Probablistic
Interene 1

I intrinsicist - Frequentist - intuition ist

I joint probability table

Belief nets

Malel Selection + Structure Discovery

\* It's about Models

Representations

Constraint

\* Probability is a saftey net

Probabilistic methods taking over computer theory
last 10 years

New thing outside statent center: hack or art

Can look back at past few years to guide our knowledge.

Or gress to that you think reflect admins attitudes

Produce joint probability table
of charactics
for each possible combos -tally up events that
tit that set of charactict
divide tally by total to get probability

OBM
But the rows grow exponentially
Lnp-problem

Roadmap

(D Joint Probability > (D) A xlows > (G) Conditional > (P) Chain Rile y
Probability > (C) Conditional > (Net

(S) Independence

Independence

Axioms

(a) \( \int \text{(a)} \leq \leq \text{(a)} \)

(b) \( \text{(a)} \)

(c) \( \te

$$P(\text{always occurs}) = 1.0$$
 $P(\text{never occurs}) = 0.0$ 

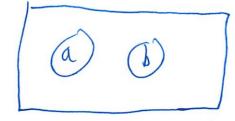
$$\bigcirc P(avb) = P(a) + P(b) - P(a \wedge b)$$

$$P(a|b) = P(anb)$$
given
 $P(b)$ 

$$P(a|b) P(b) = P(anb)$$
 and  $= P(b|a)P(a)$ 

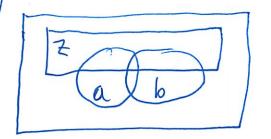
Independence

P(a/b) = P(a) when independent



Lactually that's not strictly independent

Conditional Independence
$$P(a|bnz) = P(a|Z)$$



(an do dog back tree - Mark only look at where dog barbed - then add up remaining cons where burgler is to See prob that it is burgle - it you had more details le racoon man not present it hald help make table more exact

Bugler Racon
Dog Bashs

4 possibilities: BR

P(B) = prob that burgler is around to right =0,1

P(A) = 0.5

Can add to this

Raccoon

Jrushcen

makes

noise

We might call the Police if the dog backs Day Barks 5 (all Police F 101 e call the police en may
T 1,2 Better, more complicated way to think about table Prob is conditionally independent of all non decembent given parents ie Dog Basks is conditionally ind of Trash given Brigher + Rakson (parent) So Trash does not matter for dog barbs What is Prob of encycthing together P(CADATABAR) =

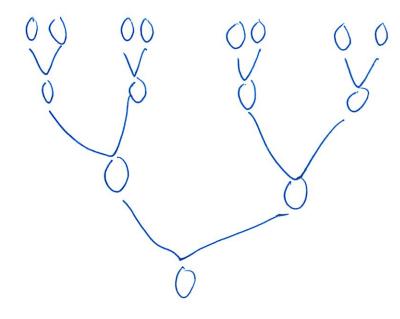
= P(C|ONTNBAR) P(DITNBAR) P(T|BAR)P(BIN) but some variables conditionally ind. of each other = P(clontnon)P(D| ThBAR)P(tiphe)P(BlanP(N) = P(C|.D) P(D|BAR) P(T|R) P(B) P(R)
Tonly has this column has 2 columns The a proiri probabilities = take probs from our relief map we drew
W if put it all in a Joint prob tuble
We would use a lot more #s 10 #s vs 2N= 25 = 32 #s

10 #s Vs  $2^{N}=20225=32$  #s

50 If P=max parents

N.  $2^{P}$  Is upper bound





8+16+8+4 Vs 2 15 36 Vs 32,000 1 much ealser to deal with

Le Since we had oir constraint about which variables in flence one another

The paper examples you look at the closer you are to actual

Gold Str Ideas (see lot po) Probabilistic model does not have constraints Saffey ret it don't have other models Bot it's more of a selled sedective safter net Floating What floats i Plastic - some things float - some talk sinks But its ceally just density Prob is that safter net it we bon 4 know density

But the figuring out we care about density

(Shipped due to MITCET meeting)

le 034 Lecture Probabilistic Influence 2

[ (alwhating/Reconstructing JPT Simulating / Acquiring

I De Obfuscation

I The Revend Dayes + Native Bajes

I, Model, Selection + Discoery

Program that discovers revenge ham -> harm lots of qu last lecture

IPT-table as of all possible values

(an calc prob of each row

Then add rows to get prob of some event

(an restrict universe (using conditional prob)

Once he have JPT can do anyting But might get too big Then we get that table/map thing
Not as flexible as JPT since made assumptions
- such as what depends on what
L(anstraint/model

But Jamatically U # of #s you need

BRITF 0 P(C) P(C) FILLS
TF 9.1 C DITF TIGHT

(3) (table must be loop-free, must be some parent indep.

Variables)

Now fill in values

So for P(D|B,R) look at the

tuble which has values for B,R

= .4.9.4.1.6

Now how to reconstruct table?

Since we have each combination

L'Simulate every possible combination

Its essentially a bias coin flipping for B with P(B=T)=1 P(B=F)=1

Same For R

So say ne got B=F R=t

Non look at that row in colon for D

FIT 15 [5] - 15 mon Flip n/ prob P(D=T)=15
P(D=F)=15

etc for rest of table Then do this lots at times As do it more your prob. get closer to tables value But in nature don't have table - instead als seve Similating off tuble is kinda pointless Will be 25 = 32 rous on the table Any computer can do (an be More complex - cald be a family tree -do you got a disease l ABCD EFGH Intrene type Pmax = max # of pwents for M=max # of entries in table < De 2 Prax upper bond

#4 = 22 Estros

n= H of variables Nom = n 2 Pmars 50 much better to have an interence table e Plus not actually 16 JPT 715 = 3200 E So it doing geneta Earnbling - 5 variables - lab fests -genolpheno type - etc Talk to your relatives - max 40 50 40 x 5 = 200 Variables

50 40 x 5 = 200 variables

JPT would be 2 200

way too hard!

Enference not 200, 22

Interesse not 200 · 22 = 800 much better!

# If con't do JPT - can use statistical sampling methods

$$P(a|b) = \underbrace{P(a,b)}_{P(b)} = P(a|b) P(b) = P(a,b) = P(b|a) P(a)$$

$$P(b|a) = \underbrace{P(a,b)}_{P(a,b)} P(b)$$

$$\underbrace{P(a|b)}_{P(a)} P(b)$$

$$\underbrace{P(a,b)}_{P(a)} P(b)$$

$$\underbrace{P(a,b)}_{P(a)} P(b)$$

$$\underbrace{P(a|b)}_{P(a)} P(b)$$

What you can you use this for?

If you have a fake + real coin biases fair

Want to say which one

(ould voite as inferce may

Or voite like this

So

F(H)

Save

P(H)

F(H)

F(

 $P(c|e_1,...,e_n|c) = \frac{P(e_1,...,e_n|c) P(c)}{P(e_1,...,e_n)}$ 

but sometimes able to make an assumption
L'that each flip is Ind of each other
Prohis Conditionally indigiven the class

- Plento P(ento) .... P(ento)
P(E)

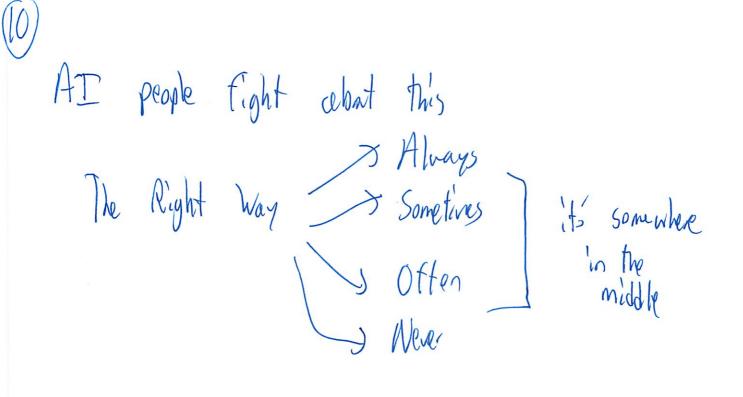
= 125 callet nieve baise when we assume and. 12 =.16 Example demonstration on compute each line is a particular com 15 Correct But what it friend suggests other model Can calc the prob of what is observed

Can do same as of coin = P(E|model 1) P(E|model 2)

Use evidence to determine which is the right mobel What gives higher probability of evidence Stories Model of story murder kill murder willy could generate a new model - perturb is it more probably than original model?

do plain hill climbing search for model,

Can do sel stories Find consistencies of human condition



# 6.034 Reitation

2 handouts

Quiz: Dec 7: SVM, Boosting, Representation

everything up to the beginning is fair game

No mega recitation this Friday

This is 2nd last recitation

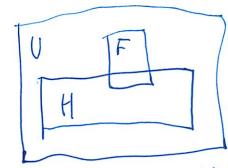
End of Boosting Problem did most of problem lest time Bid column - classitiers a, b Pich lowest error rate one Eine classifier veight based on into content  $\sqrt{=\frac{1}{2}} \ln \frac{1-E}{F}$ so end op w/ big classitter last classifler H(M= = = 1 ln 4 F + 2 ln 3 B+

Rand 3; Gets it wrong So height 7 Reveighting W- Z F W E=4/16 W= 1/6 other ones 24 So makes arithmetic eaise etc I is best clasifier on next cand

T is best clasifier on next cand might over fit a bit Probability One of best methods in last 20 years dealing al uncertanity A= Paul Revere wins B= Veuthor books clear GEP(A) & 1
ary over notation MM P(A) + Play

 $P(A \vee B) = P(A) + P(B) - P(A \wedge B)$ 

Conditional Probability



T = Have the The Headade

P(HIF) = P(HAF)

Prob

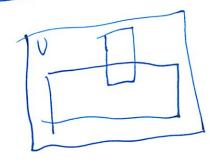
Prob

(chain ale

 $P(H,F) = P(H|F) \cdot P(F)$ 

Use in Nieue Bajes

Bayes Formula



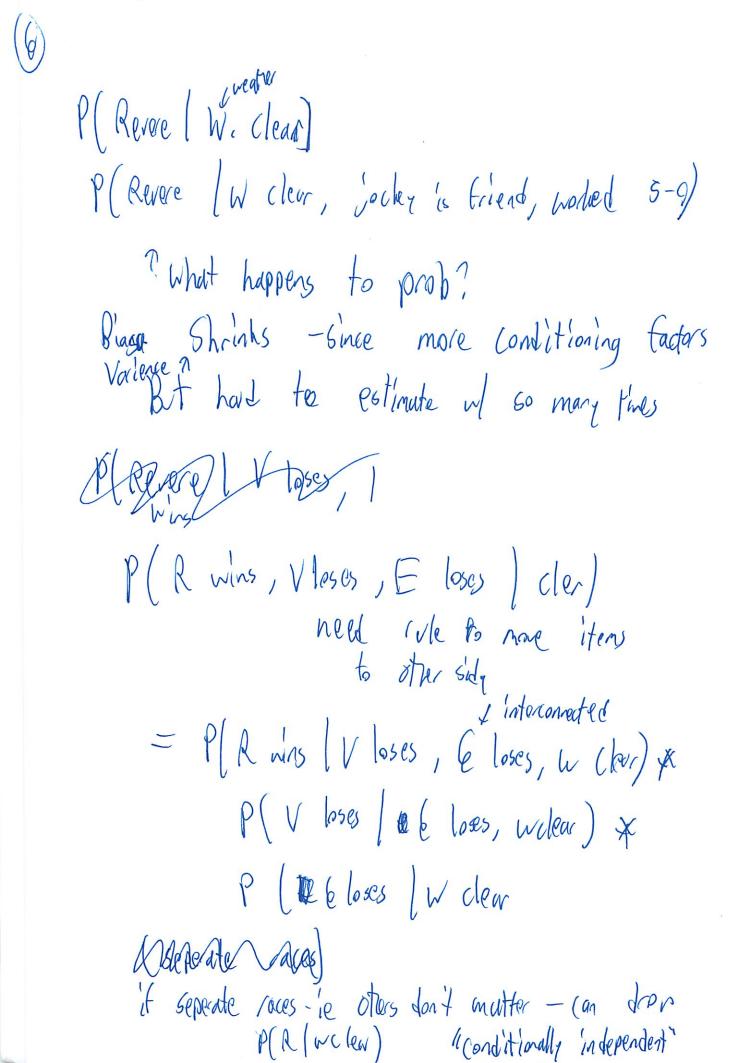


$$P(H|F) = P(H \Lambda F)$$

$$P(F) + P(H \Lambda F)$$

$$\frac{P(F|H) = P(H \land F)}{P(H \land F) + P(H)} \rightarrow \frac{P(H \land F) + P(F)}{P(F) + P(H \land F)} \cdot \frac{P(H \land F) + P(F)}{P(H \land F) + P(H)}$$
Posterier
Probability

Really is learning one of the most used learning origor than,



Naive Aages L=lable=(H,53) feat & I mentions \$ ·label = Han Feat 21 contains "buy" celationally to the conditionally and landing it Hum of each other (why its naive Bayes) ( and it ipral) more about that f, tis are not ceally ind in feal like Cavation simpler P(f, ... fn | lable = ham) P(f, [labdé = ham) x P(fz | dable = ham) Since conditional ind.

Mexicos \$ the contains by Spun Han Span hor much spane Wa even email mentions # looking (from training duta) wt it Postalo likhord Han Spom this enail contains both lable we tooking at (row) Prior . P(feat 1(A) 1) \* P(feat 2(by) (20) - lable hillihood  $14 \times 101 \times 1005 = 1000045$  $11 \times 130 \times 110 = 100363$ Then puch max Lonail is spon

# Massachvsetts Institute of Technology Department of Electrical Engineering and Computer Science 6.034 Recitation 11, Thursday, December 1

Probability & Naïve Bayes

Prof. Bob Berwick, 32D-728

### Agenda:

- 1. Finish Boosting
- 2. Probability: Axioms, Conditional Probability, Chain rule, Conditional independence, Bayes' Theorem
- 3. Naïve Bayes: another classifier (used for, e.g., Spam Asssasin)
- 4. Beyond naïve Bayes: the maximum entropy stewpot

### 1. Boosting and the Adaboost algorithm

The idea behind **boosting** is to find a weighted combination of s "weak" classifiers (classifiers that underfit the data and still make mistakes, though as we will see they make mistakes on less than  $\frac{1}{2}$  the data),  $h_1$ ,  $h_2$ ..., $h_s$ , into a **single strong** classifier, H(x). This will be in the form:

$$H(\vec{x}) = sign(\alpha_1 h_1(\vec{x}) + \alpha_2 h_2(\vec{x}) + \dots + \alpha_s h_s(\vec{x})$$

$$H(\vec{x}) = sign\left(\sum_{i=1}^s a_i h_i(\vec{x})\right)$$
where:  $H(\vec{x}) \in \{-1, +1\}, h_i(\vec{x}) \in \{-1, +1\}$ 

Recall that the sign function simply returns +1 if weighted sum is positive, and -1 if the weighted sum is negative (i.e., it classifies the data point as + or -).

Each training data point is weighted. These weights are denoted  $w_i$  for i=1, ..., n. Weights are like probabilities, from the interval (0, 1], with their sum equal to 1. BUT weights are never 0. This implies that all data points have some vote on what the classification shuld be, at all times. (You might contrast that with SVMs.)

The general idea will be to pick a single 'best' classifier h (one that has the lowest error rate when acting all alone), as an initial 'stump' to use. Then, we will **boost** the weights of the data points that this classifier **mis-classifies** (**makes mistakes on**), so as to focus on the next classifier h that does best on the re-weighted data points. This will have the effect of trying to fix up the errors that the first classifier made. Then, using this next classifier, we repeat to see if we can now do better than in the first round, and so on. In computational practice, we use the same sort of entropy-lowering function we used with ID/classifier trees: the one to pick is the one that lowers entropy the most. But usually we will give you a set of classifiers that is easier to 'see', or will specify the order.

In Boosting we always pick these initial 'stump' classifiers so that the error rate is strictly  $< \frac{1}{2}$ . Note that if a stump gives an error rate greater than  $\frac{1}{2}$ , this can always be 'flipped' by reversing the + and - classification outputs. (If the stump said -, we make it +, and vice-versa.) Classifiers with error exactly equal to  $\frac{1}{2}$  are useless because they are no better than flipping a fair coin.

## 1. Here are the definitions we will use.

### **Errors:**

The error rate of a classifier s,  $E^s$ , is simply the sum of all the weights of the training points classifier  $h_s$  gets **wrong**.

 $(1-E^s)$  is 1 minus this sum, the sum of all the weights of the training points classifier  $h_s$  gets correct.

By assumption, we have that:

$$E^{s} < \frac{1}{2}$$
 and  $(1 - E^{s}) > \frac{1}{2}$ , so  $E^{s} < (1 - E^{s})$ , which implies that  $(1 - E^{s})/E^{s} > 1$ 

### Weights:

 $\alpha_s$  is **defined** to be  $\frac{1}{2} \ln[(1 - E^s)/E^s)]$ , so from the definition of weights, the quantity inside the ln term is > 1, so all alphas must be positive numbers.

