TEST ONE Thursday Evening February 25 7:30 - 9:30 pm . The Friday class immediately following is canceled because of the evening exam.

What We Expect From You On The Exam

(1) Ability to calculate the electric field of both discrete and continuous charge distributions. We may give you a problem on setting up the integral for a continuous charge distribution, although we do not necessarily expect you to do the integral, unless it is particularly straight forward. You should be able to set up problems like: calculating the field of a small number of point charges, the field of the perpendicular bisector of a finite line of charge; the field on the axis of a ring of charge; and so on.

(2) To be able to recognize and draw the electric field line patterns for a small number of discrete charges, for example, from two point charges (of same or opposite charge)

(3) To be able to apply the principle of superposition to electrostatic problems.

(4) An understanding of how to calculate the electric potential of a discrete set of charges, that is the use of the equation $V(\mathbf{r}) = \sum_{i=1}^{N} \frac{q_i}{4\pi\varepsilon_o|\mathbf{r}-\mathbf{r}_i|}$ for the potential of N charges q_i located at positions $\mathbf{r}_i$. Also you must know how to calculate the configuration energy necessary to assemble this set of charges.

(5) The ability to calculate the electric potential given the electric field and the electric field given the electric potential, e.g. being able to apply the equations

$$\Delta V_{a\ to\ b} = V_b - V_a = -\int_a^b \mathbf{E}\cdot d\mathbf{l} \text{ and } \mathbf{E} = -\vec{\nabla} V .$$

(6) An understanding of how to use Gauss's Law. In particular, we *may* give you a problem that involves either finding the electric field of a uniformly or non-uniformly filled cylinder, slab or sphere of charge, as well as the potential associated with that electric field. You must be able to explain the steps involved in this process clearly, and in particular to argue how to evaluate $\oint\!\!\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}}$ *on every part* of the closed surface to which you apply Gauss's Law, even those parts that are zero.

(7) To be able to answer qualitative conceptual questions that require no calculation. There will be concept questions similar to those done in class.

To study for this exam (which you should DEFINITELY DO!) we suggest that you review your problem sets, in-class problems, Friday problem solving sessions, PRS in-class questions, and relevant parts of the study guide and class notes and work through multiple past exams

M'chael Plasmeier

11C

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 Spring 2010

Problem Set 3

Due: Tuesday, February 23 at 9 pm.

Hand in your problem set in your section slot in the boxes outside the door of 32-082. Make sure you clearly write your name and section on your problem set.

Text: Liao, Dourmashkin, Belcher; Introduction to E & M MIT 8.02 Course Notes.

Reading Assignments:

Week Three: Electric Potential

President's Day – M 2/15 / M Classes on T 2/16

Class 6 W03D1 T Feb 16 Electric Potential
Reading: Course Notes Sections 3.1-3.5, 3.7-3.8

Class 7 W03D02 W/R Feb 17/18 Electric Potential; Equipotential Lines and Electric Fields
Expt.1: Electric Potential; Configuration Energy;
Reading: Course Notes: Sections 3.1-3.5
Experiment: Expt. 1: Electric Potential

Class 8 W03D3 F Feb 19 PS03: Electric Potential
Reading: Course Notes: Sections 3.1-3.5, 3.7-3.8

Week Four Conductors and Capacitors

Class 9 W04D1 M/T Feb 22/3 Conductors and Insulators; Capacitance & Capacitors; Energy Stored in Capacitors;
Reading: Course Notes: Sections 4.3-4.4; 5.1-5.4, 5.9

Class 10 W04D2 W/R Feb 24/25 Exam One Review

Exam 1 Thursday Feb 25 7:30 pm –9:30 pm

W04D3 F Feb 26 No class day after exam

$V(P) - V(\infty) =$

Problem 1: Concept Questions. Explain your reasoning.

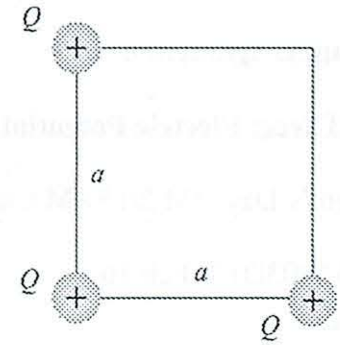
Suppose an electrostatic potential has a maximum at point P and a minimum at point M.

(a) Are either (or both) of these points equilibrium points for a negative charge? If so are they stable?

(b) Are either (or both) of these points equilibrium points for a positive charge? If so are they stable?

Problem 2: Charges on a Square

Three identical charges $+Q$ are placed on the corners of a square of side a, as shown in the figure.



(a) What is the electric field at the fourth corner (the one missing a charge) due to the first three charges?

(b) What is the electric potential at that corner?

(c) How much work does it take to bring another charge, $+Q$, from infinity and place it at that corner?

(d) How much energy did it take to assemble the pictured configuration of three charges?

Problem 3: Line of Charge

Consider a very long rod, radius R and charged to a uniform linear charge density λ.

a) Calculate the electric field everywhere outside of this rod (i.e. find $\vec{\mathbf{E}}(\vec{\mathbf{r}})$).

Review session

b) Calculate the electric potential everywhere outside, where the potential is defined to be zero at a radius $R_0 > R$ (i.e. $V(R_0) \equiv 0$)

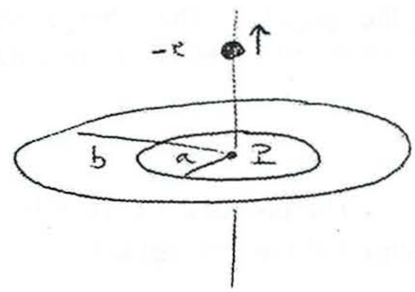
Problem 4: Estimation: High Voltage Power Lines

Estimate the largest voltage at which it's reasonable to hold high voltage power lines. Then check out this video, (http://web.mit.edu/8.02t/www/materials/ProblemSets/PS03_Video.mpeg) care of a Boulder City, Nevada power company. Air ionizes when electric fields are on the order of $3\times10^6\ \mathrm{V\cdot m^{-1}}$.

Problem 5: Charged Sphere Consider a uniformly charged sphere of radius R and charge Q. Find the electric potential difference between any point lying on a sphere of radius r and the point at the origin, i.e. $V(r)-V(0)$. Choose the zero reference point for the potential at $r=0$, i.e. $V(0)=0$. How does your expression for $V(r)$ change if you chose the zero reference point for the potential at $r=\infty$, i.e. $V(\infty)=0$.

Review session

Problem 6: Charged Washer A thin washer of outer radius b and inner radius a has a uniform negative surface charge density $-\sigma$ on the washer (note that $\sigma>0$).



a) If we set $V(\infty)=0$, what is the electric potential difference between a point at the center of the washer and infinity, $V(P)-V(\infty)$?

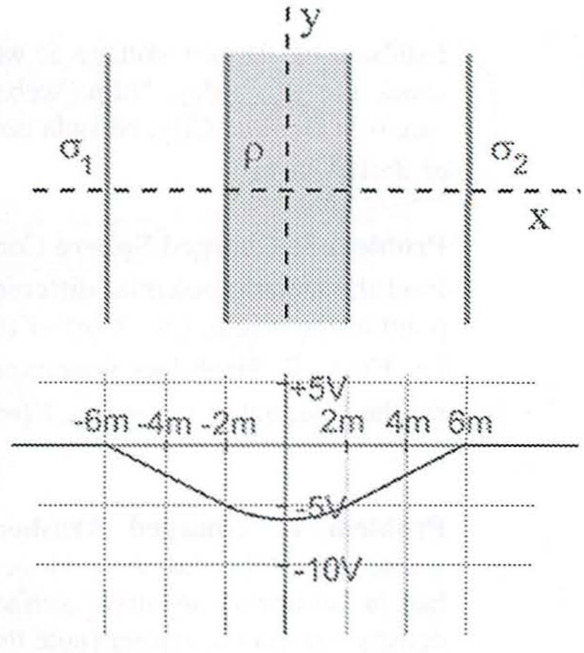
Dynamic

b) An electron of mass m and charge $q=-e$ is released with an initial speed v_0 from the center of the hole (at the origin) in the upward direction (along the perpendicular axis to the washer) experiencing no forces except repulsion by the charges on the washer. What speed does it ultimately obtain when it is very far away from the washer (i.e. at infinity)?

Problem 7: Charged Slab & Sheets

An infinite slab of charge carrying a charge per unit volume ρ has a charged sheet carrying charge per unit area σ_1 to its left and a charged sheet carrying charge per unit area σ_2 to its right (see top part of sketch). The lower plot in the sketch shows the electric potential $V(x)$ in volts due to this slab of charge and the two charged sheets as a function of horizontal distance x from the center of the slab. The slab is 4 meters wide in the x-direction, and its boundaries are located at $x = -2$ m and $x = +2$ m, as indicated. The slab is infinite in the y direction and in the z direction (out of the page). The charge sheets are located at $x = -6$ m and $x = +6$ m, as indicated.

Review solutions



(a) The potential $V(x)$ is a linear function of x in the region $-6\text{ m} < x < -2\text{ m}$. What is the electric field in this region?

(b) The potential $V(x)$ is a linear function of x in the region $2\text{ m} < x < 6\text{ m}$. What is the electric field in this region?

(c) In the region $-2\text{m} < x < 2\text{ m}$, the potential $V(x)$ is a quadratic function of x given by the equation $V(x) = \frac{5}{16}x^2 - \frac{24}{5}\text{V}$. What is the electric field in this region?

L typo 2/20/10

(d) Use Gauss's Law and your answers above to find an expression for the charge density ρ of the slab. Indicate the Gaussian surface you use on a figure.

(e) Use Gauss's Law and your answers above to find the two surface charge densities of the left and right charged sheets. Indicate the Gaussian surface you use on a figure.

Michael E Plasmeier

From: Juven Wang [juven@MIT.EDU]
Sent: Sunday, February 21, 2010 8:42 PM
To: Juven Chunfan Wang
Subject: [8.02] Fwd: [L08] Hints for 8.02t Pset 3

Hi 8.02 problem-solvers,

An updated version of hints.

========
Hints for Pset 3
========

prob 1)
check the first, second derivative of potential (or potential energy) and its sign.

equilibrium (here for static equilibrium) means that particle(charge) experiences zero net force - can stay where it was without moving.

Stable equilibrium is equilibrium with an extra condition that under small (positional) perturbation, the particle will still come back to (or be around) the equilibrium point instead of moving away.

(note: equilibrium includes stable, neutral and unstable equilibrium.)

prob 3-a)
method 1: by Gauss's law(easy), cylindrical gaussian surface method 2: Coulomb's law(tricky), integrate all charge density on an infinite long wire to get E field. or you can integrate charge density on a finite length wire(interesting and worthwhile to try), then taking the length to be infinite.

prob 3-b) potential in logarithmic log form. (the reason for not taking zero potential at infinity is because log(r) diverges as r goes to infinity, which is a bad reference.)

prob 4) the E field causes by the cable is in 1/r form outside the cable(note: though unnecessary to apply here, E field is in a linear form of r inside the cable).

The potential of the cable should be regard as potential respect to the ground, where we normally set zero potential there. Apply prob 3-b). potential difference V is in logarithmic log(r), and assume the distance from the ground to the cable is 5~10 m.

The E field caused by cable has its maximum at the radius R, say, 1~10 cm.
We like to match this maximum E field at radius R to the air-ionizing magnitude. By this relation, you can relate maximum E to a maximum V saturate the ionizing bound. Find the maximum potential V respect to the ground.

prob 5) get the E field inside the sphere(by Gauss's law), which is proportional to r. relating E field to potential difference V(r)-V(0) by doing a line integration from 0 to r.

potential difference V(\infty)-V(r) by doing a line integration from \infty to r. you need to do it by two regions since E behaves differently inside and outside the sphere.

prob 6-a)
method 1: summing over potential, contributed from each charge density on the washer.
method 2: from potential V definition, do an integration of E field from infinity to the center of washer along the symmetric axis. you have to find E field from the washer first.

method 1 and method 2 are equivalent by the fact: E field can be obtained by superposition principle.

prob 6-b)
including the electric potential energy as internal energy of the system, apply mechanical energy conservation(electric potential energy+kinetic energy).

or you can use work-energy thm if you consider electrostatic force as an external force.

prob 7) by Gauss's law and by E_x=-dV/dx figure out total net charge of two sheets and one plane is zero.
argue that the slab has negative charge. two sheets have the same positive charge.

good luck!

Juven

Michael Plasmeier
HC
8.02 PSet 3

PS 14/25
PG 15/25
other 45/50

2/20/10

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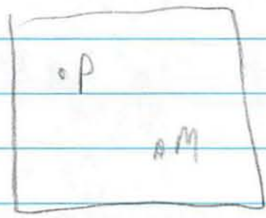
1. Concept Qu

Equilibrium =0 net force
+ if given a small push will come back



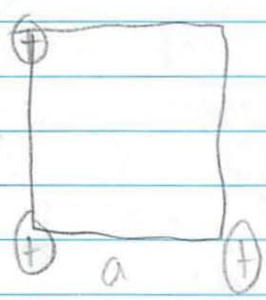
P = max
M = min

P is like the mountain peak
$\ominus$ charges will head here
- (as moving to lower U)

M is the valley
$\oplus$ charges will roll down hill here

Fint talks about 2nd deriv why?

