 - 4xy + 3y^2\,dx} + \int_0^y \cancel{2xy - 2y^2}\,dy$

$x = 0 \qquad C_1 = 0 \qquad x^2 - 4xy + 3y^2$

did remember right

2 mistakes
- flipping work + flux
- and $y=0$ $dy=0$

$\int_0^x 3y^2\,dx + \int_0^y 2xy - 2y^2\,dy$

$3xy^2\Big|_0^x + \frac{2xy^2}{2} - \frac{2y^3}{3}\Big|_0^y$

$3xy^2 + xy^2 - \frac{2}{3}y^3$

$4xy^2 - \frac{2}{3}y^3$ X −5

$x_1^2 y_1 - 2x_1y_1^2 + y_1^3$

should have # - but don't have x, y pts! so ok

c) Continue to use $a = 2,\ b = 4$, and using the earlier parts of this problem:
evaluate $\int_C \mathbf{F}\cdot d\mathbf{r} = 0$ along the path C given by: $x = \tan t,\ y = \sin 2t$, as t runs from 0 to $\pi/4$.
(This is easy, but you must show the work.)

Fundamental Theorem of calculus line integrals

well gradient field so any path is the same - if gradient field

$\int_C F\cdot dr = f(x,y)\Big|_{P_1}^{P_2}$

Could use path above

$t=0 \to (0,0)$

$t=\pi/4 \to (1,1)$

at $\frac{\pi}{4}$ $\quad x = \tan\frac{\pi}{4} = 1$

$y = \sin 2\frac{\pi}{4} = 0$ X −4

$4(1)(0)^2 - \frac{2}{3}(0)^3 = 0$

$yx^2 - 2y^2x + y^3\Big|_{0,0}^{1,1} = 0$

6/20 ran out of time

Problem 4 (20 pts: 5, 10, 5, 0)

Verify Green's theorem in the tangential (work) form for the vector field $\mathbf{F} = -y^3\,\mathbf{i} + x^3\,\mathbf{j}$ and the closed curve C consisting of the two segments of the x- and y-axes connecting the origin with the points $(a, 0)$ and $(0, a)$, plus the quarter-circle given by $x = a\cos\theta$, $y = a\sin\theta$, $0 \le \theta \le \pi/2$, $a > 0$.

That is, showing work:

(a) apply Green's theorem;

(b) evaluate the line integral (over all of C);

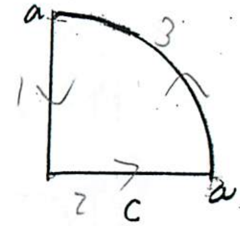
(c) evaluate the double integral;

(d) spend the next twenty minutes trying to make them agree.

(In calculating the line integral you will need the formulas on the cover sheet.)

hopefully not confused

Correction θ paramitrizing w/ θ



$\int\int_{C_3} M\,dy - N\,dx$ ← flux form

$dx = -a\sin\theta$

$dy = a\cos\theta$

$+a^3\sin^3\theta\, a\sin\theta + a^3\cos^3\theta\, a\cos\theta$

curl not calculated

~~Not Needed~~ Green not stated

$\int_0^{\pi/2}\int_0^a (-a^4\sin^4\theta - a^4\cos^4\theta)\, r\,dr\,d\theta \qquad a = r$

why double integral?

$-\frac{a^4 r^2}{2}(\sin^4\theta - \cos^4\theta)\Big|_0^a$

$-\frac{a^6}{2}(\sin^4\theta + \cos^4\theta)$

$-\frac{a^6}{2}\int_0^{\pi/2} \sin^4\theta + \cos^4\theta$

$\int \frac{(n-1)!!}{n!!} A$ 1 same

$2\,\frac{(4-1)!!}{4!!}\cdot\frac{\pi}{2}$

$2\,\frac{3(3-2)}{4(4-2)(4-4)}\cdot\frac{\pi}{2}$

$-\frac{a^6}{2}\cdot 2\,\frac{4-6}{16-8}\cdot\frac{\pi}{2}\quad \frac{-a^6\, 3\pi}{\cancel{16}\, 8}$