Let's write out the Adaboost algorithm and then run through a few iterations of an example problem.

### Adaboost algorithm

Input: training data,  $(\vec{x}_1, y_1), \dots, (\vec{x}_n, y_n)$ 

1. Initialize data point weights.

Set 
$$w_i^1 = \frac{1}{n} \ \forall i \in (1, \dots, n)$$

2. Iterate over all 'stumps': for s=1, ..., T

a. **Train base** learner using distribution  $w^s$  on training data. Get a base (stump) classifier  $h_s(x)$  that achieves the lowest error rate  $E^s$ . (In examples, these are picked from pre-defined stumps.)

b. Compute the stump weight:  $\alpha_s = \frac{1}{2} \ln \frac{(1 - E^s)}{E^s}$ 

c. Update weights (3 ways to do this; we pick Winston's method)

For points that the classifier **gets correct**,  $w_i^{s+1} = \left[\frac{1}{2} \cdot \frac{1}{1 - E^s}\right] \cdot w_i^s$ 

(Note from above that  $1-E^s > \frac{1}{2}$ , so the fraction  $1/(1-E^s)$  must be < 2, so the total factor scaling the old weight must be < 1, i.e., the weight of correctly classified points must go **DOWN** in the next round)

For points that the classifier **gets incorrect**,  $w_i^{s+1} = \left[\frac{1}{2} \cdot \frac{1}{E^s}\right] \cdot w_i^s$ 

(Note from above that  $E^s < \frac{1}{2}$ , so the fraction  $1/E^s$ ) must be > 2, so the total factor scaling the old weight must be > 1, i.e., the weight of incorrectly classified points must go UP in the next round)

### 3. Termination condition:

If s > T or if H(x) has error 0 on training data or < some error threshold, exit;

If there are no more stumps h where the weighted error is  $< \frac{1}{2}$ , exit (i.e., all stumps now have error exactly equal to  $\frac{1}{2}$ )

### 4. Output final classifier:

 $H(\vec{x}) = sign\left(\sum_{i=1}^{s} a_i h_i(\vec{x})\right)$  [this is just the weighted sum of the original stump classifiers]

**Note** that test stump classifiers that are **never** used are ones that make more errors than some preexisting test stump. In other words, if the set of mistakes stump X makes is a **superset** of errors stump Y makes, then Error(X) > Error(Y) is **always** true, no matter weight distributions we use. Therefore, we will **always** pick Y over X because it makes fewer errors. So X will **never** be used!

Let's try a boosting problem from an exam (on the other handout).

Food for thought questions.

- 1. How does the weight  $\alpha^s$  given to classifier  $h_s$  relate to the performance of  $h_s$  as a function of the error  $E^s$ ?
- 2. How does the error of the classifier  $E^s$  affect the new weights on the samples? (How does it raise or lower them?)

3. How does AdaBoost end up treating outliers?

4. Why is not the case that new classifiers "clash" with the old classifiers on the training data?

- 5. Draw a picture of the training error, theoretical bound on the true error, and the typical test error curve.
- 6. Do we expect the error of new weak classifiers to increase or decrease with the number of rounds of estimation and re-weighting? Why or why not?

### Answers to these questions:

1. How does the weight  $\alpha^s$  given to classifier  $h_s$  relate to the performance of  $h_s$  as a function of the error  $E^s$ ?

Answer: The lower the error the better the classifier h is on the (weighted) training data, and the larger the weight  $\alpha^t$  we give to the classifier output when classifying new examples.

2. How does the error of the classifier  $E^s$  affect the new weights on the samples? (How does it raise or lower them?)

Answer: The lower the error, the better the classifier h classifies the (weighted) training examples, hence the larger the increase on the weight of the samples that it classifies incorrectly and similarly the larger the decrease on those that it classifies correctly. More generally, the smaller the error, the more significant the change in the weights on the samples.

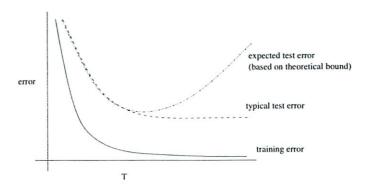
Note that this dependence can be seen indirectly in the AdaBoost algorithm from the weight of the corresponding classifier  $\alpha_t$ . The lower the error E', the larger  $\alpha_t$ , the better  $h_t$  is on the (weighted) training data.

3. How does AdaBoost end up treating outliers?

Answer: AdaBoost can help us identify outliers since those examples are the hardest to classify and therefore their weight is likely to keep increasing as we add more weak classifiers. At the same time, the theoretical bound on the training error implies that as we increase the number of base/weak classifiers, the final classifier produced by AdaBoost will classify all the training examples. This means that the outliers will eventually be "correctly" classified from the standpoint of the training data. Yet, as expected, this might lead to overfitting.

- 4. Why is not the case that new classifiers "clash" with the old classifiers on the training data? Answer: The intuition is that, by varying the weight on the examples, the new weak classifiers are trained to perform well on different sets of examples than those for which the older weak classifiers were trained on. A similar intuition is that at the time of classifying new examples, those classifiers that are not trained to perform well in such examples will cancel each other out and only those that are well trained for such examples will prevail, so to speak, thus leading to a weighted majority for the correct label.
- 5. Draw a picture of the training error, theoretical bound on the true error, and the typical test error curve.

Answer:



6. Do we expect the error of new weak classifiers to increase or decrease with the number of rounds of estimation and re-weighting? Why?

Answer: We expect the error of the weak classifiers to increase in general since they have to perform well in those examples for which the weak classifiers found earlier did not perform well. In general, those examples will have a lot of weight yet they will also be the hardest to classify correctly.

### 2. Basics of probability (review & pictures)

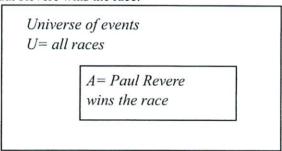
The fundamentals of probability theory: the axioms of probability. Why are these important? The power of the purse: Because while there are *other* attempts to handle the notion of 'uncertainty', e.g., 'fuzzy logic', '3-valued logic', etc., these axioms are the **only** system with the property that **if** you **gamble** with them, you **cannot** be unfairly exploited by an opponent who uses some other system (Di Finetti, 1932 theorem).

So, some first concepts.

We say that A is a random variable if A denotes an event and there is some uncertainty if A is true.

Typically, we let U denote the **universe** of all possible events (= all "possible worlds"). Then a subset of U, call it A, corresponds to the set of events in which A is true.

**Example.** Let the universe *U* be the set of all horse races. Let *Paul Revere* (abbreviation: P-R) be a horse. Then we can let *A* denote the set of racing events in which Paul Revere wins. We can draw this as a picture, where *races* labels the outer square, the universe, and the circle inside is the set of all events where Paul Revere wins the race:



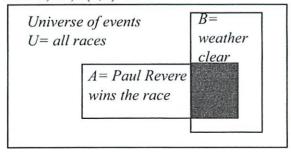
Let us denote by P(A) the fraction of events (possible worlds in the universe of events) in which A turns out true. We could spend the next 2 hours on the philosophy of possible worlds and this business. But we won't.

We will compute probabilities using an informal notion of areas (formally, we'd use measure theory).

The Universe of all events has total area 1, P(U)=1, because it denotes all the events that are true. P(A) then is the area of the smaller rectangle with respect to U (= the fraction of the total universe in which Paul Revere wins).  $P(\neg A)=1$  the races in which Paul Revere does **not** win = the set difference between U and A. From this we will posit 3 axioms regarding P(A):

- (1)  $0 \le P(A) \le 1$  [because: the area of A cannot be < 0 or > 1]
- (2) P(true)=1
- (3) P(false)=0
- (4)  $P(A \lor B) = P(A) + P(B) P(A,B)$  [where  $\lor$  means "or", i.e., either A or B must be true; + means "add together", and the comma in A, B means "and", i.e., both A and B must be true]

To see how this last axiom works, let's look at the racing universe with event A= Paul Revere wins and a second event, B= the weather is clear. The **shaded area** represents the fraction of events when **both** A and B are true, i.e., P(A,B)= true:



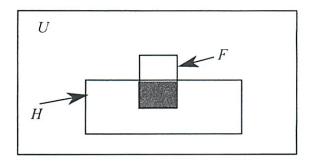
It should be apparent that in order to figure out the probability of A or B, we need to add up the areas corresponding to A and to B, but then subtract out the shaded area so that it is not counted twice. In this way, we arrive at the formula for the probability of A or B.

We next turn to the notion of conditional probability.

We let P(A|B) denote the fraction of events/possible worlds in which B is true, and then also have A true. That is, we 'shrink' the universe from U down to B, focusing in on a subset possibly more relevant to our situation, and use that as our basis to calculate probabilities.

**Example.** In the figure below, we illustrate the following situation. Let H= probability that "I have a headache"; F= probability that "I am getting the flu". These are denoted by the rectangles H and F in the figure below. Let us assume:

P(H) = 1/10; P(F)=1/40.Now let's compute the conditional probability P(H|F), i.e., the probability that I have a headache given that I have the flu. This is the fraction of flu-events that are also headache events – that is, if we just look at the rectangle F, what proportion of F overlaps with H? (The answer is 1/2). Thus, P(H|F)=1/2.



In other words, to find P(H|F), we compute:

(# worlds in which H and F are true)/(# worlds in which F is true) or, (area H and F)/(area of F), or P(H, F)/P(F)

So this is the formula for conditional probability: 
$$P(A \mid B) = \frac{P(A,B)}{P(B)} \, .$$

Note how P(B) is in the denominator here. Multiplying out, we obtain the important formula called the chain rule which we will uses in the naïve Bayes classifier:

$$P(A,B) = P(A \mid B) \cdot P(B)$$

Some other manipulations of conditional probability will be used in what follows. We consider two: (i) simplifications to the right of the conditioning bar symbol; and (ii) simplifications to the left of the conditioning bar symbol.

Simplifications to the *right* of the bar:

Suppose we have lots of conditions to impose on whether or not Paul Revere wins. For example, this could depend on not only if the weather's clear, but also whether the jockey's brother is a friend of mine, whether Paul Revere won its last race, etc. In other words:

P(Paul Revere wins | weather clear, jockey's brother a friend, P-R won last race)

Note that adding terms to the right only makes the conditions more stringent, so that this probability should get lower and lower every time we add a new factor. (Why? Think about intersection.) With more factors then, we have less bias, because we are focusing in on our particular situation, but we will have more variance, because it will become harder and harder to measure all these terms perfectly. So, sometimes we will want to reduce the number of factors to the right of the conditioning symbol to those we are more confident we can estimate; this is called *back off*. (We will see this in action soon). There is no problem in simply doing this:

P(Paul Revere wins | weather clear, jockey's brother a friend, P-R won last race)

And then of course just having P(Paul Revere wins | weather clear) remaining. But what about if there are more terms to the *left* of the bar, as in this case:

P(Paul Revere wins, Valentine loses, Epitaph loses | weather clear)

If we just care about Paul Revere, are we allowed to simply strike out the other two horses, this way?

P(Paul Revere wins, Valentine loses, Epitaph loses | weather clear)

The answer is: No! We need to carry out a more complex expansion to isolate Paul Revere on the left. To see how, let's abbreviate Paul Revere wins as R, Valentine loses as V, Epitaph loses as E, and the Weather is clear as W. Then our conditional probability:

P(Paul Revere wins, Valentine loses, Epitaph loses | weather clear)

Can be abbreviated as:

$$\frac{P(R,V,E,W)}{P(W)}$$

We can use this formula to derive the chain rule for conditional probability:

 $P(\text{Paul Revere wins, Valentine loses, Epitaph loses} \mid \text{weather clear}) = P(\text{Paul Revere wins} \mid \text{Valentine loses, Epitaph loses, weather clear}) \times P(\text{Valentine loses} \mid \text{Epitaph loses, weather clear}) \times P(\text{Epitaph loses} \mid \text{weather clear})$ 

Proof. Writing out the 3 terms:

$$\frac{P(R, V, E, W)}{P(W)} = \frac{P(R, V, E, W)}{P(V, E, W)} \times \frac{P(V, E, W)}{P(E, W)} \times \frac{P(E, W)}{P(W)}$$

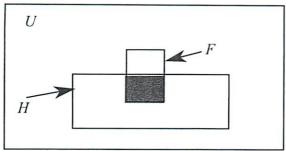
Now, supposed it is the case that the following simpler expansion holds:  $P(\text{Paul Revere wins, Valentine loses, Epitaph loses} \mid \text{weather clear}) = P(\text{Paul Revere wins} \mid \frac{\text{Valentine loses, Epitaph loses}}{\text{Valentine loses} \mid \frac{\text{Epitaph loses}}{\text{Epitaph loses}}, \text{weather clear}) \times P(\text{Epitaph loses} \mid \text{weather clear})$ 

In this case, whether Paul Revere wins or not depends only on whether the weather's clear...and not on what the other two horses do. They are irrelevant factors, so we can strike them out. In this case, when the probability is unchanged when we drop out conditioning factors, we say that the probability is conditionally independent (independent of the other horses, but still conditioned on the weather). More generally, if there are n factors f, and each factor is independent of the other, but still dependent on a condition c, we can write the following, which will be another key ingredient in our naïve Bayes classifier model:

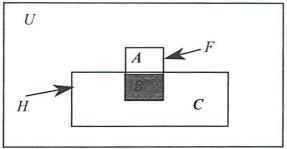
$$P(f_1, \dots, f_n \mid c) = P(f_1 \mid c) \times \dots \times P(f_n \mid c)$$

That is, we can just write out the probability as the product of the n factors, assuming they are independent from one another (the outcomes of these events do not affect the outcomes of one another); note the factors are still dependent on the outcome of event c.

OK, we come to the last ingredient we shall need, **Bayes' Law**. Again we can illustrate this with the simple picture of headache and flu as before. Recall P(H)=1/10; P(F)=1/40, P(H|F)=1/2.



Now we will **label** each of the distinct regions in this diagram, A, B, and C, as follows. A+B=area of F; B+C= area of H:



By the definition of conditional probability, P(H|F) = P(H,F)/P(F) = B/(A+B). Now consider this reasoning: one day you wake up with a headache, and you think, OMG, 50% of flus are associated with headaches, so now I have a 50-50 chance of getting the flu." Is this reasoning correct?

What we *want* to compute is: P(F|H). We already know the *other* conditional probability, that of headache given the flu. Further, by the definition of conditional probability, in terms of the regions A, B, and C, we have that: P(F|H) = B/(B+C). To find this last ratio of regions, we can take the conditional probability P(F|H) = B/(A+B), and multiply it by (A+B)/(B+C), as follows:

$$\frac{B}{B+C} = \frac{B}{A+B} \cdot \frac{A+B}{B+C} \text{ i.e.,}$$

$$P(F \mid H) = P(H \mid F) \cdot \frac{P(F)}{P(H)}$$
in our example,  $\frac{1/2 \times 1/40}{1/10} = \frac{1/80}{1/10} = \frac{1}{8}$ 

The term P(F) is called the **prior probability** (of getting the flu); the term P(H|F) is called the **likelihood**; the term P(H) is the **evidence** (e.g., that you have a headache); and the term P(F|H) is called the **posterior probability** of getting the flu (given that you have a headache). So this updated probability is a kind of learning: given the fact (data) that you indeed have a headache, how does the probability of getting the flu change? (It increases from 1/40 to 1/8.) Inverting from P(H|F) to P(F|H) is called **Bayes' Law**. It follows from a very simple manipulation of the definition of conditional probability and then application of the chain rule, i.e., that  $P(A,B) = P(A|B) \times P(B)$ :

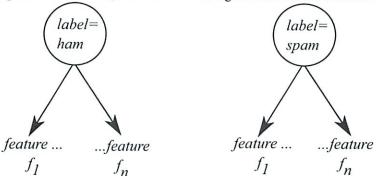
$$P(B \mid A) = \frac{P(A,B)}{P(B)}$$
 (by dfn of conditional probability)  
= 
$$\frac{P(A \mid B) \cdot P(B)}{P(B)}$$
 (by chain rule, replacing  $P(A,B)$ )

Or in words we can say this:

$$posterior = \frac{likelihood \times prior}{evidence}$$

Now let's put this all to work to build a classifier called Naïve Bayes. Like k-means and ID-trees, and Boosting, etc., this will take as input the values of some features and then output a classification label.

As our example, we will use the common, but valuable task of classifying email into 1 of 2 categories: either good email ("ham") or bad email ("spam"). The underlying probability model follows what is called a **Bayes' net**. We can imagine the following generative process: we pick a label, e.g., "ham", and given this label, email documents of this type will have a certain distribution of feature values  $f_1, \ldots, f_n$ . If we pick the other label, "spam", we will get another distribution for the feature values (hopefully distinct). So the picture looks like this, and the idea of course is that **given** a **new** email, we would like to figure out whether it is ham or spam:



Crucially, we assume that **the features are independent from one another.** (This is the "naïve" part of Naïve Bayes.) Their values depend on (are conditioned on) **only** the value of the label. That is why we draw the networks as above, with **no links** between the features, only from the label directed down to the features.

