8.02 Test 3 Review

$F = I(L \times B)$

Day 22 / 4/5

$\mathcal{E} = -\frac{d}{dt} BA\cos\theta$

" screwdriver rule "

~~when~~ fighting change

-up field ↓

So want field up

screwdriver ccw

~~current is~~

B↑ current will go ↺

So counter is ↻

falling ring slows

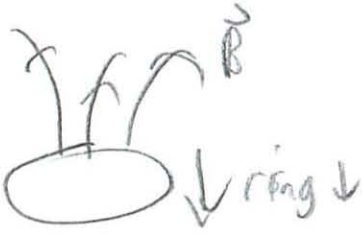
no change = no force

Coil moves up

flux up + increasing) check on this

want flux down ↻ screwdriver

electric guitars



as ring approaches dipole flux ↑

- flux = $BA\cos\theta$

$B \cdot A$

the ⊥ of B to the area

want flux to ↓

↑

So

Work more on this

TEST THREE Thursday Evening April 29 from 7:30-9:30 pm.

The Friday class immediately following is canceled because of the evening exam. Please see announcements for room assignments for Exam 3.

What We Expect From You On The Exam

1. An understanding of how to calculate magnetic fields in highly symmetric situations using Ampere's Law, e.g. as in the Ampere's Law Problem Solving Session.

2. An understanding of how to use Faraday's Law in problems involving the generation of induced EMF's. You should be able to formulate quantitative answers to questions about energy considerations in Faraday's Law problems, e.g. the power going into ohmic dissipation comes from the decreasing kinetic energy of a rolling rod, etc.

3. The ability to calculate the inductance of specific circuit elements, for example that of a long solenoid with N turns, radius a, and length L.

4. An understanding of simple circuits. For example, you should be able to set up the equations for multi-loop circuits, using Kirchhoff's Laws that include inductors. You should be able to understand and graph the solution to the differential equations for a circuit involving a battery, resistor, and inductor, and a circuit just involving a resistor and inductor. You should be able to compare and contrast RL and RC circuits, and should understand the meaning of time constants ($\tau = L/R$, $\tau = RC$)s

5. An understanding of the concept of energies stored in magnetic fields, that is $U = \frac{1}{2}LI^2$ for the total magnetic energy stored in an inductor, and $u_B = \frac{1}{2\mu_0}B^2$ for the energy density in magnetic fields. You also should review the concept of energies stored in electric fields, that is $U = \frac{1}{2}CV^2 = \frac{1}{2C}Q^2$ for the total electric energy stored in a capacitor, and $u_E = \frac{1}{2}\varepsilon_0 E^2$ for the energy density in electric fields.

6. An understanding of the nature of the *free* oscillations of an LC circuit.

To study for this exam we suggest that you review your problem sets, in-class problems, Friday problem solving sessions, PRS in-class questions, and relevant parts of the study guide and class notes.

Note: This exam will not include questions regarding undriven and driven RLC circuits but will include questions about free oscillations of LC circuits.

Review Session Test 3

Read equation sheet - putting up differential eq

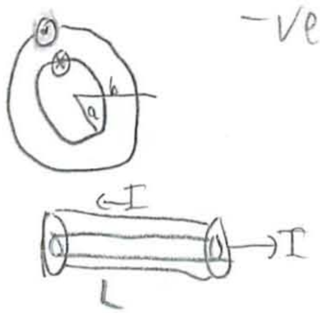
Straight foward problems - need to know which are which

- combining ideas
  - stratigies

Coverage on past exam different than before

Long coaxial cylinder

- very thin



always have current into + out of board

2D art better than 3D

Find L

$L = \frac{N\phi}{I}$

self

$\phi = BA = B \cdot \pi b^2$ ← which - a loop

$\int B\,ds = \mu_0 I_{enc}$

for each size shape

2 ways to get L

(1.) $L = \frac{\phi_{Obtained}}{I}$ ← surface integral $= \frac{\iint B \cdot da}{I}$ harder

(2) $\frac{1}{2} L I^2 = U = \iiint u_B \, dV$

(2)

$= \iiint \frac{B^2}{2\mu_0} dV$

$L = \frac{2U}{I}$

need B for both methods → Ampere's Law

$\oint B \cdot ds$	$= \mu_0 I_{enc}$
line integral	surface integral
	$\mu_0 \iint J \cdot dA$
B · lenght of loop	harder

know this fairly well - its dir and when is what

(1) Cyclinders
(2) solineids
(3) Torods
(4) Plances

different regions

(1) $r < a$ ← will be 0
(2) $a < r < b$
(3) $r > b$ ←

(know this better P-set # 1 I got wrong)

$\vec{B}$ fields tangental

slabs → parallel to surfaces ← did not know

(3)

Make a good drawing

$r < a$

How much current goes through <u>this</u> loop

$\oint B \cdot ds$	$\mu_0 I$
part that is ⊥	
$B 2\pi r$	$= 0$ nothing inside

$\vec{B} = 0$

$a < r < b$

$\int B \cdot ds$	$\mu_0 I$
$B 2\pi r$	$\mu_0 I$

$\vec{B} = \frac{\mu_0 I}{2\pi r} \hat{\theta}$ ↻

direction

I ⊗

B ↻ "screwdriver method"

right hand rule

$r > b$

$\int B \cdot ds$	$\mu_0 I$
$B 2\pi r$	$\mu_0 I - I$

$\vec{B} = 0$

Make sure to work it through

Knew it all

- but must be able to do

How harder?

- toroid (review)
- non constant $\vec{B}$ field
- solid
- B field falls off away from it

Thinking about solid vs hollow

- never thought of this
- but big diff
- these are the types of things I would confuse

Non solid $\vec{B}$ field

$J = cr\hat{k}$ $r < a$

Pick an amperian loop inside

$\oint B \cdot ds$	$\mu_0 \iint J \cdot da$
	must $\iint J$ pick da' which we integrate

r' - a ring w/ small thickness

⑤

Add the small rings

$$\mu_0 \int_0^r c r' \underbrace{2\pi r' \, dr'}_{da'}$$

$$\mu_0 \, c \, 2\pi \int_0^r r'^2 \, dr'$$

$$\mu_0 \, c \, \frac{2\pi r^3}{3}$$

Be prepared when it is $\int_a^b = \ln\left(\frac{b}{a}\right)$

$B \cdot 2\pi r$	$\mu_0 \, c \, \frac{2\pi r^3}{3}$

$$B = \frac{\mu_0 c r^2}{3} \, \theta \qquad r < a$$

I is ⊗

so ↻ $\vec{B}$

(Clockwise)

---

② Energy method

$$L = \frac{2U}{I^2} \qquad \mu_B = \frac{B^2}{2\mu_0} \qquad a < r < b$$

$$= \left(\frac{\mu_0 I}{2\pi r}\right)^2 \frac{1}{2\mu_0} = \frac{\mu_0 I^2}{8\pi^2 r^2}$$

Energy density

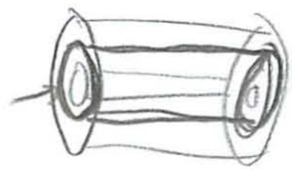
need to $\int$

(6)

$$L = \frac{2U}{I^2}$$

$$= \frac{2}{I^2} \int \frac{\mu I^2}{8\pi^2 r^2} \, dV$$

volume integral
complex to $\iiint$
pick a volume element
how much energy in there?



$$dV = 2\pi r \, dr \, l$$

$$= \frac{2}{I^2} \int_a^b \frac{\mu_0 I^2}{8\pi^2 r^2} 2\pi r \, dr \, l$$

$$= \frac{\mu_0 l}{2\pi} \int_a^b \frac{dr}{r}$$

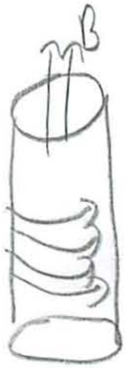
$$= \frac{\mu_0}{2\pi} l \ln\left(\frac{b}{a}\right)$$

---

(1) Normal method
- far harder

$$L = \frac{\Phi_{object}}{I} = \frac{\iint \vec{B} \cdot d\vec{a}}{I}$$

⑦



$$\Phi_{\text{total solidnoid}} = N \, \Phi_{\text{loop}}$$

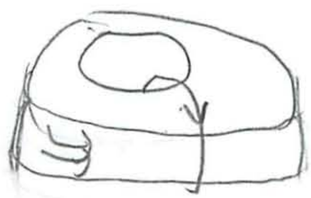
(N = # of turns)

$$= N \iint_{\text{loop}} \vec{B} \cdot d\vec{a}$$

$\vec{B}$ field through loops

toroid

(review)



bunch of wrappings

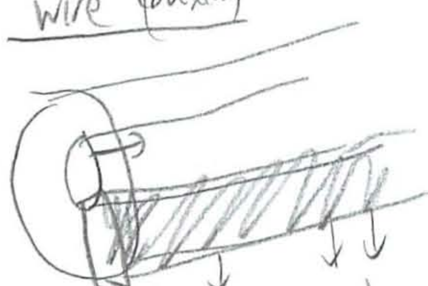
$$\Phi_{\text{total}} = N \, \Phi_{\text{loop}}$$

$$= N \iint \vec{B} \cdot d\vec{a}$$

but non uniform

must choose an area element

wire (coaxial)



$\vec{B}$ field is tangental

coming down

since perpendicular

? make rectangular loop

Side View

$\vec{B}$ field non uniform

must $\int$ it

$$\vec{B} = \frac{\mu_0 I}{2\pi r} \hat{\theta}$$

$da = l \, dr$

$$\frac{\Phi_{coaxial}}{I} = \frac{\int_a^b \frac{\mu_0 I}{2\pi r} l \, dr}{I}$$

not hard integral

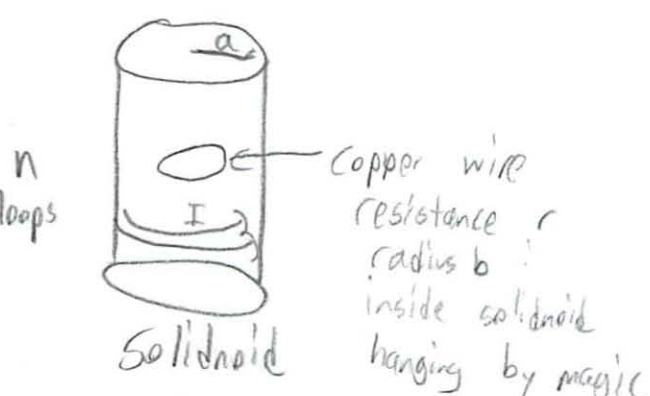
$$L = \frac{\mu_0 l \ln\left(\frac{b}{a}\right)}{2\pi}$$ ← same answer

Challenge your self to do $\vec{B}$ field of a toroid

2 diff approaches to self inductance

- 4 cases
- energy density key concept
  - scalar
  - $\int$ over space

#2 Fariday's Law + Induced current



$I(t) = ct^2$ ccw

↑ time dependent

find I induced (+ dir)

- combo of ideas

[will not consider induced current's affect on $\vec{B}$ field - total current
we are making approx]

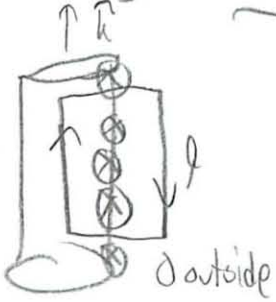
(9)

Fariday's Law

$$\mathcal{E} = \frac{-d\Phi}{dt}$$

$$I_{ind} R = -\frac{d}{dt}\iint_{\text{copper loop}} \vec{B} \cdot d\vec{a}$$

for solidnoid have n
but we are looking at flux through ring
need B field of solidnoid



Ampere's Law (assuming solidnoid ∞ long)

$$\oint B \cdot ds = \mu_0 I_{enc}$$

$$B\ell = \mu_0 \frac{N}{d} \ell I$$

only the inside one — ↑ count wires

$n = \frac{N}{d}$ $\frac{\text{\# wires}}{\text{unit length}}$

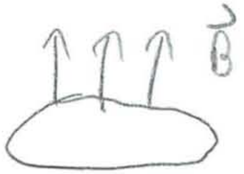
$$\vec{B} = \mu_0 \frac{N}{d} I \hat{k} \text{ up}$$

which way does induced current flow (really study this)
well how is it moving?
will oppose motion

Lenz' Law

(10)

2 step argument



B field is ↑

Φ is ↑ (same dir as B field)

flux not always ↓

must look at increasing/decreasing

(knew this from today)

(getting burned - sometimes memorable)

Φ is increasing

$\Phi_{induced}$ ↓ to oppose change

So current goes ↻ CW

Screwdriver rule

(got this now!)

Now for magnitude

- don't worry about ⊖ sign
- we delt w/ it

...

$\vec{B}$ field is uniform inside solidnoid

But I changing so $\vec{B}$ not constant

$$|\mathcal{E}_{ind}\ R| = \left(\frac{dB}{dt}\right)\pi b^2$$

(11)

$$I_{ind} = \frac{d}{dt}\left(\mu_0 \frac{N}{d} \frac{ct^2}{R} \right) \pi b^2$$

differenitate

$$= \frac{2\mu_0 NC}{dR} \pi b^2 \; t$$

Fariday's Law → changing flux

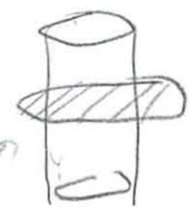
Use Ampere's Law to find $\vec{B}$ field
- is it uniform and/or constant

take deriv of $\vec{B}$

Lentz's Law direction

<u>To make harder</u>

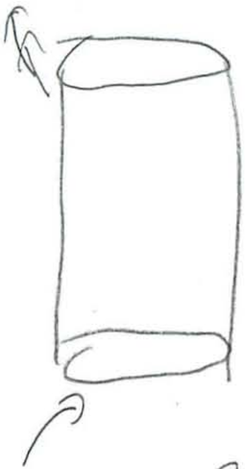
if loop was outside
- ~~no induced current~~
- ~~no B field~~



Surface area is area
- <u>is</u> $\vec{B}$ field inside solidnoid
- <u>will</u> be $\vec{B}$ field

Voltage produces $\vec{E}$ field
- drive charge in wire

(12)



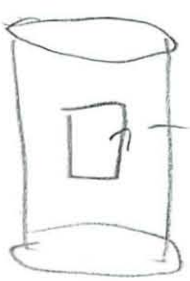
$I(t) = c t^2$

find $\vec{E}$ everywhere

$E \leftrightarrow \frac{\partial B}{\partial t}$

still foriday's law, $\frac{d\Phi}{dt}$

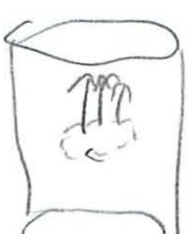
$\mathcal{E}$	$= -\frac{d\Phi_{mag}}{dt}$
$\oint \vec{E} \cdot d\vec{s}$	$= -\frac{d}{dt} \iint_{loop} \vec{B} \cdot d\vec{a}$
Foriday loop, <u>not</u> ampere's loop	
line S	surface S



but still $\vec{E}$ field

E field tangental

- chose circle
- just like ampere's law



→ is magnetic flux

$E\, 2\pi r = \left(-\frac{d}{dt} B\right) \pi r^2$

$r < a$
inside solidnoid

(13) Is E field outside?

Yes

$r > a$

$$E\, 2\pi r = -\frac{dB}{dt} \pi a^2$$

↑ could solve

Even w/o copper wire, E field still there

- due to Δ B field
- keep current steady and no induced current

$E \propto \frac{dB}{dt}$

proportional

- cal calc w/ line integral

Line integral

- field ⊥ path

Could also do loop changing area

$-B \frac{dA}{dt}$ $a(t) = a_0 t \rightarrow r = \pi a^2 t^2$

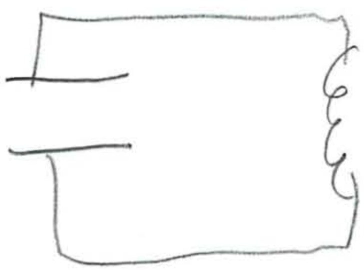
Or changing angle

$$-BA \frac{d \cos\theta}{dt}$$

(14) LC circuits

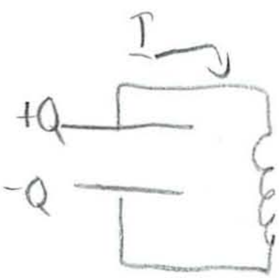
- also do LR
- open + close switches

(ok w/ this, but not differential eq)

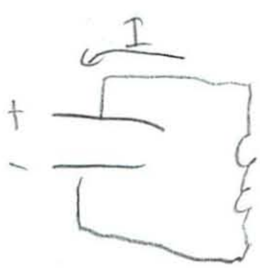


- 2 ways to think about
  - energy (easiest)
    - stored in capacitor or inductor
      E field / B field

(need to get better at energy approaches)
(I've never been good at energy)



dis. charging: $I = \frac{-dQ}{dt}$



charging: $I = \frac{dQ}{dt}$

} I (+) both pictures
- always +'s

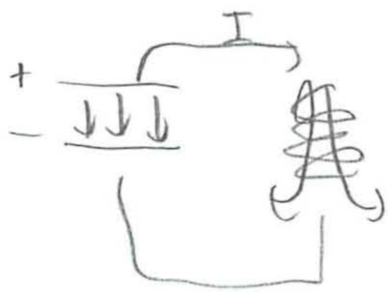
If there is a current, there is a B field

(15)

Go through a full cycle

Some energy (inital) must be added somehow

Start w/ all Energy in capicator



Current max when no charge on capicator

starts charging other plane

Capicator reverses sign

Now halfway through cycle

Now current goes other way

Back to where started

$U =$ Constant $\quad Q(t)$

$\frac{dU}{dt} = 0 \quad I(t)$

$\frac{dU}{dt} = \frac{2Q}{2C}\frac{dQ}{dt} + \frac{2}{2}LI\frac{dI}{dt}$

(working pure on board - very good - not TEAL)

(16)

$I = \frac{dQ}{dt}$

(know all of the cases
- lead / lag
- be able to do)

$= -\frac{Q}{C} I + L I \, dI \, dt = 0$

I's cancle

$\frac{Q}{C} - L\frac{dI}{dt} = 0$

↑ same as if kirchoff loop rule

Now how to get differential eq?

$\frac{dI}{dt} = \frac{-d^2Q}{dt}$ (discharging)

$\frac{Q}{C} - L\frac{d^2Q}{dt} = 0$

↑ SHM ossilator eq

$$\frac{d^2Q}{dt^2} + \frac{1}{LC} Q = 0$$

→ like spring $\frac{d^2x}{dt^2} + \frac{k}{m} x = 0$

↑ $\omega = \frac{1}{\sqrt{LC}}$

know differential eq

$I = \frac{dQ}{dt}$

$\frac{du}{dt} = \frac{Q}{C}\frac{dQ}{dt} + LI\frac{dI}{dt} = 0$ ↙ know energy eq

(17)

$$-\frac{Q}{C} I + L I \frac{dI}{dt} = 0$$

$$\frac{Q}{C} + L \frac{dI}{dt} = 0$$

$$-\frac{Q}{C} - L \frac{dI}{dt} = 0$$

Same eq

$$\frac{Q}{C} + L \frac{d^2Q}{dt} = 0$$

Same eq

- Does not matter which way you do it

Be careful to lable stuff

Make sure

1. Understand energy
2. Can go through a whole cycle
3. Talk way through cycle

After Review

4/28

~~ΔI Δt~~

$-L\frac{dI}{dt}$ in dir of current

Loop generator $= 2\frac{BA}{\quad}$ son

$BA - -BA$

and Δt is constant

so when S

its $\frac{\Delta t}{\Delta t}$ dt does not matter

$\frac{1}{2}LI^2$

↑ derivative

$Q(t) = Q_0 \cos(\omega t + \phi)$

$I = \frac{dQ}{dt}$ → deriv of this function

$= \omega_0 Q_0 \sin(\omega t + \phi)$ Elanow

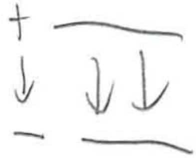
just know the directions

depends on part of cycle

I ⊖ I ⊕

Choose correct convention

$\vec{E}$



~~$\vec{E}$ field from ⊕ plate~~
~~I current~~

Fake thing

Charging I → (towards) ⊕ plate — away ⊖ plate

discharging I ← (away) ⊕ plate — toward ⊕ plate

$\mathcal{E} = L \frac{dI}{dt}$

~~Big~~ biggest when no current

inductor gives

inductor like inertia

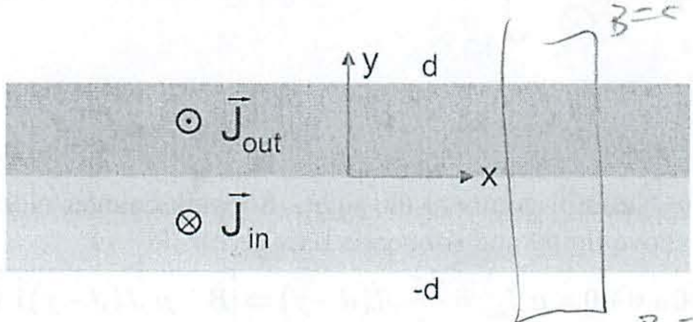
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 **Spring 2010**

Problem Set 11 Solutions

Problem 1: Current Slabs

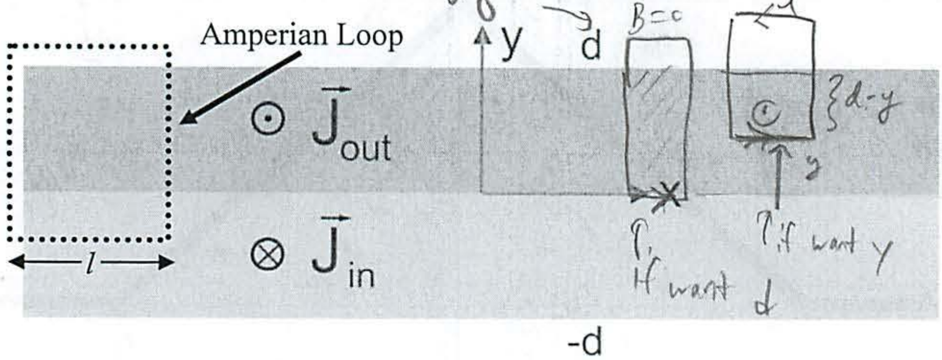
The figure below shows two slabs of current. Both slabs of current are infinite in the x and z directions, and have thickness d in the y-direction. The top slab of current is located in the region $0 < y < d$ and has a constant current density $\vec{\mathbf{J}}_{out} = J\,\hat{\mathbf{z}}$ out of the page. The bottom slab of current is located in the region $-d < y < 0$ and has a constant current density $\vec{\mathbf{J}}_{in} = -J\,\hat{\mathbf{z}}$ into the page.