Now need to do each leg + Green's theorm

$\int M\,dy \qquad \int -N\,dx$

$\int_0^a -y^3\,dy \qquad \int_0^a x^3\,dx$

$-\frac{a^4}{4} \qquad \frac{a^4}{4}$

0 both incorrect

$\int N_x + M_y$

$\int 3x^2 - 3y^2$

? ran out of time

seperate sheet

Problem 5. (10) Let $f(x, y)$ be a function with continuous second derivatives that satisfies the equation

$$f_{xx} + f_{yy} = 0.$$

Show that the field $\mathbf{F} = \nabla f$ has the same *flux* across any two differentiable paths C_1 and C_2 which start at the same point P_1, end at the same point P_2, and don't intersect.

because it is a gradiant field

$\text{Curl } \vec{F} = 0$

seperate sheet

(2)

$\cancel{N_x + M_y} = 0$

$\iint_R 0 = 0$

$N_x = f_{yx}$] if $f_{yx} + f_{xy} = 0$ then $f_{xx} + f_{yy} = 0$

$M_y = f_{xy}$

close

Create closed surface

Use Green's Theorm

$\iint M_x + N_y$

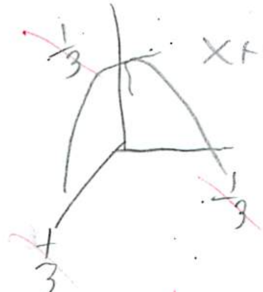
Problem 6. (10 pts.: 8,2)

(6)

Let D be the domain of finite volume in the first octant cut off by the plane $x + y + z = 1$.

a) Set up an iterated triple integral in rectangular coordinates $\int\int\int f(x, y, z)\, dz\, dy\, dx$ for the total mass of D, if the density function is given by $\delta = z$. (Give the integrand and limits.)

b) Then take only the first step in evaluating it, i.e., reduce it to a double iterated integral in x and y.



Mass $\iiint \delta\, dV$

$$\int_{0}^{1/3}\int_{0}^{\frac{1}{3}(1-z)}\int_{0}^{\cancel{\frac{1}{3}(1-z-y)}} z\, dx\, dy\, dz$$

z y x

opps

$$\int_0^1\int_0^{1-x}\int_0^{1-x-y} z\, dz\, dy\, dx$$

Oh that was wrong

it was all 1

set x constant

$y = 1 - z$

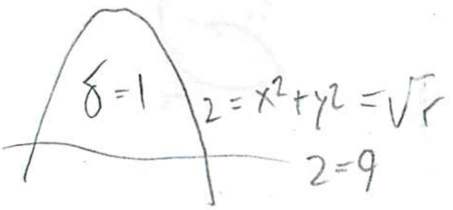
$$\int_0^{\frac{1}{3} - \frac{1}{3}z - \frac{1}{3}y} z\, dx$$

$$zx \Big|_0^{1/3 - \frac{1}{3}z - \frac{1}{3}y}$$

$$\int_0^{1/3}\int_0^{\frac{1}{3}(1-z)} z\left(\frac{1}{3} - \frac{1}{3}z - \frac{1}{3}y\right)$$

$$\frac{1}{3}z - \frac{1}{3}z^2 - \frac{1}{3}yz\; dy\, dz$$

Problem 7. (10) Let V be the finite domain in xyz-space lying between the paraboloid $z = x^2 + y^2$ and the plane $z = 9$. Taking density $\delta = 1$, set up an iterated triple integral in cylindrical coordinates (provide limits and integrand, but do not evaluate) giving its moment of inertia about the z-axis.