Now here's the idea behind the classificiation. Suppose we have estimated that 90% of our email is "ham" (OK), and that 10% is "spam". This gives us our **prior probability estimates** P(label=ham)=0.9 and P(label=spam)=0.1. That's what we can say about any new email **without** any additional information. (We'll see below how we get these estimates.)

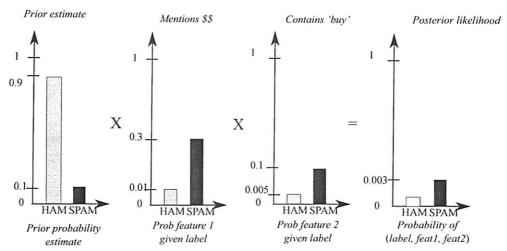
Now, when we get a new email, we will get the values of its **features** and use these to adjust the prior probabilities, as with our headache example. (In our example, to keep things simple, we will use only two features.)

So, this new email comes along: "Buy this amazing new Ginsu knife for only \$39....." Is this ham or spam? We'll assume that we use the following 2 features:

Feature 1: The email mentions money; this occurs in 30% of spam, and in 1% of ham

Feature 2: The email contains the word 'buy'; this occurs in 10% of spam, and in 0.5% of ham

We can picture our calculation as follows: our initial prior probabilities for each category are adjusted by **multiplying** the contribution **each feature** 'votes' (independently) as to how likely each category is. Then we pick the **most likely** = **biggest probability category** at the end:



So, in this case, our new email is classified as "spam" because this yields the largest posterior likelihood. Note how we got this value. It is simply this:

 $P(label) \times P(f_1 \mid label) \times P(f_2 \mid label) = P(label, f_1, f_2)$  [recall from dfn of conditional prob that:

$$\frac{P(label, f_1, f_2)}{P(label)} = P(f_1 \mid label) \times P(f_2 \mid label) \text{ IF } f_1, f_2 \text{ are independent of one another]}$$

In other words, we multiple the following out to find the label likelihood, and pick the biggest likelihood:

Prior probability of a label × Probability of feature contributions = Posterior label likelihood

In our case, for the two labels "ham" and "spam":

Prior 
$$\times$$
 Pr(feat 1 (\$)| l)  $\times$  Pr(feat 2 ('buy')|l) = Label likelihood  
Ham:  $0.9 \times 0.01 \times 0.005 = 0.000045$  (log of this likelihood: -4.34)  
Spam:  $0.1 \times 0.30 \times 0.10 = 0.00303$  (log of this likelihood: -2.52)

So, our email is more likely to be spam than ham. In fact, taking the ratios of the log likelihoods, -2.52/-4.32, the email is about 2 orders of magnitude (100x) more likely to be spam than ham. Recall that: (1) the features **must** be independent of one another; (2) we can add other features, of course...this is what a program like Spam Assassin can do, by training; and (3) one can use this method with lots more categories to **classify** documents (see the end of the handout).

Let's turn to justifying this approach probabilistically, as well as how we actually estimate the probability values above, via training, and highlighting some pitfalls.

First, why is this justified? We are computing the **maximum** probability that an input email will have a particular label (category), **given** that it has a particular set of features. We pick the label that maximizes:  $P(l=value \mid observed features)$ . Let's follow out this logic. We are maximizing the following quantity over label values:

$$\max P(label \mid features) = \max \frac{P(features, label)}{P(features)}$$
 [by dfn of conditional probability]

But note that the denominator in the expression above,  $P(features) = P(f_1, ..., f_n)$  is constant no matter what our choice of label value. So, to maximize the above quantity, it suffices to maximize the numerator:

$$\max P(features, label) = P(f_1, ..., f_n, label)$$

By the chain rule, this quantity in turn is just:

$$\max P(label) \times P(f_1, ..., f_n | label)$$

But given that the features are all independent of one another, this is the same as (recall our Paul Revere example!):

$$\max P(label) \times P(f_1 \mid label) \times ... \times P(f_n \mid label)$$
  
$$\max prior \times 'vote' f_1 \times ... \times 'vote' f_n$$

This is exactly the computation we have carried out. It remains to figure out how we 'train' our classifier – that is, how do we get the various estimates of the probabilities above? The simplest thing is just to estimate them from counts in training text, that is, known examples of ham and spam emails. These are the so-called *maximum likelihood estimates*:

$$P(label = ham) = \frac{count \ (\# \ ham \ emails)}{count(total \# \ emails)} \qquad P(label = spam) = \frac{count \ (\# \ spam \ emails)}{count(total \# \ emails)}$$

$$P(f_1 | label = ham) = \frac{count \ (\# \ ham \ emails \ mention \ \$)}{count(total \# \ ham \ emails)}$$

$$P(f_1 | label = spam) = \frac{count \ (\# \ spam \ emails \ mention \ \$)}{count(total \# \ spam \ emails)}$$

$$P(f_2 | label = ham) = \frac{count \ (\# \ ham \ emails \ contain \ "buy")}{count(total \# \ ham \ emails)}$$

$$P(f_2 | label = spam) = \frac{count \ (\# \ spam \ emails \ contain \ "buy")}{count(total \# \ spam \ emails)}$$

So this is how we get the estimates. For example, if we have 1000 emails, 900/1000 are ham, and 100/1000 are spam. Of the 100 spam emails, 30/100 mention money, and 1/100 contain 'buy'. For ham emails, 1/100 mention money and 5/1000 contain 'buy'.

Note that as the # of data samples (amount of training data) increases, then our estimates should get better; one of the properties of the maximum likelihood estimates is that they will converge to the 'true' values as the amount of data goes to infinity. (The mean approaches the true average.) But, if the # of training examples is small, our estimate will be very lousy, and have more noise (variance); there are a variety of things we can do to improve this, but that's for a machine learning course.

However, there is one particular case we should note. Suppose a particular count is actually 0 – that is, we *never* observe a particular feature associated with a particular label – this will happen especially if we keep adding more and more features. In this case, note that the entire probability product to find the likelihood will *all* be zero, just because one of the estimates is 0. So this is very bad!

There is a whole cottage industry devoted to fixing this problem, and it is called *smoothing*. It is basically the Robin Hood strategy: we rob probability mass from the rich and give it to the poor. In particular, the *simplest* smoothing strategy, invented by Laplace, is called *add*–1 *smoothing*: if a count is 0, we add 1 to it, so that, e.g., 0/100 goes to 1/100. (We must also *subtract* the appropriate probability mass, i.e., counts, from the *rest* of our estimates, so that the probabilities still add up to 1 in all.)

A second method of smoothing (probability mass redistribution) is due to Alan Turing. He figured this out when he was developing probability formulas for estimating the likelihood of finding German submarines in particular areas of the ocean. What if a submarine had *never* been observed in a particular spot? (Something that's actually quite likely!) Turing reasoned that a fairly good probability estimate of 'things never seen' would be quite close to the estimate of 'things seen *exactly* once'. This method, now called Good-Turing smoothing (only published until decades after WWII), works well but is finicky. There are whole books devoted to this subject, for machine learning and especially in natural language processing, where we quickly get word sequences never seen before.

One more thing. You may note that in our calculation we multiply together a (possibly long) string of probabilities, one for each feature. With a 1000 features, this value will quickly get very

very small. So, the usual method is to operate in log space, where multiplication is just addition, so we can maintain accuracy. (That's why we used log likelihoods above.)

#### Beyond Naïve Bayes (Optional)

OK, this method is fine so far as it goes, but it can be improved enormously. Here we will just sketch one method, known as **maximum entropy classification** that can gobble down any set of features, even if they are not independent. Yet remarkably, as first shown by Jaynes (1957), it is the most probabilistically sound method of **combining diverse features**. It rationalizes the general notion of just 'scoring' features and adding them up. We won't prove this here, but just indicate the general approach, which is now broadly used in, e.g., figuring out the part of speech labels in text. (For instance, in the sentence, *police police police*, is the first *police* a Noun or a Verb?)

1. To begin, let's assume there are now 10 labels for documents, with categories A, B, C, D, E, F, G, H, I, J. (So, e.g., category A could be travel; B sports; C business; etc.) If we know this, and **no other information** then given an email m, what is our best guess for category C (business) given this email, i.e.,  $P(C \mid m)$ ?

The maximum entropy approach would claim it is 1/10: that is, we maximize the quantity in each of the 10 bins, uniformly, by spreading out the total probability mass of 1 among 10 bins.

- 2. Now suppose I tell you that 55% of all emails are in category A, travel? Now what is the quantity P(C|m)? I think it should not be too hard to see that A gobbles up 0.55 of the probability mass, leaving 0.45 to be distributed evenly over the remaining 9 categories, or 0.05 for each of the remaining categories, including category C, business. So the maximum entropy estimate for P(C|m) is 0.05.
- 3. Now suppose I add *another* constraint: that *in addition* to the fact in (2), we know that 10% of all emails contain the word 'buy'. What is P(C|m) now? This gets harder to visualize, so we'll write it out as a table, where the first row is the probability of containing 'buy' (which thus must add up to 0.1 of all emails), and second row is the probability of not containing 'buy', which we have labeled *other* (which thus must add up to 0.9). Once again following the maximum entropy idea, since we don't know anything else about the 'contains buy' row, we should distribute its 0.1 total *evenly* among the 10 bins, thus giving 0.01 to each. Next, since *all* of category A must add up to 0.55, and since the 'contains buy' cell holds 0.01, it must be that the cell in the row labeled *other* and in column A must have the value 0.54 (so that the column total is 0.55). That leaves 0.9-0.54=0.36 for the rest of the 9 bins in the *other* row. Once again, spreading this evenly, we get 0.36/9=0.04 for each of these bins (so that each column here adds to 05). Thus we have the following table:

7 8 9 10 5 1 2 3 4 6 I J C D E F G Η A В 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 buy 0.01 0.01 0.04 0.54 0.04 0.04 0.04 0.04 0.04 0.04 0.04 0.04 other

So, why is this called maximum entropy? You should realize that by spreading out the values evenly, we are maximizing the entropy of the cell values:  $-p \log p$  summed over all entries is at a maximum. (Below we indicate why this is a good thing to do.) In any case, we are maximizing the entropy subject to the constraints specified. (We have two so far.)

4. So let's add one more constraint. Suppose that in addition, 80% of the 'buy' emails are in either category A or category C. Now we want to figure out P(C|m). Gulp! This one is much harder to figure out – in fact, in general to do this, it is like spreadsheets, but we can indicate what has to be true in our table now: the probability of the buy row, column A, plus the probability in the buy row, column C, must add up to 0.08 (80% of the 10%). That turns out to be the values 0.051 and 0.029. Since that leaves 0.020 for the rest of the bins in the buy row, these must be 0.020/8=0.0025. Since column A must still add up to 0.55, then that leaves 0.499 for row other, column A. Since

the *other* row must still sum to 0.9, we have 0.9-0.499=0.401 to distribute evenly over the rest of the *other* bins, so this is 0.401/9 = 0.0446. If we impose these constraints, you'll see that this is the answer (we don't say how we figured it out!)

	1	2	3	4	5	6	7	8	9	10
	Α	В	C	D	Е	F	G	Н	I	J
buy	0.051	.0025	0.029	.0025	.0025	.0025	.0025	.0025	.0025	.0025
other	0.499	.0446	.0446	.0446	.0446	.0446	.0446	.0446	.0446	.0446

Now we know that P(buy, C) = 0.029;  $P(C \mid buy) = 0.29$  (= 0.029/0.1);  $P(A \mid buy) = 0.51$ . This is our classifier, a *maximum entropy* classifier.

The punchline. While there are many possible distributions that could yield the three observed constraints, that 55% of the emails are in category A, that 10% of the emails contain buy, and that of these 10%, 80% are in category A or C, the **one distribution** that we picked, where we have **maximized** the entropy of the probability mass subject to these constraints, turns out to be the **only one** having the following two properties, the second one quite remarkable:

- 1. This distribution follows the form:  $P(email, label) = \frac{1}{Z(\lambda)} \exp \sum_i \lambda_i f_i(email, label)$  where the lambdas are the weights associated with each feature  $f_i$ ; the function  $f_i$  returns 1 if the feature is in the email, and 0 otherwise; and Z is a normalizing constant to make sure the probabilities all add up to 1.
- 2. This distribution maximizes the probability of the training data,  $\prod_i P(email_j, label_j)$

This is what justifies the method!

# **Problem 2: Boosting (50 points)**

After wearing Sauron's ring for several months, Frodo is rapidly losing his sanity. He fears that the ring will interfere with his better judgement and betray him to an enemy. To ensure that he doesn't put his trust into enemy hands, he flees Middle Earth in search of a way to classify his enemies from his friends. In his travels he had heard rumors of the magic of Artificial Intelligence and has decided to hire you to build him a classifier, which will correctly differentiate between his friends and his enemies. Below is all of the information Frodo remembers about the people back in Middle Earth.

ID	Name	Friend	Species	Has Magic	Part of the Fellowship	Has/Had a ring of power	Length of hair (feet)
1	Gandalf	Yes	Wizard	Yes	Yes	No	2
2	Sarumon	No	Wizard	Yes	No	No	2.5
3	Sauron	No	Wizard	Yes	No	Yes	0
4	Legolas	Yes	Elf	Yes	Yes	No	2
5	Tree-Beard	Yes	Ent	No	No	No	0
6	Sam	Yes	Hobbit	No	Yes	No	0.25
7	Elrond	Yes	Elf	Yes	No	Yes	2
8	Gollum	No	Hobbit	No	No	Yes	1
9	Aragorn	Yes	Man	No	Yes	No	0.75
10	Witch-king of Angmar	No	Man	Yes	No	Yes	2.5

# Part A: Picking Classifiers (10 points)

## A1 (6 points)

The data has a high dimensionality and so rather than trying to learn an SVM in a high dimension space you think it would be a smart approach to come up with a series of 1 dimensional stubs that can be used to construct a boosting classifier. Fill in the classifier table below. Each of the different classifiers are given a unique ID and a test returns +1 (friend) if true and -1 (enemy) if false.

Classifier	Test	Misclassified
A	Species is a Wizard	2, 3, 4, 5, 6, 7, 9
В	Species is an Elf	1, 5, 6, 9
С	Species is <b>not</b> a Man	2, 3, 8, 9
D	Does <b>not</b> have magic	1, 4, 7, 8
Е	Is <b>not</b> part of the Fellowship	1, 2, 3, 4, 6, 8, 9, 10
F	Has never owned a ring of power	2,7
G	Hair <= 1ft	1, 3, 4, 7, 8
Н	Hair <= 2 ft	3, 8
I	Friend	2, 3, 8, 10
J	Enemy	1, 4, 5, 6, 7, 9

## A2 (4 points)

Looking at the results of your current classifiers, you quickly see two more good weak classifiers (make fewer than 4 errors). What are they?

Classifier	Test	Misclassified		
K		1, 8, 10		
L				

# Part B: Build a Strong Classifier (30 points)

### **B1 (25 points)**

You realize that many of your tests are redundant and decide to move forward using only these four classifiers:  $\{B, D, F, I\}$ . Run the Boosting algorithm on the dataset with these four classifiers. Fill in the weights, classifiers, errors and alphas for three rounds of boosting. In case of ties, favor classifiers that come first alphabetically. Note: initial weights are set to be EQUAL and so

1/10 (they must add up to 1)

	Round 1		Round 2		Round 3	
w1	1/10	$h_1 = F$ (why?)	F correct:	h <sub>2</sub> =		$h_3 = $
w2	1/10	$Err = \frac{2}{10}$	F incorrect:	Err = U		Err =
w3	1/10	α =	1/16	α =		α =
w4	1/10	10 (10 (10 (10 (10 (10 (10 (10 (10 (10 (	1/16			
w5	1/10		1/16			
w6	1/10		1/16			
w7	1/10		4/16			
w8	1/10		1/16			
w9	1/10		1/16			
w10	1/10		1/16			
Err(B)	/10		4/16			1 1 2 2 2 4 1 1 1 1 1 1 1 1 1 1 1 1 1 1
Err(D)	/10		7/16			
Err(F)	2/10 WHY?		8/16			
Err(I)	/10		7/16			

## B2 (5 points)

What is the resulting classifier that you obtain after three rounds of Boosting?

$$H(x) = Sign[(1/2 ln ) * F(x) + (1/2 ln ) * + (1/2 ln ) *$$

# Part C: Boost by Inspection (10 points)

As you become frustrated that you must have picked the wrong subset of classifiers to work with, one of the 6.034 TA's, Martin, happens to walk by and sees your answer to part A1. He reminds you why the boosting algorithm works and then tells you that there is no reason to actually run boosting on this dataset. A boosted classifier of the form:

$$H(x) = Sign[h_1(x) + h_2(x) + h_3(x)]$$

can be found which solves the problem. What three classifiers  $\{h_1, h_2, h_3\}$  is Martin referring to, and why is the resulting H(x) guaranteed to classify all of the points correctly?