2. Charges on a Square



makes much more sense after reviewing

a. What is field at 4th corner

~~$E = -\nabla V$~~

~~V = superposition of the 3~~

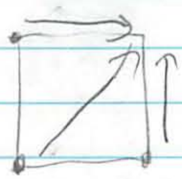
~~$O+$~~

$\rightarrow$

$E = k_e \frac{q}{r^3}$ for point charges

Superposition them
- what is r?
- distance from test charge?
- vector: from charge to observer

I am actually getting this :)



$\vec{E}_1 = -k_e \frac{q}{a} \hat{i}$

$\vec{E}_3 = -k_e \frac{q}{a} \hat{j}$

bit unsure here

$\vec{E}_2 =$ denominator is always the same
put the distance in numerator

$$-\frac{k_e q a}{\sqrt{a^2+a^2}} \hat{i} - -\frac{k_e q a}{\sqrt{a^2+a^2}} \hat{j}$$

E at $4 = E_1 + E_2 + E_3$

what about parts b, c, d?

-5

3. Line of Charge

from notes class 2

$\vec{E} = 2k_e \frac{\lambda}{s} \hat{s}$

But don't you have to use Guass' Law?
and is above for a rod w/ no radius

← don't make guassian surface ∞
interested in side

$EA = \frac{q_{inc}}{\epsilon_0}$

$A = 2\pi r L$ $\quad q = \lambda L$
$V = \pi r^2 L$

$E(2\pi r L) = \frac{\lambda L}{\epsilon_0}$

$E(r) = \frac{\lambda \cancel{L}}{\epsilon_0 2\pi r \cancel{L}} = \frac{\lambda}{\epsilon_0 2\pi r} \hat{r}$

note r is in there
so field is a function of r

Could also colomb's law - integrate all charge density λ on ∞ long wire to get E field

3b Calculate the electric potential everywhere outside

$V(R_0) = 0$

$$V = V_B - V_0 = \int_0^B \vec{E}\, ds$$

don't use ∞ as 0 since $\log(r)$ diverges at ∞

$$\int \frac{\lambda}{\varepsilon_0 2\pi r}\, dr \leftarrow r = s \text{ ?? right}$$

$$\cancel{\left.\frac{\lambda}{\varepsilon 2\pi \frac{r^2}{2}}\right|_0^R} \quad R > R_0 \leftarrow \text{So did integral wrong}$$

$$V = \cancel{\frac{\lambda}{\varepsilon_0 \pi R^2}} \leftarrow \text{where } R > R_0$$

all constant besides R

Potential in log form

$$V = \frac{\lambda}{\varepsilon_0 2\pi} \log(R) - \frac{\lambda}{\varepsilon_0 2\pi} \log(0)$$

can't have log 0

So what is it when $R = 0$

$\downarrow$

look at more examples

$$V = \frac{\lambda}{\varepsilon_0 2\pi} \log(R)$$

Or superposition of particles

$$V = k_e \sum \frac{q_i}{r_i}$$

$$V = -\int_A^B kQ \frac{\hat{r}}{r^2}\, ds$$

$$-kQ \int_A^B \frac{dr}{r^2}$$

pt charge

Hudson i
Similar to #3

4. Estimate High Voltage Power Lines

Air ionizes at $E = 3 \cdot 10^6\ V \cdot m^{-1}$

kinda similar to megavolt

E field in cable is $\frac{1}{r}$ form outside cable
unneeded to apply here as E is linear (r) inside cable

Potential diff w/ respect to ground
V in log(r)
cable → ground ≈ 5-10m

E Field caused by cable has max R at 1-10cm
Want to match E field at R w/ air ionizing magnitude. Relate max E to max V saturate bonds. Find max V respect w/ ground.

I don't get at all.

$$\vec{E} = -\nabla V = k_e \frac{q}{r^3} \qquad EA = \frac{Q_{inc}}{\epsilon_0}$$

V in log form

So similar as last problem - ∞ long wire

$$E = \int \frac{\lambda}{\epsilon_0 2\pi r}\, dr =$$

$$V = \frac{\lambda}{\epsilon_0 2\pi} \log(R)$$

$$E = \frac{\lambda}{\varepsilon_0 2\pi R} = 3 \cdot 10^6$$

$$1.88 \cdot 10^8 = \frac{\lambda}{\varepsilon_0 R}$$

electric constant $8.8 \cdot 10^{12}$

$$1.65 \cdot 10^{-4} = \frac{\lambda}{R}$$

$$1.65 \cdot 10^{-4} R = \lambda$$

~~distribution~~ of charge density

$$V_{max} = V_{Radis} - V_{Ground}$$

$$V = \frac{\lambda}{\varepsilon_0 2\pi} \log(R)$$

$$V = \frac{1.65 \cdot 10^{-4} R}{\varepsilon_0 2\pi} \log(R)$$

$8.8 \cdot 10^{12}$

prob complete wrong

$$V = 2.64 \cdot 10^{-5} R \log R$$

Get E field inside sphere - proportional to r

5. Charged Sphere

Find $V(r) - V(0)$
↑ surface ↑ origin

$V = k_e \frac{q}{r}$ superpositioned

for point charges

$V = V_B - V_A = -\int_0^r \vec{E}\, ds$

where are your answers? box them please.

$EA = \frac{\rho V}{\varepsilon_0}$

$E\, 4\pi r^2 = \frac{\rho \frac{4}{3}\pi r^3}{\varepsilon_0}$

$E_{inside} = \frac{\rho\, \cancel{4}\, \cancel{\pi} r^{\cancel{3}}}{3\varepsilon_0\, \cancel{4}\, \cancel{\pi} \cancel{r^2}} = \frac{\rho r}{3\varepsilon_0}$ ✓ proportional to r

$V = -\int_0^r \frac{\rho r}{3\varepsilon_0}\, ds$ $V(R) - V(0)$

$V = -\frac{1}{2}\frac{\rho r^2}{3\varepsilon_0} - 0$ what is ρ? (in terms of known values $Q + R$)

?, what is correct approach

How does it change for $V(\infty)$?

only true if looking at ΔV not V ← ? It does not - you just pick an arbitrary point to be 0 which you measure from

$V(\infty) - V(R)$ 2 regions E diff inside + out of sphere

$$\int_r^{\infty} \frac{\rho r}{3\epsilon_0}$$

wait why 2 regions
- just $\int$ to surface of sphere

Need to find Gauss outside

$$EA = \frac{\rho V}{\epsilon_0}$$

$$E\,4\pi r^2 = \frac{\rho \frac{4}{3}\pi r^3}{\epsilon_0}$$

$$E = \frac{\rho r}{3\epsilon_0}$$ ← but r stays constant everywhere

$$\int \frac{\rho r}{3\epsilon_0}\, ds$$

- now constant so integrate differently

$$\frac{\rho r}{3\epsilon_0} r \Big|$$

$$\frac{\rho r^2}{3\epsilon_0} - \frac{\rho r}{3\epsilon_0}\infty$$

$$V = \frac{\rho r^2}{3\epsilon_0}$$

? is that even close

but ∞

see soln

14/25

$\infty = 0$

$$\frac{Q}{4\pi\epsilon_0 r}\Big|_{r=b}^{r}$$

$$\frac{1}{4\pi\epsilon_0} Q\left(\frac{1}{r} - \frac{1}{b}\right)$$

r working with

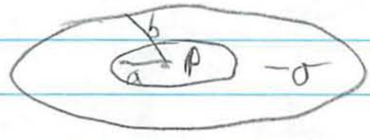
radius of shell

treat field as pt charge

Don't think need this in this problem

Method 1: summing over potential, contributed from each charge density
" 2: from potential V definition $\int_{\infty}^{center} E$. Find E in washer 1st
E = superposition

6. Charged Washer



surface chare
↳ because object is charged

a. If we set $V(\infty) = 0$ what is potential difference?

$V(P) - V(\infty)$?
↑ this is 0 because we set it

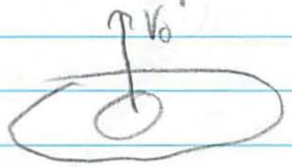
↑ so just measure electric potential from this

Is this where easy to move in 1st charge, others $-\frac{kQ}{d}$

$V(P) = \int_{\infty}^{P} \vec{E}\, ds$ - find E

See other page

b. Electron mass m charge $q = -e$ released at V_0 in upward direction only repelled by washer. What is max speed



$F = ma = \overset{-e}{q} E$
↑ find mass $- 9.1 \cdot 10^{-31}$

$W = q\Delta V = x \cdot m$

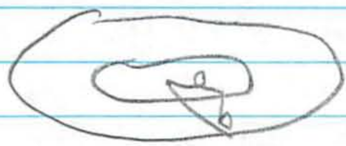
$\Delta V = -\int_{0}^{x} \vec{E}\, ds$

$a = \frac{qE}{m}$

$EA = \frac{\sigma A}{\epsilon_0}$ ← what kind of surface?

Dumashkin

5.



$\int_a^b$

ring $k\frac{q}{r}$

if you $\frac{dq}{dr}$) sum the rings

$\int_a^b \frac{k\,dq}{r}$

$\int_a^b \frac{-k\sigma\, 2\pi \cancel{r}\, dr}{\cancel{r}}$

$= \frac{1}{4\pi\epsilon_0}\,\sigma\, 2\pi (b-a)$

$= \frac{-\sigma}{2\epsilon_0}(b-a)$

-1

Include electric PE as internal energy of system
-apply mech E conservation Electric U + KE
Or work-energy theorem if electrostatic is outside force

$W = \Delta U = q\Delta V$

$$E(\pi b^2 - \pi a^2) = \frac{\sigma(\pi b^2 - \pi a^2)}{\epsilon_0}$$

right idea - where is this work?

$$E = \frac{\sigma\cancel{(\pi b^2 - \pi a^2)}}{\epsilon_0\cancel{(\pi b^2 - \pi a^2)}} = \frac{\sigma}{\epsilon_0}$$

is that right or use ring of charge from class 2 ring of charge is easier - can superimpose potential of rings

$$E = k_e Q \frac{x}{(a^2 + x^2)^{3/2}}$$

$$V = \int_0^x \frac{x}{(a^2 + x^2)^{3/2}}\, ds$$

$$V = \frac{x}{(a^2 + x^2)^{3/2}} - 0$$

$$V = \frac{xm}{q}$$

$$\frac{x}{(a^2 + x^2)^{3/2}} = \frac{xm}{-e}$$

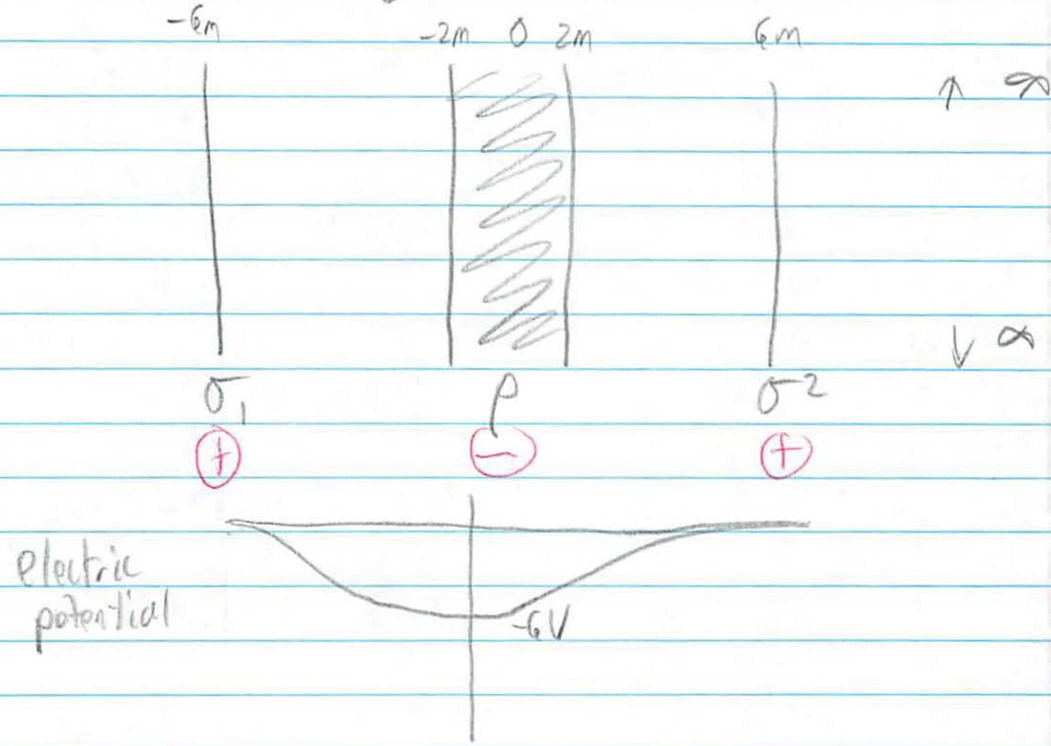
confused how to fit together

$$a = \frac{qE}{m}$$

$$a = \frac{q\left(\frac{x}{(a^2 + x^2)^{3/2}}\right)}{m}$$

15/25

7. Charged Slab + sheets



a. $V(x)$ is linear $-6m < x < -2$.