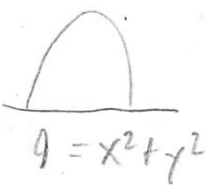
mass $\displaystyle\int_{\theta=0}^{2\pi}\int_{r=0}^{\sqrt{9}=3}\int_{z=9}^{x^2+y^2=\sqrt{r}} r^2 \; dz\, r\, dr\, d\theta$ ← did wrong order

just pase

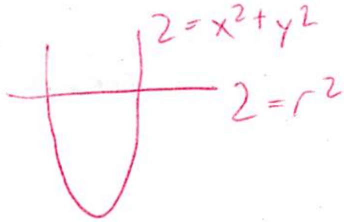
θ

5

$\displaystyle\int_{r^2}^{9}$



drew wrong



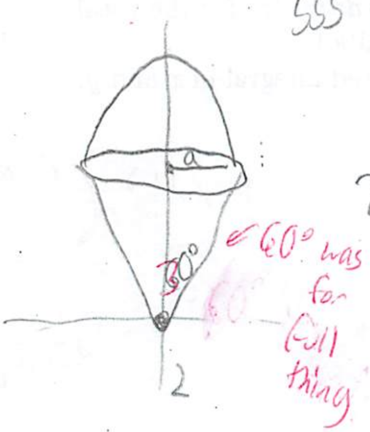
if want distance take $\sqrt{\ }$

$x^2+y^2=r^2$

not $\sqrt{r}$

Problem 8. (10) Let D be the "filled ice-cream-cone" in 3-space whose top surface is formed by a portion of the sphere of radius a centered at the origin, and whose bottom surface is a portion of the circular cone having as axis the z-axis, and whose vertex angle is 60^o. Set up a triple iterated integral in spherical coordinates which gives the volume of D (provide limits and integrand, but do not evaluate).

$\iiint \rho^2 \sin\varphi \, d\rho\, d\varphi\, d\theta$



30°

$\displaystyle\int_{\theta=0}^{2\pi}\int_{\varphi=0}^{60^\circ}\int_{\rho=\cancel{\frac{2\sqrt2}{2}}\,0}^{\cancel{\sqrt2 a + a}\; a} \rho^2 \sin\varphi \, d\rho\, d\varphi\, d\theta$

← can just say

6

$\sin 60 = \frac{a}{r}$

$r \sin 60 = a$

$r \quad \frac{a}{\frac{\sqrt2 a}{2}} = \frac{2a}{\sqrt2 a} = \frac{2\sqrt2}{2}$

$\sin 60^\circ = \frac{\sqrt2}{2}$

$\tan 60 = \frac{a}{r}$

$\tan 60 = \sqrt2$

4)

a) $\oint -y^3\,dx + x^3\,dy = \iint_A 3x^2+3y^2\,dA$

Think I screwed it up again

~~$\iint$ Flux $\iint M\,dy - N\,dx$ ← what I wrote~~

~~Work $\iint M\,dx + N\,dy$ ← what it asked for~~

~~b~~) $\oint F\hat{i}\,dx + F\hat{j}\,dy$

$\iint M\,dx + N\,dy$

?
$F\hat{i}_x$

$$\boxed{\begin{array}{lll} \text{Work} & \oint M\,dx + N\,dy \overset{GT}{=} \iint (N_x - M_y)\,dA \\ \text{Flux} & \oint M\,dy - N\,dx \overset{GT}{=} \iint (M_x + N_y)\,dA \end{array}}$$

tangent (Work) — Both Always work

normal (Flux) — Grad field or not.