(a) What is the magnetic field for $|y| > d$? Justify your answer.

Zero. The two parts of the slab create equal and opposite fields for $|y| > d$.

b) Use Ampere's Law to find the magnetic field at $y = 0$. Show the Amperian Loop that you use and give the magnitude and direction of the magnetic field.



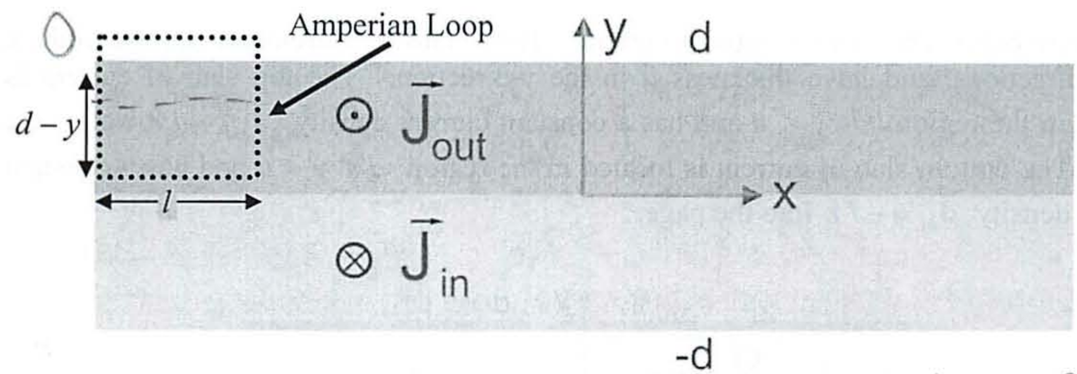
The field at $y = 0$ points to the right (both slabs make it point that way). So walk counter clockwise around the loop shown in the above figure and Ampere's Law gives:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = Bl + 0 + 0 + 0 = \frac{4\pi}{c} I_{enc} = \mu_0 (Jld) \Rightarrow \boxed{\vec{\mathbf{B}} = \mu_0 Jd\,\hat{\mathbf{i}} \text{ (to the right)}}$$

think got this – but maybe for wrong problem

c) Use Ampere's Law to find the magnetic field for $0 < y < d$. Show the Amperian Loop that you use and give the magnitude and direction of the magnetic field.

measuring from here (0) → 0



~~why ccw?~~

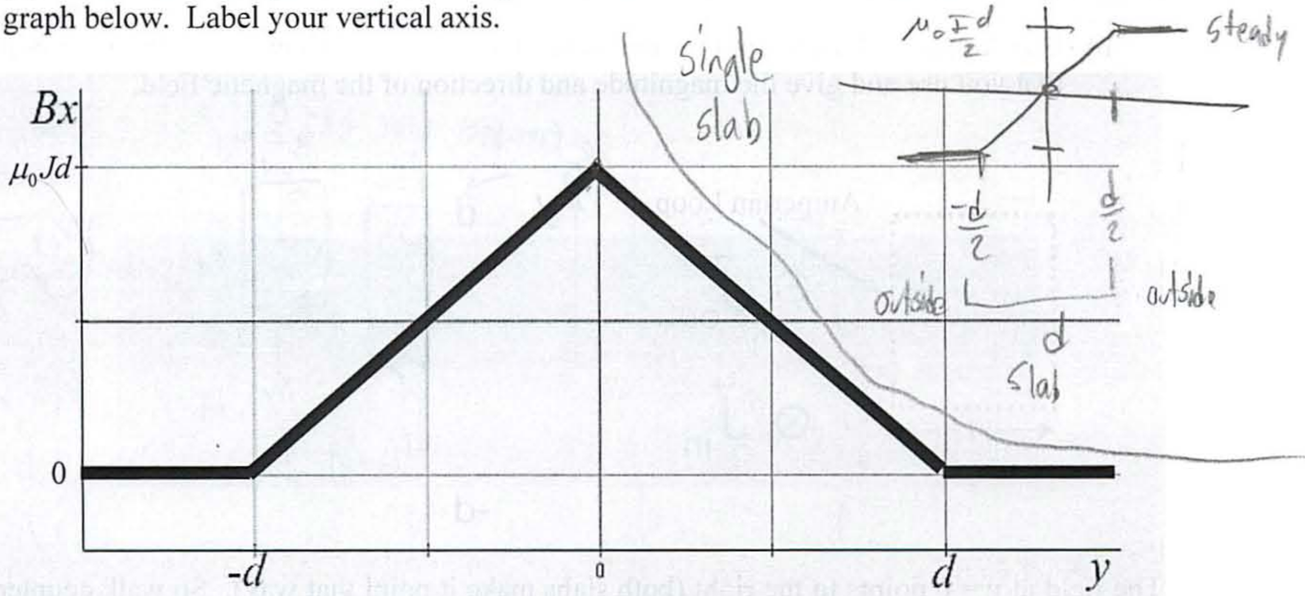
The field for $0 < y < d$ still points to the right. So walk counter clockwise around the loop shown in the above figure and Ampere's Law gives:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = Bl + 0 + 0 + 0 = \mu_o I_{enc} = \frac{4\pi}{c} Jl(d-y) \Rightarrow \boxed{\vec{\mathbf{B}} = \mu_0 J(d-y)\,\hat{\mathbf{i}} \text{ (to the right)}}$$

why d − y

OH – they measure outside in

(d) Plot the x-component of the magnetic field as a function of the distance y on the graph below. Label your vertical axis.

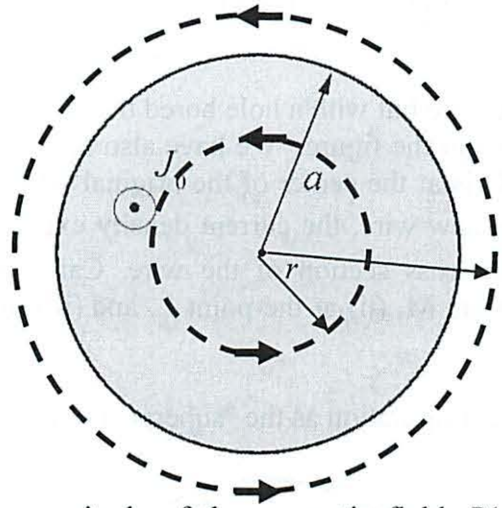


Completely got this wrong

Need to get the signs correct

Problem 2:

An infinitely long wire of radius a carries a current density J_0 which is uniform and constant. The current points "out of" the page, as shown in the figure.



a) Calculate the magnitude of the magnetic field $B(r)$ for (i) $r < a$ and (ii) $r > a$. For both cases show your Amperian loop and indicate (with arrows) the direction of the magnetic field.

The dashed lines above are the Amperian loops I will use for (i) and (ii). They both have a radius of r, and in both cases the paths are counterclockwise, as is the B field, due to a current out of the page (right hand rule).

(i) $r < a$.

From Ampere's Law:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = 2\pi r B = \mu_0 I_{penetrate} = \mu_0 J_0 \pi r^2 \Rightarrow \boxed{B = \frac{\mu_0 J_0 \pi r^2}{2\pi r} = \frac{\mu_0 J_0 r}{2} \text{ counterclockwise}}$$

(ii) $r > a$.

Now we just contain all of the current:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = 2\pi r B = \mu_0 I_{penetrate} = \mu_0 J_0 \pi a^2 \Rightarrow \boxed{B = \frac{\mu_0 J_0 \pi a^2}{2\pi r} = \frac{\mu_0 J_0 a^2}{2r} \text{ counterclockwise}}$$

this is fairly standard

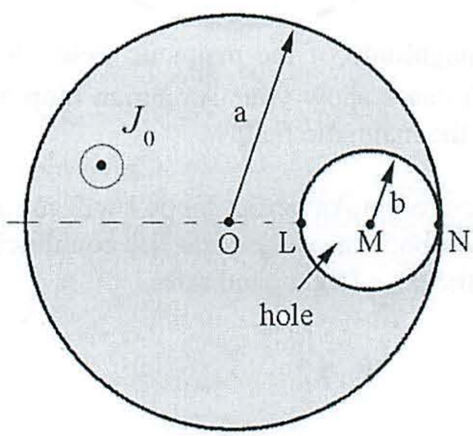
copied from notes - be able to do!

(b) What happens to the answers above if the direction of the current is reversed so that it flows "into" the page ?

If the direction of current flips then so does the direction of the magnetic field, so it is clockwise rather than counterclockwise. The magnitude of the field remains the same.

c) Consider now the same wire but with a hole bored throughout. The hole has radius b (with $2b < a$) and is shown in the figure. We have also indicated four special points: O, L, M, and N. The point O is at the center of the original wire and the point M is at the center of the hole. In this new wire, the current density exists and remains equal to J_0 over the remainder of the cross section of the wire. Calculate the magnitude of the magnetic field at (i) the point M, (ii) at the point L, and (iii) at the point N. Show your work.

Hint: Try to represent the configuration as the "superposition" of two types of wires.



The point here is that we have two wires superimposed on top of each other. The large (radius a) wire carries current out of the page while the smaller (radius b) wire carries current into the page (with the same current density). At all point L, M and N we are inside the large wire and on the right, so the counterclockwise B field is pointing up the page. What is happening from the small wire changes from place to place

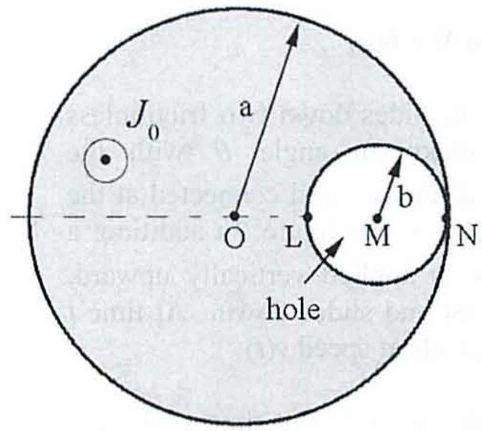
(i) the point M:

Here we are at the center of the small wire, so it contributes nothing. We are at a radius r = a - b inside the big wire, so from part (a.i) of this problem we have:

$$B = \frac{\mu_0 J_0 (a-b)}{2} \text{ up}$$

Dormashkin

very hard - perhaps too hard



(ii) at the point L:

Here we are to the left of the small wire (at a radius r = b), so the clockwise field (as we said in part b) is pointing up, just like the CCW field from the big wire We are at a radius r = a - 2b inside the big wire, so:

$$\boxed{B = \frac{\mu_0 J_0 (a-2b)}{2} + \frac{\mu_0 J_0 b}{2}\text{ up} = \frac{\mu_0 J_0 (a-b)}{2}\text{ up}}$$

(iii) at the point N:

Here we are to the right of the small wire (at a radius r = b), so the clockwise field is pointing down, opposite the CCW field from the big wire so they subtract rather than add We are at a radius r = a inside the big wire, so:

$$\boxed{B = \frac{\mu_0 J_0 a}{2} - \frac{\mu_0 J_0 b}{2}\text{ up} = \frac{\mu_0 J_0 (a-b)}{2}\text{ up}}$$

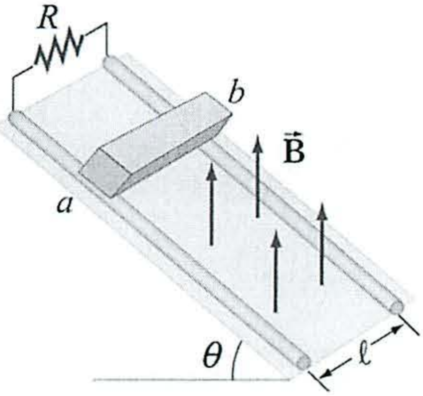
Same result everywhere - think that is what I had

A comment about people's work on this problem: I was stunned at how many people tried to do Ampere's law on the wire with a hole in it. Since the hole breaks the cylindrical symmetry of the problem you just can't do this. That is, since B is no longer constant around an Amperian centered on O, $\oint \vec{\mathbf{B}} \cdot d\vec{s} \neq 2\pi r B$. B isn't constant, so you can't just pull it out!

- did I do this

well superimposed hole over wire

Problem 3: Sliding Bar on Wedges



A conducting bar of mass m slides down two frictionless conducting rails which make an angle θ with the horizontal, separated by a distance ℓ and connected at the top by a resistor R, as shown in the figure. In addition, a uniform magnetic field $\vec{\mathbf{B}}$ is applied vertically upward. The bar is released from rest and slides down. At time t the bar is moving along the rails at speed $v(t)$.

(a) Find the induced current in the bar at time t. Which way does the current flow, from a to b or b to a?

The flux between the resistor and bar is given by

$$\Phi_B = B\ell x(t)\cos\theta$$

where $x(t)$ is the distance of the bar from the top of the rails.

Then,

$$\varepsilon = -\frac{d}{dt}\Phi_B = -\frac{d}{dt}B\ell x(t)\cos\theta = -B\ell v(t)\cos\theta$$

Because the resistance of the circuit is R, the magnitude of the induced current is

$$I = \frac{|\varepsilon|}{R} = \frac{B\ell v(t)\cos\theta}{R}$$

By Lenz's law, the induced current produces magnetic fields which tend to oppose the change in magnetic flux. Therefore, the current flows clockwise, from b to a across the bar.

(b) Find the terminal speed v_T of the bar.

At terminal velocity, the net force along the rail is zero, that is gravity is balanced by the magnetic force:

$$mg\sin\theta = IB\ell\cos\theta = \left(\frac{B\ell v_t(t)\cos\theta}{R}\right)B\ell\cos\theta$$

or

$$v_t(t) = \frac{Rmg\sin\theta}{(B\ell\cos\theta)^2}$$

After the terminal speed has been reached,

(c) What is the induced current in the bar?

$$I = \frac{B\ell v_t(t)\cos\theta}{R} = \frac{B\ell\cos\theta}{R}\left(\frac{Rmg\sin\theta}{(B\ell\cos\theta)^2}\right) = \frac{mg\sin\theta}{B\ell\cos\theta} = \frac{mg}{B\ell}\tan\theta$$

(d) What is the rate at which electrical energy is being dissipated through the resistor?

The power dissipated in the resistor is

$$P = I^2 R = \left(\frac{mg}{B\ell}\tan\theta\right)^2 R$$

(e) What is the rate of work done by gravity on the bar? The rate at which work is done is $\vec{\mathbf{F}}\cdot\vec{\mathbf{v}}$. How does this compare to your answer in (d)? Why?

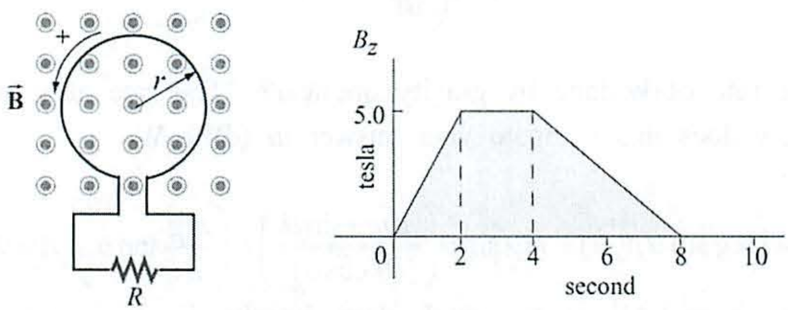
$$\vec{F}\cdot\vec{V} = (mg\sin\theta)v_t(t) = mg\sin\theta\left(\frac{Rmg\sin\theta}{(B\ell\cos\theta)^2}\right) = \left(\frac{mg}{B\ell}\tan\theta\right)^2 R = P$$

That is, they are equal. All of the work done by gravity is dissipated in the resistor, which is why the rod is not accelerating past its terminal velocity.

this is hard w/o my work
I should have scanned it in

Problem 4 EMF Due to a Time-Varying Magnetic Field

A uniform magnetic field $\vec{\mathbf{B}}$ is perpendicular to a one-turn circular loop of wire of negligible resistance, as shown in the figure below. The field changes with time as shown (the z direction is out of the page). The loop is of radius $r = 50$ cm and is connected in series with a resistor of resistance $R = 20\ \Omega$. The "+" direction around the circuit is indicated in the figure. **In order to obtain credit you must show your work; partial answers without work will not be accepted.**



(a) What is the expression for EMF in this circuit in terms of $B_z(t)$ for this arrangement?

Solution: When we choose a "+" direction around the circuit as shown in the figure above, then we are also specifying that magnetic flux out of the page is positive. (The unit vector $\hat{\mathbf{n}} = +\hat{\mathbf{k}}$ points out of the page.) Thus the dot product becomes

wtf?

$$\vec{\mathbf{B}} \cdot \hat{\mathbf{n}} = \vec{\mathbf{B}} \cdot \hat{\mathbf{k}} = B_z . \qquad (0.1)$$

From the graph, the z-component of the magnetic field B_z is given by

$$B_z = \begin{cases} (2.5\ \mathrm{T \cdot s^{-1}})t\ ; 0 < t < 2\ \mathrm{s} \\ 5.0\ \mathrm{T}\ ; 2\ \mathrm{s} < t < 4\ \mathrm{s} \\ 10\ \mathrm{T} - (1.25\ \mathrm{T \cdot s^{-1}})t\ ; 4\ \mathrm{s} < t < 8\ \mathrm{s} \\ 0\ ; t > 8\ \mathrm{s} \end{cases} . \qquad (0.2)$$

The derivative of the magnetic field is then

$$\frac{dB_z}{dt} = \begin{cases} 2.5\ \mathrm{T \cdot s^{-1}}\ ; 0 < t < 2\ \mathrm{s} \\ 0\ ; 2\ \mathrm{s} < t < 4\ \mathrm{s} \\ -1.25\ \mathrm{T \cdot s^{-1}}\ ; 4\ \mathrm{s} < t < 8\ \mathrm{s} \\ 0\ ; t > 8\ \mathrm{s} \end{cases} . \qquad (0.3)$$

The magnetic flux is therefore

$$\Phi_{magnetic} = \iint \vec{\mathbf{B}} \cdot \hat{\mathbf{n}}\, d\vec{\mathbf{A}} = \iint B_z dA = B_z \pi r^2 . (0.4)$$

The electromotive force is

$$\mathcal{E} = -\frac{d}{dt}\Phi_{magnetic} = -\frac{dB_z}{dt}\pi r^2 . \qquad (0.5)$$

So we calculate the electromotive force by substituting Eq. (0.3) into Eq. (0.5) yielding

$$\mathcal{E} = \begin{cases} -(2.5\,\mathrm{T}\cdot\mathrm{s}^{-1})\pi r^2 \,; 0 < t < 2\,\mathrm{s} \\ 0\,; 2\,\mathrm{s} < t < 4\,\mathrm{s} \\ (1.25\,\mathrm{T}\cdot\mathrm{s}^{-1})\pi r^2 \,; 4\,\mathrm{s} < t < 8\,\mathrm{s} \\ 0\,; t > 8\,\mathrm{s} \end{cases} . \qquad (0.6)$$

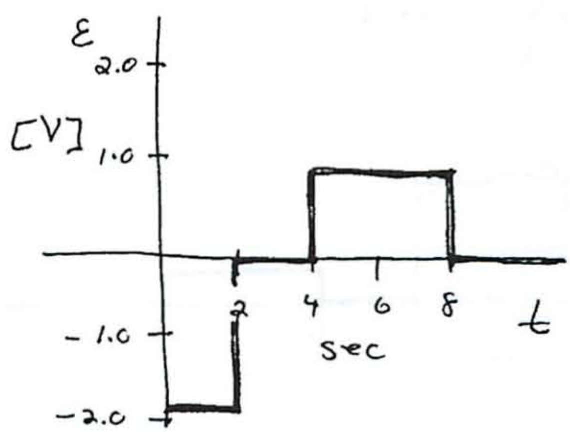
Using $r = 0.5\ \mathrm{m}$, the electromotive force is then

$$\mathcal{E} = \begin{cases} -1.96\,\mathrm{V}\,; 0 < t < 2\,\mathrm{s} \\ 0\,; 2\,\mathrm{s} < t < 4\,\mathrm{s} \\ 0.98\,\mathrm{V}\,; 4\,\mathrm{s} < t < 8\,\mathrm{s} \\ 0\,; t > 8\,\mathrm{s} \end{cases} \qquad (0.7)$$

Solution:

(b) Plot the EMF in the circuit as a function of time. Label the axes quantitatively (numbers and units). Watch the signs. Note that we have labeled the positive direction of the emf in the left sketch consistent with the assumption that positive $\vec{\mathbf{B}}$ is out of the paper.