E Field?

$$E = -\nabla V$$

→ infinite in y and z directions so don't do ???

$$E = -\frac{dV}{dx}$$

$$-\frac{0 - -5\ V}{-6 - -2\ m} = -\frac{+5}{-4} = +5/4\ V/m$$

b. $2 < x < 6$

$$E = -\frac{dV}{dx} = -\frac{-5 - 0\ V}{2 - 6\ m} = -\frac{-5}{-4} = -5/4\ V/m$$

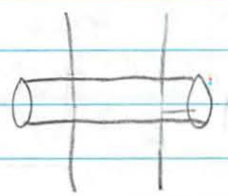
c. $-2 < x < 2$ m

$$V(x) = \frac{5}{16}x^2 - \frac{24}{5} \text{ V}$$

$$E = \frac{dV}{dx} = \frac{5}{16} \cdot 2x - 0$$

$$\frac{10}{16}x = \frac{5}{8}x \text{ V}$$

d. Use Guass' Law to find ρ of slab



Need outside + inside

~~Inside~~

$$\cancel{EA = \frac{\rho x A}{\epsilon_0}}$$

$$\cancel{E = \frac{\rho x}{\epsilon_0}}$$

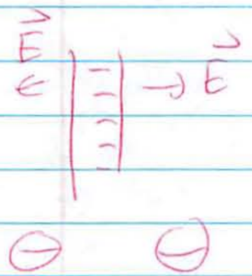
Outside

$$EA = \frac{\rho d \cancel{A}}{\epsilon_0}$$

$$E = \frac{\rho d}{\epsilon_0}$$

is this correct

$$\text{Total} = \cancel{\frac{\rho x}{\epsilon_0} - \frac{\rho x}{\epsilon_0}} + \frac{\rho d}{\epsilon_0} - \frac{\rho d}{\epsilon_0}$$



neg on both sides right?

~~do signs make it cancle~~

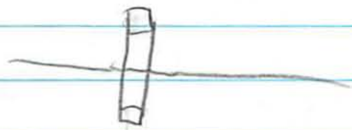
~~yeah total net charge~~ = 0

including σ σ

I should have just done outside all that matters

6. Guass's Law for sheet

Square pill box



$E_{top} A + E_{bottom} A$

$2EA$

$$2EA = \frac{\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0}$$

So $\frac{\sigma}{2\epsilon_0} - \frac{\rho d}{\epsilon_0} - \frac{\rho d}{\epsilon_0} + \frac{\sigma}{2\epsilon_0} = 0$?

Mega Volt ans

from key

1 m from ball

$V = Ed$

$V = 3 \cdot 10^6 \ V/m \cdot 1 \ m$

$V = 3.6 \cdot 10^6 V$

$\boxed{\frac{V}{m} \cdot m = V}$

More like a ball of charge

so use $\frac{kQ}{r}$

and $V \approx Ed$ still

so above ans about right

Minimum charge at breakdown strengh

$$E = \frac{kQ}{r^2}$$

Sphere = 5 cm

$$Q = \frac{r^2 E}{k}$$

$$\frac{(5cm)^2 \cdot 3 \cdot 10^6 \frac{V}{m}}{9 \cdot 10^9 \frac{Vm}{C}}$$

$$\approx 8 \cdot 10^{-7} \ C$$

$$\approx 5 \times 10^{12} e$$

field breaking down furter away so charge 20^2 lage

$$Q \approx 10^{-4} C \approx 5 \cdot 10^{14} e$$

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 Spring 2010

Problem Set 3 Solutions

Problem 1: Concept Questions. Explain your reasoning.

Suppose an electrostatic potential has a maximum at point P and a minimum at point M.

(a) Are either (or both) of these points equilibrium points for a negative charge? If so are they stable?

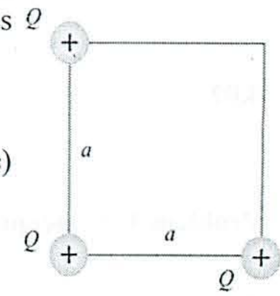
Solution: The electric field is the gradient of the potential, which is zero at both potential minima and maxima. So a negative charge is in equilibrium (feels no net force) at both P & M. However, only the maximum (P) is stable. If displaced slightly from P, a negative charge will roll back "up" hill, back to P. If displaced from M a negative charge will roll away from the potential minimum.

(b) Are either (or both) of these points equilibrium points for a positive charge? If so are they stable?

Solution: Similarly, both P & M are equilibria for positive charges, but only M is a stable equilibrium because positive charges seek low potential (this is probably the case that seems more logical since it is like balls on mountains).

Problem 2: Charges on a Square

Three identical charges $+Q$ are placed on the corners of a square of side a, as shown in the figure.



(a) What is the electric field at the fourth corner (the one missing a charge) due to the first three charges?

Solution: We'll just use superposition:

$$\vec{\mathbf{E}} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{a\hat{\mathbf{i}}}{a^3} + \frac{a\hat{\mathbf{i}} + a\hat{\mathbf{j}}}{\left(\sqrt{2}a\right)^3} + \frac{a\hat{\mathbf{j}}}{a^3}\right) = \frac{Q}{4\pi\varepsilon_0}\left(1 + 2^{-3/2}\right)\left(\hat{\mathbf{i}} + \hat{\mathbf{j}}\right)$$

(b) What is the electric potential at that corner?

Solution: A common mistake in doing this kind of problem is to try to integrate the **E** field we just found to obtain the potential. Of course, we can't do that we only found the **E** field at a single point, not as a function of position. Instead, just sum the point charge potentials from the 3 points:

$$V = \frac{1}{4\pi\varepsilon_0}\sum_{i \neq j}\frac{q_i}{r_{ij}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{a} + \frac{Q}{\sqrt{2}a} + \frac{Q}{a}\right) = \frac{Q}{4\pi\varepsilon_0 a}\left(2 + \frac{1}{\sqrt{2}}\right)$$

(c) How much work does it take to bring another charge, $+Q$, from infinity and place it at that corner?

Solution: The work required to bring a charge $+Q$ from infinity (where the potential is 0) to the corner is:

$$W = Q\Delta V = \frac{Q^2}{4\pi\varepsilon_0 a}\left(2 + \frac{1}{\sqrt{2}}\right)$$

(d) How much energy did it take to assemble the pictured configuration of three charges?

Solution: The work done to assemble three charges as pictured is the same as the potential energy of the three charges already in such an arrangement. Now, there are two pairs of charges situated at a distance of a, and one pair of charges situated at a distance of $\sqrt{2}a$, thus we have

$$W = 2\left(\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{a}\right) + \left(\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{\sqrt{2}a}\right) = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{a}\left(2 + \frac{1}{\sqrt{2}}\right)$$

Alternatively we could have started with empty space, brought in the first charge for free, the second charge in the potential of the first and so forth. We'll get the same answer.

Problem 3: Line of Charge

Consider a very long rod, radius R and charged to a uniform linear charge density λ.

a) Calculate the electric field everywhere outside of this rod (i.e. find $\vec{\mathbf{E}}(\vec{\mathbf{r}})$).

Solution: This is easily calculated using Gauss's Law and a cylindrical Gaussian surface of radius r and length l. By symmetry, the electric field is completely radial (this is a "very long" rod), so all of the flux goes out the sides of the cylinder:

$$\oiint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = 2\pi r l E = \frac{Q_{enc}}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0} \Rightarrow \boxed{\vec{\mathbf{E}} = \frac{\lambda}{2\pi r \varepsilon_0}\hat{\mathbf{r}}}$$

b) Calculate the electric potential everywhere outside, where the potential is defined to be zero at a radius $R_0 > R$ (i.e. $V(R_0) \equiv 0$)

Solution: To get the potential we simply integrate the electric field from R to wherever we want to know it (in this case r):

$$V(r) = V(r) - \underbrace{V(R_0)}_{0} = -\int_{R_0}^{r} \vec{\mathbf{E}}(\vec{\mathbf{r}}') \cdot d\vec{\mathbf{r}}' = -\int_{R_0}^{r} \frac{\lambda}{2\pi r' \varepsilon_0} dr' = -\frac{\lambda}{2\pi\varepsilon_0} \ln(r')\Big|_{R_0}^{r} = \boxed{\frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{R_0}{r}\right)}$$

Problem 4: Estimation: High Voltage Power Lines

Estimate the largest voltage at which it's reasonable to hold high voltage power lines. Then check out this video, care of a Boulder City, Nevada power company. Air ionizes when electric fields are on the order of $3\times10^6\ \mathrm{V\cdot m^{-1}}$.

Solution: In order to answer this question we have to think about what happens if we go to very high voltages. What breaks down? The problem with high voltages is that they lead to high fields. And high fields mean breakdown.

You derived the voltage and field in problem 3
:

$$E(r) = \lambda/2\pi\varepsilon_0 r;\ V(r) = (\lambda/2\pi\varepsilon_0)\ln(R_0/r) \Rightarrow \qquad V(r) = E(r)r\ln(R_0/r)$$

The strongest field, and hence breakdown, appears at $r = R \sim 1$ cm, the radius of a power line (that makes the diameter just under 1 inch – it might be 3 or 4 times that big but probably not ten times). The voltage is defined relative to some ground, either another cable (probably $R_0 \sim 1$ m away) or at the most the real ground ($R_0 \sim 10$ m away). So,

$$V_{\max} = E_{\max} R \ln(R_0/R) = (3\times10^6\ \mathrm{V\cdot m^{-1}})(1\ \mathrm{cm})\ln(10\ \mathrm{m}/1\ \mathrm{cm}) \cong \boxed{2\times10^5\ \mathrm{V}}$$

As it turns out, a typical power-line voltage is about 250 kV, about as large as we estimate here. Some high voltage lines can even go up to 600 kV though (or double that for AC voltages). They must use larger diameter cables.

By the way, you can tell that breakdown is a real concern. In humid weather (during rainstorms for example) you will sometimes hear crackling coming from the power lines. This is corona discharge, a high voltage, low current breakdown, similar to the crackling you hear from the Van de Graff generator in class. The movie is of an arc discharge, a very high current phenomenon that can be very dangerous.

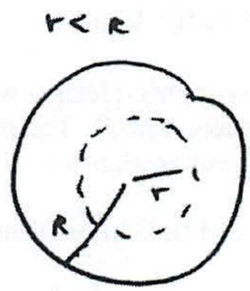
Problem 5: Charged Sphere Consider a uniformly charged sphere of radius R and charge Q. Find the electric potential difference between any point lying on a sphere of radius r and the point at the origin, i.e. $V(r)-V(0)$. Choose the zero reference point for the potential at $r=0$, i.e. $V(0)=0$. How does your expression for $V(r)$ change if you chose the zero reference point for the potential at $r=\infty$, i.e. $V(\infty)=0$.

Solution: In order to solve this problem we must first calculate the electric field as a function of r for the regions $0<r<R$ and $r>R$. Then we integrate the electric field to find the electric potential difference between any point lying on a sphere of radius r and the point at the origin. Because we are computing the integral along a path we must be careful to use the correct functional form for the electric field in each region that our path crosses.

There are two distinct regions of space defined by the charged sphere: region I: $r<R$, and region II: $r>R$. So we shall apply Gauss's Law in each region to find the electric field in that region.

For region I: $r<R$, we choose a sphere of radius r as our Gaussian surface. Then, the electric flux through this closed surface is

$$\oiint \vec{\mathbf{E}}_{\mathrm{I}} \cdot d\vec{\mathbf{A}} = E_I \cdot 4\pi r^2 .$$



The sphere has a uniform charge density $\rho = Q/(4/3)\pi R^3$. Because the charge distribution is uniform, the charge enclosed in our Gaussian surface is given by

$$\frac{Q_{enc}}{\varepsilon_0} = \frac{\rho(4/3)\pi r^3}{\varepsilon_0} = \frac{Q}{\varepsilon_0}\frac{r^3}{R^3}.$$

Now we apply Gauss's Law:

$$\oiint \vec{\mathbf{E}}_{\mathrm{I}} \cdot d\vec{\mathbf{A}} = \frac{Q_{enc}}{\varepsilon_0}.$$

to arrive at:

$$E_I \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}\frac{r^3}{R^3}.$$

which we can solve for the electric field inside the sphere

$$\vec{\mathbf{E}}_{\mathrm{I}} = E_I \hat{\mathbf{r}} = \frac{Qr}{4\pi\varepsilon_0 R^3}\hat{\mathbf{r}}, \; 0 < r < R.$$

For region II: $r > R$: we choose the same spherical Gaussian surface of radius $r > R$, and the electric flux has the same form

$$\oiint \vec{\mathbf{E}}_{\mathrm{II}} \cdot d\vec{\mathbf{A}} = E_{II} \cdot 4\pi r^2.$$

All the charge is now enclosed, $Q_{enc} = Q$, then Gauss's Law becomes

$$E_{II} \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}.$$

We can solve this equation for the electric field

$$\vec{\mathbf{E}}_{\mathrm{II}} = E_{II} \hat{\mathbf{r}} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}}, \; r > R.$$

In this region of space, the electric field falls off as $1/r^2$ as we expect since outside the charge distribution, the sphere acts as if all the charge were concentrated at the origin.

Our complete expression for the electric field as a function of r is then

$$\vec{\mathbf{E}}(r) = \begin{cases} \vec{\mathbf{E}}_{\mathrm{I}} = E_I \hat{\mathbf{r}} = \dfrac{Qr}{4\pi\varepsilon_0 R^3}\hat{\mathbf{r}}, \; 0 < r < R \\ \vec{\mathbf{E}}_{\mathrm{II}} = E_{II} \hat{\mathbf{r}} = \dfrac{Q}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}}, \; r > R \end{cases}$$

We can now find the electric potential difference between any point lying on a sphere of radius r and the origin, i.e. $V(r)-V(0)$.