١	

### Michael E Plasmeier

From:

Patrick Henry Winston <phw@MIT.EDU> Sunday, December 04, 2011 4:53 PM

Sent: To:

fa12-6.034@mit.edu

Subject:

Important note on 6.034 end game

Friends,

Tomorrow's lecture, given by Professor Nancy Kanwisher, from the Department of Brain and Cognitive Science, will address where in your brain you think various sorts of thoughts.

A substantial part of Quiz 5 on the final will come from material presented in the remaining lectures, especially this one. If you show up, and pay attention, you will do well, but because the material is not yet available in textbook or note form, you will likely find parts of Quiz 5 mysterious. References will be supplied, insofar as practicable, but the coverage will be neither complete nor efficiently connected to the lectures.

Regards, Patrick

Professor Patrick H. Winston

ariz 4 Wed ariz 5 only on final Massachusetts Institute of Technology

Room 251 | 32 Vassar Street | Cambridge, MA 02139

Email: phw@mit.edu | URL: http://people.csail.mit.edu/phw/ | Voice: 617.253.6754

Coest lecture; Wancy Kan wigher

Functional Specificily in the Human Brain of a EMPLT

\* Will be part 3 on the exam

MIT intellegence initative

- cooperation

Diain seat of mind
Diain seat of mind
Disinct regions facilities

Franology - feel bums on brain

Flourens - attacked Gall

- no specific regions

Broca - or san Jamaged brain - able to associate

Today's basic agreement on regions

Are higher level processes specific? Why do we care? - one of the most fundamental ev - makes possible a divide + conquer research strategy - lets us seen compitation better indestant - (an Closer copy humans Various ways to investigate - brain imaging I need to send oxygen when brain is processing - blood flow to that region ? - FIRT looks at charges in blood flow - put face up side donn - much harder than words up side down FMRI

- a lodring at dot is basically off

Then show faces or objects

do any parts of the brown diff between the 2? - l+ c suappel

- Don't belie blobs Llots of things that produces blobs (all it be other things') -feels lenotions to person - is it seeing or recognizing?

- very hard qu to answer - lots of alt, hypothesis Weed to test the other hypothesis -none of them hork Same image Asibe dan - very different! Does not respond to people's bodies or hands bet un internediate response for (i) Also found PPA - responds to places - spacial layars you can be is - Empty room = strong response

IBA - responds to bodies + body parts - but not faces - including stick figures Lit just same lines rundomly arranged - doesn't exist Found in me virtuily all normal brains some place Raises Questions - Specificity - Are they engaged in a specific mental process? - Origins - How do they get wired up/placed in development? - Generality - How much of the bonin is greenest specific? Specificity tace area also reporting somewhat on object that see can only see millions of neurons Also seen in monkies Lan Stick a electron in to sue measure specific neuron Data from monkles à appeals more responsive Do re need it for object recognition. Lan't tell causal relationship

Can study from patients who brain damage

Lesson able to recognize objects but completly Unable to reagnize tuces Face Recognition - sque or different? L'is a famas face Familiar Lbut not asking their name lan temp turn brain area off L transcranial Magnetic Similation - but face area too far from skull' - Can reach a second smaller area - Pertornance on face souther in charged - but most change on body perception for EBA -face Change on FBA

-TMS is unde -amazing it works at all

When doing brain surgery - when similate face orea Patient said - just for a minute you looked Lifterently hids - same at age 5 - very good at face recognition Face recognition genetici - It different in identical twins and fraternal toxing -or are you maggingly social so you learn faces (an fest 1-3 day old interns -See how long they look - it less time - then its the some -1-3 day olds are good at face recognition - even different angles - but not pside down Also w/ monhies who never saw faces -Same as regular adult monthies All This shows very strong gene cole

Bt some things of experience - (eading is fairly cecont in humans - natural selection has not the let reading area grow Heas stronger when people read different languages Plany open - Why do some things get own - (an regions mae ovo? lafter injury - Han do areas work together for real bl world (could not copy cost)

# 6.034 Recitation

l, hardat

Quiz bach

288

Thorney

Z 37

Adregate

234

1. Naire Bayes Exemple

2. Google translate + Bayos

3. Be smooth

2. Chain Rule

$$\frac{P(A \cap B)}{P(B)} = P(A \setminus B) P(B)$$

3. Consitional in dependence

if fi are ind from each other

I tratifes conditionally each other Naive Bayes - find max prob of label (cat) given features from assumed ind P(c|f1, ..., fn) = orgmaxp(c). P(f1, ..., fn/c) P(f, m, tn) by Bayes Ng max ( iterate of all cs. See which finds merximing can revite ul Bayes (see abae) = np(c). P(Rt, intol) = oromany P(c) D[ PP(f, lc) P(f, lc) - ... P(f, lc)] "Since light Conditionally ind

Example = 30 student

Dorn = (East, Vest, FSILG)

3 qus

- Pylo 37

- Fereign Lary 3 T

- Good Shape JF

	Pylo	FL	65	协	
East	8/10	1/10	3/10	lo	e does not add to 10
West	3/10	6/10	3/10	lo	<i>'° 10</i>
FSIL6	1/10	3/10	8/10	10	
	7.11				

Padds to 10

$$P(C) = \frac{1}{3}$$
 point prob - knowing nothing  $\frac{10}{30}$ 

which down she should they be?

$$\begin{array}{lll}
& = P(C=East) & P(PY/no \mid EC) & P(7FL \mid EC) & P(765 \mid Ed) \\
& = \frac{1}{3} & \frac{8}{10} & (1-\frac{1}{10}) & (1-\frac{3}{10})
\end{array}$$

65 = F

= prob that you are in EC Given your responses to surem

$$= P(C=West)$$

$$= \frac{1}{3} \quad \text{o} \quad \left[\frac{3}{10} \quad \text{o} \quad \left(1-\frac{3}{10}\right)\right]$$

Now as multiply each through

Find largest value - J EC.

So student most likely From EC

If ans all tre

Lighter can just look at table

See WC has largest

But how to actually get these values?

boogle can detect which larg you are triping. Uses below nieve Bayes.

It has a lot of examples of text in various larguage.

Coogle has a lot of text to do this eight

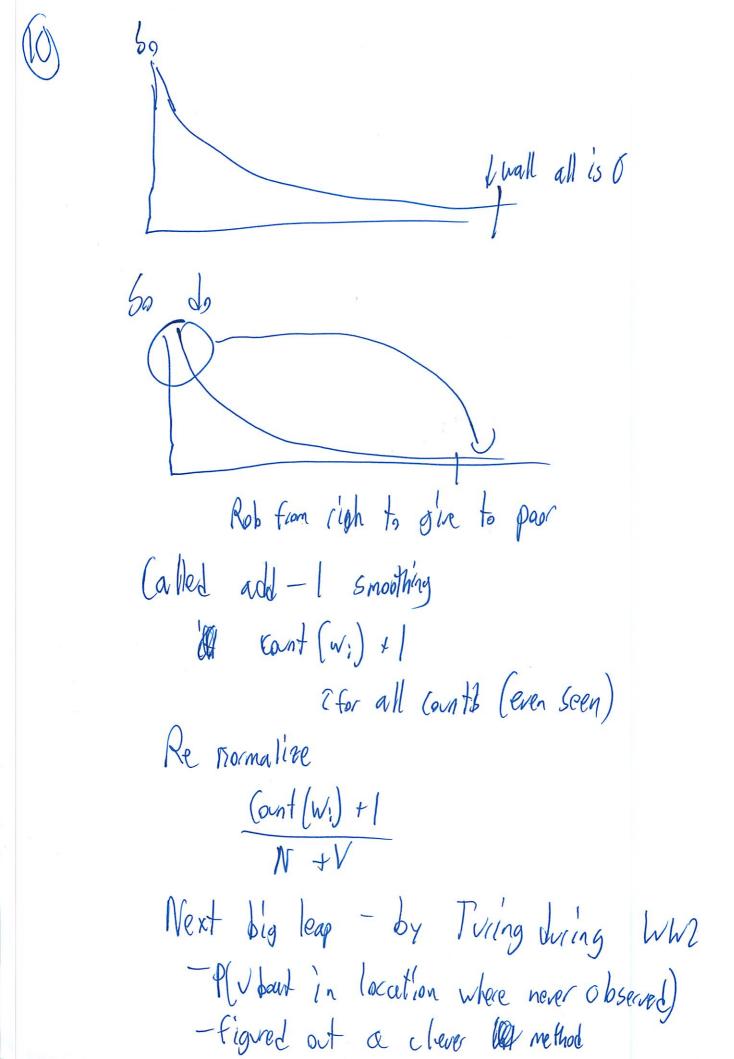
So how does Google translate?	
English — > French  French  Trench  Trench	
Sentance: George Bush is not an idial Result: 6.B. n'est pas un idial.  But it	
Sentance = 6,B n'est pas un idiat  Result i GB is an idiat	
A 60 Sentanco !	ar con
CAPPle eats the boy) Result i Boy eats the apple	

Voing Bayes rules at lots of examples Lno real understanding of language
P(E F*) = argmax E P(E). P(F*/E)  Endish Senture
So why does the wrong example come at Since George Bush is an idiot appears much more frequently!
P(E') = Is idial ( ) lot more frequently P(E'') = Is NOT idiant ) likely to appear
They try to patch these. But sheer size aludes you them

N-grams P(E)
The Sky is blue, Ash P(Sky | Previous word = The) Call & (shy ) P.V. = The) p(shy) & Epilgram Larger than P(The 16 kg) 2-gran P ( blue | the sky is) Google has 5-gram and 6-gram for many pairs of languages

F le cie est due bleu | E the sty is blue V=# 8f distinct vocab/vord types 36 h-40 h for English Speakers Li25 5-6 learn 10-12 words every day So 5-gran = V, V, V, V, V = V5 But it one is 0 - then whole multiple is 0 LNeed a fix & Smoothing Sometimes called Robin Hood Solution # 8 b sound ing -

the a order "the sky" Epairs to



2 # of times something appeals uniquely ronly once before that it never appeared Then it appeared ong Then not aguin celested to per all theory Called good - Turing fix He teaches 6.863 T natural lang 6.549 /7.33 Evolutionary blology

There will be a pour peri - Question 3

# Massachvsetts Institute of Technology Department of Electrical Engineering and Computer Science 6.034 Recitation 12, Thursday, December 8

Naïve Bayes & the Holy Grail

Prof. Bob Berwick, 32D-728

#### Agenda:

0. Probability revie

1. Naïve Bayes: another classifier (used for, e.g., Spam Asssasin)

2. How Google does translation

3. Beyond naïve Bayes: the maximum entropy stewpot

#### 0. Basics of probability (review & pictures)

The fundamentals of probability theory: the axioms of probability. Why are these important? The power of the purse: Because while there are *other* attempts to handle the notion of 'uncertainty', e.g., 'fuzzy logic', '3-valued logic', etc., these axioms are the **only** system with the property that **if** you **gamble** with them, you **cannot** be unfairly exploited by an opponent who uses some other system (Di Finetti, 1932 theorem).

So, some first concepts.

We say that A is a random variable if A denotes an event and there is some uncertainty if A is true.

Typically, we let U denote the **universe** of all possible events (= all "possible worlds"). Then a subset of U, call it A, corresponds to the set of events in which A is true.

**Example.** Let the universe *U* be the set of all horse races. Let *Paul Revere* (abbreviation: P-R) be a horse. Then we can let *A* denote the set of racing events in which Paul Revere wins. We can draw this as a picture, where *races* labels the outer square, the universe, and the circle inside is the set of all events where Paul Revere wins the race:

Universe of events  $U= all \ races$   $A= Paul \ Revere$ wins the race

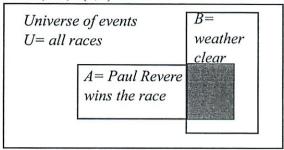
Let us denote by P(A) the fraction of events (possible worlds in the universe of events) in which A turns out true. We could spend the next 2 hours on the philosophy of possible worlds and this business. But we won't.

We will compute probabilities using an informal notion of areas (formally, we'd use measure theory).

The Universe of all events has total area 1, P(U)=1, because it denotes all the events that are true. P(A) then is the area of the smaller rectangle with respect to U (= the fraction of the total universe in which Paul Revere wins).  $P(\neg A)=1$  the races in which Paul Revere does **not** win = the set difference between U and A. From this we will posit 3 axioms regarding P(A):

- (1)  $0 \le P(A) \le 1$  [because: the area of A cannot be < 0 or > 1]
- (2) P(true)=1
- (3) P(false)=0
- (4)  $P(A \lor B) = P(A) + P(B) P(A,B)$  [where  $\lor$  means "or", i.e., either A or B must be true; + means "add together", and the comma in A, B means "and", i.e., both A and B must be true]

To see how this last axiom works, let's look at the racing universe with event A= Paul Revere wins and a second event, B= the weather is clear. The **shaded area** represents the fraction of events when **both** A and B are true, i.e., P(A,B)= true:



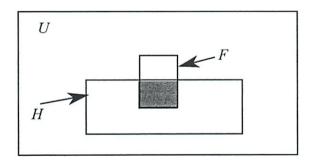
It should be apparent that in order to figure out the probability of A or B, we need to add up the areas corresponding to A and to B, but then subtract out the shaded area so that it is not counted twice. In this way, we arrive at the formula for the probability of A or B.

We next turn to the notion of conditional probability.

We let P(A|B) denote the fraction of events/possible worlds in which B is true, and then also have A true. That is, we 'shrink' the universe from U down to B, focusing in on a subset possibly more relevant to our situation, and use that as our basis to calculate probabilities.

**Example.** In the figure below, we illustrate the following situation. Let H= probability that "I have a headache"; F= probability that "I am getting the flu". These are denoted by the rectangles H and F in the figure below. Let us assume:

P(H) = 1/10; P(F)=1/40. Now let's compute the conditional probability P(H|F), i.e., the probability that I have a headache given that I have the flu. This is the fraction of flu-events that are also headache events – that is, if we just look at the rectangle F, what proportion of F overlaps with H? (The answer is 1/2). Thus, P(H|F)=1/2.



In other words, to find P(H|F), we compute:

(# worlds in which H and F are true)/(# worlds in which F is true) or, (area H and F)/(area of F), or P(H, F)/P(F)

So this is the formula for conditional probability:

$$P(A \mid B) = \frac{P(A,B)}{P(B)}.$$

Note how P(B) is in the denominator here. Multiplying out, we obtain the important formula called the **chain rule** which we will uses in the naïve Bayes classifier:

$$P(A,B) = P(A \mid B) \cdot P(B)$$

Some other manipulations of conditional probability will be used in what follows. We consider two: (i) simplifications to the *right* of the conditioning bar symbol |; and (ii) simplifications to the *left* of the conditioning bar symbol.

Simplifications to the right of the bar:

Suppose we have *lots* of conditions to impose on whether or not Paul Revere wins. For example, this could depend on not only if the weather's clear, but also whether the jockey's brother is a friend of mine, whether Paul Revere won its last race, etc. In other words:

P(Paul Revere wins | weather clear, jockey's brother a friend, P-R won last race)

With more factors then, we have less *bias*, because we are focusing in on our particular situation, but we will have more *variance*, because it will become harder and harder to measure all these terms perfectly. So, sometimes we will want to reduce the number of factors to the right of the conditioning symbol to those we are more confident we can estimate; this is called *back off*. (We will see this in action soon). There is no problem in simply doing this:

P(Paul Revere wins | weather clear, jockey's brother a friend, P-R won last race)

And then of course just having P(Paul Revere wins | weather clear) remaining. But what about if there are more terms to the *left* of the bar, as in this case:

P(Paul Revere wins, Valentine loses, Epitaph loses | weather clear)

Note that if we *add* terms to the left the probability should get lower and lower every time we add a new factor. (Why? Think about intersection.) If we just care about Paul Revere, are we then allowed to simply strike out the other two horses, this way?