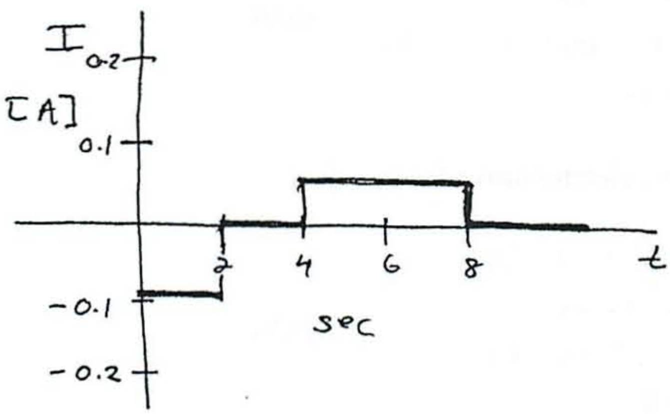
Solution:



(c) Plot the current I through the resistor R. Label the axes quantitatively (numbers and units). Indicate with arrows on the sketch the *direction* of the current through R during each time interval.

Solution: The current through the resistor ($R = 20\,\Omega$) is given by

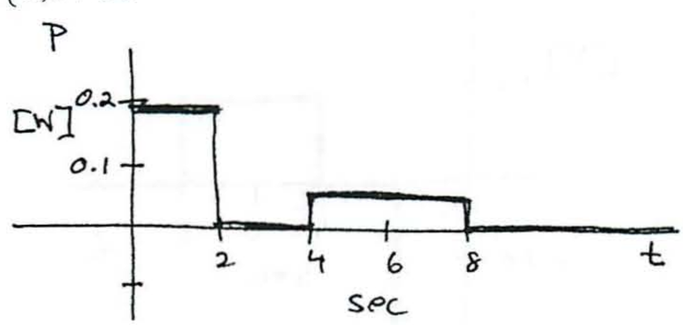
$$I = \frac{\mathcal{E}}{R} = \begin{cases} -9.8\times10^{-2}\ \mathrm{A}\,;0 < t < 2\ \mathrm{s} \\ 0\,;2\ \mathrm{s} < t < 4\ \mathrm{s} \\ 4.9\times10^{-2}\ \mathrm{A}\,;4\ \mathrm{s} < t < 8\ \mathrm{s} \\ 0\,;t > 8\ \mathrm{s} \end{cases}. \qquad (0.8)$$



(d) Plot the power dissipated in the resistor as a function of time.

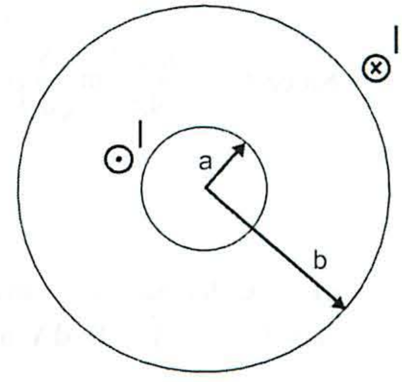
Solution: The power dissipated in the resistor is given by

$$P = I^2 R = \begin{cases} 1.9\times10^{-1}\ \mathrm{W}\,;0 < t < 2\ \mathrm{s} \\ 0\,;2\ \mathrm{s} < t < 4\ \mathrm{s} \\ 4.8\times10^{-2}\ \mathrm{W}\,;4\ \mathrm{s} < t < 8\ \mathrm{s} \\ 0\,;t > 8\ \mathrm{s} \end{cases}. \qquad (0.9)$$



Problem 5: Inductor

An inductor consists of two very thin conducting cylindrical shells, one of radius a and one of radius b, both of length h. Assume that the inner shell carries current I out of the page, and that the outer shell carries current I into the page, distributed uniformly around the circumference in both cases. The z-axis is out of the page along the common axis of the cylinders and the r-axis is the radial cylindrical axis perpendicular to the z-axis.



a) Use Ampere's Law to find the magnetic field between the cylindrical shells. Indicate the direction of the magnetic field on the sketch. What is the magnetic energy density as a function of r for $a < r < b$?

The enclosed current I_{enc} in the Ampere's loop with radius r is given by

$$I_{\text{enc}} = \begin{cases} 0, & r < a \\ I, & a < r < b \\ 0, & r > b \end{cases}$$

Applying Ampere's law, $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B(2\pi r) = \mu_0 I_{\text{enc}}$, we obtain

$$\vec{\mathbf{B}} = \begin{cases} 0, & r < a \\ \dfrac{\mu_0 I}{2\pi r}\hat{\boldsymbol{\varphi}}, & a < r < b \text{ (counterclockwise in the figure)} \\ 0, & r > b \end{cases}$$

The magnetic energy density for $a < r < b$ is

$$u_B = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0}\left(\frac{\mu_0 I}{2\pi r}\right)^2 = \boxed{\frac{\mu_0 I^2}{8\pi^2 r^2}}$$

It is zero elsewhere.

b). Calculate the inductance of this long inductor recalling that $U_B = \frac{1}{2} L I^2$ and using your results for the magnetic energy density in (a).

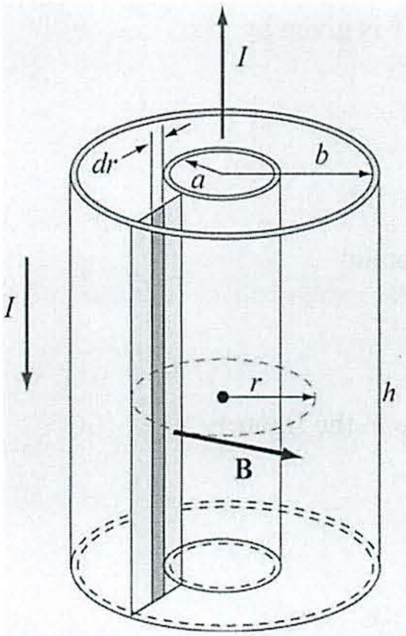
The volume element in this case is $2\pi r h dr$. The magnetic energy is :

$$U_B = \int_V u_B d\text{Vol} = \int_a^b \left(\frac{\mu_0 I^2}{8\pi^2 r^2}\right) 2\pi h\, r dr = \frac{\mu_0 I^2 h}{4\pi}\ln\left(\frac{b}{a}\right)$$

Since $U_B = \dfrac{\mu_0 I^2 l}{4\pi}\ln\left(\dfrac{b}{a}\right) = \dfrac{1}{2}LI^2$, the inductance is

$$\boxed{L = \frac{\mu_0 h}{2\pi}\ln\left(\frac{b}{a}\right)}$$

c) Calculate the inductance of this long inductor by using the formula $\Phi = LI = \int_{\text{open surface}} \vec{\mathbf{B}}\cdot d\vec{\mathbf{A}}$ and your results for the magnetic field in (a). To do this you must choose an appropriate open surface over which to evaluate the magnetic flux. Does your result calculated in this way agree with your result in (b)?



The magnetic field is perpendicular to a rectangular surface shown in the figure. The magnetic flux through a thin strip of area $dA = l dr$ is

$$d\Phi_B = BdA = \left(\frac{\mu_0 I}{2\pi r}\right)(h\, dr) = \frac{\mu_0 I h}{2\pi r}dr$$

Thus, the total magnetic flux is

$$\Phi_B = \int d\Phi_B = \int_a^b \frac{\mu_0 I h}{2\pi r}dr = \frac{\mu_0 I h}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 I h}{2\pi}\ln\left(\frac{b}{a}\right)$$

Thus, the inductance is

$$L = \frac{\Phi_B}{I} = \boxed{\frac{\mu_0 h}{2\pi}\ln\left(\frac{b}{a}\right)}$$

which agrees with that obtained in (b).

don't think I got that

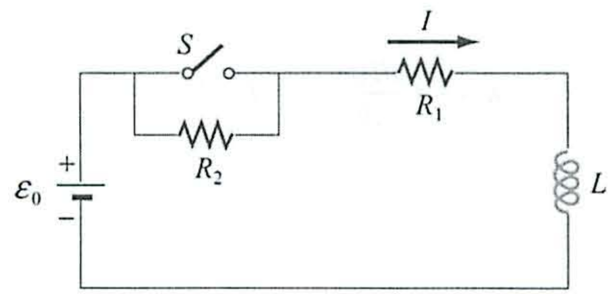
- go to review session
- do math hw real fast

Review notes → - do pratice problems + correct instantly

- review PRS for today
- dir of Lenz's law

Problem 6: Trying to open the switch on an *RL* Circuit

The *LR* circuit shown in the figure contains a resistor R_1 and an inductance L in series with a battery of emf ε_0. The switch S is initially closed. At t = 0, the switch S is opened, so that an additional very large resistance R_2 (with $R_2 \gg R_1$) is now in series with the other elements.



(a) If the switch has been *closed* for a long time before t = 0, what is the steady current I_0 in the circuit?

There is no induced emf before t = 0. Also, no current is flowing on R_2. Therefore,

$$I_0 = \frac{\varepsilon_0}{R_1}$$

(b) While this current I_0 is flowing, at time t = 0, the switch S is opened. Write the differential equation for $I(t)$ that describes the behavior of the circuit at times $t \geq 0$. Solve this equation (by integration) for $I(t)$ under the approximation that $\varepsilon_0 = 0$. (Assume that the battery emf is negligible compared to the total emf around the circuit for times just after the switch is opened.) Express your answer in terms of the initial current I_0, and R_1, R_2, and L.

The differential equation is

$$\varepsilon_0 - I(t)(R_1 + R_2) = L\frac{dI(t)}{dt}$$

Under the approximation that $\varepsilon_0 = 0$, the equation is

$$-I(t)(R_1 + R_2) = L\frac{dI(t)}{dt}$$

The solution with the initial condition $I(0) = I_0$ is given by

$$I(t) = I_0 \exp(-\frac{(R_1 + R_2)}{L} t)$$

(c) Using your results from (b), find the value of the total emf around the circuit (which from Faraday's law is $-L dI/dt$) just after the switch is opened. Is your assumption in (b) that ε_0 could be ignored for times just after the switch is opened OK?

$$\varepsilon = -L \frac{dI(t)}{dt}\bigg|_{t=0} = I_0 (R_1 + R_2)$$

Since $I_0 = \dfrac{\varepsilon_0}{R_1}$,

$$\varepsilon = \frac{\varepsilon_0}{R_1}(R_1 + R_2) = \left(1 + \frac{R_2}{R_1}\right)\varepsilon_0 >> \varepsilon_0 \quad (\because R_2 >> R_1)$$

Thus, the assumption that ε_0 could be ignored for times just after the switch is open is OK.

(d) What is the magnitude of the potential drop across the resistor R_2 at times $t > 0$, just after the switch is opened? Express your answers in terms of ε_0, R_1, and R_2. How does the potential drop across R_2 just after $t = 0$ compare to the battery emf ε_0, if $R_2 = 100 R_1$?

The potential drop across R_2 is given by

$$\Delta V_2 = \frac{R_2}{R_1 + R_2}\varepsilon = \left(\frac{R_2}{R_1 + R_2}\right)\left(1 + \frac{R_2}{R_1}\right)\varepsilon_0 = \frac{R_2}{R_1}\varepsilon_0$$

If $R_2 = 100 R_1$,

$$\Delta V_2 = 100\,\varepsilon_0$$

This is why you have to open a switch in a circuit with a lot of energy stored in the magnetic field very carefully, or you end up very dead!!

Problem 7: *LC* Circuit

An inductor having inductance L and a capacitor having capacitance C are connected in series. The current in the circuit increase linearly in time as described by $I = Kt$. The capacitor initially has no charge. Determine

(a) the voltage across the inductor as a function of time,

The voltage across the inductor is

$$\varepsilon_L = -L\frac{dI}{dt} = -L\frac{d}{dt}(Kt) = -LK$$

(b) the voltage across the capacitor as a function of time, and

Using $I = \dfrac{dQ}{dt}$, the charge on the capacitor as a function of time may be obtained as

$$Q(t) = \int_0^t I dt' = \int_0^t Kt' dt' = \frac{1}{2}Kt^2$$

Thus, the voltage drop across the capacitor as a function of time is

$$\Delta V_C = -\frac{Q}{C} = -\frac{Kt^2}{2C}$$

(c) the time when the energy stored in the capacitor first exceeds that in the inductor.

The energies stored in the capacitor and the inductor are

$$U_C = \frac{1}{2}C(\Delta V_C)^2 = \frac{1}{2}C\left(-\frac{Kt^2}{2C}\right)^2 = \frac{K^2t^4}{8C}$$

$$U_L = \frac{1}{2}LI^2 = \frac{1}{2}L(Kt)^2 = \frac{1}{2}LK^2t^2$$

The two energies are equal when

$$\frac{K^2t'^4}{8C} = \frac{1}{2}LK^2t'^2 \quad \Rightarrow t' = 2\sqrt{LC}$$

Therefore, $U_C > U_L$ when $t > t'$.

Problem 8: *LC* Circuit

(a) Initially, the capacitor in a series *LC* circuit is charged. A switch is closed, allowing the capacitor to discharge, and after time *T* the energy stored in the capacitor is one-fourth its initial value. Determine *L* if *C* and *T* are known.

The energy stored in the capacitor is given by

$$U_C(t) = \frac{Q(t)^2}{2C} = \frac{(Q_0 \cos \omega_0 t)^2}{2C} = \frac{Q_0^2}{2C} \cos^2 \omega_0 t$$

Thus,

$$\frac{U_C(T)}{U_C(0)} = \frac{\cos^2 \omega_0 T}{\cos^2 (0)} = \frac{\cos^2 \omega_0 T}{1} = \frac{1}{4} \quad \Rightarrow \cos \omega_0 T = \frac{1}{2}$$

which implies that $\omega_0 T = \frac{\pi}{3} \text{rad} = 60°$. Therefore, with $\omega_0 = \frac{1}{\sqrt{LC}}$, we obtain

$$T = \frac{\pi}{3\omega_0} = \frac{\pi}{3}\sqrt{LC} \quad \Rightarrow \quad L = \frac{1}{C}\left(\frac{3T}{\pi}\right)^2$$

(b) A capacitor in a series *LC* circuit has an initial charge Q_0 and is being discharged. The inductor is a solenoid with *N* turns. Find, in terms of *L* and *C*, the flux through each of the *N* turns in the coil at time *t*, when the charge on the capacitor is *Q*(*t*).

We can do this two ways, either is acceptable. First,we can make the explicit assumption that $Q(t) = Q_0 \cos \omega_0 t$ and the total flux through the inductor is $LI = L\frac{dQ}{dt} = -L\omega_0 Q_0 \sin \omega_0 t$

Therefore the flux through one turn of the inductor at time *t* is $\Phi_{\text{one turn}} = -\frac{L\omega_0 Q_0}{N} \sin \omega_0 t$

or in terms of *L* and *C*, $\Phi_{\text{one turn}} = -\sqrt{\frac{L}{C}}\frac{Q_0}{N} \sin \omega_0 t$. Or second, we can simply leave *Q*(*t*) as an unspecified function of time and write (using the same arguments as above) that $\Phi_{\text{one turn}} = \frac{L}{N}\frac{dQ}{dt}$.

(c) An *LC* circuit consists of a 20.0-mH inductor and a 0.500-μF capacitor. If the maximum instantaneous current is 0.100 A, what is the greatest potential difference across the capacitor?

The greatest potential difference across the capacitor when $U_{C\max} = U_{L\max}$, or

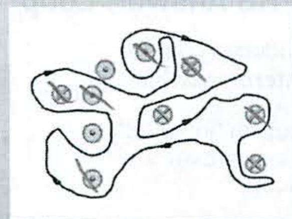
$$\frac{1}{2} C V_{C\max}^2 = \frac{1}{2} L I_{\max}^2 \quad \Rightarrow \quad V_{C\max} = \sqrt{\frac{L}{C}} I_{\max} = \sqrt{\frac{(20.0\,\text{mH})}{(0.500\,\mu\text{F})}} (0.100\text{A}) = 20 \text{ V}$$

PRS Review

4/27 class

Redo for practice 4/28

PRS: Ampere's Law



nets out

Integrating B around the loop shown gives us:

0% 1. a positive number

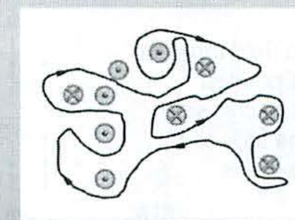
0% 2. a negative number

0% (3.) zero

:15

plain screwdriver method

PRS Answer: Ampere's Law



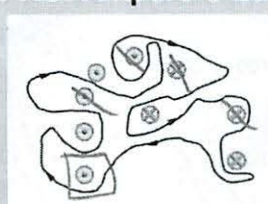
Answer: 3. Total penetrating current is zero, so

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc} = 0$$

P31- 2

✓

PRS: Ampere's Law



Integrating B around the loop shown gives us:

0% 1. a positive number

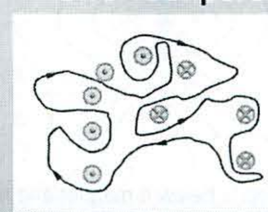
0% (2.) a negative number

0% 3. zero

15

thumb up but arrow other way

PRS Answer: Ampere's Law



Answer: 2. $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} < 0$

Net penetrating current is out of the page, so field is counter-clockwise (opposite path)

P31- 4

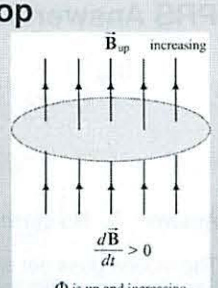
✓

much better since review session

PRS: Loop

0

The magnetic field through a wire loop is pointed upwards and *increasing* with time. The induced current in the coil is



0% (1) Clockwise as seen from the top

0% 2. Counterclockwise

P31- 5

matters what dir it is going

so B↑ will cause ↻

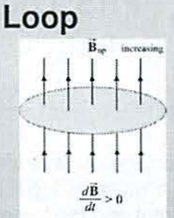
so flux will be ↺

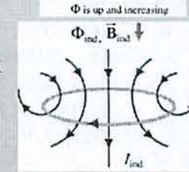
PRS Answer: Loop

Answer: 1. Induced current is **clockwise**

This produces an "induced" B field pointing down over the area of the loop.

The "induced" B field opposes the increasing flux through the loop – Lenz's Law





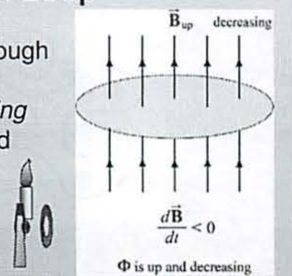
P31- 6

✓

↑ don't get this pic

PRS: Loop

The magnetic field through a wire loop is pointed upwards and *decreasing* with time. The induced current in the coil is



0% 1. Clockwise as seen from the top

0% 2. Counterclockwise

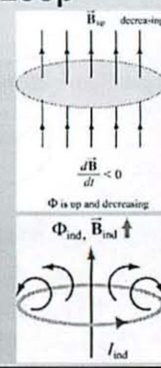
decreasing is like ↓ ↷
want other way ↶

PRS Answer: Loop

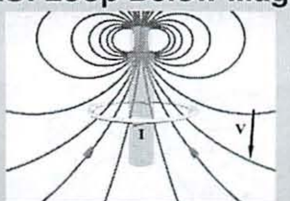
Answer: 2. Induced current is **counterclockwise**

This produces an "induced" B field pointing up over the area of the loop.

The "induced" B field opposes the decreasing flux through the loop – making up for the loss – Lenz's Law



PRS: Loop Below Magnet



A conducting loop is below a magnet and moving downwards. This induces a current as pictured. The *I ds x B* force on the coil is

0% 1. Up

0% 2. Down

0% 3. Zero

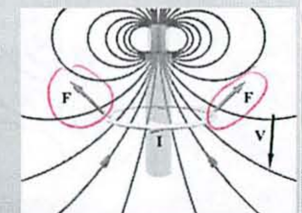
I(L×B)

B field ↑ but decreasing
will always move up

PRS Answer: Loop Below Magnet

Answer: 1. Force is Up

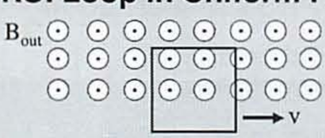
Lenz' Law:
Must oppose motion – force is up



More detail:
Induced current is counter-clockwise to oppose drop in upward flux.
This looks like a dipole facing upward, so it is attracted to the other dipole

I don't know all the explanation just that falling experiment

PRS: Loop in Uniform Field



A rectangular wire loop is pulled thru a uniform B field penetrating its top half, as shown. The induced current and the force and torque on the loop are:

% 1. Current CW, Force Left, No Torque

% 2. Current CW, No Force, Torque Rotates CCW

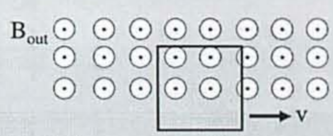
% 3. Current CCW, Force Left, No Torque

% 4. Current CCW, No Force, Torque Rotates CCW

% 5. No current, force or torque

even, no change

PRS Answer: Loop in Uniform Field



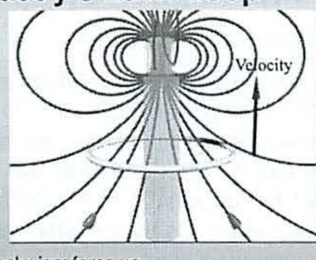
Answer: 5. No current, force or torque

The motion does not change the magnetic flux, so Faraday's Law says there is no induced EMF, or current, or force, or torque.

Of course, if we were pulling at all up or down there would be a force to oppose that motion.