We begin by considering values of r such that $0<r<R$. We shall calculate the potential difference by calculating the line integral

$$V(r)-V(0)=-\int_{r'=0}^{r'=r}\vec{\mathbf{E}}_{\mathrm{I}}\cdot d\vec{\mathbf{r}}'\ ;\ 0<r<R$$

We use as integration variable r' and integrate from $r'=0$ to $r'=r$:

$$V(r)-V(0)=-\int_{r'=0}^{r'=r}\frac{Qr'}{4\pi\varepsilon_0 R^3}\hat{\mathbf{r}}\cdot dr'\hat{\mathbf{r}}=-\int_{r'=0}^{r'=r}\frac{Qr'}{4\pi\varepsilon_0 R^3}dr'=-\frac{Qr^2}{8\pi\varepsilon_0 R^3};\ 0<r<R$$

For $r>R$: we are taking a path form the origin through regions I and regions II and so we need to use both functional forms for the electric field in the appropriate regions. The potential difference between any point lying on a sphere of radius $r>R$ and the origin is given by the line integral expression

$$V(r)-V(0)=-\int_{r'=0}^{r'=R}\vec{\mathbf{E}}_{\mathrm{I}}\cdot d\vec{\mathbf{r}}'-\int_{r'=R}^{r'=r}\vec{\mathbf{E}}_{\mathrm{II}}\cdot d\vec{\mathbf{r}}'\ ;\ r>R\,.$$

Using our results for the electric field we get that

$$V(r)-V(0)=-\int_{r'=0}^{r'=R}\frac{Qr'}{4\pi\varepsilon_0 R^3}\hat{\mathbf{r}}\cdot dr'\hat{\mathbf{r}}-\int_{r'=R}^{r'=r}\frac{Q}{4\pi\varepsilon_0 r'^2}\hat{\mathbf{r}}\cdot dr'\hat{\mathbf{r}}\ ;\ r>R$$

This becomes

$$V(r)-V(0)=-\int_{r'=0}^{r'=R}\frac{Qr'}{4\pi\varepsilon_0 R^3}dr'-\int_{r'=R}^{r'=r}\frac{Q}{4\pi\varepsilon_0 r'^2}dr'\ ;\ r>R$$

Integrating yields

$$V(r)-V(0)=-\frac{Qr'^2}{8\pi\varepsilon_0 R^3}\bigg|_{r'=0}^{r'=R}+\frac{Q}{4\pi\varepsilon_0 r'}\bigg|_{r'=R}^{r'=r}\ ;\ r>R$$

Substituting in the endpoints yields

$$V(r)-V(0)=V(r)-V(0)=-\frac{Q}{8\pi\varepsilon_0 R}+\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r}-\frac{1}{R}\right);\ r>R$$

A little algebra then yields

$$V(r)-V(0)=\frac{Q}{4\pi\varepsilon_0 r}-\frac{3Q}{8\pi\varepsilon_0 R};\ r>R$$

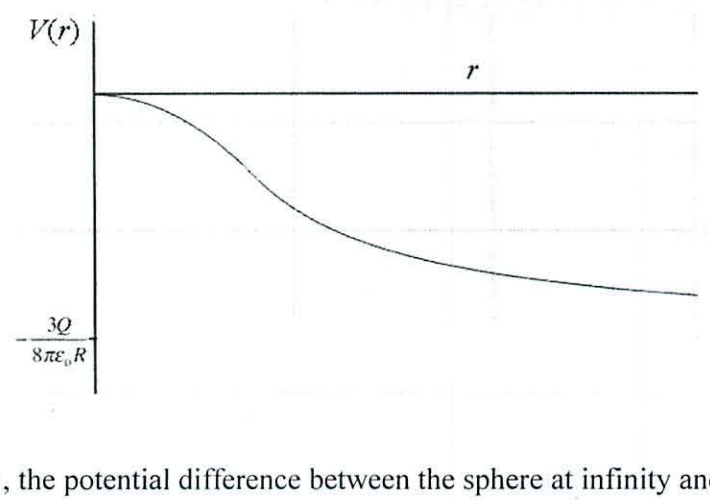
Thus the electric potential difference between any point lying on a sphere of radius r and the origin (where $V(0)=0$) is given by

$$V(r)-V(0)=\begin{cases}-\dfrac{Qr^2}{8\pi\varepsilon_0 R^3};\ 0<r<R\\[2ex] \dfrac{Q}{4\pi\varepsilon_0 r}-\dfrac{3Q}{8\pi\varepsilon_0 R};\ r>R\end{cases}$$

When we set $V(0)=0$, we have an expression for the electric potential function

$$V(r)=\begin{cases}-\dfrac{Qr^2}{8\pi\varepsilon_0 R^3};\ 0<r<R\\[2ex] \dfrac{Q}{4\pi\varepsilon_0 r}-\dfrac{3Q}{8\pi\varepsilon_0 R};\ r>R\end{cases}$$

We plot $V(r)$ vs. r in the figure below. Note that the graph of the electric potential function is continuous at $r=R$.



When we set $r=\infty$, the potential difference between the sphere at infinity and the origin is

$$V(\infty) - V(0) = -\frac{3Q}{8\pi\varepsilon_0 R}.$$

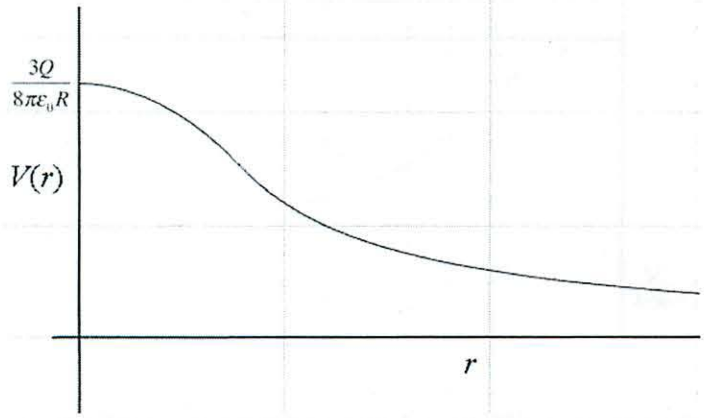
If we had chosen the zero reference point for the electric potential at $r = \infty$, with $V(\infty) = 0$. The with that choice, we have that $V(0) = \frac{3Q}{8\pi\varepsilon_0 R}$. Therefore using our results above the new form for the potential function is

$$V(r) = \begin{cases} V(0) - \frac{Qr^2}{8\pi\varepsilon_0 R^3}; & 0 < r < R \\ V(0) + \frac{Q}{4\pi\varepsilon_0 r} - \frac{3Q}{8\pi\varepsilon_0 R}; & r > R \end{cases}$$

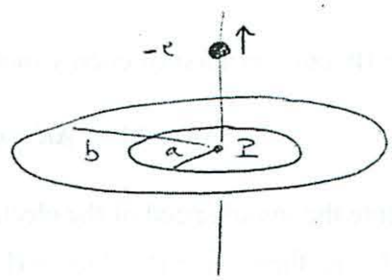
This amounts to just adding the constant $\frac{3Q}{8\pi\varepsilon_0 R}$ to the above results for the potential function $V(r)$ giving

$$V(r) = \begin{cases} \frac{3Q}{8\pi\varepsilon_0 R} - \frac{Qr^2}{8\pi\varepsilon_0 R^3}; & 0 < r < R \\ \frac{Q}{4\pi\varepsilon_0 r}; & r > R \end{cases}.$$

In the above expression we can easily check that $V(\infty) = 0$. Equivalently we shift our previous graph up by $3Q/8\pi\varepsilon_0 R$ as shown in the graph below.



Problem 6: Charged Washer A thin washer of outer radius b and inner radius a has a uniform negative surface charge density $-\sigma$ on the washer (note that $\sigma > 0$).

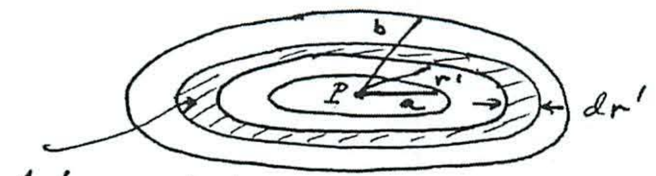


a) If we set $V(\infty) = 0$, what is the electric potential difference between a point at the center of the washer and infinity, $V(P) - V(\infty)$?

Solution: The potential difference $V(P) - V(\infty)$ between infinity and the point P at the center of the washer is given by

$$V(P) - V(\infty) = \int_{source} \frac{k(-\sigma)da'}{|\vec{\mathbf{r}} - \vec{\mathbf{r}}'|}$$

Choose as an integration element a ring of radius r' and width dr' with charge $dq' = (-\sigma)da'$ where $da' = 2\pi r' dr'$.



$dq' = -\sigma\, da' = -\sigma\, 2\pi r' dr'$

Because the field point P is at the origin $\vec{\mathbf{r}} = \vec{\mathbf{0}}$ and the vector from the origin to the any point on the ring is $\vec{\mathbf{r}}' = r'\hat{\mathbf{r}}$, therefore in the above expression the distance from the integration element, the ring, to the field point P is

$$\frac{1}{|\vec{\mathbf{r}} - \vec{\mathbf{r}}'|} = \frac{1}{r'}$$

So the integral becomes

$$V(P) - V(\infty) = \int_{source} \frac{k(-\sigma)da'}{|\vec{\mathbf{r}} - \vec{\mathbf{r}}'|} = \int_{r'=a}^{r'=b} \frac{k(-\sigma)2\pi r' dr'}{r'} = -k\sigma 2\pi(b - a)$$

b) An electron of mass m and charge $q = -e$ is released with an initial speed v_0 from the center of the hole (at the origin) in the upward direction (along the perpendicular axis to the washer) experiencing no forces except repulsion by the charges on the washer. What speed does it ultimately obtain when it is very far away from the washer (i.e. at infinity)?

Solution: By conservation of energy (note that $V(\infty) - V(P) = k\sigma 2\pi(b-a) > 0$)

$$0 = \Delta K + \Delta U = \Delta K + q(V(\infty) - V(P)) = \Delta K - ek\sigma 2\pi(b-a)\,.$$

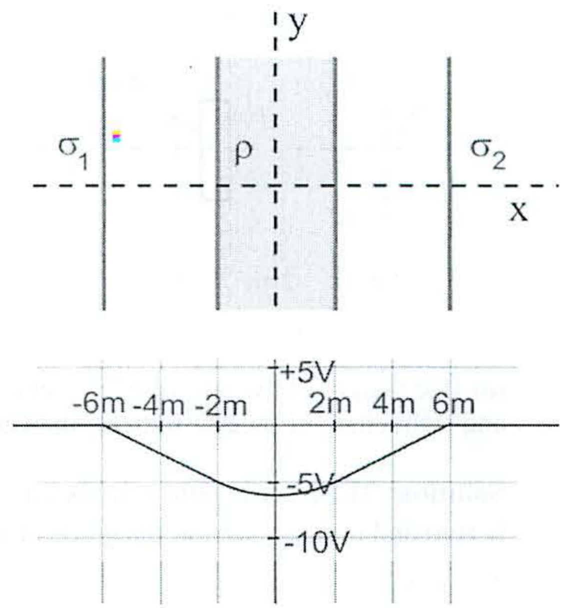
If we denote the initial speed of the electron by v_0 and the speed of the electron when it is very far away by v_f then $\Delta K = (1/2)mv_f^2 - (1/2)mv_0^2$. Hence

$$(1/2)mv_f^2 - (1/2)mv_0^2 = ek\sigma 2\pi(b-a) > 0\,.$$

We can now solve for the final speed of the electron when it is very far away from the washer

$$v_f = \sqrt{v_0^2 + ek\sigma 4\pi(b-a)/m}\,.$$

Problem 7: Charged Slab & Sheets



An infinite slab of charge carrying a charge per unit volume ρ has a charged sheet carrying charge per unit area σ_1 to its left and a charged sheet carrying charge per unit area σ_2 to its right (see top part of sketch). The lower plot in the sketch shows the electric potential $V(x)$ in volts due to this slab of charge and the two charged sheets as a function of horizontal distance x from the center of the slab. The slab is 4 meters wide in the x-direction, and its boundaries are located at $x = -2$ m and $x = +2$ m, as indicated. The slab is infinite in the y direction and in the z direction (out of the page). The charge sheets are located at $x = -6$ m and $x = +6$ m, as indicated.

(a) The potential $V(x)$ is a linear function of x in the region $-6\text{ m} < x < -2\text{ m}$. What is the electric field in this region?

Solution:

$$\vec{\mathbf{E}} = -\frac{\partial V}{\partial x}\hat{\mathbf{i}} = -\frac{\Delta V}{\Delta x}\hat{\mathbf{i}} - \frac{-5\text{ V}}{4\,m} = 1.25\frac{\text{V}}{m}\hat{\mathbf{i}}$$

(b) The potential $V(x)$ is a linear function of x in the region $2\text{ m} < x < 6\text{ m}$. What is the electric field in this region?

Solution:

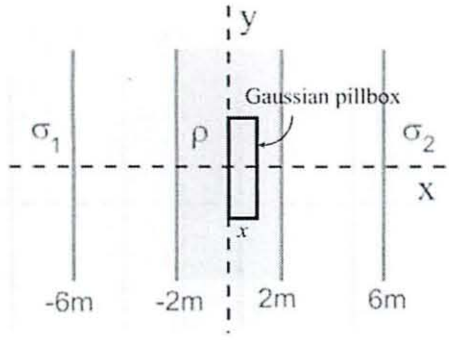
$$\vec{\mathbf{E}} = -\frac{\partial V}{\partial x}\hat{\mathbf{i}} = -\frac{\Delta V}{\Delta x}\hat{\mathbf{i}} = -\frac{5\text{ V}}{4\,m} = -1.25\frac{\text{V}}{m}\hat{\mathbf{i}}$$

(c) In the region $-2\text{m} < x < 2\text{ m}$, the potential $V(x)$ is a quadratic function of x given by the equation $V(x) = \frac{5}{16}x^2\frac{\text{V}}{m^2} - \frac{25}{4}\text{V}$. What is the electric field in this region?