P(Paul Revere wins, Valentine loses, Epitaph loses | weather clear)

The answer is: No! We need to carry out a more complex expansion to isolate Paul Revere on the left. To see how, let's abbreviate Paul Revere wins as R, Valentine loses as V, Epitaph loses as E, and the Weather is clear as W. Then our conditional probability:

P(Paul Revere wins, Valentine loses, Epitaph loses | weather clear)

Can be abbreviated as:

$$\frac{P(R,V,E,W)}{P(W)}$$

We can use this formula to derive the chain rule for conditional probability:

 $P(\text{Paul Revere wins, Valentine loses, Epitaph loses} \mid \text{weather clear}) = P(\text{Paul Revere wins} \mid \text{Valentine loses, Epitaph loses, weather clear}) \times P(\text{Valentine loses} \mid \text{Epitaph loses, weather clear}) \times P(\text{Epitaph loses} \mid \text{weather clear})$ 

Proof. Writing out the 3 terms:

$$\frac{P(R,V,E,W)}{P(W)} = \frac{P(R,V,E,W)}{P(V,E,W)} \times \frac{P(V,E,W)}{P(E,W)} \times \frac{P(E,W)}{P(W)}$$

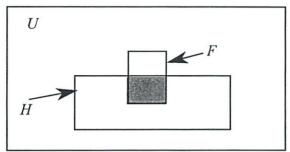
Now, supposed it is the case that the following simpler expansion holds:  $P(\text{Paul Revere wins, Valentine loses, Epitaph loses} \mid \text{weather clear}) = P(\text{Paul Revere wins} \mid \text{Valentine loses, Epitaph loses, weather clear}) \times P(\text{Valentine loses} \mid \text{Epitaph loses, weather clear}) \times P(\text{Epitaph loses} \mid \text{weather clear})$ 

In this case, whether Paul Revere wins or not depends only on whether the weather's clear...and not on what the other two horses do. They are irrelevant factors, so we can strike them out. In this case, when the probability is unchanged when we drop out conditioning factors, we say that the probability is conditionally independent (independent of the other horses, but still conditioned on the weather). More generally, if there are n factors f, and each factor is independent of the other, but still dependent on a condition c, we can write the following, which will be another key ingredient in our naïve Bayes classifier model:

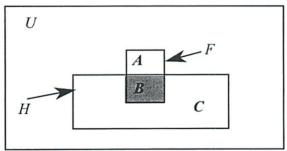
$$P(f_1,...,f_n \mid c) = P(f_1 \mid c) \times ... \times P(f_n \mid c)$$

That is, we can just write out the probability as the product of the n factors, assuming they are independent from one another (the outcomes of these events do not affect the outcomes of one another); note the factors are still dependent on the outcome of event c.

OK, we come to the last ingredient we shall need, **Bayes' Law**. Again we can illustrate this with the simple picture of headache and flu as before. Recall P(H)=1/10; P(F)=1/40, P(H|F)=1/2.



Now we will **label** each of the distinct regions in this diagram, A, B, and C, as follows. A+B=area of F; B+C= area of H:



By the definition of conditional probability, P(H|F) = P(H,F)/P(F) = B/(A+B). Now consider this reasoning: one day you wake up with a headache, and you think, OMG, 50% of flus are associated with headaches, so now I have a 50-50 chance of getting the flu." Is this reasoning correct?

What we *want* to compute is: P(F|H). We already know the *other* conditional probability, that of headache given the flu. Further, by the definition of conditional probability, in terms of the regions A, B, and C, we have that: P(F|H) = B/(B+C). To find this last ratio of regions, we can take the conditional probability P(F|H) = B/(A+B), and multiply it by (A+B)/(B+C), as follows:

$$\frac{B}{B+C} = \frac{B}{A+B} \cdot \frac{A+B}{B+C} \text{ i.e.,}$$

$$P(F \mid H) = P(H \mid F) \cdot \frac{P(F)}{P(H)}$$
in our example,  $\frac{1/2 \times 1/40}{1/10} = \frac{1/80}{1/10} = \frac{1}{8}$ 

The term P(F) is called the **prior probability** (of getting the flu); the term P(H|F) is called the **likelihood**; the term P(H) is the **evidence** (e.g., that you have a headache); and the term P(F|H) is

called the **posterior probability** of getting the flu (given that you have a headache). So this updated probability is a kind of learning: given the fact (data) that you indeed have a headache, how does the probability of getting the flu change? (It increases from 1/40 to 1/8.) Inverting from P(H|F) to P(F|H) is called **Bayes' Law.** It follows from a very simple manipulation of the definition of conditional probability and then application of the chain rule, i.e., that  $P(A,B)=P(A|B)\times P(B)$ :

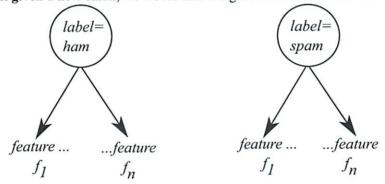
$$P(B \mid A) = \frac{P(A,B)}{P(B)}$$
 (by dfn of conditional probability)  
= 
$$\frac{P(A \mid B) \cdot P(B)}{P(B)}$$
 (by chain rule, replacing  $P(A,B)$ )

Or in words we can say this:

$$posterior = \frac{likelihood \times prior}{evidence}$$

Now let's put this all to work to build a classifier called **Naïve Bayes**. Like k-means and ID-trees, and Boosting, etc., this will take as input the values of some **features** and then output a classification **label**.

As our example, we will use the common, but valuable task of classifying email into 1 of 2 categories: either good email ("ham") or bad email ("spam"). The underlying probability model follows what is called a **Bayes' net**. We can imagine the following generative process: we pick a label, e.g., "ham", and given this label, email documents of this type will have a certain distribution of feature values  $f_1, \ldots, f_n$ . If we pick the other label, "spam", we will get another distribution for the feature values (hopefully distinct). So the picture looks like this, and the idea of course is that **given** a **new** email, we would like to figure out whether it is ham or spam:



Crucially, we assume that the features are independent from one another. (This is the "naïve" part of Naïve Bayes.) Their values depend on (are conditioned on) only the value of the label. That is why we draw the networks as above, with no links between the features, only from the label directed down to the features.

Now here's the idea behind the classification. Suppose we have estimated that 90% of our email is "ham" (OK), and that 10% is "spam". This gives us our **prior probability estimates** P(label=ham)=0.9 and P(label=spam)=0.1. That's what we can say about any new email **without any additional information.** (We'll see below how we get these estimates.)

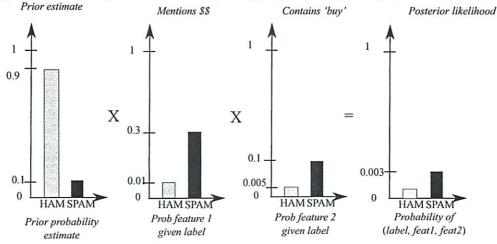
Now, when we get a new email, we will get the values of its **features** and use these to adjust the prior probabilities, as with our headache example. (In our example, to keep things simple, we will use only two features.)

So, this new email comes along: "Buy this amazing new Ginsu knife for only \$39....." Is this ham or spam? We'll assume that we use the following 2 features:

Feature 1: The email mentions money; this occurs in 30% of spam, and in 1% of ham

Feature 2: The email contains the word 'buy'; this occurs in 10% of spam, and in 0.5% of ham

We can picture our calculation as follows: our initial prior probabilities for each category are adjusted by **multiplying** the contribution **each feature** 'votes' (independently) as to how likely each category is. Then we pick the **most likely = biggest probability category** at the end:



So, in this case, our new email is classified as "spam" because this yields the largest posterior likelihood. Note how we got this value. It is simply this:

 $P(label) \times P(f_1 | label) \times P(f_2 | label) = P(label, f_1, f_2)$  [recall from dfn of conditional prob that:

$$\frac{P(label, f_1, f_2)}{P(label)} = P(f_1 \mid label) \times P(f_2 \mid label) \text{ IF } f_1, f_2 \text{ are independent of one another]}$$

In other words, we multiple the following out to find the label likelihood, and pick the biggest likelihood:

Prior probability of a label × Probability of feature contributions = Posterior label likelihood

In our case, for the two labels "ham" and "spam":

Prior 
$$\times$$
 Pr(feat1 (\$)| I)  $\times$  Pr(feat 2 ('buy')|I) = Label likelihood  
Ham:  $0.9 \times 0.01 \times 0.005 = 0.000045$  (log of this likelihood: -4.34)  
Spam:  $0.1 \times 0.30 \times 0.10 = 0.00303$  (log of this likelihood: -2.52)

So, our email is more likely to be spam than ham. In fact, taking the ratios of the log likelihoods, -2.52/-4.32, the email is about 2 orders of magnitude (100x) more likely to be spam than ham. Recall that: (1) the features **must** be independent of one another; (2) we can add other features, of course...this is what a program like Spam Assassin can do, by training; and (3) one can use this method with lots more categories to **classify** documents (see the end of the handout).

Let's turn to justifying this approach probabilistically, as well as how we actually estimate the probability values above, via training, and highlighting some pitfalls.

First, why is this justified? We are computing the **maximum** probability that an input email will have a particular label (category), **given** that it has a particular set of features. We pick the label that maximizes:  $P(l=value \mid observed features)$ . Let's follow out this logic. We are maximizing the following quantity over label values:

$$\max P(label \mid features) = \max \frac{P(features, label)}{P(features)}$$
 [by dfn of conditional probability]

But note that the denominator in the expression above,  $P(features) = P(f_1, ..., f_n)$  is constant no matter what our choice of label value. So, to maximize the above quantity, it suffices to maximize the numerator:

$$\max P(features, label) = P(f_1, ..., f_n, label)$$

By the chain rule, this quantity in turn is just:

$$\max P(label) \times P(f_1, ..., f_n | label)$$

But given that the features are all independent of one another, this is the same as (recall our Paul Revere example!):

$$\max P(label) \times P(f_1 \mid label) \times ... \times P(f_n \mid label)$$
  
$$\max prior \times 'vote' f_1 \times ... \times 'vote' f_n$$

Putting this down as a formula, we have:

$$\arg\max_{C} P(C|f_1,\ldots,f_n) = \arg\max_{C} \frac{P(C)\prod_{i=1}^n P(f_i|C)}{P(f_1,\ldots,f_n)} = \arg\max_{C} P(C)\prod_{i=1}^n P(f_i|C)$$

This is exactly the computation we have carried out. It remains to figure out how we 'train' our classifier – that is, how do we get the various estimates of the probabilities above? The simplest thing is just to estimate them from counts in training text, that is, known examples of ham and spam emails. These are the so-called *maximum likelihood estimates*:

$$P(label = ham) = \frac{count \ (\# \ ham \ emails)}{count(total \# \ emails)} \qquad P(label = spam) = \frac{count \ (\# \ spam \ emails)}{count(total \# \ emails)}$$

$$P(f_1 | label = ham) = \frac{count \ (\# \ ham \ emails \ mention \ \$)}{count(total \# \ ham \ emails)}$$

$$P(f_1 | label = spam) = \frac{count \ (\# \ spam \ emails \ mention \ \$)}{count(total \# \ spam \ emails)}$$

$$P(f_2 | label = ham) = \frac{count \ (\# \ ham \ emails \ contain \ "buy")}{count(total \# \ ham \ emails)}$$

$$P(f_2 | label = spam) = \frac{count \ (\# \ spam \ emails \ contain \ "buy")}{count(total \# \ spam \ emails)}$$

So this is how we get the estimates. For example, if we have 1000 emails, 900/1000 are ham, and 100/1000 are spam. Of the 100 spam emails, 30/100 mention money, and 1/100 contain 'buy'. For ham emails, 1/100 mention money and 5/1000 contain 'buy'.

Note that as the # of data samples (amount of training data) increases, then our estimates should get better; one of the properties of the maximum likelihood estimates is that they will converge to the 'true' values as the amount of data goes to infinity. (The mean approaches the true average.) But, if the # of training examples is small, our estimate will be very lousy, and have more noise (variance); there are a variety of things we can do to improve this, but that's for a machine learning course.

#### A second worked example:

MIT decides to use surveys to determine how to sort students into into dorms. They decide to use Naive Bayes and survey data from current residents to classify where to put future students.

To collect this "training data:, they surveyed 30 random students. Each surveyed student is asked to fill out a simple questionaire with 3 true/false questions.

- 0. Which dorm do you live in: {East Campus, West Campus, or FSILG}
- 1. Are a Pyro i.e. do you enjoy performing feats with fire (or inadvertently trigger fire alarms)?
- 2. Are you a foreign student or do you like studying foreign languages?
- 3. Are you in Good shape?

Here are the results. It turns out that our random survey gave us exactly 10 students from each dorm group.

	Pyro	ForeignLang	GoodShape	# surveyed
East Campus	8/10	1/10	3/10	10 10/30
West Campus	3/10	6/10	3/10	10 10/30
FSILG	1/10	3/10	8/10	10 10/30

What can you do this data? We can use these counts to make estimates of the following probabilities:

P(C) (the prior probability of being in any dorm)

 $P(f_i | C)$  (the likelihood of having one of the 3 features given being in a particular dorm)

E.g. 
$$P(\text{Pyro}=True \mid C=\text{East Campus}) = 8/10$$
  $P(\text{Language} = True \mid C=\text{FSILG}) = 3/10$ 

Now we can use these probability estimates to classify new students by applying Bayes rule, i.e., our formula:

$$\arg \max_{C} P(C|f_1, \dots, f_n) = \arg \max_{C} P(C) \prod_{i=1}^{n} P(f_i|C)$$

Question 1: where would a new student who loves foreign languages most likely be classified if they filled in their incoming survey as follows:

Pyro = True ForeignLang = False GoodShape = False

To do this, we compute  $P(C_i| \text{Pyro}=True, \text{ForeignLang}=False, \text{Goodshape}=False)$  for all three possible campuses, and find the largest one! (That is what the "arg max" part means.)

```
For C= East campus:
```

 $\operatorname{argmax} P(C=\operatorname{East} \mid P=T, F=F, G=F)$ 

=  $\operatorname{argmax} P(C=\operatorname{East}) * [P(P=T \mid C=\operatorname{East}) P(L=F \mid C=\operatorname{East}) P(G=F \mid C=\operatorname{East})]$ 

= (10/30) \* [(8/10) (1-1/10)(1-3/10)] = 1/3\*[(8\*9\*3)/1000] = 1/3 \* [216/1000]

= 0.072000

For C= West campus:

 $\operatorname{argmax} P(C=\operatorname{West} \mid P=T, F=F,G=F)$ 

=  $\operatorname{argmax} P(C=\operatorname{West}) * [P(P=T \mid C=\operatorname{West}) P(L=F \mid C=\operatorname{West}) P(G=F \mid C=\operatorname{West})]$ 

= (10/30) \* [ (3/10) (1-6/10) (1-3/10) ]

= 1/3 \* [3\*4\*7/1000] = 1/3 \* [84/1000]

= 0.028000

#### For *C*= FSILG:

 $\operatorname{argmax} P(C=FSILG \mid P=T,F=F,G=F)$ 

= P(C=FSILG)\*[P(P=T|C=FSILG) P(L=FC=FSILG) P(G=F|C=FSILG)]

= (10/30) \* [(1/10) (1-3/10)(1-8/10)]

= 1/3\*[(1\*7\*2\*)/1000 = 1/3\*[14/1000] = 1/3[14/1000]

= 0.004667

The largest value for such a student (Pyros true, all other attributes, false) is East Campus.

Question 2. What about an all-round student who checks all the boxes in the incoming survey?  $P(C=? \mid \text{Pyro} = True, \text{ForeignLang} = True, \text{GoodShape} = True)$ 

```
P(C=East | P=T, F=T, G=T)
= P(C=East) * [P(P=T|C=East)P(L=T|C=East)P(G=T|C=East)]
= 10/30* [(8/10) (1/10)(3/10)]
= 1/3 * [8*1*3/1000] = 1/3 * [24/1000]
= 0.008000

P(C=West Campus| P=T, F=T, G=T)
= P(C=West) * [P(P=F|C=West) P(L=T|C=West) P(G=T|C=West)]
= 10/30 * [(3/10) (6/10)(3/10)]
= 1/3 * [(3*6 *3/1000) = 1/3 *[54/1000]
=

P(C=FSILG| P=T, F=T,G=T)
= P(C=FSILG) * [P(P=T|C=FSILG)P(L=T|C=FSILG)P(G=T|C=FSILG)]
= (1/3) * [(1/10)(3/10)(8/10)]
= 1/3 * [1*3*8/1000] = 1/3 * [24/1000]
= 0.008000
```

The maximum C is West Campus.

In Naive Bayes, the P(C=some value) is also known as the "prior". Knowledge about the prior probabilities can help us distinguish what proportion to assign to each class. In our case we got lucky and it just happened that each campus got 10 students, so the prior in this case is *Uniform*.

#### Estimation & its discontents

There is at least one particular case about estimating the probabilities from data counts that we should note. Suppose a particular count is actually 0 – that is, we *never* observe a particular feature associated with a particular label – this will happen especially if we keep adding more and more features. In this case, note that the entire probability product to find the likelihood will *all* be zero, just because one of the estimates is 0. So this is very bad!

There is a whole cottage industry devoted to fixing this problem, and it is called *smoothing*. It is basically the Robin Hood strategy: we rob probability mass from the rich and give it to the poor. In particular, the *simplest* smoothing strategy, invented by Laplace, is called *add*–1 *smoothing*: if a count is 0, we add 1 to it, so that, e.g., 0/100 goes to 1/100. (We must also *subtract* the appropriate probability mass, i.e., counts, from the *rest* of our estimates, so that the probabilities still add up to 1 in all.)