PRS: Faraday's Law: Loop

A coil moves up from underneath a magnet with its north pole pointing upward. The current in the coil and the force on the coil:



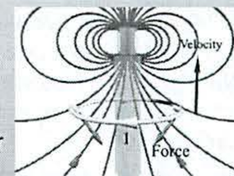
- 0% 1. Current clockwise; force up
- 0% 2. Current counterclockwise; force up
- 0% 3. Current clockwise; force down
- 0% 4. Current counterclockwise; force down

B field ↑ increasing ↻
opposes ↷ force opposes ⊥

PRS Answer: Faraday's Law: Loop

Answer: 3. Current is clockwise; force is down

The clockwise current creates a self-field downward, trying to offset the increase of magnetic flux through the coil as it moves upward into stronger fields (Lenz's Law).

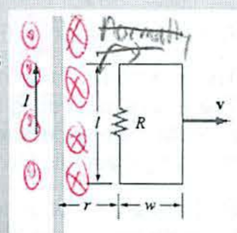


The I dl x B force on the coil is a force which is trying to keep the flux through the coil from increasing by slowing it down (Lenz's Law again).

✓

PRS: Circuit

A circuit in the form of a rectangular piece of wire is pulled away from a long wire carrying current *I* in the direction shown in the sketch. The induced current in the rectangular circuit is



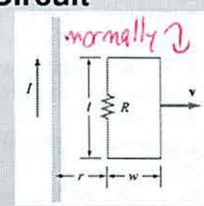
- 0% 1. Clockwise
- 0% 2. Counterclockwise
- 0% 3. Neither, the current is zero

if not moving 0 current
Well what is B from wire? ⊗ Screwdriver method - duh
~~⊥ to current~~ ~~guessing CCW~~ like 1/r away from wire

PRS Answer: Circuit

Answer: 1. Induced current is **clockwise**

B due to I is into page; the flux through the circuit due to that field decreases as the circuit moves away. So the induced current is clockwise (to make a B into the page) obvious



Note: I_{ind} *dl* x *B* force is left on the left segment and right on the right, but the force on the left is bigger. So the net force on the rectangular circuit is to the left, again trying to keep the flux from decreasing by slowing the circuit's motion

B-S law

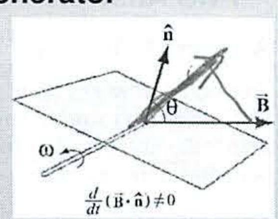
✗

yes is normally current through

$F = I(L \times B)$ - decreasing - so still ↷

PRS: Generator

A square coil rotates in a magnetic field directed to the right. At the time shown, the current in the square, when looking down from the top of the square loop, will be

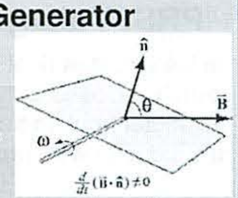


- 0% 1. Clockwise
- 0% 2. Counterclockwise
- 0% 3. Neither, the current is zero
- 0% 4. I don't know

PRS Answer: Generator

Answer: 1. Induced current is **counterclockwise**

Flux through loop decreases as normal rotates away from B. To try to keep flux from decreasing, induced current will be CCW, trying to keep the magnetic flux from decreasing (Lenz's Law)



Note: I_{ind} *dl* x *B* force on the sides of the square loop will be such as to produce a torque that tries to stop it from rotating (Lenz's Law).

✗

Normally ↻ from B does not ask induced

will be induced current

But is it ↑ or ⊥ — must always ask

PRS: Stopping a Motor

Consider a motor (a loop of wire rotating in a B field) which is driven at a constant rate by a battery through a resistor.
Now grab the motor and prevent it from rotating. What happens to the current in the circuit?

0%	1. Increases
0%	2. Decreases
0%	3. Remains the Same
0%	4. I don't know

Burns out

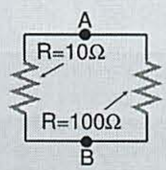
PRS Answer: Stopping a Motor

Answer: 1. **Increases**

When the motor is rotating in a magnetic field an EMF is generated which opposes the motion, that is, it reduces the current. When the motor is stopped that back EMF disappears and the full voltage of the battery is now dropped across the resistor – the current increases. For some motors this increase is very significant, and a stalled motor can lead to huge currents that burn out the windings (e.g. your blender).

PRS: Faraday Circuit

A magnetic field B penetrates this circuit outwards, and is increasing at a rate such that a current of 1 A is induced in the circuit (which direction?).
The potential difference VA-VB is:



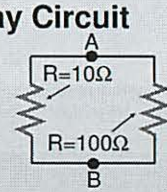
0%	1. +10 V
0%	2. -10 V
0%	3. +100 V
0%	4. -100 V
0%	5. +110 V
0%	6. -110 V
0%	7. +90 V
0%	8. -90 V
0%	9. None of the above

Remembering a lot from class

Obi hated this qu skip

PRS Answer: Faraday Circuit

Answer: 9. None of the above



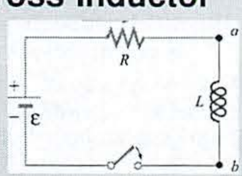
The question is meaningless. There is no such thing as potential difference when a changing magnetic flux is present.

By Faraday's law, a non-conservative E is induced (that is, its integral around a closed loop is non-zero). Non-conservative fields can't have potentials associated with them.

Omit

PRS: Voltage Across Inductor

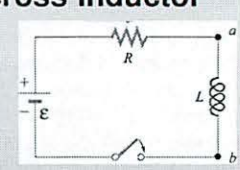
In the circuit at right the switch is closed at $t = 0$. A voltmeter hooked across the inductor will read:



0%	1. $V_L = \varepsilon e^{-t/\tau}$ increasing
0%	2. $V_L = \varepsilon(1-e^{-t/\tau})$ decreasing
0%	3. $V_L = 0$
0%	4. I don't know

PRS Answer: V Across Inductor

Answer: 1. $V_L = \varepsilon e^{-t/\tau}$



The inductor "works hard" at first, preventing current flow, then "relaxes" as the current becomes constant in time.

Although "voltage differences" between two points isn't completely meaningful now, we certainly can hook a voltmeter across an inductor and measure the EMF it generates.

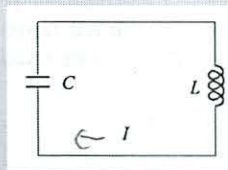
Wrong direction

$\mathcal{E} - IR - L\frac{dI}{dt} = 0$ current will slowly ↑

$L\frac{dI}{dt} = V_L = \mathcal{E} - IR$

PRS: LC Circuit

Consider the LC circuit at right. At the time shown the current has its maximum value. At this time

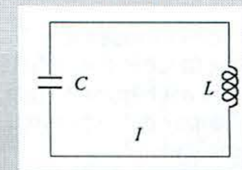


- 0% 1. The charge on the capacitor has its maximum value (min value)
- 0% 2. The magnetic field is zero
- 0% 3. The electric field has its maximum value
- 0% 4. The charge on the capacitor is zero ✓
- 0% 5. Don't have a clue

:00

PRS Answer: LC Circuit

Answer: 4. The current is maximum when the charge on the capacitor is zero



Current and charge are exactly 90 degrees out of phase in an ideal LC circuit (no resistance), so when the current is maximum the charge must be identically zero.

P31-26

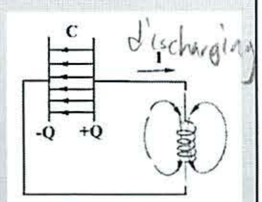
✓

kinda remember
3 remember is if capacitor is full
max mag field max value

PRS: LC Circuit

0

In the LC circuit at right the current is in the direction shown and the charges on the capacitor have the signs shown. At this time,



- 0% 1. I is increasing and Q is increasing
- 0% 2. I is increasing and Q is decreasing
- 0% 3. I is decreasing and Q is increasing
- 0% 4. I is decreasing and Q is decreasing
- 0% 5. Don't have a clue

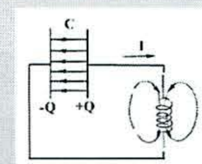
P31-27

I ↑ Q ↓

PRS Answer: LC Circuit

Answer: 2. I is increasing;
Q is decreasing

With current in the direction shown, the capacitor is discharging (Q is decreasing).



But since Q on the right plate is positive, I must be increasing. The positive charge *wants* to flow, and the current will increase until the charge on the capacitor changes sign. That is, we are in the first quarter period of the discharge of the capacitor, when Q is decreasing and positive and I is increasing and positive.

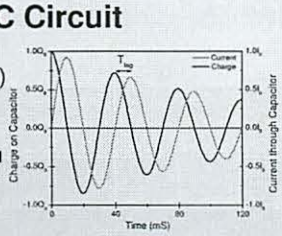
P31-28

✓

PRS: LC Circuit

They tell you anyway →

The plot shows the charge on a capacitor (black curve) and the current through it (red curve) after you turn off the power supply. If you put a core into the inductor what will happen to the time T_{Lag}?

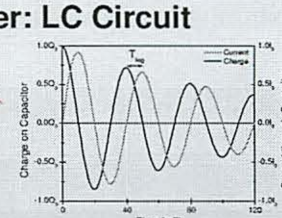


- 0% 1. It will increase
- 0% 2. It will decrease
- 0% 3. It will stay the same
- 0% 4. I don't know

0

PRS Answer: LC Circuit

Answer:
1. T_{Lag} will increase



Putting in a core increases the inductor's inductance and hence decreases the natural frequency of the circuit. Lower frequency means longer period. The phase will remain at 90º (a quarter period) so T_{Lag} will increase.

P31-30

✗

current lagging charge
Q ← dominates
I
L → 0

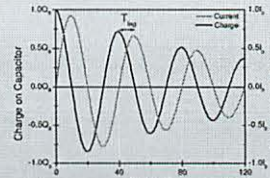
core ↑ L
so what happens to I

well T_{Lag} → stays same

↑L ↓ω
$\omega = \frac{1}{\sqrt{LC}}$
so ↓ω = ↑T
remember this

PRS: LC Circuit

If you increase the resistance in the circuit what will happen to rate of decay of the pictured amplitudes?

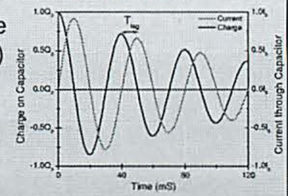


0% 1. It will increase (decay more rapidly)
0% 2. It will decrease (decay less rapidly)
0% 3. It will stay the same
0% 4. I don't know

:00

PRS Answer: LC Circuit

Answer: 1. It will increase (decay more rapidly)



Resistance is what dissipates power in the circuit and causes the amplitude of oscillations to decrease. Increasing the resistance makes the energy (and hence amplitude) decay more rapidly.

Pratice test

Physics 8.02 **Exam Three** **Spring 2009**

last year

Please Remove this Tear Sheet from Your Exam

Some (possibly useful) Relations:

$$d\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_o}\frac{dq}{r^2}\hat{\mathbf{r}}$$

$$\oiint_{\text{closed surface}} \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \frac{Q_{\text{inside}}}{\varepsilon_o}$$

$d\vec{\mathbf{A}}$ points from inside to outside

$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}} = -\frac{d}{dt}\iint \vec{\mathbf{B}}\cdot d\vec{\mathbf{A}} \qquad \varepsilon = -\frac{d\Phi_{\text{total}}}{dt}$$

$$\Delta V_{\text{moving from } a \text{ to } b} = V_b - V_a = -\int_a^b \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}}$$

$$\vec{\mathbf{B}} = \frac{\mu_o}{4\pi}\frac{q\,\vec{\mathbf{v}}\times\hat{\mathbf{r}}}{r^2} \quad |\vec{\mathbf{v}}| << c \qquad d\vec{\mathbf{B}} = \frac{\mu_o I}{4\pi}\frac{d\vec{\mathbf{s}}\times\hat{\mathbf{r}}}{r^2}$$

where $\hat{\mathbf{r}}$ points **from** source **to** observer

$$\oiint_{\substack{closed\\surface}} \vec{\mathbf{B}}\cdot d\vec{\mathbf{A}} = 0$$

$$\oint_{\text{contour}} \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = \mu_o\left(I_{\text{through}} + \varepsilon_0\frac{d\Phi_E}{dt}\right)$$

where I_{through} is the current flowing through any open surface bounded by the contour:

$$I_{\text{through}} = \iint_{\text{open surface}} \vec{\mathbf{J}}\cdot d\vec{\mathbf{A}}$$

ds is right-handed with respect to dA

$$u_E = \frac{1}{2}\varepsilon_0 E^2 \qquad u_B = \frac{B^2}{2\mu_o}$$

$$\vec{\mathbf{F}} = q\left(\vec{\mathbf{E}} + \vec{\mathbf{v}}\times\vec{\mathbf{B}}_{ext}\right) \qquad d\vec{\mathbf{F}} = I\,d\vec{\mathbf{s}}\times\vec{\mathbf{B}}_{ext}$$

$$F_{\text{cent.}} = mv^2/r$$

$$\vec{\boldsymbol{\mu}} = IA\,\hat{\mathbf{n}}$$

$$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}}\times\vec{\mathbf{B}} \qquad U = -\vec{\boldsymbol{\mu}}\cdot\vec{\mathbf{B}}$$

$$\Delta V = IR \qquad R = \frac{\rho L}{A}$$

$$P_{\text{ohmic heating}} = I\Delta V = I^2 R = \frac{\Delta V^2}{R}$$

$$C = \frac{Q}{\Delta V} \qquad U = \frac{1}{2}C\Delta V^2 = Q^2/2C$$

$$L = \frac{N\Phi_{\text{B,self,sgl coil}}}{I} \qquad \varepsilon_{\text{back}} = -L\frac{dI}{dt}$$

$$U_L = \tfrac{1}{2}LI^2$$

$$\omega = 2\pi f = 2\pi/T \qquad k = 2\pi/\lambda$$

$$c = \lambda/T = \lambda f = \omega/k = (\mu_0\varepsilon_0)^{-1/2}$$

$$E_0 = v_{\text{light}}B_0 \qquad \hat{\mathbf{E}}\times\hat{\mathbf{B}} = \hat{\mathbf{p}}$$

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}}\times\vec{\mathbf{B}} \qquad P_{\text{absorb}} = \frac{S}{c};\; P_{\text{reflect}} = \frac{2S}{c}$$

Interference

2 slit interference: $d\sin\theta = m\lambda$ Constructive

1 slit diffraction: $a\sin\theta = m\lambda$ Destructive

Far field: $\sin\theta \approx y/L$

Cross-products of unit vectors:

$$\hat{\mathbf{i}}\times\hat{\mathbf{i}} = \hat{\mathbf{j}}\times\hat{\mathbf{j}} = \hat{\mathbf{k}}\times\hat{\mathbf{k}} = 0$$

$$\hat{\mathbf{i}}\times\hat{\mathbf{j}} = \hat{\mathbf{k}} \quad \hat{\mathbf{j}}\times\hat{\mathbf{k}} = \hat{\mathbf{i}} \quad \hat{\mathbf{k}}\times\hat{\mathbf{i}} = \hat{\mathbf{j}}$$ ✓ oh cool

Some potentially useful numbers

$$k_e = \frac{1}{4\pi\varepsilon_0} = 9\times10^9\,\frac{\text{N m}^2}{\text{C}^2} \qquad \mu_o = 4\pi\times10^{-7}\,\frac{\text{T m}}{\text{A}}$$

Breakdown of air	E ~ 3 x 10^6 V/m
Earth's B Field	B ~ 5 x 10^{-5} T = 0.5 Gauss
Speed of light	c = 3 x 10^8 m/s
Light (violet to red)	λ = 400 nm to 700 nm
Electron charge	e = 1.6 x 10^{-19} C
Avogadro's number	N_A = 6.02 x 10^{23} mol^{-1}
Calories	1 cal = 10^{-3} Cal = 4.184 J

this year differential eq

4/28
Practice

8.02 Exam Three Spring 2009

FAMILY (last) NAME

GIVEN (first) NAME

Student ID Number

Your Section:

___L01 MW 9 am **____L02 MW 11 am** **____L03 MW 1 pm** **____L04 MW 3 pm**

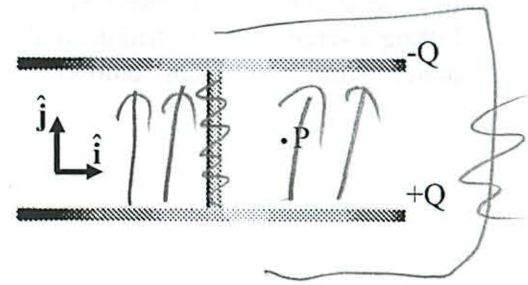
___L05 TR 9 am **____L06 TR 11 am** **____L07 TR 1 pm** **____L08 TR 3 pm**

Your Group (e.g. 9C): ______________

Problem	**Score**	**Grader**
1 (20 points)		
2 (5 points)		
3 (25 points)		
4A (15 points)		
4B (15 points)		
4C (20 points)		
TOTAL		

Problem 1: Five Short Questions. Circle your choice for the correct answer.

Question A (4 points out of 20 points):
Consider a circular parallel plate capacitor, initially charged to ±Q, then discharged through a resistor connecting the centers of the two plates, as pictured at right. A standard analytic problem is to calculate the E & B fields and the Poynting vector at point P. Which of the following statements is true?

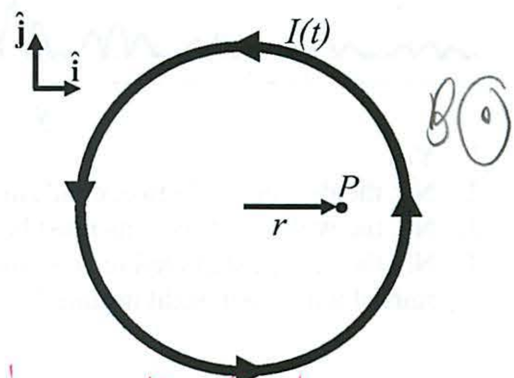


not on test

1) The Poynting vector at P points to the left $(-\hat{\mathbf{i}}$ direction$)$ not covered
2) The Electric Field at P, calculated by Faraday's law, points up $(+\hat{\mathbf{j}}$ direction$)$
3) Using Ampere's Law to calculate the B field at P, you would find that both the displacement current and physical current lead to B fields out of the page, so you need to sum their effects
4) None of the above

no B field

Question B (4 points out of 20 points):
Consider a standard solenoid, driven by a current supply to have a linearly decreasing counterclockwise current $I(t)$ when viewed from above, as pictured at right. A standard analytic problem is to calculate the E & B fields and the Poynting vector at point P. Which of the following statements is true?



omit

1) The Poynting vector at P points to the left $(-\hat{\mathbf{i}}$ direction$)$
2) The Electric Field at P, calculated by Faraday's law, points up $(+\hat{\mathbf{j}}$ direction$)$
3) Using Ampere's Law to calculate the B field at P, you would find that both the displacement current and physical current lead to B fields out of the page, so you need to sum their effects
4) None of the above

oh there is a E field

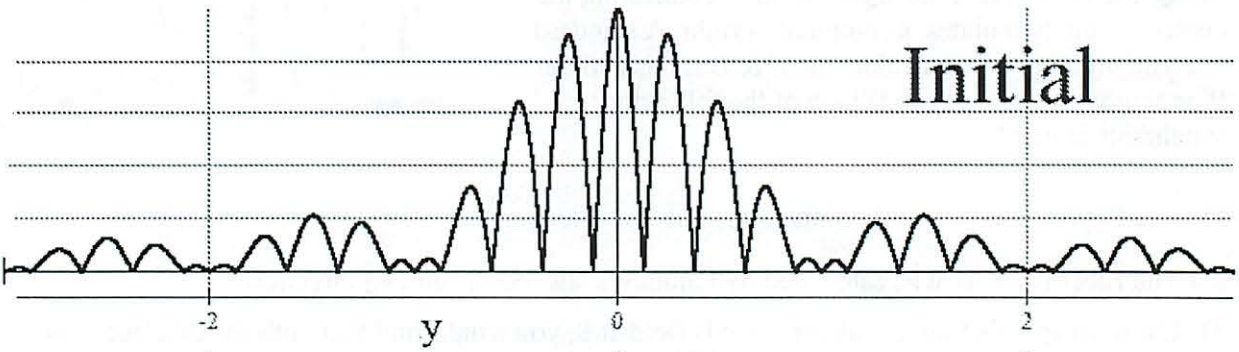
could be

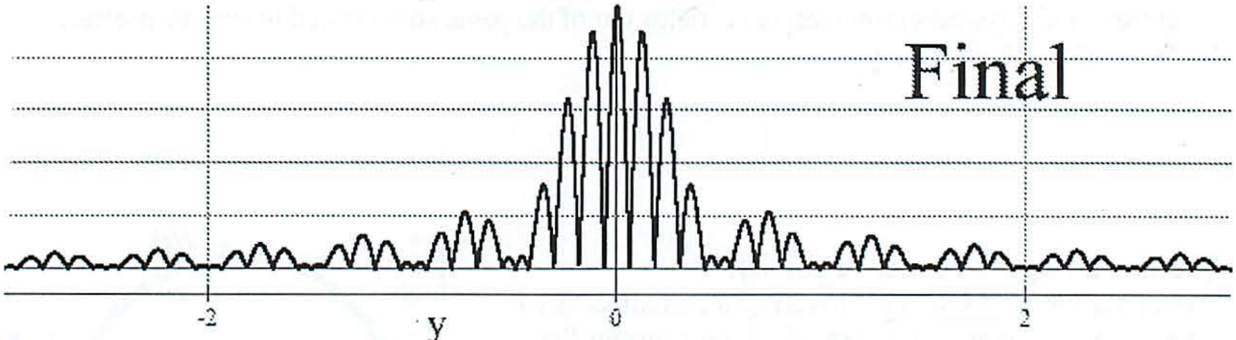
induced current

$\mathcal{E} = \int E \cdot dl = -\frac{d\Phi}{dt}$

Faraday's law

Question C (4 points out of 20 points):
In experiment six you observed an "Initial" intensity pattern for light coming from two slits and hitting a screen. If you had used a green laser rather than a red one, would you have seen a pattern similar to "Final" below?