the fundemental problem I had on the test was screwing up these 4

M

$$\int M\,dx + N\,dy = \iint (N_x - M_y)\,dA$$

$M = F\hat{i}$ — one equation

b) Now break into parts

$C_1 = y=0 \quad dy=0 \quad \int=0$

$C_3 = x=0 \quad dx=0 \quad \int=0$

$\rightarrow$

$C_2 = \quad x = a\cos\theta \qquad dx = -a\sin\theta$

$y = a\sin\theta \qquad dy = a\cos\theta$

✓ had this

$$\int_{C2} = \int_0^{\pi/2} -a^3\sin^3\theta\,(-a\sin\theta) + a^3\cos^3\theta\,(a\cos\theta)\,d\theta$$

not double
integral
just along
line

$$a^4 \int_0^{\pi/2} (\sin^4\theta + \cos^4\theta)\,d\theta$$

$$a^4\left(\frac{3\cdot 1}{4\cdot 2}\,\frac{\pi}{2} + \frac{3\cdot 1}{4\cdot 2}\,\frac{\pi}{2}\right)$$

$$\frac{3\pi}{8}\,a^4$$

?what did I screw up here

remember !! = every oter one

$n = 4$

$n - 1 = 3$

$(n-1)!! = (4-1)\cdot(4-1-2) = 3\cdot 1$

$n!! = 4\cdot 2$

this was close but I
screwed it up

No it was actually right
except I did not do 16 - 8
really stupid!

c) Here double integral

$= \iint_R 3x^2 + 3y^2 \, dA$

but w/ these #

$\int_0^{\pi/2} \int_0^a 3r^2 \cdot r \, dr \, d\theta$

- use polar

$= \frac{3r^4}{4} \Big|_0^a \cdot \frac{\pi}{2}$

$\frac{3\pi}{8} a^4$

---

5. $\nabla f = \langle f_x, f_y \rangle$

Did not know these rules

flux of $\vec{\nabla} f$ over C

$= \int_C M\,dy - N\,dx$

$M = f_x \quad N = f_y$

$\oint M\,dy - N\,dx$

div

$\iint_R f_{xx} + f_{yy} \, dA$

$$\int_{C_1' + C_2} = -\int_{C_1} + \int_{C_2} = 0$$

$$\int_{C_1} = \int_{C_2}$$

yeah that it is a - proof

4/27

work $\oint M dx + N dy = \iint (N_x - M_y) dA$

flux $\oint M dy - N dx = \iint (M_x + N_y) dA$

$\nabla f = \langle f_x, f_y \rangle$

flux of $\vec{\nabla} f$ over C

$= \int M dy - N dx$

$M = f_x$

$N = f_y$

$\oint (M_x + N_x) dA$

$\cancel{\oint (M + N) dA}$?

$M_x = f_{xx}$ +0

$M_y = f_{yy}$

or could do + −

they just throw in

$\int_{C_1} + \int_{C_2} = 0 \qquad \int_{C_2} = -\int_{C_1}$ etc

Now my mistakes w/ $\iiint$ integrals

$x + y + z = 1$

vertex $(1,0,0)$

$(0,0,1)$

$(0,1,0)$

then # were wrong in limit

and flipped $dx\,dy\,dz$

-stupid

Remember mass = $\iiint \delta \, dV$

inertia ~~mass~~ $\dfrac{\iiint \text{around what } \delta}{\text{mass}}$

---

for other one wrote parabola wrong

$z = x^2 + y^2$ - will be up instead of down

Last one problem written ~~wrong~~ poorly

and then just $\int_0^a$

18.02 Exam 3 Solns Spring 2010

1 a) $\vec{F} = -\dfrac{\langle x, y\rangle}{\sqrt{x^2+y^2}}$

b) $\displaystyle\int_C \vec{F}\cdot d\vec{r} = \underset{\text{on } C_1}{1} + \underset{\text{on } C_2}{0} = 1$

flux over $C = \underset{\text{on } C_1}{0} + \underset{\text{on } C_2}{\frac{\pi}{2}\cdot 1} = \frac{\pi}{2}$

2 $\displaystyle\oint_C (y^3+2y)\,dx + x(3y^2-1)\,dy$

$\displaystyle = \iint_R (N_x - M_y)\,dA$

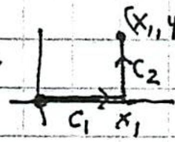
$\displaystyle = \iint_R (3y^2-1)-(3y^2+2)\,dA$