A second method of smoothing (probability mass redistribution) is due to Alan Turing. He figured this out when he was developing probability formulas for estimating the likelihood of finding German submarines in particular areas of the ocean. What if a submarine had *never* been observed in a particular spot? (Something that's actually quite likely!) Turing reasoned that a fairly good probability estimate of 'things never seen' would be quite close to the estimate of 'things seen *exactly* once'. This method, now called Good-Turing smoothing (only published until decades after WWII), works well but is finicky. There are whole books devoted to this subject, for machine learning and especially in natural language processing, where we quickly get word sequences never seen before.

One more thing. You may note that in our calculation we multiply together a (possibly long) string of probabilities, one for each feature. With a 1000 features, this value will quickly get very,

very small. So, the usual method is to operate in log space, where multiplication is just addition, so we can maintain accuracy. (That's why we used log likelihoods above.)

#### Beyond Naïve Bayes (Optional)

OK, this method is fine so far as it goes, but it can be improved enormously. Here we will just sketch one method, known as **maximum entropy classification** that can gobble down any set of features, even if they are not independent. Yet remarkably, as first shown by Jaynes (1957), it is the most probabilistically sound method of **combining diverse features**. It rationalizes the general notion of just 'scoring' features and adding them up. We won't prove this here, but just indicate the general approach, which is now broadly used in, e.g., figuring out the part of speech labels in text. (For instance, in the sentence, *police police police*, is the first *police* a Noun or a Verb?)

1. To begin, let's assume there are now 10 labels for documents, with categories A, B, C, D, E, F, G, H, I, J. (So, e.g., category A could be travel; B sports; C business; etc.) If we know this, and **no other information** then given an email m, what is our best guess for category C (business) given this email, i.e.,  $P(C \mid m)$ ?

The maximum entropy approach would claim it is 1/10: that is, we maximize the quantity in each of the 10 bins, uniformly, by spreading out the total probability mass of 1 among 10 bins.

- 2. Now suppose I tell you that 55% of all emails are in category A, travel? Now what is the quantity P(C|m)? I think it should not be too hard to see that A gobbles up 0.55 of the probability mass, leaving 0.45 to be distributed evenly over the remaining 9 categories, or 0.05 for each of the remaining categories, including category C, business. So the maximum entropy estimate for P(C|m) is 0.05.
- 3. Now suppose I add *another* constraint: that *in addition* to the fact in (2), we know that 10% of all emails contain the word 'buy'. What is P(C|m) now? This gets harder to visualize, so we'll write it out as a table, where the first row is the probability of containing 'buy' (which thus must add up to 0.1 of all emails), and second row is the probability of not containing 'buy', which we have labeled *other* (which thus must add up to 0.9). Once again following the maximum entropy idea, since we don't know anything else about the 'contains buy' row, we should distribute its 0.1 total *evenly* among the 10 bins, thus giving 0.01 to each. Next, since *all* of category A must add up to 0.55, and since the 'contains buy' cell holds 0.01, it must be that the cell in the row labeled *other* and in column A must have the value 0.54 (so that the column total is 0.55). That leaves 0.9-0.54=0.36 for the rest of the 9 bins in the *other* row. Once again, spreading this evenly, we get 0.36/9=0.04 for each of these bins (so that each column here adds to 05). Thus we have the following table:

	8									
	1	2	3	4	5	6	7	8	9	10
	A	В	С	D	Е	F	G	Н	I	J
buy	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
other	0.54	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04

So, why is this called maximum entropy? You should realize that by spreading out the values evenly, we are maximizing the entropy of the cell values:  $-p \log p$  summed over all entries is at a maximum. (Below we indicate why this is a good thing to do.) In any case, we are maximizing the entropy subject to the constraints specified. (We have two so far.)

4. So let's add one more constraint. Suppose that in addition, 80% of the 'buy' emails are in either category A or category C. Now we want to figure out P(C|m). Gulp! This one is much harder to figure out – in fact, in general to do this, it is like spreadsheets, but we can indicate what has to be true in our table now: the probability of the buy row, column A, plus the probability in the buy row, column C, must add up to 0.08 (80% of the 10%). That turns out to be the values 0.051 and 0.029. Since that leaves 0.020 for the rest of the bins in the buy row, these must be 0.020/8=0.0025. Since column A must still add up to 0.55, then that leaves 0.499 for row other, column A. Since

the *other* row must still sum to 0.9, we have 0.9-0.499=0.401 to distribute evenly over the rest of the *other* bins, so this is 0.401/9 = 0.0446. If we impose these constraints, you'll see that this is the answer (we don't say how we figured it out!)

	1	2	3	4	5	6	7	8	9	10
	A	В	С	D	Е	F	G	Н	I	J
buy	0.051	.0025	0.029	.0025	.0025	.0025	.0025	.0025	.0025	.0025
other	0.499	.0446	.0446	.0446	.0446	.0446	.0446	.0446	.0446	.0446

Now we know that P(buy, C) = 0.029;  $P(C \mid buy) = 0.29$  (= 0.029/0.1);  $P(A \mid buy) = 0.51$ . This is our classifier, a *maximum entropy* classifier.

The punchline. While there are many possible distributions that could yield the three observed constraints, that 55% of the emails are in category A, that 10% of the emails contain buy, and that of these 10%, 80% are in category A or C, the **one distribution** that we picked, where we have **maximized** the entropy of the probability mass subject to these constraints, turns out to be the **only one** having the following two properties, the second one quite remarkable:

- 1. This distribution follows the form:  $P(email, label) = \frac{1}{Z(\lambda)} \exp \sum_i \lambda_i f_i(email, label)$  where the lambdas are the weights associated with each feature  $f_i$ ; the function  $f_i$  returns 1 if the feature is in the email, and 0 otherwise; and Z is a normalizing constant to make sure the probabilities all add up to 1.
- 2. This distribution maximizes the probability of the training data,  $\prod_{i} P(email_{j}, label_{j})$

This is what justifies the method!

Everyone does part 5

Not really time to do all 5

If near a cutoff, then might upprade

One of the head topics
He thinks had

Leprobability ditfinit to understand.

Hunars don't have intilitie sense

Not all students agree

Silver Star

\*\*Bayes' Rule i P(AIB) P(B) = P(B|A) P(A)

\*\*Independence: If ind., P(AIB) = P(A)

\*\*Expanding i P(A) = P(AIB) P(B) + P(A[.7B]) P(7B)

\*\*Explaining away

\*\*Naivete

Bayes Things depend lare inthemal by
Also $P(A \cap B) = P(A) P(B)$ White if ind:
Expanding Away - lots ef things are inrelated
MINIE (1) (2) (2) (1) (2) (3) (4) (4) (5) (6) (7) (6) (7) (7) (7) (7) (7) (7) (7) (7) (7) (7
But don't really use Its since can leave staff in terms of tomaly
Q) P(MNHNZNVNQ) =
Can use some of the previous rules to try + figure or if all ind just multiply together Lbut they are not!

b) P(2|V) =We know P(2|V,M)Only want P(2|V)Use expanding = P(2|V,M)P(M) + P(2|V,7M)P(7M)

expanding again = P(Z/V)P(V)

That can't real that
but answer to pravious Since M. V are ind.  $= \left(P\left(\frac{1}{2} \mid V, m\right)P(m) + P\left(\frac{1}{2} \mid V, 7m\right)P(m)\right)P(V)$ + P(Z|M,7V) P(M) P(7V) + P(Z|7M,7V) P(7M) P(7W) d | P(v|z) =Use Bayes Rle  $= P(\pm |\lambda) P(\lambda)$ e) P(V/Z) P(V/Z,7M) P(V/Z,M) Rank least to greatest (AB

(5)

We hant to figure out virus

Lif Zombie - pretty likely is a virus

p(virus) goes up after seeing Zombie

Lit one oth cause is there, less of a chance of the other cause being there (explaining away)

Thieve Bayes

Trying to bild model that is obserable
bot make assumption that all dep, on world madel

20mble 6 8 9 10

Healthy 10 15 12 1 20

Tchamidis-all ind of each other



# \* Hidden world model - observable daty

New gry Eparu. Zombie or not?

We see W=T

that S=T

N=T

Do nath ... P(Z | WASANA7B)

Not a zombie (Sta . 9 . 8 . 6 . to

23 . 12 . 10 . 10 . 19

25 . 12 . 10 . 19

20 . 10 . 19

20 . 10 . 19

6,004 Quiz 4 Deproit

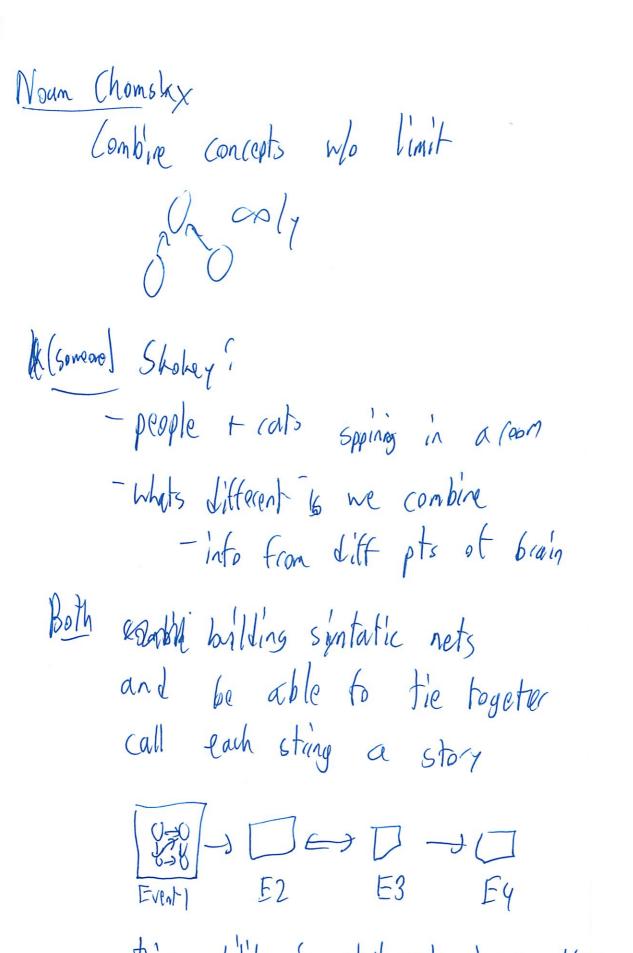
Actually thought I did pretty good
finished very early -20-25 min left
Caching was harded but I think I might
of got it

Cest I think I got Prediction 24-28 G.034 Lecture
The Right Way
Five Hypotheses

The Right Vay [ Inner Language Mypothesis 1 Strong Story Hypothesis D Directed Reception Hypothesis D Social Animal Hypothesis [ Exotic Blained Mypothesis Engineering \* Jalh & Look (Notes very bad today \* Draw - tire( - lecture unstructured) \* Collaberate

A Guest Lecture: Min Well what do you think & Why Applications of AI - not engioning

De Original your of AI! inderstand human intellegence ( Working on slides, will show afterwards) What motivates him? Member of Navel Sci Board Visits orgitangs at a 200 - Vsing a fool Why are we different? Paleo anthropoliquits Im we made some tools as naderitals for tens of thousands of years in Southern Africa - we became different - making immlery
- " Scelputures
- painting cares Is it since we are sómbolic? Or significantly simbolic



This ability is what makes humans different

Inner Lang Myp (See slides)

Strong Story Hypothesis the Mechanisms that enable.

Types of stories

- fairy tales

- ce ligion

- law

- business (all case stilies)

- Math (follow recipe - special case of story)

Other AI people should think story First Linabes the reasoning possible

What to do about it?
1. Charactaire behavior - not a specific method yet
2. Formulate Computational problems
3. Propose Computational Solutions
L'différence from a comme them typical psychologist  Y. Exploratory Implementation  Common souse lang 5, Principles
If someone hills you, Then you become dead
Reflective lung reverge) XX's having yy leads to yy's having xx
If you de can't build it, you don't understand it (" or was it other way around)
2 cultures thinking about MacBeth Story -2 sep. personas knowledge bases reflective large

Program reading background knowledge Then reading story

> bias i cererge someon other i sensless violence Aer.

One: Situational Marchet (1027)
Other: dispositional Something made Marcheth Crazy

If -then roles make a graph

LMae buch on grath to see what happens

Moved a Rissian war monument

So their network was hacked

Can System Find out what happened?

One view i revenge teaching you a lesson

Little thing on side looks at cesponse

Little

\* Story is figured out above what the words say \*

"Elaboration" graph
White - written
'Grey - elaboration added

Most of story is us hallowingting - filling in the gaps

Minshey 6 levels of thinking

Book: Emotion machines

L. Self concious reflective trinking - what do other think about my

2. Self reflective trinking - This will be were - I don't do that

3. Reflective trinking - how trink about your trinking

4. Deliberative Trinking - if I argor you, you'll hill me Genesis

5. Leaved reflex - if I kill you, you' dead

6. Inate reflex

What's New 1)-Story was alignment tanalogy
- browing someone knows. - 4 Tevery mathematical is cational -just get a model of their decion having alignment comes from bio -adapted to doal W stories let offinsive tegyptions Loon political, not military reason behind (1) Story telling Story - persona retells story to other persona - knowing how it worky

- knowing how it worky
target propossy

how much detail to include

It, the spoon teeding level But thow for to generalize L(on give It the Mes National reconsiliation after civil new - need to have believe otherside has legitimite por Concept for discover, - we were told xx horms xy Ly hams xx is "ceverge" when we were young How do we wan extract this concept from stories' Redo Carons arrows

Now more paths
Thinh a boat the prob of taking each path
Lean use Tieve Bayes
a priori based on Size

Event sequences PHD thesis Refine matels over + over

Social Ahimal Hyp -we develop outer larg b/c of

- Need someone to say it to

- it amplitles everything clse

- we substitute education for genes

Directed Perception Hyp (see slides

Cat drinking looks different from human drinking

Ashi How many Countries in Africa communicous equater?

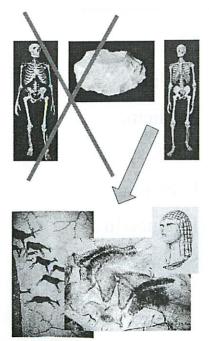
Need a map

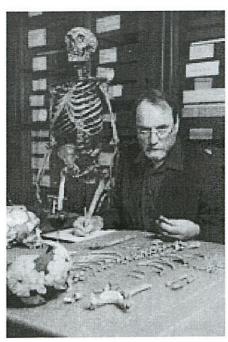
ferceptual level - Visual processing - learning to recognize a jump Bit can also produce video to its that! If This is he was no think what I do We make our sthes smarter by talking danting tak notes

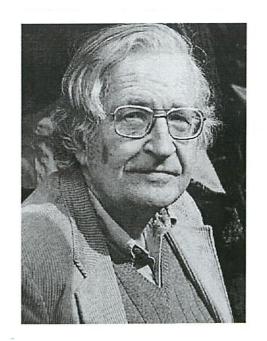
(sou won 4 look as) Collebrate - engages
everything Thes subject makes You smarter, Dothese Things!

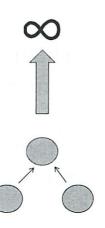
# The Right Way: Five Hypotheses



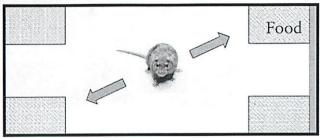












# The Inner Language Hypothesis

We are different because we have a symbolic inner language

# The Strong Story Hypothesis

The mechanisms that enable us humans to tell, understand, and recombine stories separate our intelligence from that of other primates. Fairy and folk tales

Religious parables

Ethnic narratives

History

Literature

Experience

News

Law

**Business** 

Medicine

Defense

Diplomacy

Engineering

Science

# The Strong Story Hypothesis

The mechanisms that enable us humans to tell, understand, and recombine stories separate our intelligence from that of other primates.

#### Commonsense level:

If someone kills you, then you become dead.

#### • Reflective level:

Description of "revenge": xx's harming yy leads to yy's harming xx.

A thane is a kind of noble. Macbeth and Macduff are thanes. Lady Macbeth is Macbeth's wife and Lady Macbeth is greedy. Duncan, who is Macduff's friend, is the king, and Macbeth is Duncan's successor. Macbeth defeated a rebel. Macbeth's success made Duncan become happy. Witches had visions and talked with Macbeth. Duncan rewarded Macbeth because Duncan became happy. Lady Macbeth is greedy. Lady Macbeth is Macbeth's wife. Macbeth wants to become king because Lady Macbeth persuaded Macbeth to want to become the king. Macbeth murders Duncan. Then, Lady Macbeth becomes crazy. Lady Macbeth kills herself. Dunsinane is a castle and Burnham Wood is a forest. Burnham Wood goes to Dunsinane. Then, Macduff fights with Macbeth. Then, Macduff kills Macbeth. Macduff had unusual birth. The witches's predictions came true.