1. Yes
2. No, the distance d between the slits must have changed in going from Initial to Final
3. No, the width a of the slits must have changed in going from Initial to Final
4. No, the change depicted results from a change in wavelength the other direction (as if we had started with green light in Initial and moved to red light in Final)

Question D (4 points out of 20 points):

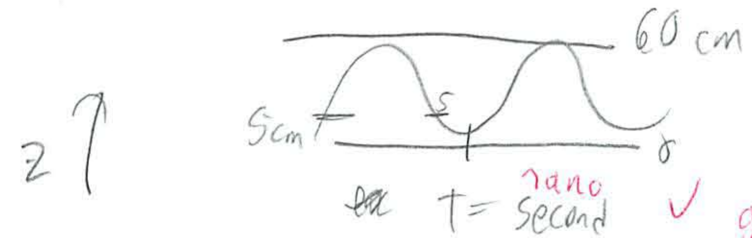
Take another look at the "Initial" intensity pattern above (in Question C). What can be deduced about the ratio of the distance between slits d to the width of each slit a?

1. $d/a = 8$
2. $d/a = 6$
3. $d/a = 5$
4. $d/a = 4$
5. $d/a = 3$
6. d/a can not be determined from the intensity pattern alone

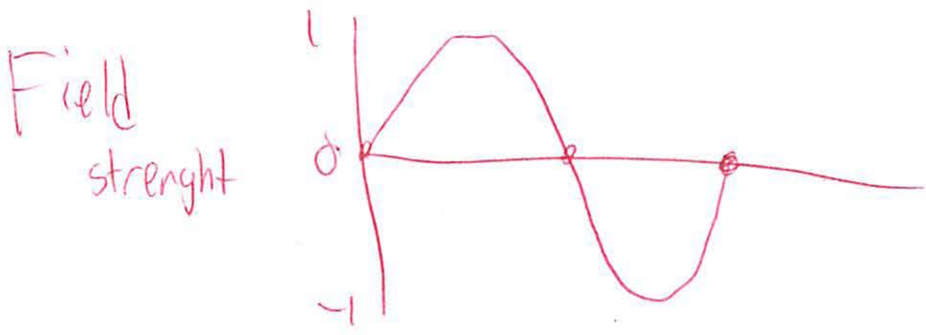
Question E (4 points out of 20 points):
Two perfectly conducting sheets are placed in vacuum parallel to the xy plane at z = 0 and z = 60 cm. A plane electromagnetic wave is generated in the region between the sheets, and continual reflection off of the sheets sets up a standing wave. It is found that a proton held between the sheets on the z-axis at z = 5 cm feels a reversal of force every half a nanosecond. That is, the force will be in the $+\hat{\mathbf{i}}$ direction for half a nanosecond, then in the $-\hat{\mathbf{i}}$ direction for half a nanosecond, and then in the $+\hat{\mathbf{i}}$ direction again, and so forth.

How many places on the z-axis can the proton be placed in between (not touching) the two sheets so that it would at no time feel a force?

1) 9
2) 8
3) 7
4) 6
5) 5
6) 4
7) 3
8) 2
9) 1
10) 0
11) More than 9
12) There is not enough information given to determine the right answer



✓ got it



* did not know was on z axis

~this was awful do other section |

Problem 2: Back of the Envelope Calculation (5 Pts)
As always, you are not given enough information to exactly determine the answer to this question. Make your best estimates for unknowns, clearly indicating what your estimates are (e.g. Radius R ~ ….) NO CREDIT will be given for simply guessing a final numerical answer from scratch. It must be properly motivated (i.e. write equations!)

The average power density of sunlight hitting the earth is about 1 kW/m^2. About how much force would it take for you to hold an electron at rest in light of the sun's rays?

not coverage I think

I guess you could have done it

Problem 3: Electromagnetic Plane Wave (25 pts)

A perfect conductor fills the xy plane (at z = 0 m). An electromagnetic plane wave traveling normal to the conductor approaches it in vacuum from above (z > 0). It has a wavelength of 300 nm. At time t = 0 the wave just reaches the conductor, and the magnetic field happens to be a maximum at the conductor's surface, pointing purely in the positive x direction. The power density carried in the wave at the surface of the conductor at this time is 120π W/m^2.

a) Identify a variable name (using the conventional labels is good!) and numerical value for each of the following quantities (feel free to leave factors of π in your answers):

coverage?

~~[illegible]~~

– guess

Could do but not a focus

Frequency = ____ = ______________________

Angular Frequency = ____ = ______________________

Velocity = ____ = ______________________

Wave number = ____ = ______________________

E Field Amplitude = ____ = ______________________

B Field Amplitude = ____ = ______________________

Also tell us the Direction of Propagation = ______________________

Problem 3: Electromagnetic Plane Wave ***continued…***

In answering the following **use the variable names rather than the numeric quantities** you arrived at in part (a).

b) Write an equation for the position and time dependent incoming Magnetic Field

c) Write an equation for the position and time dependent incoming Electric Field

d) Now the light source is very slightly (negligibly) tilted, so that the light is no longer hitting the surface perfectly normal. The surface is also split in two, with the half in the $x > 0$ part of the plane pushed downwards (so that it sits at $z < 0$ while the $x < 0$ part remains at $z = 0$). How far down will the $x > 0$ half of the conductor need to be pushed before an observer looking down at the reflected light would first see an interference minimum (destructive interference between the light bouncing off the part at $z = 0$ and that off the part at $z < 0$)? (HINT: We want a numerical answer here – we gave you the numbers you'll need above)

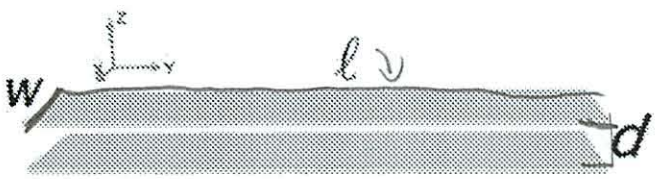
This test is like very different

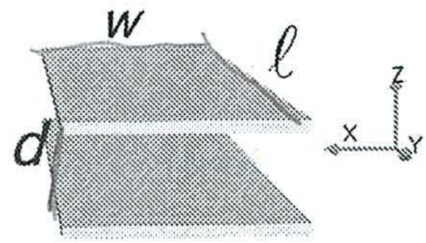
half the test

Problem 4: Transmission Line (50 pts)

The rest of this exam is an extended question dealing with transmission lines. There are a variety of transmission lines used in the world. A simple example is two wires running next to each other with current flowing one direction in one and the opposite in the other. Another example that you considered in the sample exam of problem set 11 was the coaxial cable, where current flowed up the inside wire and back along the outer shield.

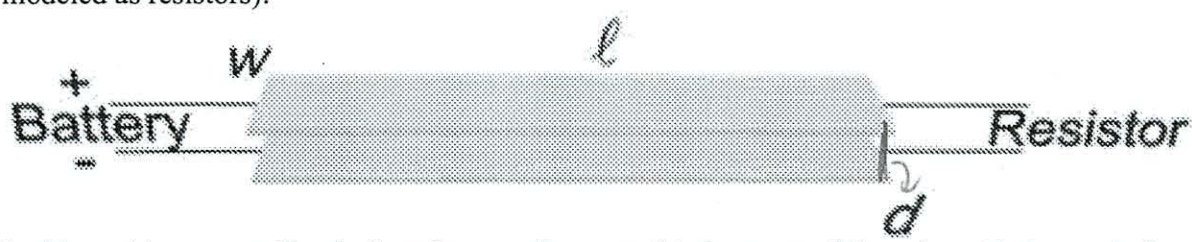
In this problem you will calculate the properties of a microstrip transmission line. It consists of two thin parallel plates of width w and length ℓ, separated by a small distance d (they are typically held apart by a dielectric, but to make your life simple let's just pretend there is air between the plates). It is shown both in side view and front view below.





The dimensions are such that **you should assume that any fields created by the transmission line are confined to the region between the two plates.**

We use transmission lines to carry power from batteries or power supplies to loads (typically modeled as resistors):



In this problem you will calculate the capacitance and inductance of the microstrip transmission line and then study energy flow at DC.

NOTE: PLEASE READ THIS CAREFULLY

In several parts of this problem you will be asked to calculate something that will require the use of one of Maxwell's equations. Make sure that you state the name of the equation and the write it in the form that you plan to use it before you do that part. Be explicit in the calculations and draw and label anything that you need to use. I will not provide any further drawings. Please duplicate drawings from this page (simplified to remove the perspective of course) when you think they will be useful.

Do not forget to give both magnitude and direction of vector quantities.

Feel free to tear out this page so that you do not have to continually turn back to it.

no q's on this page.

Problem 4A: Capacitance of the Microstrip Transmission Line *continued*

Voltage b/w $\Delta V = Ed = \frac{Qd}{wl\epsilon_0}$

Capitance $C = \frac{Q}{\Delta V} = \frac{wl\epsilon}{d}$

Review Grass's Law

- flux through a surface

$\phi = \vec{E} \cdot \vec{A}$

$= EA\cos\theta$

pill box

- EA that counts

$q_{inc} = \sigma A$

constant away from shape

Problem 4A: Capacitance of the Microstrip Transmission Line (15 points)

In the first two parts of this problem (A and B) we will consider the transmission line in isolation (no battery or load resistor).

Calculate the capacitance of the transmission line.

this is what I need help in

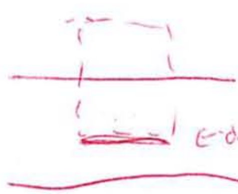
did not really cover

Calc $\vec{E}$ field between the plates

Guass's Law $\oint\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$

?not on test but should be able to do

- yeah from past exams - likely not be on



Pill box

- end cap area A

$E A = \frac{Q_{enc}}{\epsilon_0}$

$Q_{ence} = Q \frac{A}{wl} = 1 ? \quad wl = 1$

?where in did they get this

$E = \frac{Q}{\epsilon_0 wl}$

+ Maxwells eq tie together

Problem 4B: Inductance of the Microstrip Transmission Line (15 points)

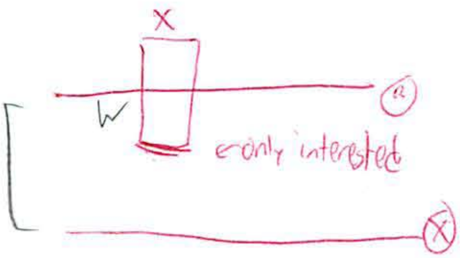
Calculate the inductance of the transmission line.

L — how calculate this again

Calculate B

then what?

know from p-set



like solenoid

- but free space here

guessing

Amperien Loop

$\oint B \cdot ds = \mu_0 I_{enc}$

$Bx = \mu_0 \frac{x}{w} I$

only what interested in

$B = \frac{\mu_0 I}{w}$

oh w/ energy method!

$U_B = \frac{B^2}{2\mu_0} wld$

↑ energy density · volume — uniform mag field — otherwise ∫∫

$= \left(\frac{\mu I}{w}\right)^2 \frac{wld}{2\mu_0}$

$= \frac{1}{2} \boxed{\frac{\mu_0 ld}{w}} I^2$

$= \frac{1}{2} L I^2$

or $L = \frac{\Phi}{I}$

$= \frac{Bld}{I}$

$= \frac{\mu_0 I}{w} \frac{ld}{I}$

$= \frac{\mu_0 ld}{w}$

↑ much more familiar

~~have to know I here~~

Problem 4B: Inductance of the Microstrip Transmission Line *continued*

Problem 4C: DC Power Transmission with the Microstrip Transmission Line (20 points)

We now connect the transmission line to a battery (EMF ε) on the left and a resistor (resistance R) on the right, as pictured at the beginning of this problem. We are interested in what happens a long time after this connection has been made (after any transient behavior has passed). Make sure that your answers below only involve the variables we have provided

LC circuits

(a) What is the electric field between the plates? HINT: This is much easier than you probably think now that the battery fixes the potential difference between the plates.

$\varepsilon = \oint \vec{E} + \vec{v} \times \vec{B} \, dr$

had

and it is the differential - don't have to take

$E = \frac{d\varepsilon}{dt} = \frac{-\varepsilon}{d}$ ← over area, not time

what is eletric field

$\Delta V = -\int_A^B E \cdot ds$ just did not do it right

$V = -Ed \rightarrow E = -\frac{V}{d}$ duh

(b) What is the magnetic field between the plates? HINT: You probably already did this, at least in part, in 4B. Feel free to make use of your previous result, but make sure that it isn't clearly wrong (e.g. wrong units). Clearly wrong answers will be penalized both there and here.

$B = \oint B \cdot dr = \mu_0 I_{enc}$

– or ~~maxwell $\oint E \cdot dr = -\frac{d}{dt} \iint$~~

well previus ans

$B = -\frac{\mu_0}{W} I = -\frac{\mu_0}{W} \frac{\varepsilon}{R}$

? found lastline ↑ How know I

well kirk off

Problem 4C: DC Power Transmission with the Microstrip Transmission Line *continued*

(c) What is the Poynting vector between the plates? Does the direction make sense?

Skip

(d) Integrate the Poynting vector over a relevant area (be clear what this is and explain why) and show that the result simplifies to what you would expect given the meaning of the Poynting vector. If you were unable to obtain an expression for the Poynting vector, you should still answer this question qualitatively – explain what the relevant area is and what result you expect.

Skip

So that pratice test did not go so well

Physics 8.02 **Exam Three Solutions** **Spring 2009**

Please Remove this Tear Sheet from Your Exam

Some (possibly useful) Relations:

$$d\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_o}\frac{dq}{r^2}\hat{\mathbf{r}}$$

$$\underset{\text{closed surface}}{\oiint} \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \frac{Q_{\text{inside}}}{\varepsilon_o}$$

$d\vec{\mathbf{A}}$ points from inside to outside

$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}} = -\frac{d}{dt}\iint \vec{\mathbf{B}}\cdot d\vec{\mathbf{A}} \qquad \varepsilon = -\frac{d\Phi_{\text{total}}}{dt}$$

$$\Delta V_{\text{moving from } a \text{ to } b} = V_b - V_a = -\int_a^b \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}}$$

$$\vec{\mathbf{B}} = \frac{\mu_o}{4\pi}\frac{q\,\vec{\mathbf{v}}\times\hat{\mathbf{r}}}{r^2} \quad |\vec{\mathbf{v}}| << c \qquad d\vec{\mathbf{B}} = \frac{\mu_o I}{4\pi}\frac{d\vec{\mathbf{s}}\times\hat{\mathbf{r}}}{r^2}$$

where $\hat{\mathbf{r}}$ points **from** source **to** observer

$$\underset{\substack{closed\\surface}}{\oiint} \vec{\mathbf{B}}\cdot d\vec{\mathbf{A}} = 0$$

$$\underset{\text{contour}}{\oint} \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = \mu_o\left(I_{\text{through}} + \varepsilon_0\frac{d\Phi_E}{dt}\right)$$

where I_{through} is the current flowing through any open surface bounded by the contour:

$$I_{\text{through}} = \underset{\text{open surface}}{\iint} \vec{\mathbf{J}}\cdot d\vec{\mathbf{A}}$$

ds is right-handed with respect to dA

$$u_E = \frac{1}{2}\varepsilon_0 E^2 \qquad u_B = \frac{B^2}{2\mu_o}$$

$$\vec{\mathbf{F}} = q\left(\vec{\mathbf{E}} + \vec{\mathbf{v}}\times\vec{\mathbf{B}}_{ext}\right) \qquad d\vec{\mathbf{F}} = I\,d\vec{\mathbf{s}}\times\vec{\mathbf{B}}_{ext}$$

$$F_{\text{cent.}} = mv^2/r$$

$$\vec{\boldsymbol{\mu}} = IA\,\hat{\mathbf{n}}$$

$$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}}\times\vec{\mathbf{B}} \qquad U = -\vec{\boldsymbol{\mu}}\cdot\vec{\mathbf{B}}$$

$$\Delta V = IR \qquad R = \frac{\rho L}{A}$$

$$P_{\text{ohmic heating}} = I\Delta V = I^2 R = \frac{\Delta V^2}{R}$$

$$C = \frac{Q}{\Delta V} \qquad U = \frac{1}{2}C\Delta V^2 = Q^2/2C$$

$$L = \frac{N\Phi_{\text{B,self,sgl coil}}}{I} \qquad \varepsilon_{\text{back}} = -L\frac{dI}{dt}$$

$$U_L = \tfrac{1}{2}LI^2$$

$$\omega = 2\pi f = 2\pi/T \qquad k = 2\pi/\lambda$$

$$c = \lambda/T = \lambda f = \omega/k = (\mu_0\varepsilon_0)^{-1/2}$$

$$E_0 = v_{\text{light}}B_0 \qquad \hat{\mathbf{E}}\times\hat{\mathbf{B}} = \hat{\mathbf{p}}$$

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}}\times\vec{\mathbf{B}} \qquad P_{\text{absorb}} = \frac{S}{c};\ P_{\text{reflect}} = \frac{2S}{c}$$

Interference

2 slit interference: $d\sin\theta = m\lambda$ Constructive

1 slit diffraction: $a\sin\theta = m\lambda$ Destructive

Far field: $\sin\theta \approx y/L$

Cross-products of unit vectors:

$$\hat{\mathbf{i}}\times\hat{\mathbf{i}} = \hat{\mathbf{j}}\times\hat{\mathbf{j}} = \hat{\mathbf{k}}\times\hat{\mathbf{k}} = 0$$

$$\hat{\mathbf{i}}\times\hat{\mathbf{j}} = \hat{\mathbf{k}} \quad \hat{\mathbf{j}}\times\hat{\mathbf{k}} = \hat{\mathbf{i}} \quad \hat{\mathbf{k}}\times\hat{\mathbf{i}} = \hat{\mathbf{j}}$$

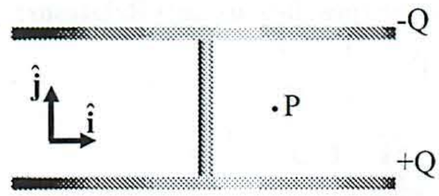
Some potentially useful numbers

$$k_e = \frac{1}{4\pi\varepsilon_0} = 9\times10^9\,\frac{\text{N m}^2}{\text{C}^2} \qquad \mu_o = 4\pi\times10^{-7}\,\frac{\text{T m}}{\text{A}}$$

Breakdown of air	$E \sim 3 \times 10^6$ V/m
Earth's B Field	$B \sim 5 \times 10^{-5}$ T = 0.5 Gauss
Speed of light	$c = 3 \times 10^8$ m/s
Light (violet to red)	λ = 400 nm to 700 nm
Electron charge	$e = 1.6 \times 10^{-19}$ C
Avogadro's number	$N_A = 6.02 \times 10^{23}$ mol^{-1}
Calories	1 cal = 10^{-3} Cal = 4.184 J

Problem 1: Five Short Questions. Circle your choice for the correct answer.

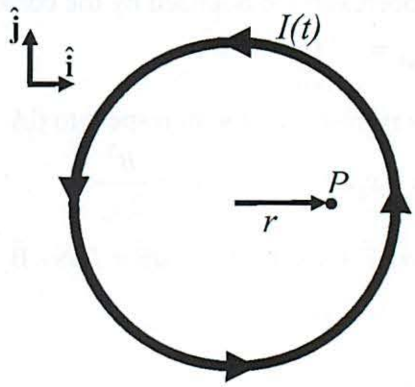
Question A (4 points out of 20 points):
Consider a circular parallel plate capacitor, initially charged to ±Q, then discharged through a resistor connecting the centers of the two plates, as pictured at right. A standard analytic problem is to calculate the E & B fields and the Poynting vector at point P. Which of the following statements is true?



1) The Poynting vector at P points to the left $\left(-\hat{i}\text{ direction}\right)$
2) The Electric Field at P, calculated by Faraday's law, points up $\left(+\hat{j}\text{ direction}\right)$
3) Using Ampere's Law to calculate the B field at P, you would find that both the displacement current and physical current lead to B fields out of the page, so you need to sum their effects
4) None of the above

The electric field at P is up, but by Gauss's Law, not Faraday's (there is no changing magnetic flux to generate an E field in that way). Current flows up the wire from the +Q to the –Q charge (discharging). This generates a magnetic field into the page at point P. The displacement current, being negative, points opposite the direction of the E field (down), meaning it generates a B field in the opposite direction (out of the page). But an Amperian loop through P won't contain as much displacement current as physical current, so the net field is into the page, meaning the Poynting vector is to the left.

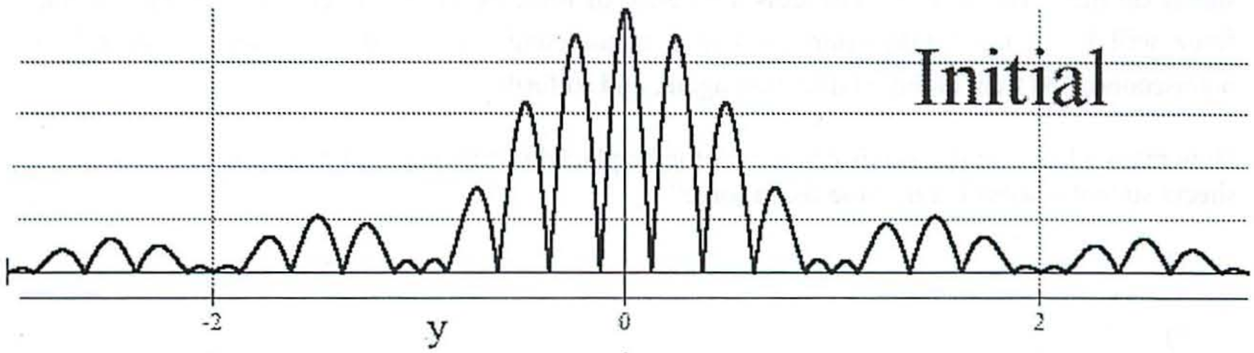
Question B (4 points out of 20 points):
Consider a standard solenoid, driven by a current supply to have a linearly decreasing counterclockwise current $I(t)$ when viewed from above, as pictured at right. A standard analytic problem is to calculate the E & B fields and the Poynting vector at point P. Which of the following statements is true?