$\displaystyle = \iint_R -3\,dA = -3\cdot\underset{\text{area}}{(\sqrt{2})^2} = -6$

3 $\dfrac{\partial M}{\partial y} = a(x-2y)$ $\quad\dfrac{\partial N}{\partial x} = 2x - by$

a) $a(x-2y) = 2x - by \Rightarrow \begin{array}{l} a = 2 \\ b = 2a = 4\end{array}$

b) Method 1:



$\displaystyle f(x_1,y_1) = \int_{C_1+C_2} 2y(x-y)\,dx + (x^2-4xy+3y^2)\,dy$

$\displaystyle\int_{C_1} = 0 + 0$ (since $y=0$, $dy=0$)

$\displaystyle\int_{C_2} = 0 + \int_0^{y_1}(x_1^2 - 4x_1y + 3y^2)\,dy$
(dx=0)

$= x_1^2y_1 - 2x_1y_1^2 + y_1^3$

Method 2:

$f_x = 2y(x-y)$ $\qquad f_y = x^2 - 4yx + g'(y)$

$f = yx^2 - 2y^2x + g(y)$ $\qquad = x^2 - 4xy + 3y^2$

$\therefore g' = 3y^2, \quad g = y^3$

$f = yx^2 - 2y^2x + y^3$ (+C unnecessary)

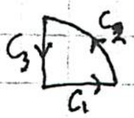
c) $t = 0 \Rightarrow (x,y) = (0,0)$ $\qquad$ use F.T.Calc for line integrals

$t = \pi/4 \Rightarrow (x,y) = (1,1)$

$yx^2 - 2y^2x + y^3\Big]_{(0,0)}^{(1,1)} = 0 - 0 = 0$

4 a) $\displaystyle\oint_C -y^3\,dx + x^3\,dy = \iint_R 3x^2 + 3y^2\,dA$

b) On $C_1: y=0, dy=0$ $\quad\displaystyle\int_{C_1} = 0$



$C_3: x=0, dx=0$ $\quad\displaystyle\int_{C_3} = 0$

C_2: $x = a\cos\theta$ $\quad y = a\sin\theta$

$dx = -a\sin\theta$ $\quad dy = a\cos\theta$

$\displaystyle\int_{C_2} = \int_0^{\pi/2} -a^3\sin^3\theta(-a\sin\theta) + a^3\cos^3\theta\cdot(a\cos\theta)\,d\theta$

$\displaystyle = a^4\int_0^{\pi/2}(\sin^4\theta + \cos^4\theta)\,d\theta =$

$\displaystyle = a^4\left(\frac{3\cdot 1}{4\cdot 2}\cdot\frac{\pi}{2} + \frac{3\cdot 1}{4\cdot 2}\cdot\frac{\pi}{2}\right) = \frac{3\pi a^4}{8}$

c) $\displaystyle\iint_R 3x^2+3y^2\,dA = \int_0^{\pi/2}\int_0^a 3r^2\cdot r\,dr\,d\theta$

(use polar coords) $\displaystyle = \frac{3r^4}{4}\Big]_0^a\cdot\pi/2$

$\displaystyle = \frac{3\pi a^4}{8}$

5 $\vec{\nabla} f = \langle f_x, f_y\rangle$

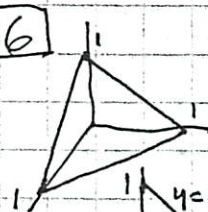
Flux of $\vec{\nabla} f$ over C (any C) $\displaystyle = \int_C M\,dy - N\,dx$

$M = f_x, \; N = f_y$

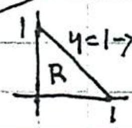
$\displaystyle\oint_{C_1'+C_2} M\,dy - N\,dx = \iint_R \underset{\operatorname{div}(\vec{\nabla} f)}{f_{xx}+f_{yy}}\,dA = 0$