Solution: In the region inside the slab, the electric field is

$$\vec{\mathbf{E}} = -\frac{\partial V}{\partial x}\hat{\mathbf{i}} = \left[-\frac{5}{8}\frac{\text{V}}{m^2}\right]x\hat{\mathbf{i}}$$

(d) Use Gauss's Law and your answers above to find an expression for the charge density ρ of the slab. Indicate the Gaussian surface you use on a figure.



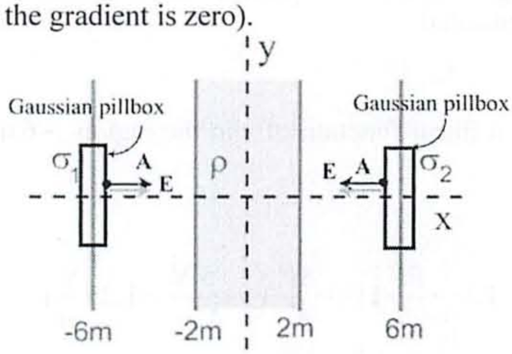
Solution:

$$\oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = EA = \left[-\frac{5}{8}\frac{\text{V}}{m^2}\right] xA = \frac{q_{in}}{\varepsilon_0} = \frac{\rho x A}{\varepsilon_0}$$

$$\Rightarrow \rho = \left[-\frac{5}{8}\frac{\text{V}}{m^2}\right]\varepsilon_0$$

(e) Use Gauss's Law and your answers above to find the two surface charge densities of the left and right charged sheets. Indicate the Gaussian surface you use on a figure.

Solution: The electric field vanishes in the regions $x > 6$ m and $x < -6$ m (the electric potential is zero and remains zero so the gradient is zero).



Using Gauss's law with the Gaussian pillboxes indicated in the figure, we have

$$\oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = EA = \left[\frac{5}{4}\frac{\text{V}}{m}\right] A = \frac{q_{in}}{\varepsilon_0} = \frac{\sigma_1 A}{\varepsilon_0}$$

$$\Rightarrow \sigma_1 = \left[\frac{5}{4}\frac{\text{V}}{m}\right]\varepsilon_0$$

In a similar manner, $\sigma_2 = \frac{5}{4}\frac{\text{V}}{m}\varepsilon_0$.

A common mistake is to think that the sign must flip because the electric field sign flips. Note that because the area vector of the Gaussian pillbox also flips direction this is NOT true. It is very important to draw pictures and show the vector directions. If the vectors ($\vec{\mathbf{E}}$ and $d\vec{\mathbf{A}}$) are in the same direction then the dot product (and the enclosed charge) is positive.

Physics Review Exam

2/23

Colomb's Law

- electrostatic interaction b/w charged particles

$$F = k_e \frac{q_1 q_2}{r^2}$$

vector r_{21} from $2 \rightarrow 1$

Why is this seeming easy to me

Reviewed on weekend

Like math

- I think I know it, but can't do it or run into trouble w/ subtilies

What should I review

- do pratice problems

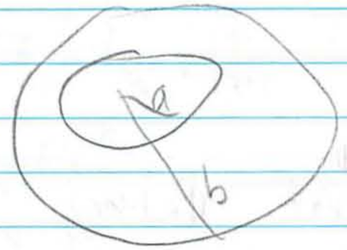
This semester its not just 1 class per day

- lots of work on weekend
- small work weekdays

After fixing p-set seems really hard

Rings of charge
Dumaskin

2/23



~~ring~~ $\frac{kq}{r}$ ~~at~~ pt charge at 0

dq dr) sum rings

each ring is = distance away from center so sum

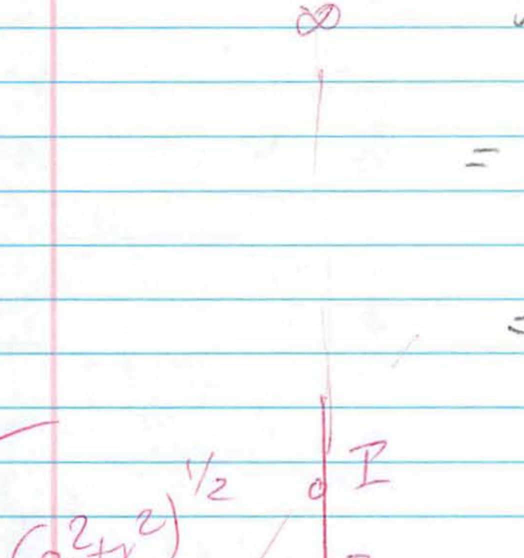
$$\int_a^b \frac{k\,dq}{r} \quad q = \sigma A$$

$$\int_a^b \frac{k\sigma 2\pi r\,dr}{r}$$

$$= \frac{1}{4\pi\epsilon_0}\sigma 2\pi (b-a)$$

$$= \frac{\sigma}{2\epsilon_0}(b-a)$$

for 1 ring



$$= -\int_\infty^P \frac{kq_s\,dr}{r^2}$$

$$\phi(P) - \phi(\infty) = \frac{kdq_s}{r_{sp}}$$

add charges around in a circle

$$= \int_{ring} \frac{k\,dq_s}{(r^2+z^2)^{1/2}}$$

$$= \frac{k}{(r^2+z^2)^{1/2}}\int_{ring} dq_s = \frac{kq_{ring}}{(r^2+z^2)^{1/2}}$$

then add rings up

r becomes variable
-each a different distance
from center

*

Dumaskin Review Session

2/23

1. Discrete charges = Sources

Classic qu

Source distribution

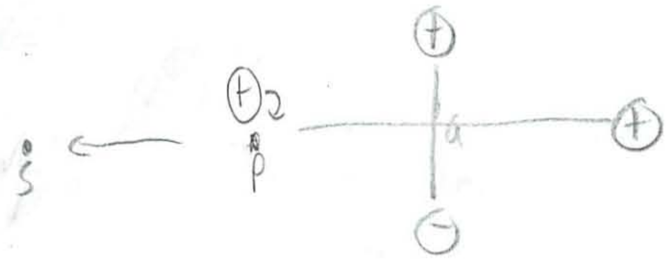
- electric field
- potential difference

$\vec{V}(F) - \vec{V}(P)$

reference pt

- how much energy does it take to assemble source

If place additional charge near these sources



What is force on q at pt P

$F_q = Q\,\vec{E}_s(P)$

Move q from P to S

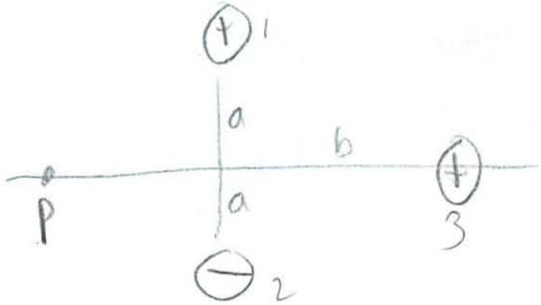
$\Delta U = U(S) - U(P) = Q\,\Delta V_s$

potential

If release Q from rest, at P, what is its Speed at S

$\Delta k + \Delta U = 0$

(2)

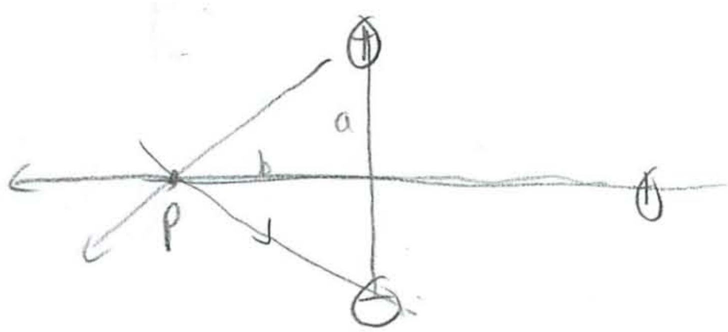


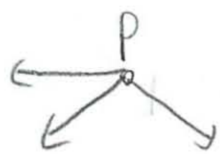
✱ E from + to −

- superposition + vector addition

$$\vec{E}_p = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

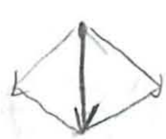
- can draw vectors knowing + −
- or use $\hat{r}$ q formula approch





all same lenght

$$E_3 = -\hat{\imath} \rightarrow \frac{kq}{d^2} = \frac{kq}{(2b)^2}\,\hat{\imath}$$

$E_1 + E_2$  $-\hat{\jmath} \rightarrow \left(2|\vec{E}_1|\cos\theta \;-\hat{\jmath}\right)$

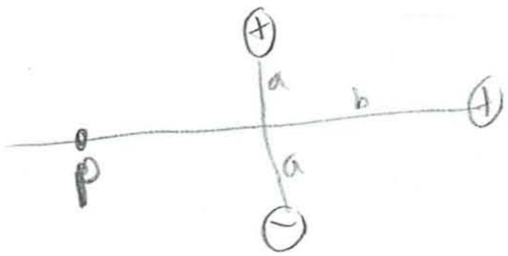
$$|E_1| = \frac{kq}{d^2} = \frac{kq}{\sqrt{a^2+b^2}^{\,2}} = \frac{kq}{a^2+b^2}$$

$\cos\theta =$  $= \dfrac{a}{\sqrt{a^2+b^2}}$

$$\vec{E}(P) = \frac{2kq}{(a^2+b^2)} \frac{a}{a^2+b^2}(-\hat{j}) - \frac{kq}{4b^2}\hat{\imath}$$

* now about adding vectors
* de nom always same
* could decompose into $\hat{\imath}$ $\hat{j}$
* So I was <u>wrong</u> w/ writing d in numerator
  - or it only worked here

Scalar Potential



single charge

$$V(P) - V(\infty) = \frac{k q_s}{r_{sp}}$$

↑ integral of electric field

$$V(P) - V(\infty) = -\int_{\infty}^{P} \vec{E} \cdot ds = -\int \frac{k q_s}{r^2} d$$

on straight li
only ....

dir of path by dir of points

$$= \frac{k q_s}{r}\Big|_{\infty}^{r_{sp}}$$

(5) Difference √ factor out

$$V(s) - V(P) = \frac{kq}{b}\left(\frac{1}{3} - \frac{1}{2}\right) = \frac{kq}{6b}$$

$$\Delta U = Q\,\Delta V_s = -\frac{Q\,kq}{6b}$$

$$\Delta k + \Delta U = 0$$

$$\Delta k = -\Delta U = \frac{Q\,kq}{6b}$$

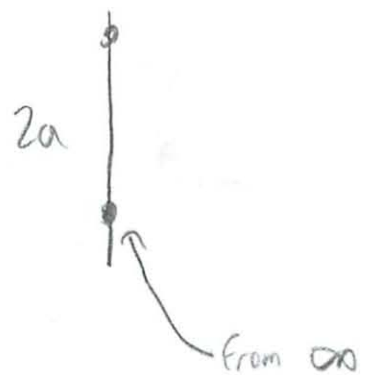
$$\frac{1}{2} m v_f^2 - 0 = \frac{Q\,kq}{6b}$$

$$v_f = \sqrt{\frac{2Q\,kq}{6mb}}$$

How much energy to assemble these charges

First is free

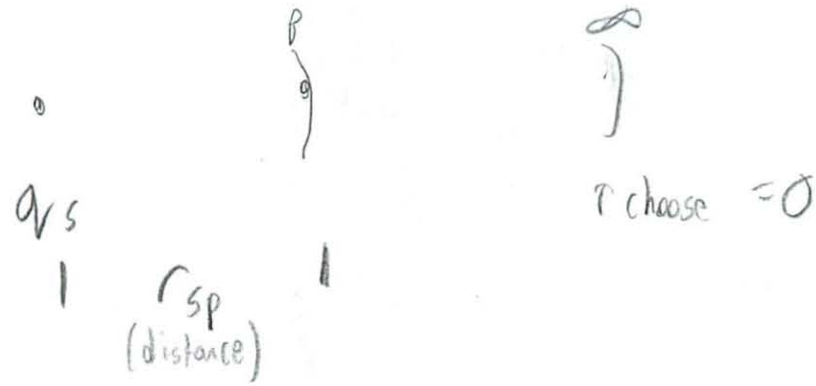
2nd



$$\Delta U_2 = -q\left(V(P) - V(\infty)\right)$$

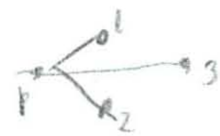
$$= -q\,\frac{kq}{2a}$$

(4)



Can do superposition + add them

$V(\infty) = 0$



$V(P) = V_1(P) + V_2(P) + V_3(P)$ - scalar, no vectors

$$\frac{kq}{(a^2+b^2)^{1/2}} - \frac{kq}{(a^2+b^2)^{1/2}} + \frac{kq}{2b} \leftarrow \frac{kq}{r} \text{ not } \frac{kq}{r^2}$$

$$= \frac{kq}{2b}$$

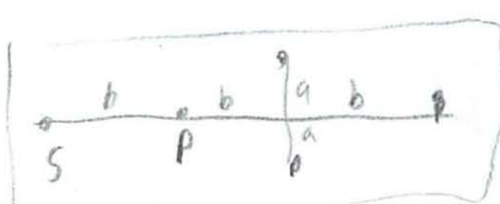
If $V(\infty) \neq 0$

then have to add it for each charge

$V(\infty) + \frac{kq}{r}$

If move $P \to S$

Remember P_1 P_2 cancle



$V(S) = \frac{kq}{3b}$ by itself

⑥ Bring 3rd in
- ~~Now~~ Must sum energy w/ 1 and energy w/ 2

$$\Delta U = \Delta U_{12} + \Delta U_{13} + \Delta U_{23}$$

$$= (-q)\frac{kq}{2a} + q\frac{kq}{\sqrt{a^2+b^2}} + \frac{(q)(k)(-q)}{\sqrt{a^2+b^2}}$$

$$= -\frac{kq^2}{2a}$$

↑ ⊖ sign means does it on own
energy stored in config

How to use Gauss' Law

3 types of problems

spheres
cylinders
planes

- or combos, or concentrics
slabs, planes

need to know how to choose right surface
! - draw pic
- where is charge inclosed

straighten out ρ, λ, σ
is it just σV or do I need to integrate
↑ σ constant ↑ σ varing

↑ do example of this

(7) No conductors on exam

↑ E field is 0

but would not have to know

Guass' Law, potential difference, PE difference

1. Must be enough symmetry
2. Find E everywhere

$\rho = \frac{\text{charge}}{\text{Volume}}$

So $5 \to \frac{C}{m^4}$ do the units!