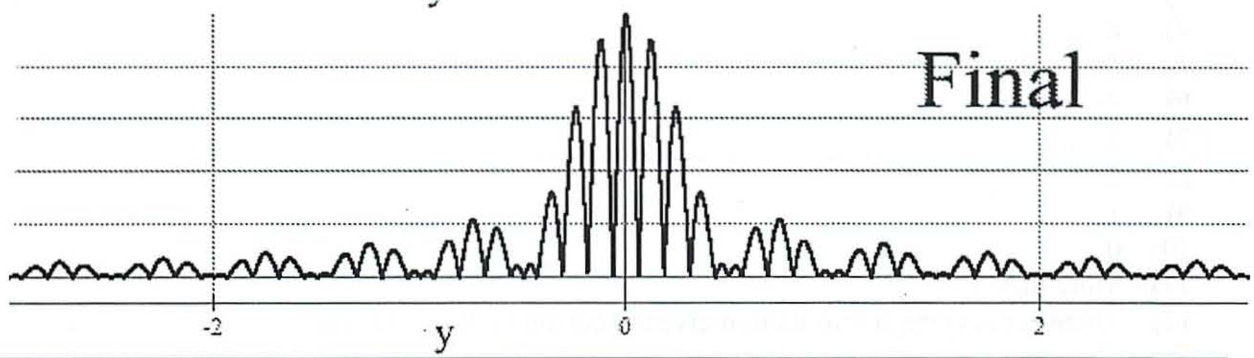


1) The Poynting vector at P points to the left $\left(-\hat{i}\text{ direction}\right)$
2) The Electric Field at P, calculated by Faraday's law, points up $\left(+\hat{j}\text{ direction}\right)$
3) Using Ampere's Law to calculate the B field at P, you would find that both the displacement current and physical current lead to B fields out of the page, so you need to sum their effects
4) None of the above

By Ampere's law the B field is out of the page. There is no displacement current here (no changing E field). Since B is decreasing, Faraday's law says there is a CCW E field (same direction as the current), so pointing up at P). The Poynting vector is to the right (energy is leaving the solenoid).

Question C (4 points out of 20 points):
In experiment six you observed an "Initial" intensity pattern for light coming from two slits and hitting a screen. If you had used a green laser rather than a red one, would you have seen a pattern similar to "Final" below?





1. Yes
2. No, the distance d between the slits must have changed in going from Initial to Final
3. No, the width a of the slits must have changed in going from Initial to Final
4. No, the change depicted results from a change in wavelength the other direction (as if we had started with green light in Initial and moved to red light in Final)

The angles of both the interference maxima and diffraction minima both shrink by the same amount, meaning the wavelength shrank.

Question D (4 points out of 20 points):

Take another look at the "Initial" intensity pattern above (in Question C). What can be deduced about the ratio of the distance between slits d to the width of each slit a?

1. $d/a = 8$
2. $d/a = 6$
3. $d/a = 5$
4. $d/a = 4$
5. $d/a = 3$
6. d/a can not be determined from the intensity pattern alone

Note that the m=4 interference maximum is "missing" because it lies at the same location as the n=1 diffraction minimum. So equate those two angles and you get the ratio d/a = 4.

Question E (4 points out of 20 points):
Two perfectly conducting sheets are placed in vacuum parallel to the xy plane at z = 0 and z = 60 cm. A plane electromagnetic wave is generated in the region between the sheets, and continual reflection off of the sheets sets up a standing wave. It is found that a proton held between the sheets on the z-axis at z = 5 cm feels a reversal of force every half a nanosecond. That is, the force will be in the $+\hat{\mathbf{i}}$ direction for half a nanosecond, then in the $-\hat{\mathbf{i}}$ direction for half a nanosecond, and then in the $+\hat{\mathbf{i}}$ direction again, and so forth.

How many places on the z-axis can the proton be placed in between (not touching) the two sheets so that it would at no time feel a force?

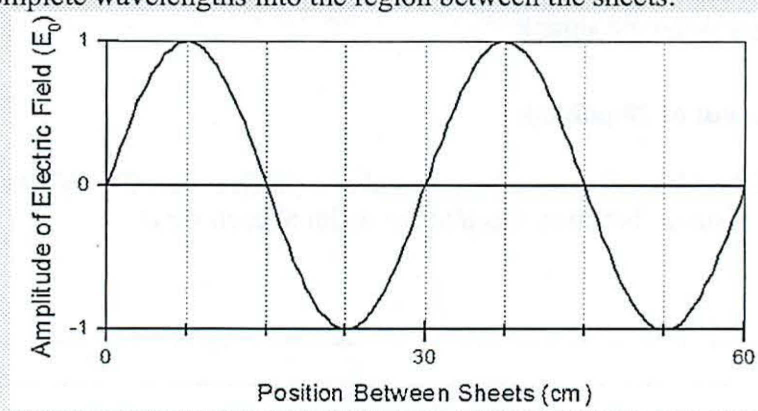
1) 9
2) 8
3) 7
4) 6
5) 5
6) 4
7) 3
8) 2
9) 1
10) 0
11) More than 9
12) There is not enough information given to determine the right answer

That the direction of force changes every half a nanosecond tells us that the period is a nanosecond. From that we can get the wavelength:

$$\lambda f = \lambda / T = c$$

$$\lambda = cT = \left(3\times10^{8}\text{ m s}^{-1}\right)\left(10^{-9}\text{s}\right) = 0.3\text{ m} = 30\text{ cm}$$

So we get two complete wavelengths into the region between the sheets:



Note that there are 3 nodes (where the field strength is always zero) between the two sheets!

Problem 2: Back of the Envelope Calculation (5 Pts)
As always, you are not given enough information to exactly determine the answer to this question. Make your best estimates for unknowns, clearly indicating what your estimates are (e.g. Radius R ~ ….) NO CREDIT will be given for simply guessing a final numerical answer from scratch. It must be properly motivated (i.e. write equations!)

The average power density of sunlight hitting the earth is about 1 kW/m^2. About how much force would it take for you to hold an electron at rest in light of the sun's rays?

The "power density" is the magnitude of the Poynting vector, from which we can get the magnitude of the Electric field:

$$\langle \vec{\mathbf{S}} \rangle = \left\langle \frac{\vec{\mathbf{E}} \times \vec{\mathbf{B}}}{\mu_0} \right\rangle = \frac{1}{2\mu_0} E_0 B_0 = \frac{E_0^2}{2\mu_0 c}$$

$$E_0 = \sqrt{2\mu_0 c \langle \vec{\mathbf{S}} \rangle} \approx \sqrt{2\left(4\pi \times 10^{-7} \frac{\mathrm{T\,m}}{\mathrm{A}}\right)\left(3\times 10^8 \mathrm{\,m\,s^{-1}}\right)\left(1\,\frac{\mathrm{kW}}{\mathrm{m}^2}\right)} = \sqrt{\left(24\pi \times 10^4 \frac{\mathrm{T}}{\mathrm{A}}\right)\left(\frac{1}{\mathrm{s}}\right)\left(\frac{\mathrm{N \cdot m}}{\mathrm{s}}\right)} \approx 850 \frac{\mathrm{N}}{\mathrm{C}}$$

To do the math, 24π ~ 75, which is between 64 and 81, so I assume that the square root is halfway between as well (not bad, it is really closer to 8.7). To get the units, I always get rid of Tesla using F = qvB. So the T turns into another N, and eliminates the charge (A*s) and the velocity (m/s). The factor of ½ came from the time average – in this kind of a problem it is fine to drop that.

Anyway, the question was what force we'd apply. Of course the force is sinusoidal, so if you were clever you could say the average force was zero and be done with it. But I was really asking about the magnitude of the sinusoidal force:

$$F = qE_0 \approx 1.6\times 10^{-19}\mathrm{C} \cdot 850 \frac{\mathrm{N}}{\mathrm{C}} \approx \boxed{1.4\times 10^{-16}\mathrm{N}}$$

This is a pretty darn small force, but that's good (otherwise our electrons would be flying all over the place every time we walked outside in the sun). If you wanted to see how small such a force is, you could compare it to the Coulomb force on an electron in, for example, a Hydrogen atom. There we have:

$$F = \frac{kq^2}{r^2} \approx \frac{\left(9\times 10^9 \mathrm{Nm^2C^{-2}}\right)\left(1.6\times 10^{-19}\mathrm{C}\right)^2}{\left(10^{-10}\mathrm{m}\right)^2} \approx 2.5\times 10^{-8}\mathrm{N}$$

This is eight orders of magnitude bigger – the force of the sunlight is just tiny.

Please note: If you tried to use radiation pressure on this problem it is just wrong. As many of you noted, you have no idea what the "area" of an electron is (we consider it to be a point particle, it certainly does not have a radius of a nm or larger as some of you seem to think – that is 10 times the size of an atom!). Also, as always, way too much math (too many equations) and not nearly enough units. If you write a physical quantity, make sure a unit is attached.

Problem 3: Electromagnetic Plane Wave (25 pts)

A perfect conductor fills the xy plane (at z = 0 m). An electromagnetic plane wave traveling normal to the conductor approaches it in vacuum from above (z > 0). It has a wavelength of 300 nm. At time t = 0 the wave just reaches the conductor, and the magnetic field happens to be a maximum at the conductor's surface, pointing purely in the positive x direction. The power density carried in the wave at the surface of the conductor at this time is 120π W/m^2.

a) Identify a variable name (using the conventional labels is good!) and numerical value for each of the following quantities (feel free to leave factors of π in your answers):

It is in vacuum, so the velocity is c. The wavelength λ is given, from which we can get the wave number $k = 2\pi/\lambda$. The frequency we get from $\lambda f = c$, from which we can get the angular frequency $\omega = 2\pi f$. Finally, the amplitudes we get from the power density (Poynting vector). Since the B field is a maximum at that point we will calculate it in terms of the B field amplitude: $S = \frac{1}{\mu_0} E_0 B_0 = \frac{cB_0^2}{\mu_0} \Rightarrow B_0 = \sqrt{\frac{\mu_0 S}{c}}$ and then E_0 is just c times bigger.

$$B_0 = \sqrt{\frac{\mu_0 S}{c}}$$

$$= \sqrt{\frac{\left(4\pi \times 10^{-7} \frac{\text{Tm}}{\text{A}}\right)\left(120\pi \frac{\text{W}}{\text{m}^2}\right)}{\left(3 \times 10^8 \text{ m s}^{-1}\right)}}$$

$$= \sqrt{\frac{\left(40\pi \frac{\text{T}\cdot\text{m}}{\text{A}}\right)\left(40\pi \frac{\text{N}\cdot\text{m}}{\text{s}\cdot\text{m}^2}\right)}{\left(\text{m s}^{-1}\right)}} \times 10^{-8}$$

$$= 40\pi \times 10^{-8}\ \text{T}$$

Frequency = f = 10^{15} Hz

Angular Frequency = ω = $2\pi \times 10^{15}$ s^{-1}

Velocity = c = 3×10^8 m/s

Wave number = k = π/150 nm^{-1}

E Field Amplitude = E_0 = 120π N/C

B Field Amplitude = B_0 = $40\pi \times 10^{-8}$ T

The direction of propagation we get from the fact that it is approaching z = 0 from above, so it is moving downward:

Also tell us the Direction of Propagation = $-\hat{k}$

Problem 3: Electromagnetic Plane Wave ***continued...***

In answering the following **use the variable names rather than the numeric quantities** you arrived at in part (a).

b) Write an equation for the position and time dependent incoming Magnetic Field

We get the direction ($\hat{\mathbf{i}}$) from the statement that at the surface of the conductor the magnetic field is pointing purely in the positive x direction. The amplitude we calculated above and labeled B_0 and the argument of the cos is such that it is a maximum at z=0, t=0 and that it is traveling in the –z direction.

$$\vec{\mathbf{B}} = \hat{\mathbf{i}} B_0 \cos(kz + \omega t)$$

c) Write an equation for the position and time dependent incoming Electric Field

We get the direction ($\hat{\mathbf{j}}$) from the fact that $\hat{\mathbf{E}} \times \hat{\mathbf{B}} = \hat{\mathbf{j}} \times \hat{\mathbf{i}} = \hat{\mathbf{S}} = -\hat{\mathbf{k}}$. The amplitude we calculated above and labeled E_0 and the rest is the same as above:

$$\vec{\mathbf{E}} = \hat{\mathbf{j}} E_0 \cos(kz + \omega t)$$

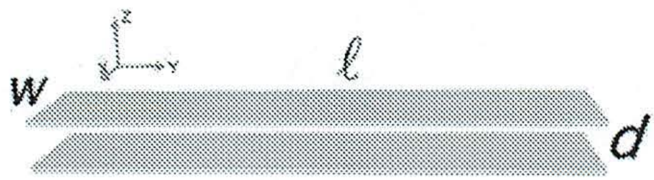
d) Now the light source is very slightly (negligibly) tilted, so that the light is no longer hitting the surface perfectly normal. The surface is also split in two, with the half in the $x > 0$ part of the plane pushed downwards (so that it sits at $z < 0$ while the $x < 0$ part remains at $z = 0$). How far down will the $x > 0$ half of the conductor need to be pushed before an observer looking down at the reflected light would first see an interference minimum (destructive interference between the light bouncing off the part at $z = 0$ and that off the part at $z < 0$)? (HINT: We want a numerical answer here – we gave you the numbers you'll need above)

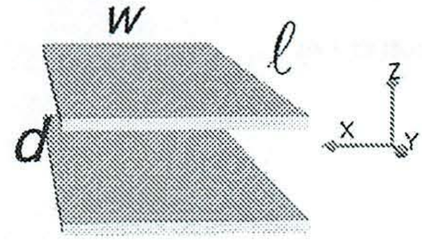
The extra path distance the light travels to bounce off the lower plate is twice the distance it is pushed down. This gives destructive interference when that distance is half a wavelength. So:

$$2d = \frac{\lambda}{2} \Rightarrow \boxed{d = \frac{\lambda}{4} = 75 \text{ nm}}$$

Problem 4: Transmission Line (50 pts)
The rest of this exam is an extended question dealing with transmission lines. There are a variety of transmission lines used in the world. A simple example is two wires running next to each other with current flowing one direction in one and the opposite in the other. Another example that you considered in the sample exam of problem set 11 was the coaxial cable, where current flowed up the inside wire and back along the outer shield.

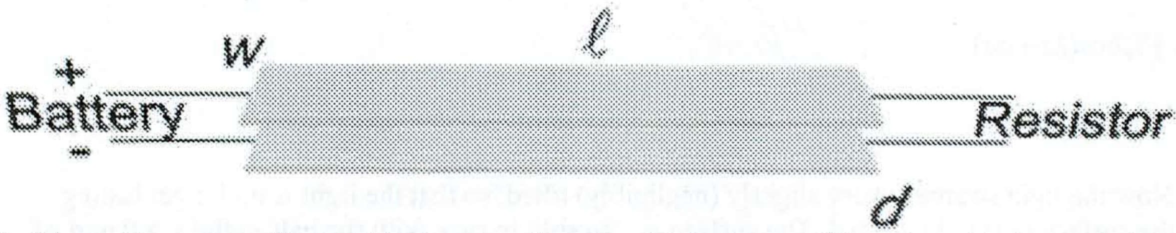
In this problem you will calculate the properties of a microstrip transmission line. It consists of two thin parallel plates of width w and length ℓ, separated by a small distance d (they are typically held apart by a dielectric, but to make your life simple let's just pretend there is air between the plates). It is shown both in side view and front view below.





The dimensions are such that **you should assume that any fields created by the transmission line are confined to the region between the two plates.**

We use transmission lines to carry power from batteries or power supplies to loads (typically modeled as resistors):



In this problem you will calculate the capacitance and inductance of the microstrip transmission line and then study energy flow at DC.

<u>NOTE: PLEASE READ THIS CAREFULLY</u>

In several parts of this problem you will be asked to calculate something that will require the use of one of Maxwell's equations. Make sure that you state the name of the equation and the write it in the form that you plan to use it before you do that part. Be explicit in the calculations and draw and label anything that you need to use. I will not provide any further drawings. Please duplicate drawings from this page (simplified to remove the perspective of course) when you think they will be useful.

Do not forget to give both magnitude and direction of vector quantities.

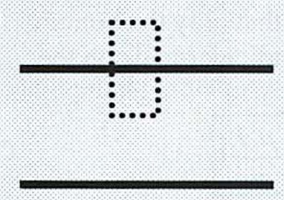
Feel free to tear out this page so that you do not have to continually turn back to it.

Problem 4A: Capacitance of the Microstrip Transmission Line (15 points)

In the first two parts of this problem (A and B) we will consider the transmission line in isolation (no battery or load resistor).

Calculate the capacitance of the transmission line.

STEP 1: Place ±Q on the plates and calculate the electric field between them



We have a charge +Q on the top plate so an electric field will be created pointing downwards. We will use Gauss's Law to calculate the electric field between the plates: $\oiint_{\text{Pillbox}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{\text{enc}}}{\varepsilon_o}$

We use a Gaussian pillbox with end cap area A. The only surface of the pillbox we care about is the one between the plates. The field runs perpendicular to the area vector on the sides (doesn't penetrate them) and the field is zero outside because the fields from the two plates cancel there.

$$\oiint_{\text{Pillbox}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = EA = \frac{Q_{\text{enc}}}{\varepsilon_o} = \frac{Q}{\varepsilon_o}\frac{A}{w\ell} \Rightarrow \boxed{\vec{\mathbf{E}} = -\frac{Q}{w\ell\varepsilon_o}\hat{\mathbf{k}}}$$

STEP 2: Calculate the voltage difference between them

$$\Delta V = Ed = \frac{Qd}{w\ell\varepsilon_o}$$

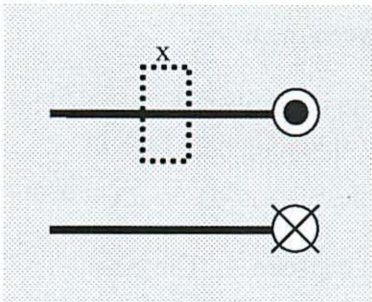
STEP 3: Calculate the capacitance

$$C = \frac{Q}{\Delta V} = \frac{w\ell\varepsilon_o}{d}$$

Problem 4B: Inductance of the Microstrip Transmission Line (15 points)

Calculate the inductance of the transmission line.

STEP 1: Place current $\pm I$ on the plates and calculate the magnetic field between them



We have a current I flowing out the top plate and in the bottom plate meaning that a magnetic field is created between the two plates pointing to the right ($-\hat{\mathbf{i}}$ direction). The field is zero outside by cancellation. We use Ampere's Law with the Amperian loop pictured at left and note that only the bottom leg contributes (B=0 at top and is perpendicular to ds on the sides):

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = Bx = \mu_o I_{enc} = \mu_o \frac{x}{w} I \Rightarrow \boxed{\vec{\mathbf{B}} = -\frac{\mu_o I}{w} \hat{\mathbf{i}}}$$

STEP 2: Calculate the inductance

We will use energy to calculate the inductance. The magnetic field is uniform so we can just multiply the energy density by the volume:

$$U_B = \frac{B^2}{2\mu_o} \cdot w\ell d = \left(\frac{\mu_o I}{w}\right)^2 \frac{w\ell d}{2\mu_o} = \frac{1}{2} \frac{\mu_o \ell d}{w} I^2 = \frac{1}{2} L I^2 \Rightarrow \boxed{L = \frac{\mu_o \ell d}{w}}$$

Alternatively you could have used flux, with the flux penetrating the area normal to the field direction (i.e. the area ℓd). So:

$$L = \frac{\Phi_B}{I} = \frac{B\ell d}{I} = \left(\frac{\mu_o I}{w}\right) \frac{\ell d}{I} = \boxed{\frac{\mu_o \ell d}{w}}$$

Problem 4C: DC Power Transmission with the Microstrip Transmission Line (20 points)

We now connect the transmission line to a battery (EMF ε) on the left and a resistor (resistance R) on the right, as pictured at the beginning of this problem. We are interested in what happens a long time after this connection has been made (after any transient behavior has passed). Make sure that your answers below only involve the variables we have provided

(a) What is the electric field between the plates? HINT: This is much easier than you probably think now that the battery fixes the potential difference between the plates.

$$\vec{\mathbf{E}} = -\frac{\varepsilon}{d}\hat{\mathbf{k}}$$

(b) What is the magnetic field between the plates? HINT: You probably already did this, at least in part, in 4B. Feel free to make use of your previous result, but make sure that it isn't clearly wrong (e.g. wrong units). Clearly wrong answers will be penalized both there and here.

$$\vec{\mathbf{B}} = -\frac{\mu_o}{w} I\hat{\mathbf{i}} = -\frac{\mu_o}{w}\frac{\varepsilon}{R}\hat{\mathbf{i}}$$

(c) What is the Poynting vector between the plates?

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}}\times\vec{\mathbf{B}} = \frac{1}{\mu_0}\left(-\frac{\varepsilon}{d}\hat{\mathbf{k}}\right)\times\left(-\frac{\mu_o}{w}\frac{\varepsilon}{R}\hat{\mathbf{i}}\right) = \frac{\varepsilon^2}{R}\frac{1}{wd}\hat{\mathbf{j}}$$

(d) Integrate the Poynting vector over a relevant area and show that the result simplifies to what you would expect given the meaning of the Poynting vector. If you were unable to obtain an expression for the Poynting vector, you should still answer this question qualitatively – explain what the relevant area is and what result you expect.