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## What's New

✓ Story alignment and analogy

The Israelis know to defeat the		The Israelis believe the	•••
Egyptians.	Egyptians know they defeat the Egyptians.	Egyptians not to attack them.	
The USA knows to defeat the viet cong		The USA believes the viet cond not to attack it.	The viet cong attacks the US
gyptians.	Egyptians know they defeat the		The Egyptians attack the Israelis.
riet cong.	cong knows thelefeats the viet		The viet constacks the USA
T	he USA knows to defeat the let cong.  he Israelis know to defeat the gyptians.  he USA knows to defeat the et cong.	Egyptians  He USA knows to defeat the tet cong  He Israelis know to defeat the The Israelis know that the Egyptians.  Egyptians know they defeat the Egyptians know they defeat the Egyptians.  The USA knows to defeat the The USA knows that the viet	Egyptians The USA knows to defeat the iet cong The USA believes the viet cong not to attack it.  The USA believes the viet cong not to attack it.  The Israelis know to defeat the Egyptians know they defeat the Egyptians not to attack them. Egyptians The USA knows to defeat the at cong.  The USA knows to defeat the cong knows that the viet cong doesn't attack it.

The USA knows that the viet cong knows it defeats the viet cong.

The Egyptians attack the Israelis

The Israelis know the Egyptians prepare to attack them.	The Israelis know to defeat the Egyptians.	The Israelis know that the Egyptians know they defeat the Egyptians.	The Israelis believe the Egyptians not to attack them.	***
The USA knows that the viet cong prepares to attack it	The USA knows to defeat the viet cong.		The USA believes the viet con- not to attack it.	The viet cong attacks the USA
The Israelis know that the Egyptians prepare to attack them	The Israelis know to defeat the Egyptians	The Israelis know that the Egyptians know they defeat the Egyptians.	The Israelis believe the Egyptians not to attack them.	The Egyptians attack the Israelis.
The USA knows that the viet cong prepares to attack it.	The USA knows to defeat the viet cong.		The USA believes that the viet oong doesn't aftack if	The viet cong attacks the USA

# What's New

- Story alignment and analogy
- ✓ Story telling story

Duncan is a person. Lady Macbeth is a person. Macduff is a person. Macbeth is a person. A thane is a noble.

Macbeth is a thane. Macduff is a thane. Lady Macbeth is Macbeth's wife. Lady Macbeth is greedy. Duncan is the king. Macbeth is Duncan's successor. Duncan is Macduff's friend. Macbeth defeats a rebel. Appear is a success. Macbeth has a success. Witches talk with Macbeth. Witches have visions. Duncan rewards Macbeth because Duncan becomes happy. Macbeth wants to become king because Lady Macbeth persuades Macbeth to want to become king. Macbeth murders Duncan.

Duncan becomes dead. Macbeth becomes king.

Duncan is a person. Lady Macbeth is a person. Macduff is a person. Macbeth is a person. A thane is a noble. Macbeth is a thane. Macduff is a thane. Lady Macbeth is greedy. Macbeth defeats a rebel. Appear is a success. Macbeth has a success. Witches talk with Macbeth. Witches have visions. Duncan rewards Macbeth because Duncan becomes happy. Macbeth wants to become king because Lady Macbeth persuades Macbeth to want to become king. Macbeth murders Duncan.

Duncan becomes dead because if a person murders another person, the other person becomes dead.

Duncan is a person. Lady Macbeth is a person. Macduff is a person. Macbeth is a person. A thane is a noble. Macbeth is a thane. Macduff is a thane. Lady Macbeth is greedy. Macbeth defeats a rebel. Appear is a success. Macbeth has a success. Witches talk with Macbeth. Witches have visions. Duncan rewards Macbeth because Duncan becomes happy. Macbeth wants to become king because Lady Macbeth persuades Macbeth to want to become king.

Duncan becomes dead because Macbeth murders Duncan. Macbeth becomes king because Duncan becomes dead, Duncan is king, and Macbeth is Duncan's successor.

- Spoon feeding
- Explanation
- Explanation with intervention
- X intervenes to prevent Y from acting
- X understands Y's point of view
- · X negotiates with Y
- X explains situation to Y in Y's terms
- X teaches Y how to interpret situation
- · X shapes Y's reaction

## What's New

- Story alignment and analogy
- Story telling story
- ✓ Concept discovery

In 1998, Afghan terrorists bombed the U.S.'s embassy in Cairo, killing over 200 people and 12 Americans. Two weeks later, The U.S. retaliated for the bombing with cruise missile attacks on the terrorist's camps in Afghanistan, which were largely unsuccessful. The terrorists claimed that the bombing was a response to America torturing Egyptian terrorists several months earlier.

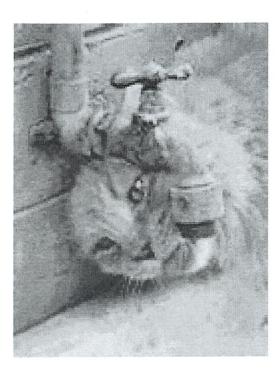
In early 2010, Google's servers were attacked by Chinese hackers. As such, Google decided to withdraw from China, removing its censored search site and publically criticizing the Chinese policy of censorship. In response, a week later China banned all of Google's search sites.

# The Social Animal Hypothesis

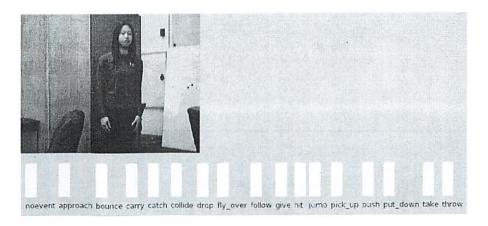
We developed an outer language because we are social animals

# The Directed Perception Hypothesis

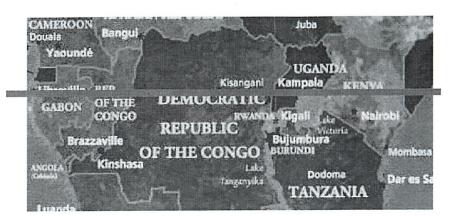
The mechanisms that enable us humans to direct and hallucinate with our perceptual faculties separate our intelligence from that of other primates.



The perceptual level



# Thinking about the Equator



40

Quiz', people thought was fair
Final's each section will cover same lectures set - might be a

TAIT'M not very good at probability

$$P(I,H,V,T,S,C) =$$

$$= I(I) P(H) P(T) P(W|I) P(S|I,T,H) P(C,T)$$

$$= M h (I-X) W g )$$

$$= P(S|I) =$$

$$= P(S,I) is a simpler way$$

$$= e+K+g+s but Bayes wle-need pwww.s...$$

$$= P(S,T,H)+P(S,T,H)+P(S,T,H)+P(S,T,H)$$

 $= \frac{P(w)P(\overline{H})}{P(w)}$ 

New pablem

5 M		MW	Dos	
PC , I	F=1 Sh=17 CW=12	k=,3 C=,3 MWF=,2 PE=,2	,3	
MBC /c	3 F= .4 Sch= ,4 (w= ,14)	K=M. (= AM) MF-04 PE:	.3 1	
TOLL	11 F=15	ELY MF	5 14	
ST	50	hall M	=12 =15 P=11 12 F=12	

P(5 | T, F, C) = Very similar to Zombie, Tatered clothes, Ate brains example but not just true, false

PC  $\frac{M}{14}$  + Means T=.4 F=.6(Ohhhh !)

call ind of 5 No 5 is dep on each of the valubles but each of M, F, C are Ind at each other That is what nieve Bayes means!

We can't just read it off Have all the probabilities in the other direction

So do Bayes ale

$$= P(\overline{M}, F, C \mid S) P(S)$$

$$P(\overline{M}, V \mid F, C)$$

We show still can't pull #s off chart
$$= P(\overline{n}|S) P(F|S) P(C|S) P(S)$$

$$P(\overline{m}|P(F) P(C))$$

take the maximum

$$P(\overline{n}, F, c) =$$

$$= P(\overline{m}, F, c) + P(\overline{m},$$

12/14

(last lecture)

Dhe Fifth Hypothesis

The Sheet

The Final

What Next

Bonefil Edea

\* Poweful Edea

There is a kind of engineering in our heads
which we are nearly not aware of

Normal systems

D-1 D-D-D-

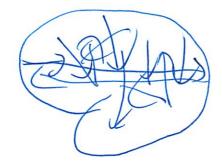
2)

Textbooks

Simple model of Brain



but sliff is all over the place



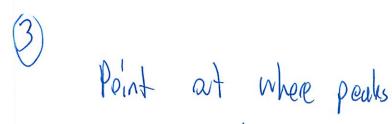
Speech

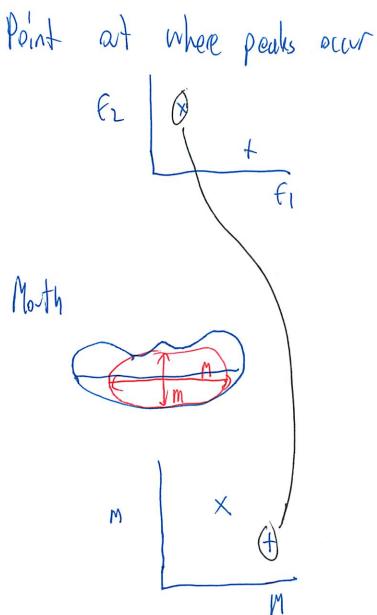
Intersity 22 freq

Smooth sut

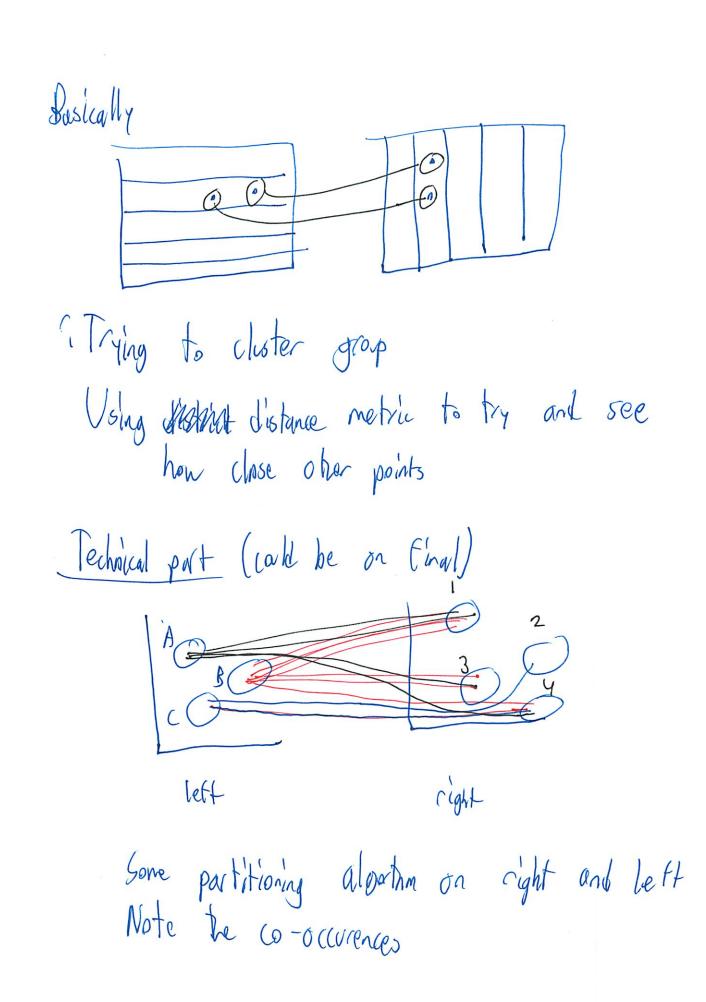
I the

the





But when we were little we didn't learn in graphs One side can assist the otherside



What work you merge? Aml B-not C Then alternate sides merging regions Can also messocross gestures, etc Try to find not Zebra Finch lean maiting call? Not known if it actually norks

Review We've seen a lot of methods from different people Integration program to impublished work (an Minh of AI as - Eng displine-building statt - Sci displine - understanding how Biz -abort making new things possible Why is AI different, -larg for procedures Then ways to make models - entored detail - opportunity to experiment - Upper bounds

D'if than other subjects because its computational Can quantity knowledge needed to solve poblem How do you do it? - Characteire behavior - Formulate computational problems - Propose « Solutions - Implement exploratory systems - Crystallze at the principles 1, Rules + Search 2. Games + Constraints 3. NN, NN, ID trees 4. SVM, Boosting Si Prob Interenes

Part 3s on all sections (except perhaps 1)
Limight be from a diff lecture

Open book, calculator, etc as always Bring a clock No compters

What's Next

Esp fen AI classes next semestor

Do you have any UROPO!

No unless you are persistent
Beruich's class to evolution

Winston's possible Class

- (eating primary sames
- Communications

- Undergrand Gride says lots of quizzes
- hacking the underground gride
Har to Speak talk IAP

Feb 3 11 AM

(9) Still some isoves

Lots of schools you can go
Lgo for the prof you like
apprenticeship
Grad schools only care about their
Hon will you contribute to their program
People interested often in staff not good at

Big Qv
Is AI Vsell?
Lyes

What are the powerful ideas?

(an thus be truly smart?

Are we close?

Chinese Room Argument some gry translating Eng > Chinese Computers like this? Is system not smarti Winstons; But humans are like This too! Momentus Fallacy M (see slike) Powerful Ideas - 6001 representations make you small (missed cest) Really Powerful Ideas

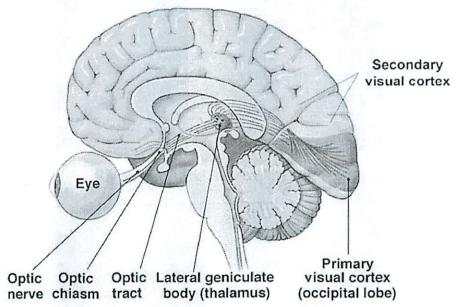
- You can change the world - Only you can to it - You can't to it alone - Your or bligged to 1 !t

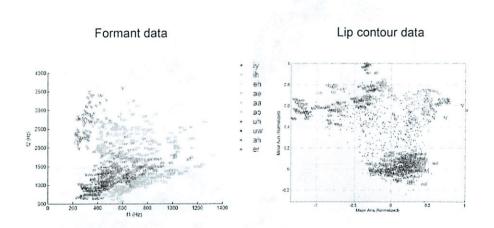
- Your or bligged to do it - 7-8 people toled to get in instead

## The Exotic Engineering Hypothesis

# 6.034 Farewell Address 2011

There is a kind of engineering in our heads about which we are nearly clueless.

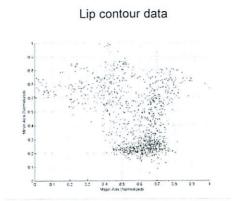


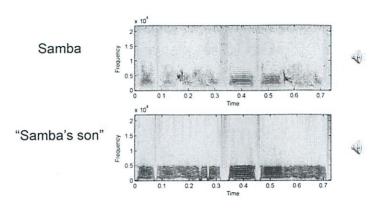






Formant data



















$$\int \frac{x^4}{(1-x^2)^{5/2}} dx = \frac{1}{3} \tan^3(\arcsin x)$$
$$-\tan(\arcsin x)$$
$$+\arcsin x$$

Duncan is a person. Lady Macbeth is a person. Macduff is a person. Macbeth is a person. A thane is a noble. Macbeth is a thane. Macduff is a thane. Lady Macbeth is greedy. Macbeth defeats a rebel. Appear is a success. Macbeth has a success. Witches talk with Macbeth. Witches have visions. Duncan rewards Macbeth because Duncan becomes happy. Macbeth wants to become king because Lady Macbeth persuades Macbeth to want to become king. Macbeth murders Duncan.

Duncan becomes dead because if a person murders another person, the other person becomes dead.

## Scientific Perspective

Artificial Intelligence is about understanding stuff with

Representations

Methods

Architectures

## **Engineering Perspective**

Artificial Intelligence is about building stuff with

Representations

Methods

Architectures

## The Business Perspective

	Saves Money	Creates New Opportunity
Information Gatherers		<b>V</b>
Blunder Stoppers		~
Novice Workers		
Expert Workers	*	·

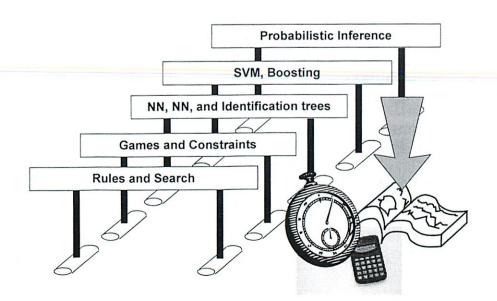
#### What Does AI Offer That Is Different

- A language for procedures
- New ways to make models
- Enforced detail
- Opportunities to experiment
- Upper bounds

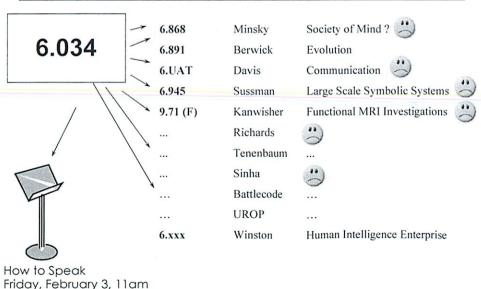
#### How do you do it?