↳ make $5 = k$

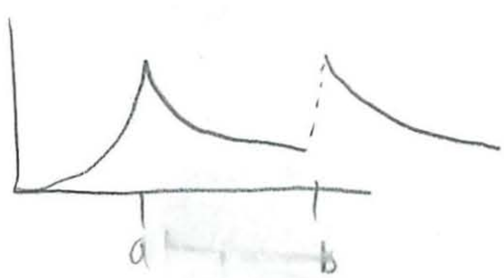
$\rho = kr \quad r < a$

$= 0$ elsewhere

$$\vec{E} = \begin{cases} E_I & 0 < r < a \\ E_{II} & a < r < b \\ E_{III} & r > b \end{cases}$$

piecewise function

don't just add



↓

8) $0 < r < a$

$\oiint \vec{E} \cdot d\vec{a}$	$\dfrac{q_{inc}}{\varepsilon_0}$
$E_1 \cdot 4\pi r^2$	$\dfrac{1}{\varepsilon_0} \int \rho \, dv$

how much is pointing out - just surface area here

what is the dV

$$\underbrace{dV = 4\pi r'^2}_{\text{area}} \underbrace{dr}_{\text{thickness}} \text{ of shell}$$

$$\rho \, dV = hr' \, 4\pi r'^2 \, dr$$

$$\int_0^r \rho \, dV = \int_0^r hr' \, 4\pi r'^2 \, dr$$

radius guassian surface you picked

1 D integral

$$q_{inc} = \int_0^r hr' \, 4\pi r'^2 \, dr$$

$$= h 4\pi \int_0^r r'^3 \, dr'$$

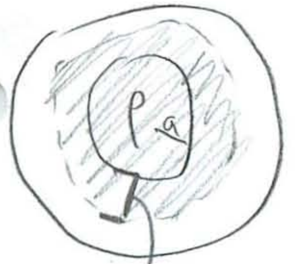
$$= \frac{h 4\pi r^4}{4}$$

$$= h r^4$$

$E_1 4\pi r^2$	$\dfrac{h\pi r^4}{\varepsilon_0}$

$$\boxed{E_1 = \frac{h}{\varepsilon_0} r^2 \hat{r} \quad 0 < r < a}$$

(9) $a < r < b$

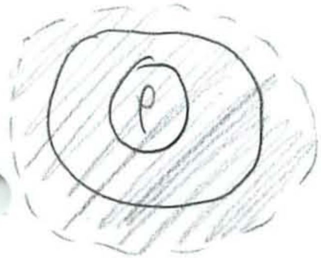


no charge here nothing to integrate here

$\oint E \cdot da$	$\frac{q_{inc}}{\varepsilon_0}$
$E_2 4\pi r^2$	$\frac{1}{\varepsilon_0}\int_0^a h r' 4\pi r'^2 \, dr'$
$E_2 =$	$\frac{1}{4\pi r^2 \varepsilon_0} h 4\pi \frac{a^4}{4}$ ← may be wrong
$E_2 =$	$\frac{h a^4}{4\varepsilon_0}\frac{1}{r^2}\hat{r} \quad a < r < b$ ← right

↑ inverse square

$r < b$



$E \, da$	$\frac{q_{inc}}{\varepsilon_0}$
~~$E_3 4\pi r^2$~~	~~$\frac{1}{\varepsilon_0}\int_0^b h r' 4\pi r'^2 \, dr'$~~
~~$E_3 =$~~	~~$\frac{1}{4\pi r^2 \varepsilon_0}\frac{h 4\pi b^4}{4}$~~
~~$E_3 =$~~	~~$\frac{h b^4}{4\varepsilon_0}\frac{1}{r^2}\hat{r}$~~

$$\boxed{E_3 = \frac{h a^4}{4\varepsilon_0}\frac{1}{r^2}\hat{r} + \frac{Q}{4\pi\varepsilon_0}\frac{1}{r^2}\hat{r}}$$ superposition argument

or traditional

$E_3 4\pi r^2$	$\frac{1}{\varepsilon_0}\left(\int_0^a h r' 4\pi r'^2 \, dr' + Q\right)$
$E_3 =$	$\frac{1}{4\pi\varepsilon_0}\frac{1}{r^2}\left(\frac{h 4\pi a^2}{4} + Q\right)$

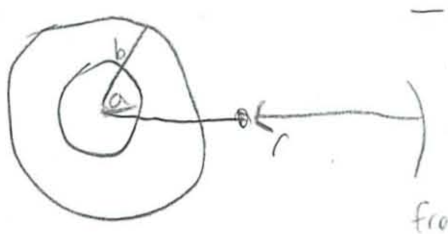
not finding gradients

(10) If need $V(R)$ everywhere in space
- choose where to have it 0
- this is the difficult part

can do ∞ or 0
- does not matter
- will choose $V(\infty)=0$

$$V(r) - \underbrace{V(\infty)}_{0} = \begin{cases} \_\_\_ & r > b \\ \_\_\_ & a < r < b \\ \_\_\_ & r < a \leftarrow \text{hardest} \end{cases}$$

$\underline{r > b}$



$$-\int_{\infty}^{r} \vec{E}_3 \cdot ds$$

from $\infty \to r$

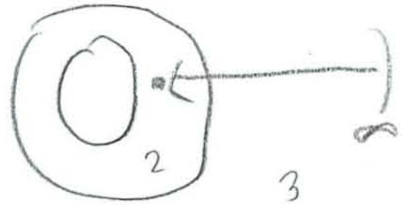
$$= -\int_{\infty}^{r} \frac{Q_{inc}}{4\pi\epsilon_0} \frac{dr'}{r'^2}$$

$$\boxed{-\int^{r} \frac{dr}{r^2} = \frac{1}{r}}$$

looks just like formula for pt charge

$$= \frac{Q_{inc}}{4\pi\epsilon_0} \frac{1}{r}$$

(11) $a < r < b$



going through regions 3 and 2

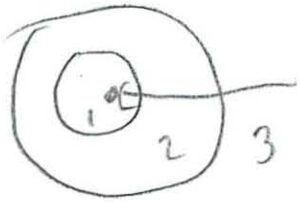
$$V(r) - V(\infty) = -\int_{\infty}^{b} \vec{E_3} \cdot ds - \int_{b}^{r} \vec{E_2} \cdot \vec{ds}$$

$$= -\int_{\infty}^{b} \frac{Q_{inc}}{4\pi\epsilon_0} \frac{\pi}{r^2} dr - \int_{b}^{r} \frac{ha^4}{4\epsilon_0} \frac{1}{r^2} dr$$

$$= \frac{Q_{enc}}{4\pi\epsilon_0} \frac{1}{b} + \frac{ha^4}{4\epsilon_0} \left(\frac{1}{r} - \frac{1}{b}\right)$$

$Q_{inc} = h\pi a^4 + Q$ ← is a shortcut so don't have to rewrite

$r < a$



$$V(r) - V(\infty) = -\int_{\infty}^{b} \vec{E_3} \cdot ds - \int_{b}^{a} \vec{E_2} \cdot ds - \int_{a}^{r} \vec{E_1} \cdot ds$$

---

the guassian surface is the variable

get E field for each part (are vectors)

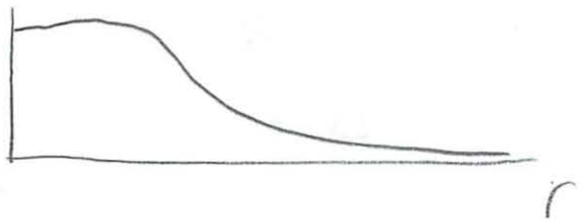
The potential difference transverses a path

- need E field for each region

(12) $V(0) = V(\infty) + \Delta V_{0\infty}$

↑ you calculate

$r(\infty) = 0$



$\infty \rightarrow$

Just where you start from

$r(0) = 0$

$\infty \rightarrow$

Be able to do this for planes + cylinders

↑ easy

↑ bit tricker

Not doing find E from V

Class 10: Outline

Hour 1 & 2:

Review

Concept Review / Overview

PRS Questions – possible on exam

Sample Exam

Exam Thursday: 7:30 – 9:30 pm

See announcements page for section room assignments

Exam One: Review

Class 13: Outline

Hour 1:

Concept Review / Overview

PRS Questions – possible on exam

Hour 2:

Sample Exam

Exam Thursday: 7:30 – 9:30 pm

Exam 1 Topics

- Fields (visualizations)
- Electric Field & Potential
  - Discrete Point Charges
  - Continuous Charge Distributions
  - Symmetric Distributions – Gauss's Law
- Conductors and Insulators

General Exam Suggestions

- You should be able to complete every problem
  - If you are confused, ask
  - If it seems too hard, think some more
  - Look for hints in other problems
  - If you are doing math, you're doing too much
- Read directions completely (before & after)
- Write down what you know before starting
- Draw pictures, define (label) variables
  - Make sure that unknowns drop out of solution
- Don't forget units!

Have time to think

no hard math

- pictures + units

~~Area should usually~~

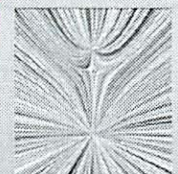
What You Should Study

- Review Friday Problem Solving (& Solutions)
- Review In Class Problems (& Solutions)
- Review PRS Questions (& Solutions)
- Review Problem Sets (& Solutions)
- Review PowerPoint Presentations
- Review Relevant Parts of Study Guide (& Included Examples)
- Do **Sample Exams** (online under Exam Prep)

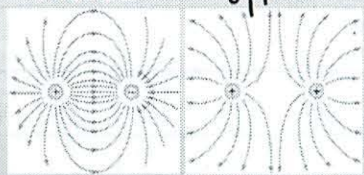
different

Fields

opposite



Grass Seeds
Know how to read



Field Lines
Know how to draw

- Field line density tells you field strength
- Lines have tension (want to be straight)
- Lines are repulsive (want to be far from other lines)
- Lines begin and end on sources (charges) or ∞

P10 - 7

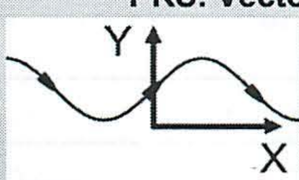
from ⊕ → ⊖

PRS Questions: Fields

P10 - 8

PRS: Vector Field

:20



The field line at left corresponds to the vector field:

1. $\vec{\mathbf{F}}(x,y)=\sin(x)\,\hat{\mathbf{i}}+\hat{\mathbf{j}}$
2. $\vec{\mathbf{F}}(x,y)=\hat{\mathbf{i}}+\sin(x)\,\hat{\mathbf{j}}$
3. $\vec{\mathbf{F}}(x,y)=\cos(x)\,\hat{\mathbf{i}}+\hat{\mathbf{j}}$
4. $\vec{\mathbf{F}}(x,y)=\hat{\mathbf{i}}+\cos(x)\,\hat{\mathbf{j}}$
5. I don't know

0% 0% 0% 0% 0%

1 2 3 4 I don't know

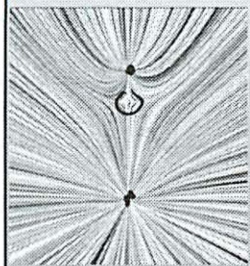
P10 - 9

at origin has (+,+) sin would have 0

↑ x component constant

PRS: Grass Seeds

20



Closer ↑

The vector field at left is created by:

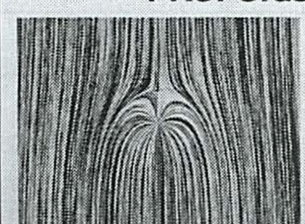
0% 1. Two sources (equal strength)
0% 2. Two sources (top stronger)
0% 3. Two sources (bottom stronger)
0% 4. Source & Sink (equal strength)
0% 5. Source & Sink (top stronger)
0% 6. Source & Sink (bottom stronger)
0% 7. I don't know

P10-10

* remember identical so repel
- could also be 2 sinks

PRS: Grass Seeds

20

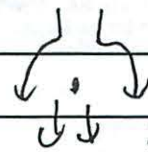


Here there is an initial downward flow.