The relevant area is the cross-sectional area of the transmission line, *wd*. The Poynting vector is uniform so we can just multiply rather than integrate:

$$\iint \vec{\mathbf{S}}\cdot d\vec{\mathbf{A}} = SA = \frac{\varepsilon^2}{R} = \text{Power dissipated by the resistor}$$

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 **Spring 2008**

Exam Three: Equation Sheet

Force Laws: $\vec{\mathbf{F}} = q(\vec{\mathbf{E}}_{ext} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}_{ext})$

$d\vec{\mathbf{F}} = I\, d\vec{\mathbf{s}} \times \vec{\mathbf{B}}_{ext}$

Current:
$I = \iint \vec{\mathbf{J}} \cdot d\vec{\mathbf{a}}$

Source equations:

$$d\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_o} \frac{dq}{r^2} \hat{\mathbf{r}}$$

$$d\vec{\mathbf{B}} = \frac{\mu_o I}{4\pi} \frac{d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$

Maxwell's Equations:

$$\oiint \vec{\mathbf{E}} \cdot d\vec{\mathbf{a}} = \frac{Q_{inside}}{\varepsilon_o}$$

$$\oiint \vec{\mathbf{B}} \cdot d\vec{\mathbf{a}} = 0$$

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{r}} = -\frac{d}{dt} \iint \vec{\mathbf{B}} \cdot d\vec{\mathbf{a}}$$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{r}} = \mu_o I_{\text{enc}} + \mu_o \varepsilon_0 \frac{d}{dt} \iint \vec{\mathbf{E}} \cdot d\vec{\mathbf{a}}$$

Electromotive Force:

$$\mathcal{E} = \oint \left(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}} \right) \cdot d\vec{\mathbf{r}}$$

Electric Potential Difference:

$$\Delta V_{a \text{ to } b} = V_b - V_a = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\mathbf{r}}$$

$$V = \sum_{i=1}^{N} \frac{q_i}{4\pi\varepsilon_o \left| \vec{\mathbf{r}} - \vec{\mathbf{r}}_i \right|}$$

Capacitance: $C = Q / \Delta V$

$U_C = \frac{1}{2} Q^2 / C = \frac{1}{2} C (\Delta V)^2$

Ohm's Law: $\Delta V = I\, R$

$\vec{\mathbf{J}} = \sigma_c \vec{\mathbf{E}}$ where σ_c is the conductivity

$\vec{\mathbf{E}} = \rho_r \vec{\mathbf{J}}$ where ρ_r is the resistivity

Inductance: $L = N\Phi_B / I$

$\mathcal{E}_{\text{back}} = -L dI / dt \qquad U_L = \frac{1}{2} L I^2$

Energy Density Stored in Fields:
$u_E = \frac{1}{2} \varepsilon_0 E^2 \;;\; u_B = \frac{1}{2} B^2 / \mu_0$

Capacitors in Parallel: $C_{eq} = C_1 + C_2 + \cdots$

Capacitors in Series: $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$

Resistors in Parallel: $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$

Resistors in Series: $R_{eq} = R_1 + R_2 + \cdots$

$\oint E = -\frac{d\phi}{dt}$

Joule Heating:
$P_{\text{Joule}} = I \Delta V = I^2 R = \Delta V^2 / R$

AC Circuits: $\omega_0 = 1/\sqrt{LC}$;

$X_L = \omega L \;;\; X_C = 1/\omega C \;;\; X_R = R$

Series RLC **:**

$Z = \sqrt{R^2 + X^2} = \sqrt{R^2 + (X_L - X_c)^2}$;

$\tan\phi = (X_L - X_C)/R \;;\; V_0 = I_0 Z$

Some potentially useful numbers:

$\mu_o = 4\pi \times 10^{-7}\, \text{T} \cdot \text{m} \cdot \text{A}^{-1}$

$k_e = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\, \text{N} \cdot \text{m}^2 \cdot \text{C}^{-2}$

$c = 3 \times 10^8\, \text{m} \cdot \text{s}^{-1} \qquad e = 1.6 \times 10^{-19}\, \text{C}$

$F = I\vec{a} \times B \quad = I(L \times B)$??

↑ what is ?
not in notes

8.02 Exam Three Spring 2008

FAMILY (last) NAME

GIVEN (first) NAME

Student ID Number

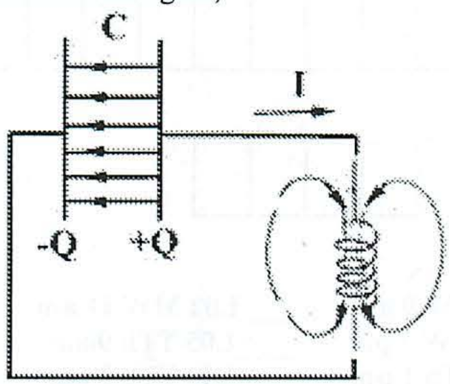
Your Section: ____**L01 MW 9 am** ____**L02 MW 11 am** ____**L03 MW 1 pm**
____**L04 MW 3 pm** ____**L05 TTh 9 am** ____**L06 TTh 11 am**
____**L07 TTh 1 pm** ____**L08 TTh 3 pm**

Your Group (e.g. 10A): ____________

	Score	**Grader**
Section 1 (25 points)		
Problem 1 (25 points)		
Problem 2 (25 points)		
Problem 3 (25 points)		
TOTAL		

Section I: (25 points) Five Concept Questions. Please circle your answers.

Question 1 (5 points): In an LC circuit, the electric and magnetic fields are shown in the figure. At the moment depicted in the figure,



was pps

discharging

Q↓ I↑

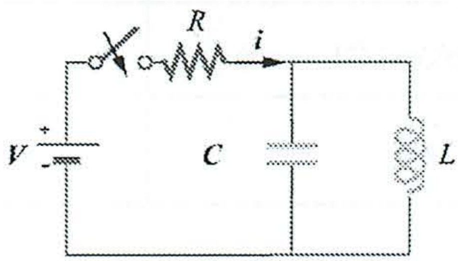
a) the current in the circuit is increasing and the charge on the positive plate of the capacitor is decreasing

b) the current in the circuit is increasing and the charge on the positive plate of the capacitor is increasing

c) the current in the circuit is decreasing and the charge on the positive plate of the capacitor is increasing

d) the current in the circuit is decreasing and the charge on the positive plate of the capacitor is decreasing

Question 2 (5 points): The switch on the circuit shown below is closed at $t = 0$. Let i denote the current through the resistor. Consider the current $i(t = 0^{+})$ and the current $i(t = \infty)$.



a) $i(t = 0^{+}) - i(t = \infty) > 0$.

b) $i(t = 0^{+}) - i(t = \infty) = 0$. same

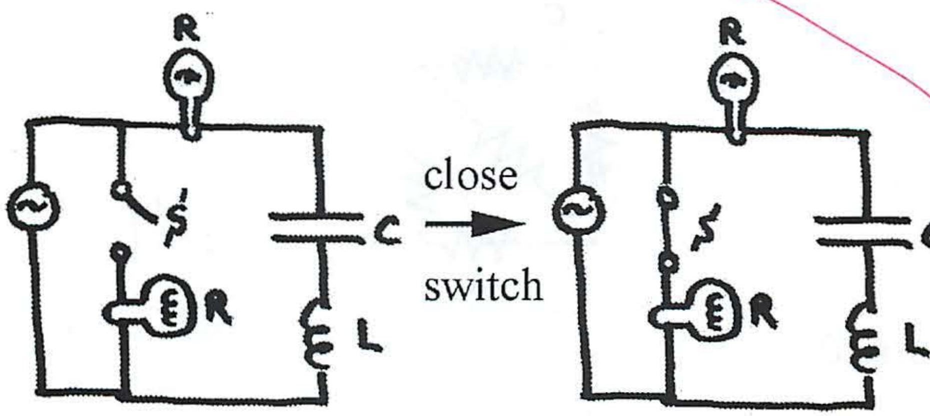
c) $i(t = 0^{+}) - i(t = \infty) < 0$.

i at 0 would a flow + fill capacitor so $\mathcal{E} - IR - \frac{Q_i}{C}$

i at ∞ would all go through inductor — i same

Question 3 (5 points): A driven RLC circuit uses a lightbulb for the resistor and is driven at 1 Hz, below the resonance frequency of the circuit, so that the flashing of the bulb can easily be seen. If a second light bulb is placed in parallel to the power supply, this bulb will flash:

driven not on



a) At the same times as the first bulb

b) Just before the first bulb

c) Just after the first bulb

d) Not at all

On then other one when the other one not on

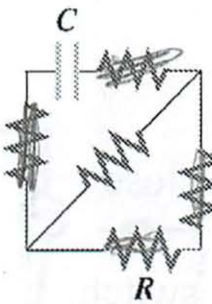
Before switch closed → circuit in resonance
current ✓ leads voltage
← how do you know what dominates

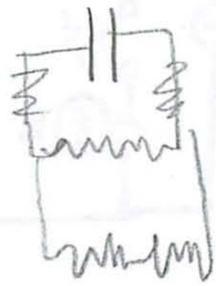
When switch closed current through 2nd lags behind first in phase

– ? still don't really get

and isn't what comes 1st kinda arbitrary

Question 4 (5 points): What is the value of the time constant τ that governs the time dependence of currents and charges in the RC circuit shown below? Assume that all resistors have a common value R of the resistance and that the capacitor is initially charged.





a) $\tau = (5/8)RC$.

b) $\tau = (8/5)RC$.

c) $\tau = (3/8)RC$.

d) $\tau = (8/3)RC$.

e) $\tau = 5RC$.

f) $\tau = (1/5)RC$.

g) $\tau = (5/2)RC$.

h) $\tau = (2/5)RC$.

$\frac{Q}{C} - 2IR -$ what was in parallel?

$R_{eq} = 2R + \frac{R}{3} = \frac{8}{3}$

$\frac{1}{IR} + \frac{1}{I2R}$ $\frac{R}{3}$

almost had that but afraid to write it

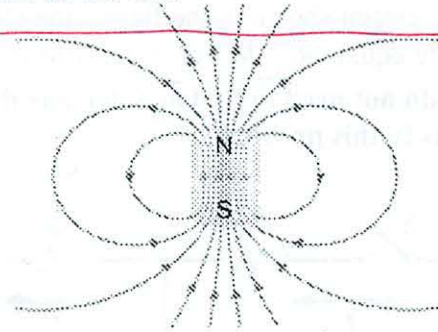
~~$\frac{2}{2IR} + \frac{1}{2R} \frac{2}{4IR}$~~ almost had it

$\tau = \frac{1}{RC} = \frac{8}{3}RC$

? no clue what #

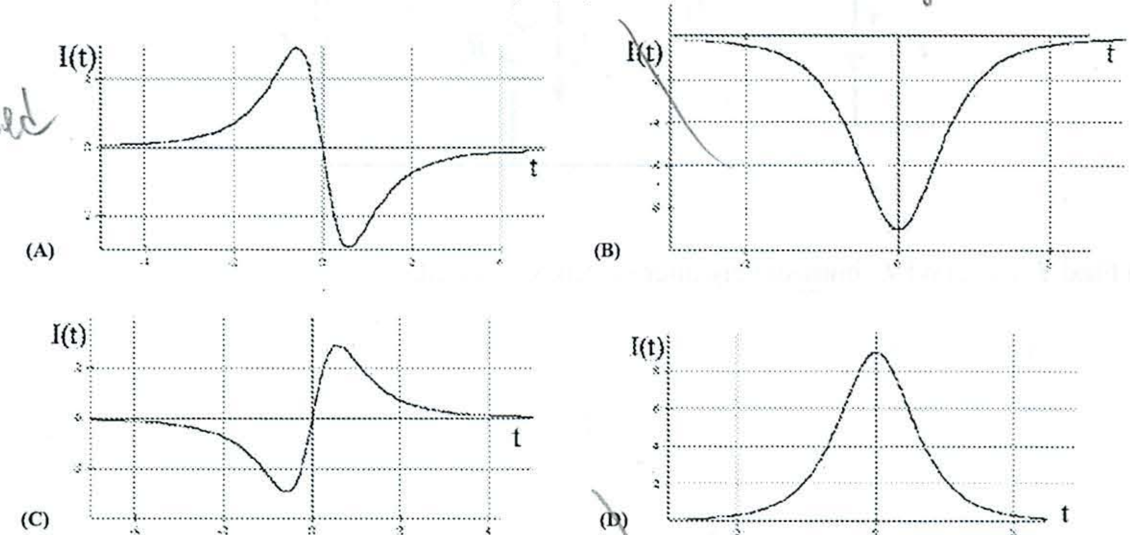
Question 5 (5 points) In Experiment 6, Faraday's Law, a coil moves up from underneath a magnet (with its north pole pointing upward as shown in the diagram) and then back to its starting point. In the figures below, positive current is in the counterclockwise direction when looking down on the coil.

↑ did not see



thought this shape not allowed

I induced



Moving from below to above and back, you measured a *current* of:

- a) A then A
- b) C then C
- c) A then C
- d) C then A
- e) B then B
- f) D then D
- g) B then D
- h) D then B

otherwise correct

what dir

opps I defined ⊕ down again

B↑ increasing will induce ↻ ⊕ then B↑ decreasing induce ↺ ⊖

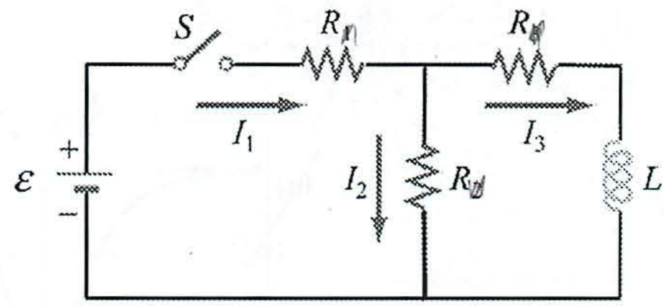
B↑ increasing induce ↻ ⊕

B↑

A → A

Section II: Three Analytic Problems. Answers without work receive no credit.

Problem 1 (25 points): In the circuit shown in the figure, the electromotive force of the battery is ε, all the resistors are equal, $R_1 = R_2 = R_3 = R$, and the inductance in the right loop of the circuit is L. **You do not need to find or solve any differential equations in order to answer the questions in this problem.**



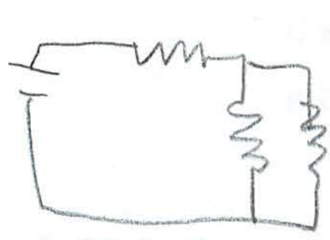
(a) Find I_1, I_2, and I_3 immediately after switch S is closed.

$I_3 = 0$

$I_1 = I_2$ $\varepsilon - I(R_1 + R_2) = 0$

$I_1 = \frac{\varepsilon}{2R} = I_2$ ✓

(b) Find I_1, I_2, and I_3 a long time later.



$R_{eq} = R + \frac{1}{R} + \frac{1}{R}$

$\frac{2}{2R} = \frac{1}{2}R$ what is this again?

$R + \frac{R}{2}$ know this!

calculate for each loop

$\frac{3R}{2}$

$I_2 = I_3$

$I_1 =$??? think it current same everywhere

no

$\varepsilon - IR_{eq} = 0$

no induced ε ✓

$\varepsilon - I_1 R - I_2 R = 0$

$I_2 R - I_3 R = 0$

$I_1 = I_2 + I_3$

combine w/ junction rule →

$I_1 = \frac{2R\mathcal{E}}{3R^2} = \frac{2\mathcal{E}}{3R}$

$I_2 = I_3 = \frac{\mathcal{E}}{3R}$

think I almost did this - but not fully good to seperate it

did not actually find

(c) What is the energy stored in the inductor a long time later?

$U_L = \frac{1}{2} L I^2$? I_3 ?

$L = \frac{\Phi}{I}$

$\Phi = BA\cos\theta$

$\frac{1}{2}\frac{\Phi}{I} I^2$

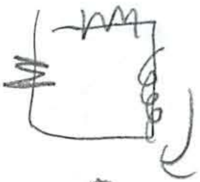
$\frac{1}{2} BAI$

? $\oint B\,ds = \mu_0 I_{enc}$? how do we know?

they did not find L (how was I supposed to know)

(d) A long, long time later, switch S is opened again. Find I_1, I_2, and I_3 immediately after switch S is opened again.

discharging



$L\frac{dI_3}{dt} - I_3(R_1 + R_2) = 0$

$I_2 = -I_3$

? don't have to solve diff eq - immediately

$I_1 = 0$

I_3 same as before ✓

(e) Find I_1, I_2, and I_3 a long time after switch S is opened.

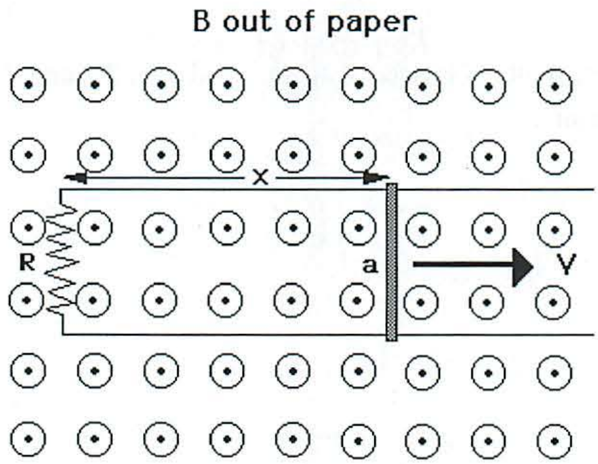
0 all

✓

this is much more up my alley

Problem 2 (25 points):

A conducting rod of mass m slides on frictionless rails in a uniform magnetic field $\vec{\mathbf{B}}$. The magnetic field is directed out of the page. The rails are a distance a apart, and are connected by a resistor with resistance R. At time t, the rod is a distance $x(t)$ from the resistor, moving with a speed $v(t) = dx(t)/dt$ (see sketch).



~~no induced current since B constant~~

area ↑

(a) What is the magnetic flux Φ_B at time t through the circuit consisting of the resistor, the rod, and the intervening rails, in terms of the quantities given?

$\Phi = BA = \vec{B}\, a x$ ✓

(b) What is the emf in the circuit at time t, in terms of the quantities given? What is the current? Show on the sketch clearly (by means of arrows) the direction of this current.

$\mathcal{E} = \frac{-d\Phi}{dt} = -B\, a \frac{dx}{dt}$

$-B a\, v(t)$

forgot

normally ↺ (increasing)

now ↻

$I = \frac{\mathcal{E}}{R} = \frac{Bav}{R}$

← did not realize Kirkoff (simple)

this test a lot more fair

(c) What is the force $\vec{\mathbf{F}}$ on the rod at time t, in terms of the quantities given? Give its magnitude, and indicate its direction on the sketch.

induced force never in notes - could not find in notes

So resistance

I was always bad at this calc

$F = \vec{I}(L \times B) = I a \times \vec{B} = I a B - \hat{i}$

inductant

do I have to calculate

fill in

Opposite pulling force

←

$\frac{Bav}{R} a B$

$F = \frac{B^2 a^2 v}{R}$

(d) What is the rate of energy dissipation at time t due to ohmic heating in the resistor, in terms of the quantities given?

here again

$P = I^2 R$

I not F

$\left(\frac{B a v}{R}\right)^2 R$

$\frac{B^2 a^2 v^2}{R}$

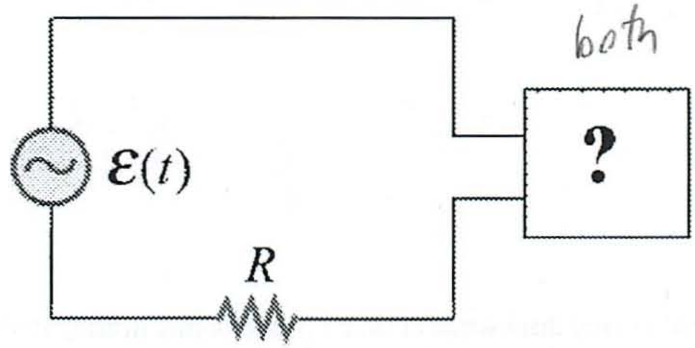
learn this pg

Problem 3 (25 points): I don't think on test nope

The circuit shown below contains an AC generator which provides a source of sinusoidally varying emf $\mathcal{E} = \mathcal{E}_0 \sin \omega t$, a resistor with resistance $R = 6\,\Omega$, and a "black box", which contains ***either*** an inductor ***or*** a capacitor, ***or both***. The amplitude of the driving emf is $\mathcal{E}_0 = 6$ volt. We measure the current in the circuit at an angular frequency $\omega = 2\ \text{rad}\cdot\text{s}^{-1}$ and find that it is exactly in phase with the driving emf. We measure the current in the circuit at an angular frequency $\omega = 1\ \text{rad}\cdot\text{s}^{-1}$ and find that it is out of phase from the driving emf by exactly $\pi/4$ radians.

$\omega = \frac{1}{\sqrt{LC}}$ V_L I V_C

both



a) What does the black box contain – an inductor or a capacitor, or both? Explain your reasoning. Does current lead or lag at $\omega = 1\ \text{rad}\cdot\text{s}^{-1}$?

both ✓ Since if changes ω

current will ~~lag~~ leads & capicator stronger ✓

ω below resonace

I drew phasor wrong

L on top! → V_L → I V_C

b) What is the numerical value of the capacitance *or* of the inductance, *or of both*, as the case may be? Indicate units. Your answer(s) will involve simple fractions only, you will not need a calculator to find the value(s).

got a peak at ans

$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$ & know $X_L = \omega L$

$X_C = \frac{1}{\omega C}$

$\frac{\pi}{4} = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right) = -1$

leave like that

$\tan\frac{\pi}{4} = -1$

Thought something was fishy

c) What is ratio of the amplitudes of the current $\frac{I_0(\omega = 2\ \mathrm{rad \cdot s^{-1}})}{I_0(\omega = 1\ \mathrm{rad \cdot s^{-1}})}$?