- Characterize behavior
- Formulate computational problems
- Propose computational solutions
- Implement exploratory systems
- Crystallize out the principles

## What Might Be on the Final



## Winston's Picks



#### INTERESTED IN EVOLUTIONARY BIOLOGY? 6.049J/7.33J





Professor Dave Bartel
Professor Robert C. Berwick
Tues, Thurs 11–12:30pm (56-154)
First Class: Tuesday, February 7
Prereq: 7.03; 6.00, 6.01; or permission of instructor

An undergraduate elective in the new Course 6/7 degree in Computer Science & Molecular Biology

MIT's only undergraduate course devoted entirely to evolutionary biology

- What does evolutionary biology says about life, genomics, and drug discovery?
- . Is Richard Dawkins right? Is everything explained by "selfish genes"?
- · Has there been natural selection for a language gene?
- . How can maximizing fitness lead to evolutionary extinction?
- · Did humans ever mate with Neanderthals?

Distinguished guest lecturers including:

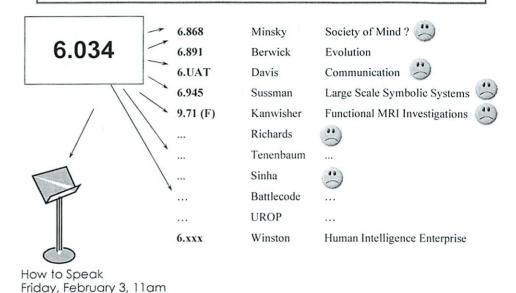
 Dr. Ian Tattersall, Curator of the American Museum of Natural History, New York, on human evolution and paleontology

Catalog Description: Explores and illustrates how evolution explains biology, with an emphasis on computational model building for analyzing evolutionary data. Covers key concepts of biological evolution, including adaptive evolution, neutral evolution, evolution of sex, genomic conflict, speciation, phylogeny and comparative methods, Life's history, coevolution, human evolution, and evolution of disease.

#### 6.XXX Benefits

- Understand the great ideas of the great thinkers and how they got them
- Learn how to extract and evaluate ideas from original, sometimes opaque sources
- Learn how to package your own ideas and expose their greatness

#### Winston's Picks



## 6.XXX Packaging Topics

Abstracts Business plans

Proposals Press releases

Slide presentations Job interviews

Promotion letters Study briefs

Letters of complaint Terms of reference

Trip reports Panel discussions

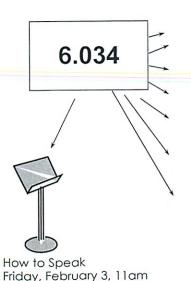
Elevator talks How to threaten people

Openings

## From the Underground Guide

Exams were described as "incredibly difficult," "brutal," and "frustrating." They were graded harshly and "covered topics not taught in the class."

#### Winston's Picks



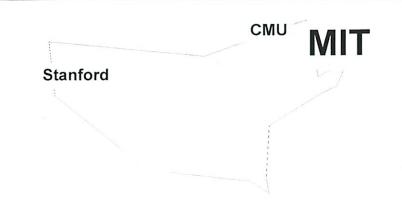
## From the Underground Guide

Officially, Winston has never confirmed or denied that there are quizzes for this class. His students seem to take after him --- comments were evenly split between complaints of brutal weekly 9:30AM quizzes and a "7-hour final", and denial of any and all testing. We at the UG aren't quite sure what to make of this.

#### The Issues

- What can we know about the physical world?
- How do we handle abstract worlds?
- What can we imagine and why?
- How do we discover order in our perceptions?
- How do experience and culture guide thinking?
- How do symbols ground out in perception?
- How do our faculties learn to communicate?
- Why are human computers so robust?

#### Where Can You Go Next



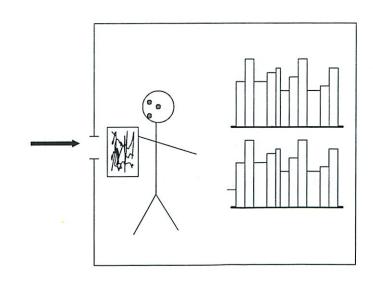
## The Big Questions

- Is AI useful?
- What are the powerful ideas?
- Can they be truly smart?
- Are we close?

## Where Can You Go Next

U. British Columbia CMU U. Washington Wisconsin Michigan Harvard Northwestern Stanford Tufts Purdue **Brandeis** Cal Tech **Brown** Cornell Berkeley **UCLA** Johns Hopkins UCSD **Hunter College** Georgia Tech USC U. Maryland ••• U. Massachusetts U. Pennsylvania

#### The Chinese-Room Argument



## The Homunculus Fallacy

- It cannot be in the program
- It cannot be in the computer
- Therefore, it cannot be at all

## The Powerful Ideas

- Good representations make you smarter
- Sleep makes you smarter
- You cannot learn unless you almost know
- You think with mouths, eyes, and hands
- The Strong Story Hypothesis

### The Biggest Issue

- Are people too smart?
- Are people smart enough?

#### The Staff

Avril Kenney

**Bob Berwick** 

Adam Mustafa

Randy Davis

Caryn Krakauer

Erek Speed

David Broderick

Gary Planthaber

Mark Seifter

The Rolling Stones

Peter Brin

The Black Eyed Peas

Tanya Kortz

...

# A Really Powerful Idea

- You can change the world
- Only you can do it
- You can't do it alone
- You are obliged to do it



## Grades for Michael E Plasmeier:

Lab Average: 5.0

Labs Started/Completed: 6

#### lab5

Started: 2011-11-19 23:58:51 Ended: 2011-11-20 00:04:55

Lab Grade:

5.0

Test was Run to Completion:

YES

Submissions: 6

- <u>lab5\_theplaz\_MIT\_EDU\_2011Nov19-221547.tar.bz2</u>
- lab5 theplaz MIT EDU 2011Nov19-224123.tar.bz2
- <u>lab5\_theplaz\_MIT\_EDU\_2011Nov19-233023.tar.bz2</u>
- <u>lab5\_theplaz\_MIT\_EDU\_2011Nov19-234937.tar.bz2</u>
- lab5\_theplaz\_MIT\_EDU\_2011Nov19-235421.tar.bz2
- <u>lab5\_theplaz\_MIT\_EDU\_2011Nov20-000217.tar.bz2</u>

#### lab4

Started: 2011-10-31 19:44:41 Ended: 2011-10-31 19:50:34

Lab Grade:

5.0

Test was Run to Completion:

YES

Submissions: 8

- lab4 theplaz MIT EDU 2011Oct30-214528.tar.bz2
- <u>lab4\_theplaz\_MIT\_EDU\_2011Oct31-173341.tar.bz2</u>
- <u>lab4 theplaz MIT EDU 2011Oct31-180921.tar.bz2</u>

- lab4 theplaz MIT EDU 2011Oct31-182048.tar.bz2
- lab4 theplaz MIT EDU 2011Oct31-192142.tar.bz2
- lab4 theplaz MIT EDU 2011Oct31-192927.tar.bz2
- lab4 theplaz MIT EDU 2011Oct31-192954.tar.bz2
- lab4 theplaz MIT EDU 2011Oct31-194532.tar.bz2

#### lab3

Started: 2011-10-14 22:56:04 Ended: 2011-10-14 23:04:41

Lab Grade:

5.0

Test was Run to Completion:

YES

Submissions: 13

- <u>lab3</u> theplaz MIT EDU 2011Oct03-211514.tar.bz2
- lab3 theplaz MIT EDU 2011Oct03-215856.tar.bz2
- lab3 theplaz MIT EDU 2011Oct03-223949.tar.bz2
- <u>lab3\_theplaz\_MIT\_EDU\_2011Oct03-230221.tar.bz2</u>
- <u>lab3 theplaz MIT EDU 2011Oct10-165956.tar.bz2</u>
- lab3 theplaz MIT EDU 2011Oct10-235302.tar.bz2
- lab3 theplaz MIT EDU 2011Oct11-012455.tar.bz2
- <u>lab3 theplaz MIT EDU 2011Oct14-211926.tar.bz2</u>
- lab3 theplaz MIT EDU 2011Oct14-224620.tar.bz2
- lab3 theplaz MIT EDU 2011Oct14-225526.tar.bz2
- lab3 theplaz MIT EDU 2011Oct14-225606.tar.bz2
- lab3 theplaz MIT EDU 2011Oct14-230736.tar.bz2
- <u>lab3 theplaz MIT EDU 2011Oct14-231611.tar.bz2</u>

#### lab2

Started: 2011-09-25 02:02:02

Ended: 2011-09-25 02:02:17

Lab Grade:

5.0

Test was Run to Completion:

YES

Submissions: 6

• lab2 theplaz MIT EDU 2011Sep25-003250.tar.bz2

- lab2 theplaz MIT EDU 2011Sep25-010213.tar.bz2
- lab2 theplaz MIT EDU 2011Sep25-011148.tar.bz2
- lab2 theplaz MIT EDU 2011Sep25-012911.tar.bz2
- lab2 theplaz MIT EDU 2011Sep25-015649.tar.bz2
- lab2 theplaz MIT EDU 2011Sep25-020205.tar.bz2

#### lab0

Started: 2011-09-16 22:01:22 Ended: 2011-09-16 22:01:29

Lab Grade:

5.0

Test was Run to Completion:

YES

Submissions: 15

- lab0 theplaz MIT EDU 2011Sep16-172228.tar.bz2
- lab0 theplaz MIT EDU 2011Sep16-180550.tar.bz2
- <u>lab0 theplaz MIT EDU 2011Sep16-180615.tar.bz2</u>
- lab0 theplaz MIT EDU 2011Sep16-180641.tar.bz2
- lab0 theplaz MIT EDU 2011Sep16-193148.tar.bz2
- lab0 theplaz MIT EDU 2011Sep16-193543.tar.bz2
- lab0 theplaz MIT EDU 2011Sep16-193627.tar.bz2
- lab0 theplaz MIT EDU 2011Sep16-193728.tar.bz2
- lab0 theplaz MIT EDU 2011Sep16-194032.tar.bz2
- lab0 theplaz MIT EDU 2011Sep16-194827.tar.bz2
- lab0 theplaz MIT EDU 2011Sep16-195054.tar.bz2
- lab0 theplaz MIT EDU 2011Sep16-211514.tar.bz2
- lab0 theplaz MIT EDU 2011Sep16-214410.tar.bz2
- lab0 theplaz MIT EDU 2011Sep16-215138.tar.bz2
- lab0 theplaz MIT EDU 2011Sep16-220124.tar.bz2

#### lab1

Started: 2011-09-20 01:46:20 Ended: 2011-09-20 01:46:30

Lab Grade:

5.0

Test was Run to Completion:

YES

Submissions: 6

- lab1 theplaz MIT EDU 2011Sep19-220902.tar.bz2
- lab1 theplaz MIT EDU 2011Sep19-224442.tar.bz2
- lab1 theplaz MIT EDU 2011Sep20-001854.tar.bz2
- lab1 theplaz MIT EDU 2011Sep20-013703.tar.bz2
- lab1 theplaz MIT EDU 2011Sep20-014622.tar.bz2
- lab1 theplaz MIT EDU 2011Sep20-014905.tar.bz2

# Reference material and playlist

From 6.034 Fall 2011

Final

Most of the readings come from Patrick Winston's AI textbook (third edition), which exists as a physical book (http://www.amazon.com/Artificial-Intelligence-3rd-Winston/dp/0201533774/), but is also available on the internet (http://courses.csail.mit.edu/6.034f/ai3/) (and there's a table of contents here (http://people.csail.mit.edu/phw/Books/AITABLE.HTML)).

# **Topics and Playlist 2011**

September	Day	Торіс	Quiz#	Playlist
7	Wed	What it's all about	1	This could be the last time, Stones
12	Mon	Goal trees and symbolic integration (http://courses.csail.mit.edu/6.034f /ai3/saint.pdf)	1	You can get it if you really want it, Jimmy Cliff
14	Wed	Goals and rule-based systems (pp.53-60) (http://courses.csail.mit.edu/6.034f/ai3/ch3.pdf)	1	Engineer's Song, Chorallaries
19	Mon	Basic search (http://courses.csail.mit.edu/6.034f/ai3/ch4.pdf)	1	Searchin', Stones
23	Fri	Optimal search (http://courses.csail.mit.edu/6.034f/ai3/ch5.pdf)	1	Route 66, Stones
26	Mon	Games (http://courses.csail.mit.edu/6.034f/ai3/ch6.pdf)	2	It's Only Rock and Roll, Stones
28	Wed	Quiz 1	-	-
October	Day	Topic	Quiz#	Playlist
3	Wed	Constraints in drawings (http://courses.csail.mit.edu/6.034f /ai3/ch12.pdf)	2	I Can't Get No Satisfaction, Stones
5	Wed	Constraints in maps and resource allocation	2	Paint it Black, Stones
12	Wed	Constraints in object recognition (http://courses.csail.mit.edu/6.034f /ai3/ch26.pdf)	2	The First Time I Saw your Face, Presley
14	Fri	Nearest neighbor learning (http://courses.csail.mit.edu/6.034f /ai3/ch19.pdf) /Sleep (http://courses.csail.mit.edu/6.034f/sleep.pdf)	3	ABC song, Ray Charles et al.

17	Mon	Identification tree learning (http://courses.csail.mit.edu/6.034f /ai3/ch21.pdf)	3	Romanian national anthem, Desteapta-te române!
19	Wed	Neural net learning (http://courses.csail.mit.edu/6.034f/ai3/netmath.pdf)	3	19th Nervous Breakdown, Stones
24	Mon	Genetic algorithms (http://courses.csail.mit.edu/6.034f/ai3/ch25.pdf)	3	Let's spend the night together, Stones
26	Wed	Quiz 2	-	-
31	Mon	Learning in sparse spaces (http://courses.csail.mit.edu/6.803/pdf/yip.pdf)	3	You talk too much, Peas
November	Day	Торіс	Quiz#	Playlist
2	Wed	Support-vector machines (http://courses.csail.mit.edu/6.034f /ai3/SVM.pdf), SVM (and Boosting) Notes (http://ai6034.mit.edu/fall11/images /SVM_and_Boosting.pdf)	4	Get a little help from my friends, Beatles
7	Mon	Learning from near misses (http://courses.csail.mit.edu/6.034f /ai3/ch16.pdf)	3	Imma Be Rocking that Body, Peas
9	Wed	Boosting (Winston and Ortiz notes) (http://courses.csail.mit.edu/6.034f /ai3/boosting.pdf), Boosting (Shapiri paper) (http://courses.csail.mit.edu/6.034f/ai3/msri.pdf)	4	Workin' together, Ike and Tina Turner
14	Mon	Frames and representation (http://courses.csail.mit.edu/6.034f/ai3/ch9.pdf)	4	Selections from the Black Watch, aka The Ladies from Hell
16	Wed	Quiz 3	-	-
21	Mon	Slides (http://courses.csail.mit.edu/6.034f /ai3/Emotionmachine.pdf) GPS, SOAR (http://courses.csail.mit.edu/6.034f /ai3/SOAR.pdf), Subsumption (http://courses.csail.mit.edu/6.034f /ai3/Subsumption.pdf), Society of Mind (http://web.media.mit.edu/~minsky/eb5.html)	4	Thus spake Zarathustra, Strauss
23	Wed	The AI Business	- 10 miles 1	Money, money, ABBA
28	Mon	Probabilistic inference I (http://courses.csail.mit.edu/6.034f /ai3/bayes.pdf)	5	Oh No, Not You Again, Stones
30	Wed	Probabilistic inference II (http://courses.csail.mit.edu/6.034f /ai3/bayes.pdf)	5	Tumbling Dice, Stones

December	Day	Торіс	Quiz#	Playlist	
5	Mon	Watching the brain at work, less than you want to know (http://courses.csail.mit.edu/6.034f /ai3/Kanwisher2010.pdf) Watching the brain at work, more than you want to know (http://web.mit.edu/bcs/nklab /publications.shtml)	5	Happy, Stones	,
7	Wed	Quiz 4	-	-	
12	Mon	Slides (http://courses.csail.mit.edu/6.034f /ai3/Rightway.pdf) Hypotheses: more than you want to know (http://courses.csail.mit.edu/6.034f /ai3/Submitted.pdf)	5	Ode to Joy, Ninth Symphony, Beethoven	
14	Wed	Slides (http://courses.csail.mit.edu/6.034f/ai3/Farewell2011.pdf) Cross-modal clustering: less and more than you want to know (http://courses.csail.mit.edu/6.034f/ai3/short-coen.pdf)	Cross modal clustering, remarks, discussion of the final	5	Don't stop, Stones

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