↑ need to know

0% 1. The point is a source
0% 2. The point is a sink
0% 3. I don't know

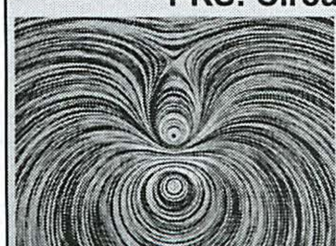
P10-11



source as well

PRS: Circulation

20



These two circulations are in:

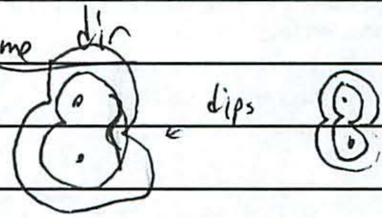
0% 1. The same direction (e.g. both clockwise)
0% 2. Opposite directions (e.g. one cw, one ccw)
0% 3. I don't know

P10-12

field lines appear to add in the middle of them

same dir

dips



lines in middle = opposite

PRS: Vector Field

20

The grass seeds field plot at left is a representation of the vector field:

0% 1. $\vec{F}(x,y) = x^2\hat{i} + y^2\hat{j}$

0% 2. $\vec{F}(x,y) = y^2\hat{i} + x^2\hat{j}$

0% 3. $\vec{F}(x,y) = \sin(x)\hat{i} + \cos(y)\hat{j}$

0% 4. $\vec{F}(x,y) = \cos(x)\hat{i} + \sin(y)\hat{j}$

0% 5. I don't know

P10-13

X going in ↑ direction (x^2)

Y going → dir (x^2)

PRS: Electric Field

:20

Two opposite charges are placed on a line as shown below. The charge on the right is three times larger than the charge on the left. Other than at infinity, where is the electric field zero?

0% 1. Between the two charges

0% 2. To the right of the charge on the right ✓

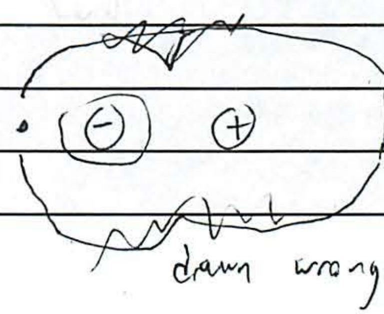
0% 3. To the left of the charge on the left

0% 4. The electric field is nowhere zero

0% 5. Not enough info – need to know which is positive

0% 6. I don't know

P10-14



PRS: Field Lines

Electric field lines show:

1. Directions of forces that exist in space at all times.
2. Directions in which charges on those lines will accelerate.
3. Paths that charges will follow.
4. More than one of the above.
5. I don't know.

Remember: Don't pick up until you are ready to answer

P10-15

= force per unit charge

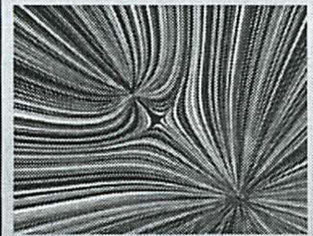
field in dir of force, acc

1. is also correct kinda

 Force is tangent to field lines

show dir ⊕ acc

PRS: Force

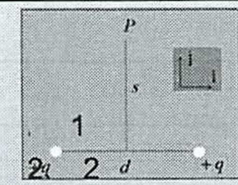


The force between the two charges is:

- 0% 1. Attractive
- 0% 2. Repulsive
- 0% 3. Can't tell without more information
- 0% 4. I don't know

P10-16

PRS: Equal Charges



Electric field at P is:

1. $\vec{E} = \frac{2k_e q s}{\left[s^2 + \frac{d^2}{4}\right]^{3/2}}\hat{j}$
2. ~~$\vec{E} = -\frac{2k_e q d}{\left[s^2 + \frac{d^2}{4}\right]^{3/2}}\hat{i}$~~
3. $\vec{E} = \frac{2k_e q d}{\left[s^2 + \frac{d^2}{4}\right]^{3/2}}\hat{j}$
4. $\vec{E} = -\frac{2k_e q s}{\left[s^2 + \frac{d^2}{4}\right]^{3/2}}\hat{i}$
5. I Don't Know

s →

d →

P10-17

Remeber denom same, put "dir" in numerator

E superposition

Should be ↑

↑ will cancle

look at limiting cases

if $s = 0$ → field will go to 0 ~~correct~~ so 1

PRS: 5 Equal Charges

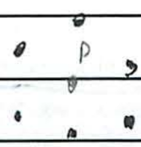
0

Six equal positive charges q sit at the vertices of a regular hexagon with sides of length R. We remove the bottom charge. The electric field at the center of the hexagon (point P) is:

1. $\vec{E} = \frac{2kq}{R^2}\hat{j}$
2. $\vec{E} = -\frac{2kq}{R^2}\hat{j}$
3. $\vec{E} = \frac{kq}{R^2}\hat{j}$
4. $\vec{E} = -\frac{kq}{R^2}\hat{j}$
5. $\vec{E} = 0$
6. I Don't Know

P10-18

know → ↑



and its 1 q

could look at pairs ✓

✓ got right

also like adding ⊖ charge at bottom

dipole moment

$\vec{p}$ = charge × displacement

(review this

PRS: Dipole Field

As you move to large distances r away from a dipole, the electric field will fall-off as:

0% 1. $1/r^2$, just like a point charge

0% 2. More rapidly than $1/r^2$

0% 3. More slowly than $1/r^2$

0% 4. I Don't Know

P10-19

exist a lot in nature

as you go far away - just looks like a single charge

$\frac{1}{r^3}$ - quenches it

$k_e q\left(\frac{x}{(x^2+(y-a)^2)^{3/2}}-\frac{x}{(x^2+(y-a)^2)^{3/2}}\right)$

$\hookrightarrow \frac{\hat{r}}{r^2}=\frac{\vec{r}}{r^3}$

study more

PRS: Dipole in Non-Uniform Field

A dipole sits in a non-uniform electric field E

Due to the electric field this dipole will feel:

0% 1. force but no torque

0% 2. no force but a torque

0% 3. both a force and a torque

0% 4. neither a force nor a torque

:00

P10-20

dipole wants to rotate to align w/ a field

E Field and Potential: Creating

A point charge q creates a field and potential around it:

$$\vec{\mathbf{E}}=k_e\frac{q}{r^3}\vec{\mathbf{r}};\ V=k_e\frac{q}{r}$$

Use superposition for systems of charges

They are related:

$$\vec{\mathbf{E}}=-\nabla V;\ \Delta V\equiv V_B-V_A=-\int_A^B\vec{\mathbf{E}}\cdot d\vec{\mathbf{s}}$$

P10-21

or superposition

field = force / unit charge

$E=\frac{kq_r}{r^3}$ $V=k_e\frac{q}{r}$ ← if $V(\infty)$ is 0

E Field and Potential: Creating

Discrete set of point charges:

$$\vec{\mathbf{E}} = k_e \frac{q}{r^3}\vec{\mathbf{r}};\ V = k_e \frac{q}{r}$$

Add up from each point charge

Continuous charge distribution:

$$d\vec{\mathbf{E}} = k_e \frac{dq}{r^3}\vec{\mathbf{r}};\ dV = k_e \frac{dq}{r}$$

Break charged object into small pieces, *dq*, and integrate

by integrating

P10-22

Continuous Sources: Charge Density

Charge Densities:

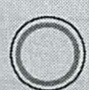
$$\lambda = \frac{Q}{L} \qquad \sigma = \frac{Q}{A} \qquad \rho = \frac{Q}{V}$$

$$dQ = \lambda\, dL \qquad dQ = \sigma\, dA \qquad dQ = \rho\, dV$$

Don't forget your geometry:

$dL = dx$

$dL = R d\theta$



$dA = 2\pi r dr$

$dV_{cyl} = 2\pi r l dr$

$dV_{sphere} = 4\pi r^2 dr$

P10-23

Symmatry

E Field and Potential: Creating

Discrete set of point charges:

$$\vec{\mathbf{E}} = k_e \frac{q}{r^2}\hat{\mathbf{r}};\ V = k_e \frac{q}{r}$$

Add up from each point charge

Continuous charge distribution:

$$d\vec{\mathbf{E}} = k_e \frac{dq}{r^2}\hat{\mathbf{r}};\ dV = k_e \frac{dq}{r}$$

Break charged object into small pieces, *dq*, and integrate

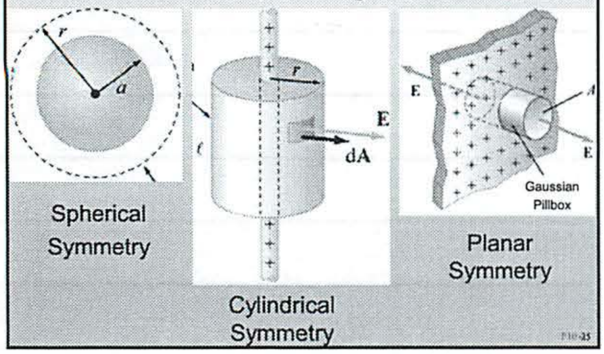
Symmetric charged object:

$$\oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{in}}{\varepsilon_0};\ \Delta V \equiv -\int \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

Use Gauss' law to get E everywhere, then integrate to get V

P10-24

Gauss's Law: $\oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{in}}{\varepsilon_0}$



P10-25

Came from experiment

pillbox = cylinder - but use top

Whats the symmetry to make it easy to use

E Field and Potential: Effects

If you put a charged particle, *q*, in a field:

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}}$$

To move a charged particle, *q*, in a field:

$$W = \Delta U = q\Delta V$$

change in potential

P10-26

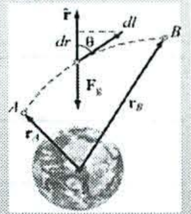
PRS Questions: Electric Fields and Potential

P10-27

PRS: Sign of W_g

Thinking about the sign and meaning of this…

$$W_g = GMm\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$



Moving from r_A to r_B:

- 0% 1. W_g is positive – we do work
- 0% 2. W_g is positive – gravity does work
- 0% 3. W_g is negative – we do work
- 0% 4. W_g is negative – gravity does work
- 0% 5. I don't know

P10-28

W_g ⊖ $\quad U_B - U_A \quad A \to B \quad \int_A^B$

W_g = work of gravity field = − we work do

PRS: Masses in Potentials

Consider 3 equal masses sitting in different gravitational potentials:

A) Constant, zero potential
B) Constant, non-zero potential
C) Linear potential ($V \propto x$) but sitting at $V = 0$

Which statement is true?

- 0% 1. None of the masses accelerate
- 0% 2. Only B accelerates
- 0% 3. Only C accelerates
- 0% 4. All masses accelerate, B has largest acceleration
- 0% 5. All masses accelerate, C has largest acceleration
- 0% 6. I don't know

P10-29

Potential is like height

If height constant - does not acc. only if changing

knew that, it tricked me

here we go again

PRS: Positive Charge

Place a positive charge in an electric field. It will accelerate from

- 0% 1. higher to lower *electric potential*; lower to higher *potential energy*
- 0% 2. higher to lower *electric potential*; higher to lower *potential energy*
- 0% 3. lower to higher *electric potential*; lower to higher *potential energy*
- 0% 4. lower to higher *electric potential*; higher to lower *potential energy*

P10-30

All high → low PE

(+) high → low V $\quad -\int \vec{E} \cdot d\vec{s}$

↑ neg, so V ↓

PRS: Negative Charge

Place a negative charge in an electric field. It will accelerate from

0% 1. higher to lower *electric potential*; lower to higher *potential energy*
0% 2. higher to lower *electric potential*; higher to lower *potential energy*
0% 3. lower to higher *electric potential*; lower to higher *potential energy*
0% 4. lower to higher *electric potential*; higher to lower *potential energy*

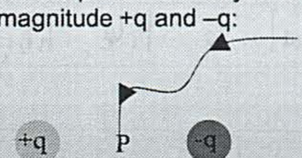
P10-31

read carefully

PRS: Two Point Charges

0

The work done in moving a positive test charge from infinity to the point P midway between two charges of magnitude +q and –q:



0% 1. is positive.
0% 2. is negative.
0% 3. is zero.
0% 4. can not be determined – not enough info is given.
0% 5. I don't know

P10-32

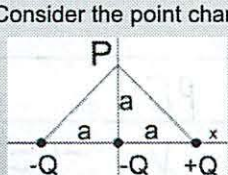
*Scalar
integral paths don't matter

can add the points - see P
The 2 cancel out, so 0

depends otherwise where 0 is

PRS: E from V

Consider the point charges you looked at earlier:



$$\boxed{V(P) = -kQ/a}$$

You calculated V(P). From that can you derive E(P)?

0% 1. Yes, its kQ/a^2 (up)
0% 2. Yes, its kQ/a^2 (down)
0% 3. Yes in theory, but I don't know how to take a gradient
0% 4. No, you can't get E(P) from V(P)
0% 5. I don't know

P10-33

$V = \frac{kQ}{a}$

can add / superposition

Only left is origin

You don't know whats around field
- just b/c know E at 1 pt, need spatial dependence

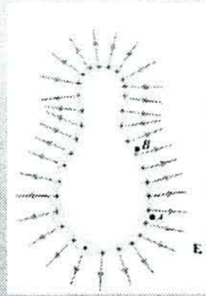
Conductors in Equilibrium

Conductors are equipotential objects:

1) E = 0 inside
2) E perpendicular to surface
3) Net charge inside is 0
4) Excess charge on surface

$$E = \sigma / \varepsilon_0$$

5) Shielding – inside doesn't "talk" to outside



P10-34

SAMPLE EXAM:

P10-35

+ −

? if −

+ • +

$V = \frac{kQ}{r} + \frac{kQ}{r}$

this Q → −Q

$P = \frac{kQ^2}{r} + \frac{kQ^2}{r}$ = amt of + work you do

would − work (automatic)

+ you are forcing it to do that

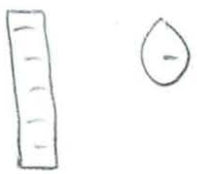
you are doing + work

Day 10

2/24

How to approch problems

1.