Opps very complex

Same $\frac{1}{1} = 1$

Complex as well

— not copying — see ans sheet

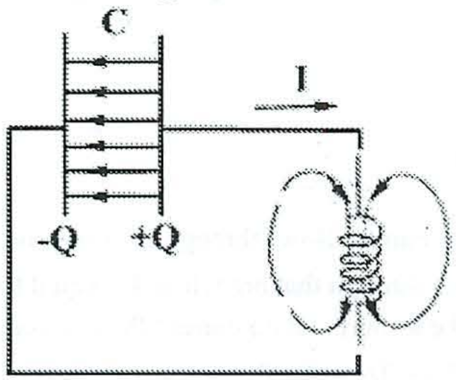
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 Spring 2008

Exam Three: Solutions

Section I: (25 points) Five Concept Questions. Please circle your answers.

Question 1 (5 points): In an LC circuit, the electric and magnetic fields are shown in the figure. At the moment depicted in the figure,



a) the current in the circuit is increasing and the charge on the positive plate of the capacitor is decreasing

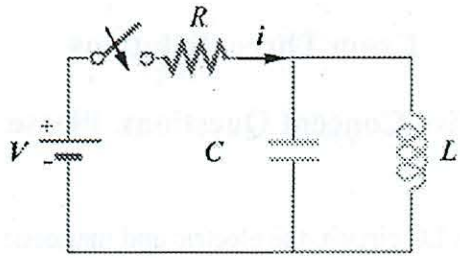
b) the current in the circuit is increasing and the charge on the positive plate of the capacitor is increasing

c) the current in the circuit is decreasing and the charge on the positive plate of the capacitor is increasing

d) the current in the circuit is decreasing and the charge on the positive plate of the capacitor is decreasing

Solution: (a). As the charge on the capacitor decreases the current increases.

Question 2 (5 points): The switch on the circuit shown below is closed at $t = 0$. Let i denote the current through the resistor. Consider the current $i(t = 0^+)$ and the current $i(t = \infty)$.



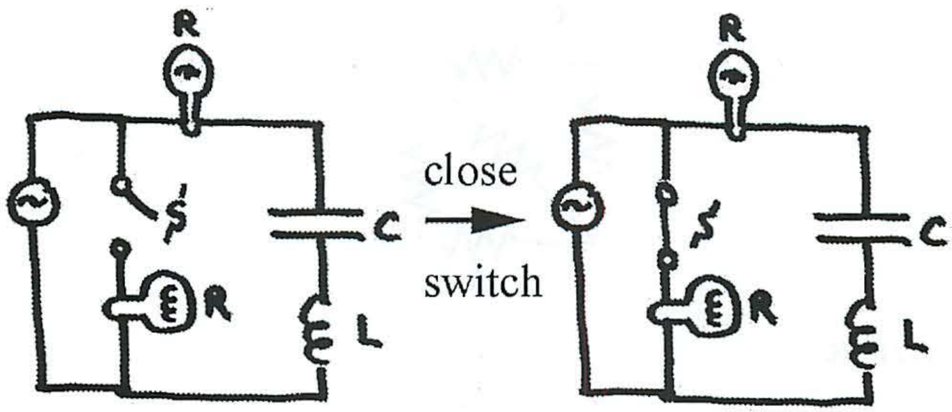
a) $i(t = 0^+) - i(t = \infty) > 0$.

b) $i(t = 0^+) - i(t = \infty) = 0$.

c) $i(t = 0^+) - i(t = \infty) < 0$.

Solution: (b) At $i(t = 0^+)$, no current flows through the inductor and the capacitor acts as a short so all the current flows through that branch and is equal to $i(t = 0^+) = V / R$. At $i(t = \infty)$, the inductor acts like a short and no current flows through the capacitor so $i(t = \infty) = V / R$. Thus $i(t = 0^+) - i(t = \infty) = 0$.

Question 3 (5 points): A driven RLC circuit uses a lightbulb for the resistor and is driven at 1 Hz, below the resonance frequency of the circuit, so that the flashing of the bulb can easily be seen. If a second light bulb is placed in parallel to the power supply, this bulb will flash:



a) At the same times as the first bulb

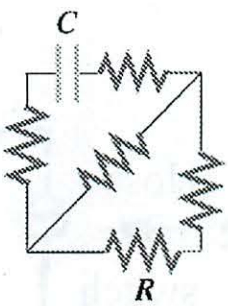
b) Just before the first bulb

c) Just after the first bulb

d) Not at all

Solution: (c). Before the switch is closed the circuit, the circuit is drive below resonance frequency so the current leads the voltage. When the switch is closed, the current and the voltage throught eh second light bulb are in phase lagging behind the current throught eh first light bulb, so the second light bulb goes off just after the first light bulb.

Question 4 (5 points): What is the value of the time constant τ that governs the time dependence of currents and charges in the RC circuit shown below? Assume that all resistors have a common value R of the resistance and that the capacitor is initially charged.



a) $\tau = (5/8)RC$.

b) $\tau = (8/5)RC$.

c) $\tau = (3/8)RC$.

d) $\tau = (8/3)RC$.

e) $\tau = 5RC$.

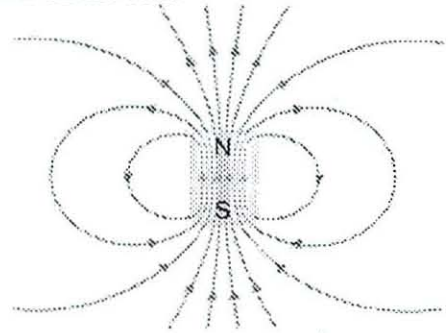
f) $\tau = (1/5)RC$.

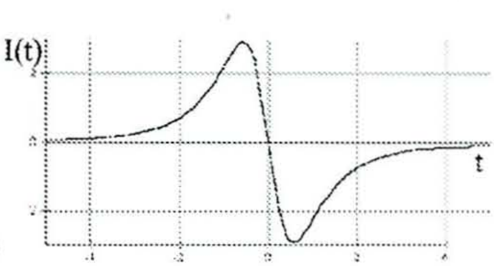
g) $\tau = (5/2)RC$.

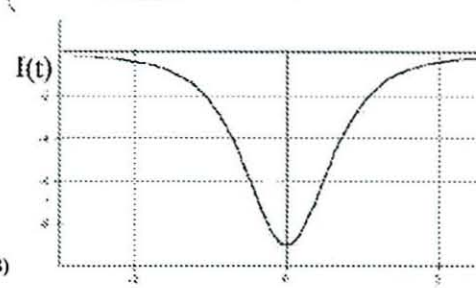
h) $\tau = (2/5)RC$.

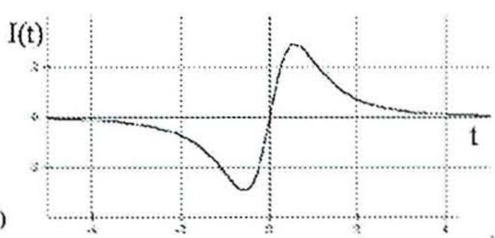
Solution: (d). The three resistors in parallel have an equivalent resistance of $R_{eq,parallel} = R/3$. The two series elements have an equivalent resistance of $R_{eq,series} = 2R$. So the total equivalent resistance is the sum of these two $R_{eq} = 2R + R/3 = (8/3)R$. Thus the time constant is $\tau = R_{eq}C = (8/3)RC$.

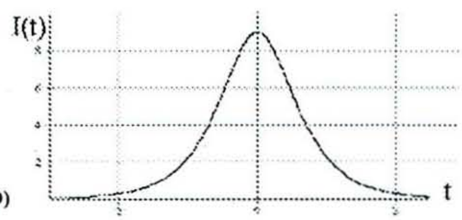
Question 5 (5 points) In Experiment 6, Faraday's Law, a coil moves up from underneath a magnet (with its north pole pointing upward as shown in the diagram) and then back to its starting point. In the figures below, positive current is in the counterclockwise direction when looking down on the coil.



(A)


(B)


(C)


(D)


Moving from below to above and back, you measured a *current* of:

a) A then A

b) C then C

c) A then C

d) C then A

e) B then B

f) D then D

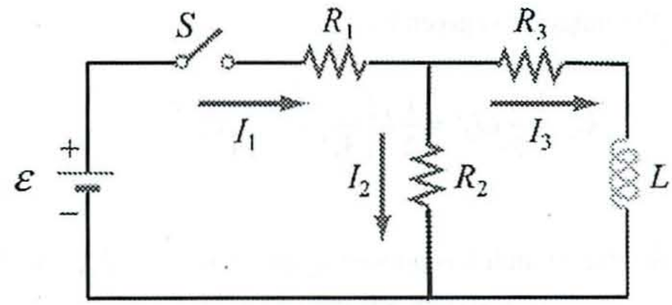
g) B then D

h) D then B

Solution: (b). When the coil is moving up but before it reaches the center of the magnet, it encounters flux up and increasing so the induced flux is down and hence the current flows clockwise when seen from above and thus is negative. After the coil passes the center of the magnet, the flux is up and decreasing so the induced flux is up corresponding to a counterclockwise current which is positive. So the graph of the current vs. time for this part is (c). When the coil is above the magnet and moves down, the flux is up and increasing so the induced flux is down corresponding to a clockwise current. After the coil passes the magnet, the flux is up and decreasing so the induced flux is up corresponding to a counterclockwise current which is positive so once again the graph of the current vs time is (c).

Section II: Three Analytic Problems. Answers without work receive no credit.

Problem 1 (25 points): In the circuit shown in the figure, the electromotive force of the battery is ε, all the resistors are equal, $R_1 = R_2 = R_3 = R$, and the inductance in the right loop of the circuit is L. **You do not need to find or solve any differential equations in order to answer the questions in this problem.**



(a) Find I_1, I_2, and I_3 immediately after switch S is closed.

The current through the inductor is zero at t=0.because the self-induced emf prevents the current from rising abruptly. Therefore, $I_3 = 0$ and $I_1 = I_2$.

Applying the Kirchhoff's loop rule to the left loop yields

$$I_1 = I_2 = \frac{\varepsilon}{R_1 + R_2} = \frac{\varepsilon}{2R}$$

(b) Find I_1, I_2, and I_3 a long time later.

Now, there is no induced emf. Therefore, Kirchhoff's loop rule gives

$$\varepsilon - I_1 R - I_2 R = 0$$

for the left loop, and

$$I_2 R - I_3 R = 0$$

for the right loop. Combining the two equations with the junction rule $I_1 = I_2 + I_3$, we obtain

$$I_1 = \frac{2R\varepsilon}{3R^2} = \frac{2\varepsilon}{3R}$$

$$I_2 = I_3 = \frac{R\varepsilon}{3R^2} = \frac{\varepsilon}{3R}$$

(c) What is the energy stored in the inductor a long time later?

The energy stored in the inductor is given by

$$U_B = \frac{1}{2}LI_3^{\,2} = \frac{1}{2}L\left(\frac{\varepsilon}{3R}\right)^2 = \frac{1}{18}L\left(\frac{\varepsilon}{R}\right)^2$$

(d) A long, long time later, switch S is opened again. Find I_1 , I_2, and I_3 immediately after switch S is opened again.

The current through R_1 is zero, i.e., $I_1 = 0$. This implies that $I_1 = I_2 + I_3 = 0$. On the other hand, the right loop now forms a decaying RL circuit and I_3 starts to decrease, but initially is unchanged. Thus, I_2 immediately after the switch is opened reverses and I_3 stays the same.

$$I_3 = \frac{\varepsilon}{3R}$$

$$I_2 = -\frac{\varepsilon}{3R}$$

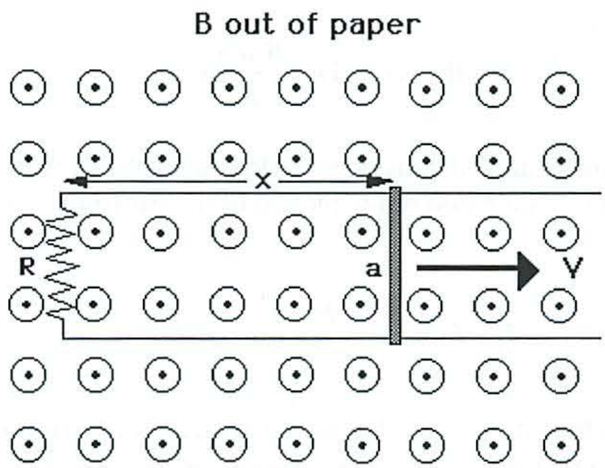
(e) Find I_1 , I_2, and I_3 a long time after switch S is opened.

A long time after the switch has been closed, all the currents will be zero. That is,

$$I_1 = I_2 = I_3 = 0$$

Problem 2 (25 points):

A conducting rod of mass m slides on frictionless rails in a uniform magnetic field $\vec{\mathbf{B}}$. The magnetic field is directed out of the page. The rails are a distance a apart, and are connected by a resistor with resistance R. At time t, the rod is a distance $x(t)$ from the resistor, moving with a speed $v(t) = dx(t)/dt$ (see sketch).



(a) What is the magnetic flux Φ_B at time t through the circuit consisting of the resistor, the rod, and the intervening rails, in terms of the quantities given?

Solution: At time t, the flux through the loop is given by

$$\Phi_B = \iint \vec{\mathbf{B}} \cdot d\vec{\mathbf{a}} = Bxa$$

Note: That we have chosen the unit normal out of the page and thus the flux is positive.

(b) What is the emf in the circuit at time t, in terms of the quantities given? What is the current? Show on the sketch clearly (by means of arrows) the direction of this current.

The electromotive force is equal to the negative of the time derivative of the flux

$$\varepsilon = -\frac{d\Phi_B}{dt} = -Ba\frac{dx}{dt} = -Bav$$

The magnitude of the current is equal to

$$I = \frac{|\varepsilon|}{R} = \frac{Bav}{R}.$$

The flux is out of the page and increasing so the induced flux must be into the page. By the right hand rule this correspond to a current flowing in the clockwise direction .

(c) What is the force $\vec{\mathbf{F}}$ on the rod at time t, in terms of the quantities given? Give its magnitude, and indicate its direction on the sketch.

The induced force is given by

$$\vec{\mathbf{F}} = I\vec{\mathbf{a}} \times \vec{\mathbf{B}} = IaB(-\hat{\mathbf{i}}) = \frac{B^2 a^2 v}{R}(-\hat{\mathbf{i}})$$

The current is directed downwards, the magnetic fields out of the page so the cross product points in the opposite direction of the motion of the bar (opposing force). So the induced force is

$$\vec{\mathbf{F}} = IaB(-\hat{\mathbf{i}}) = \frac{B^2 a^2 v}{R}(-\hat{\mathbf{i}})$$

Note: The problem states that at time t the bar moves with a speed $v(t)$ however it doesn't explicitly mention that there are no other forces acting on the bar. There is no additional external force pulling the bar in the direction of the velocity of the rod. There are in fact two other forces acting on the bar, the gravitational force pointing downwards and the normal force of the rails on the bar acting upwards. Since these forces add to several, the force on the rod at time t is only the induced force.

(d) What is the rate of energy dissipation at time t due to ohmic heating in the resistor, in terms of the quantities given?

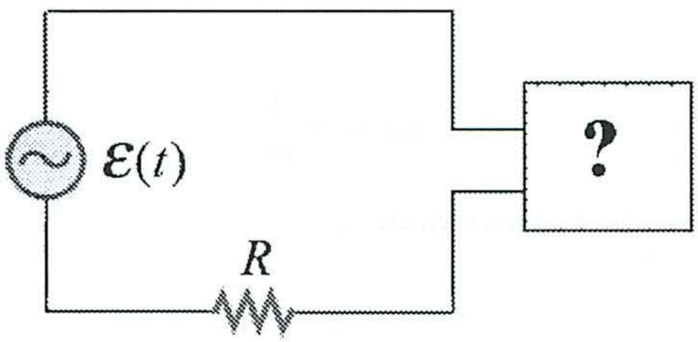
The rate of energy dissipation is given by

$$P = I^2 R = \frac{B^2 a^2 v^2}{R}$$

Final 4

Problem 3 (25 points):

The circuit shown below contains an AC generator which provides a source of sinusoidally varying emf $\mathcal{E} = \mathcal{E}_0 \sin\omega t$, a resistor with resistance $R = 6\,\Omega$, and a "black box", which contains ***either*** an inductor ***or*** a capacitor, ***or both***. The amplitude of the driving emf is $\mathcal{E}_0 = 6$ volt. We measure the current in the circuit at an angular frequency $\omega = 2\ \text{rad}\cdot\text{s}^{-1}$ and find that it is exactly in phase with the driving emf. We measure the current in the circuit at an angular frequency $\omega = 1\ \text{rad}\cdot\text{s}^{-1}$ and find that it is out of phase from the driving emf by exactly $\pi/4$ radians.



a) What does the black box contain – an inductor or a capacitor, or both? Explain your reasoning. Does current lead or lag at $\omega = 1\ \text{rad}\cdot\text{s}^{-1}$?

Solution: Since the current is exactly in phase with the inductor at $\omega = 2\ \text{rad}\cdot\text{s}^{-1}$, the circuit is resonance and therefore the box must contain both an inductor and capacitor in series. For $\omega = 1\ \text{rad}\cdot\text{s}^{-1}$, the driving angular frequency is below resonance, therefore the circuit is acting capacitively, which means the current leads the driving emf.

b) What is the numerical value of the capacitance *or* of the inductance, *or of both*, as the case may be? Indicate units. Your answer(s) will involve simple fractions only, you will not need a calculator to find the value(s).

When $\omega = 1\ \text{rad}\cdot\text{s}^{-1}$, the phase angle $\phi = -\pi/4$. So $\tan\phi = \tan(-\pi/4) = -1$. Since

$$\tan\phi = \frac{X_L - X_C}{X_R} = \frac{\omega L - \dfrac{1}{\omega C}}{R} = -1.$$

We have that for $\omega = 1\ \text{rad}\cdot\text{s}^{-1}$

$$L - \frac{1}{C} = -R.$$

We can divide through by L in the above equation yielding

$$1\ \text{rad}\cdot\text{s}^{-1} - \frac{1}{LC(1\ \text{rad}\cdot\text{s}^{-1})} = -\frac{R}{L}$$

We also have the resonance condition at $\omega = 2\ \text{rad}\cdot\text{s}^{-1}$,

$$\omega = 2\ \text{rad}\cdot\text{s}^{-1} = \frac{1}{\sqrt{LC}}.$$

Thus

can find #

$$\omega^2 = 4\ \text{rad}^2\cdot\text{s}^{-2} = \frac{1}{LC}$$

Substituting that in the above equation yields

for L, C

$$1\ \text{rad}\cdot\text{s}^{-1} - 4\ \text{rad}\cdot\text{s}^{-1} = -\frac{R}{L}.$$

(and not $\vec{E}, \vec{B}$)

Solving for L then yields

$$L = \frac{R}{3\ \text{rad}\cdot\text{s}^{-1}} = \frac{6\,\Omega}{3\ \text{rad}\cdot\text{s}^{-1}} = 2\ \text{H}.$$

From the resonance condition

F = farads
– capacitor

$$4\ \text{rad}^2\cdot\text{s}^{-2}C = \frac{1}{L(4\ \text{rad}^2\cdot\text{s}^{-2})} = \frac{1}{(2\,\Omega)(4\ \text{rad}^2\cdot\text{s}^{-2})} = \frac{1}{8}\ \text{F}.$$

c) What is ratio of the amplitudes of the current $\frac{I_0(\omega = 2\ \text{rad}\cdot\text{s}^{-1})}{I_0(\omega = 1\ \text{rad}\cdot\text{s}^{-1})}$?

At resonance $\omega = 2\ \text{rad}\cdot\text{s}^{-1}$:

$$I_0(\omega = 2\ \text{rad}\cdot\text{s}^{-1}) = \frac{\varepsilon_0}{R} = \frac{6\ \text{V}}{6\,\Omega} = 1\ \text{A}.$$

Below resonance

* keep forgetting

$I = \frac{\varepsilon}{R}$ (simple kirkoff)

$$I_0(\omega = 1\ \mathrm{rad \cdot s^{-1}}) = \frac{\varepsilon_0}{\left(R^2 + (X_L - X_C)^2\right)^{1/2}} = \frac{\varepsilon_0}{\left(R^2 + (\omega L - \frac{1}{\omega C})^2\right)^{1/2}}.$$

T: again flirting

From the phase condition $\omega L - \frac{1}{\omega C} = -R$, the amplitude at $\omega = 1\ \mathrm{rad \cdot s^{-1}}$ becomes

$$I_0(\omega = 1\ \mathrm{rad \cdot s^{-1}}) = \frac{\varepsilon_0}{\left(R^2 + R^2\right)^{1/2}} = \frac{\varepsilon_0}{\sqrt{2}R} = \frac{1}{\sqrt{2}}\ \mathrm{A}.$$

Therefore the ratio of the amplitudes of the current is

$$\frac{I_0(\omega = 2\ \mathrm{rad \cdot s^{-1}})}{I_0(\omega = 1\ \mathrm{rad \cdot s^{-1}})} = \frac{\varepsilon_0 / R}{\varepsilon_0 / \sqrt{2}R} = \sqrt{2}.$$

amplitudes I_0 will differe

$I = \frac{\mathcal{E}}{R}$ ← have to do R eq

$X_R = R$

$X_L = \omega L$

$X_C \ \frac{1}{\omega C}$