$\displaystyle\int_{C_1'+C_2} = -\int_{C_1} + \int_{C_2} = 0, \quad\therefore \int_{C_1} = \int_{C_2}$

6



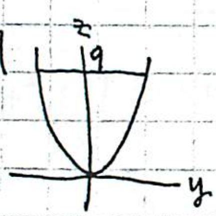
$\displaystyle\int_0^1\int_0^{1-x}\int_0^{1-x-y} z\,dz\,dy\,dx$



Inner: $\displaystyle\frac{1}{2}z^2\Big]_0^{1-x-y}$

$\displaystyle\therefore \int_0^1\int_0^{1-x}\frac{1}{2}(1-x-y)^2\,dy\,dx$

$\boxed{7}$



$z = x^2 + y^2$

$z = r^2$

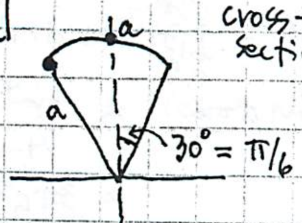
(cross-section)

$$\int_0^{2\pi}\int_0^3\int_{r^2}^9 r^2 \cdot dz \cdot \underbrace{r\,dr\,d\theta}$$

moment of inertia about z-axis

dA in polar coords

$\boxed{8}$



cross-section

eqn cone: $\varphi = \pi/6$

eqn cap: $\rho = a$

$$\text{Vol} = \int_0^{2\pi}\int_0^{\pi/6}\int_0^a \rho^2 \sin\varphi \, d\rho\, d\varphi\, d\theta$$

integrand = dV in spherical coords

Practice Test

4/27

I get #1 now!

When F + C same dir, just multiply · dot product

F · lenght of C

remember diff work + flux

flux $\perp$ matters $\parallel$ 0
work $\parallel$ matters $\perp$ 0

work $\oint M\,dx + N\,dy = \iint_R N_x - M_y \, dA$

$\oint M\,dy - N\,dx = \iint M_x + N_y \, dA$

2. Oh FTC line integral

$\int F \cdot dr = x^2 + 4y^2 \Big|_{(1,1)}^{(2,2)}$

So if grad field $\int F \cdot dr$ is FTC line integrals

locus = set of paths that satisfy a condition

solve for missing variables

$M_x = N_y$

Practice test seems much easier

flux -calc $\int M dy - N dx = \iint_C (M_x + N_y) dA$

#4

~~? Greens~~ ~~? normal~~

?line integral ?greens

$\int M dy - N dx$

x start and end

No greens make it a closed curve
and then

$\int_{C_1+C_2}$

then $= \iint (M_x + N_y) dA$

So must make sure to make it a closed curve
and then $\iint (M_x + N_y) dA$

The prove only depends on area
- find work and it depends

$\iint 2$ area

- everything else drops out

6. There ~~was no other~~ were no holes qu on real test

Curl $= \oint M\,dx + N\,dy = \iint N_x - M_y \, dA$ = work

do it cones at 0

7. Tripple $\int$

$3x + 2y + z = 1$

$x = \frac{1}{3}$ $\quad (\frac{1}{3}, 0, 0)$

$(0, \frac{1}{2}, 0)$

Make sure know other stuff $\quad (0, 0, 1)$

$\underbrace{\rho \sin \varphi}\, d\rho\, d\varphi\, d\theta$

r

know how to convert

Moment of inertia

$\iiint \boxed{}$ around what $\to r^2$

~~$\iiint$ =mass~~ only for com $\qquad$ $com = \frac{\text{inertia}}{\text{mass}}$

I always do for better on Makeup

- need this focus on main
- main exam is same as pratice

only final left

Post test collection

- went fast
- could not remember how to do certain problems

That problem w/ paramitization was FTC

original test had the paramitrerization too

- Parabola should be right, unless flip $\int_{r^2}^{q}$

breaking into parts

- should have paramatrized x + y
- think I started to — then found dx + dy

and can actually solve

actually solving w 2x integral think I got

Hopefully well enough to pass!