Use Guasses Law
- but not inclosed

- do as much goes tangelt through surface

$\Phi = 0$; not enclosed

- but its flux - still some

One of the 6 sides of a cube

$$\frac{-q}{6\varepsilon_0}$$

⊖ charge

$\frac{1}{6}$ of cube

2. Diapoles are just charges
- subject to Colomb's law

on the left will
left + clockwise (D)

4. Equipotential lines — topo map

~~opposite~~ + smaller

Same sign

* don't screw up → same sige = opposit charges

4.

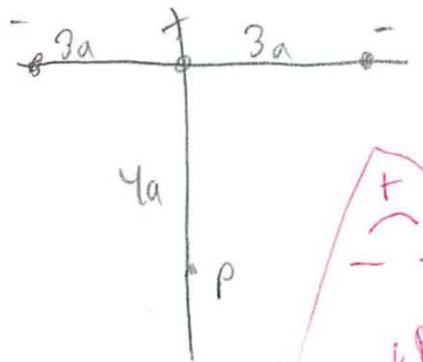
repell or

-2 ← see that larger

3 charges equidistant

I did not read all of the answers to see — missed

5.



$$\vec{E} = \frac{kqq}{r^2} = \frac{k\,-q\,3a}{(3a^2+4a^2)}\uparrow + \frac{k\,-q\,3a}{(3a^2+4a^2)}\uparrow + kq$$

always add does not matter

always charge to point!

6.

$$\frac{1}{r^2}\,\frac{\lambda R}{2\epsilon_0}\,\frac{z}{\left(R^2+z^2\right)^{3/2}}$$

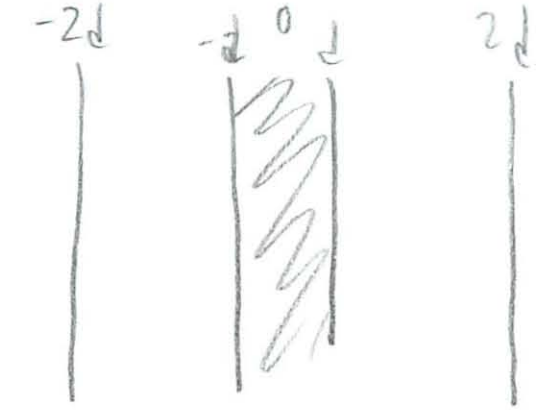
All charge same distance almost like pt charge

Only vertical component that survives

what happens when $z \to 0$

$\boxed{\vec{E} = \frac{kq}{r^2}}$ $\boxed{\frac{\ell}{\ell^3} = \frac{1}{\ell^2}}$

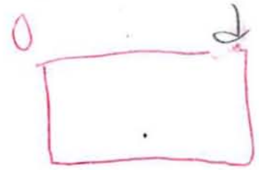
(3)



not drawn to scale

pill box

take advantage of symmetry



$V(-d) = 0$

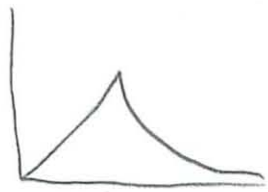
Straight forward - read it later

* Think about what you need to solve problem

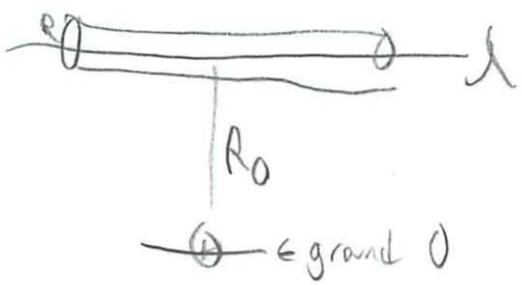
①

Office Hrs

2/24



Problem 4 Pset 3 Power Lines



$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$ from doing cylinder

$$V = \frac{-\lambda}{2\pi\epsilon_0}\ln(r) + C$$

↳ $0 = \frac{-\lambda}{2\pi\epsilon_0}\ln R_0 + C$

has to sum to 0

$C = \frac{\lambda}{2\pi\epsilon_0}\ln(R_0)$

$$V = \frac{-\lambda}{2\pi\epsilon_0}\ln\left(\frac{R}{r_0}\right)$$ ← put together

+ flip

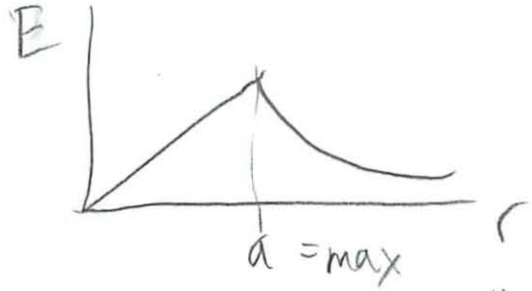
$$V = \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{r_0}{R}\right)$$

Remember

$\ln r - \ln(R_0) = \ln\frac{r}{R_0}$

② Estimate radius of line

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$



estimate radius

$$\boxed{\frac{\lambda}{2\pi\epsilon_0 a} = 10^6 \text{ V/m}}$$

$$V = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{R_0}{a}$$

$$\uparrow = rE$$

$$V = rE \ln \frac{R_0}{r}$$

potential — function of E

0 at ground

$$\boxed{V = 1\text{cm} \left(10^6 \text{ V/m}\right) \ln\left(\frac{10\text{m}}{1\text{cm}}\right)}$$

You learn more from doing 1 problem slowly than lots of problems fast

3

E field points from (+) to (−)

Voltage (+) charge goes to lower potential
(−) charge goes to higher potential

~~When summing config energy~~

~~Summing config energy~~ from charge to point measuring
Calcing E field at P

---

equipotential $\perp$ field lines

againts E field potential $\uparrow$
$\uparrow$ E must be (−)

Potential = work to do to move (+) charge
potential $\uparrow$ have to do work

Config energy $\neq$ calc field at P

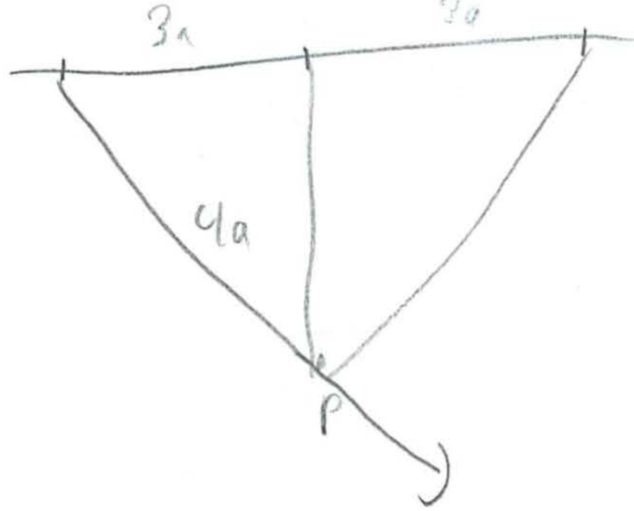
#5 from class tody

$$E = \frac{kq(\vec{r} - \vec{r}')}{|r - r'|^3}$$

r = where ~~ma~~ measuring P
r' = charge where

apply 3 times

(4.)



from charge → measuring

$\vec{r} = -4a\hat{j}$ ← measuring

$\vec{r}' = -3a\hat{i}$ ← charge

Using origin to measure from

$\vec{r} - \vec{r}' = -4a\hat{j} - -3a\hat{i}$

$= -4a\hat{j} + 3a\hat{i}$

$|\vec{r} - \vec{r}'| = \sqrt{16a^2 + 9a^2}$

$5a$

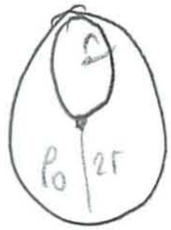
$\frac{1}{r^3} = \frac{\hat{r}}{r^2} = \frac{\vec{r}}{r^3}$

since $\frac{r}{r^3} = \frac{1}{r^2}$ to get units to work out

$$\frac{k(-1)(-4a\hat{j} + 3a\hat{i})}{(5a)^3}$$

now do this for other 2 vectors

⑤ Cavity Problem Pset 2



Super position fully charged + empty

$$\vec{E} \text{ field} = \vec{E}_{large} + \vec{E}_{small}$$

$$E = \frac{\rho}{3\epsilon_0} \vec{r}_{center_1} + \frac{-\rho_0 \vec{r}_{center_2}}{3\epsilon_0}$$

$$\frac{\rho \, r_{center}}{3\epsilon_0} +$$

Vector → $\vec{r}_{center2} = \vec{r}_{center_1} + \vec{R}$ ⤴ the radius

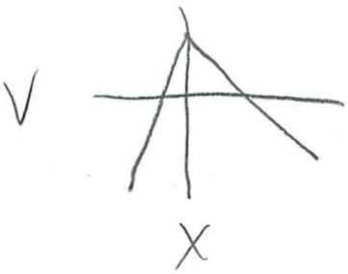
have = + opposite charge
So <u>not</u> like comparing volumes

E from V/Gradient

Dumaskin did not do, so I will

$E = -\nabla V$

differentiate each part



$x > 0$

magnitude of E smaller

(since not as steep)

as $x < 0$

* Don't get tricked by concept qu where the ans is IDK b/c I have to look around

* And it is the − gradient

---

* units $E = \frac{V}{m}$

Other Review

Collecting E from charges $\frac{1}{r^3}$

$F = \frac{kqQ}{r^2}$

$E = k\frac{Q}{r^2}$ $\hat{r}$ from charge to observer $= \frac{kQr}{r^3}$

(2) $F = qE$

$p = qd$

$\vec{\tau} = \vec{p} \times \vec{E}$) don't think used much

$E = k \sum \frac{q}{r^2}$

Config E - moving pt in

- work

- $\frac{kq}{r}$ + summing

$W = qV$

Not finding E at a pt

$U = qE$

E falls off

Diapole $\frac{1}{r^3}$

Pt $\frac{1}{r^2}$

Line $\frac{1}{r}$

Plane 1 - constant

Physics 8.02 **Equation Sheet Exam One**

Please Remove this Tear Sheet from Your Exam

$$\vec{\mathbf{E}} = \frac{q}{4\pi\varepsilon_o r^2}\hat{\mathbf{r}} = \frac{q}{4\pi\varepsilon_o r^3}\vec{\mathbf{r}}$$

$\hat{\mathbf{r}} = \frac{\vec{\mathbf{r}}}{r}$ *points* **from** *source* q **to** *observer*

$$\vec{\mathbf{E}}_{\text{many point charges}} = \sum_{i=1}^{N} \frac{q_i}{4\pi\varepsilon_o \left|\vec{\mathbf{r}} - \vec{\mathbf{r}}_i\right|^3}(\vec{\mathbf{r}} - \vec{\mathbf{r}}_i)$$

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \frac{1}{4\pi\varepsilon_0}\int_{source} \frac{dq}{\left|\vec{\mathbf{r}} - \vec{\mathbf{r}}'\right|^2}\hat{\mathbf{r}}$$

$$\vec{\mathbf{F}}_q = q\vec{\mathbf{E}}_{source}$$

$$\oiint_{\substack{closed\\surface}} \vec{\mathbf{E}} \cdot \mathbf{d}\vec{\mathbf{A}} = \frac{Q_{\text{enc}}}{\varepsilon_o}$$

$\mathbf{d}\vec{\mathbf{A}}$ points from inside to outside

$$\oint_{\substack{closed\\path}} \vec{\mathbf{E}} \cdot \mathbf{d}\vec{\mathbf{s}} = 0$$

$$\Delta V_{moving\ from\ a\ to\ b} = V_b - V_a = -\int_a^b \vec{\mathbf{E}} \cdot \mathbf{d}\vec{\mathbf{s}}$$

$$\Delta U = q\Delta V$$

$$V_{\text{point charge}} = \frac{q}{4\pi\varepsilon_o r};\ V(\infty) = 0$$

$$V_{\text{many point charges}} = \sum_{i=1}^{N} \frac{q_i}{4\pi\varepsilon_o \left|\vec{\mathbf{r}} - \vec{\mathbf{r}}_i\right|};\ V(\infty) = 0$$

$$V(\vec{\mathbf{r}}) = \frac{1}{4\pi\varepsilon_0}\int_{source} \frac{dq}{\left|\vec{\mathbf{r}} - \vec{\mathbf{r}}'\right|};\ V(\infty) = 0$$

$$U = \sum_{all\ pairs} \frac{q_i q_j}{4\pi\varepsilon_o \left|\vec{\mathbf{r}}_i - \vec{\mathbf{r}}_j\right|};\ U(\infty) = 0$$

$$U = \frac{1}{2}\varepsilon_o \iiint_{all\ space} E^2 dV_{vol}$$

$E_r = -\dfrac{\partial V}{\partial r}$ for spherical symmetry,

$$\vec{\mathbf{E}} = -\vec{\nabla} V$$

$$E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z}$$

$$C = \frac{|Q|}{|\Delta V|} \qquad U = \frac{1}{2}C\Delta V^2 = \frac{Q^2}{2C}$$

Circumferences, Areas, Volumes:

1) The area of a circle of radius r is πr^2
   Its circumference is $2\pi r$

2) The surface area of a sphere of radius r is $4\pi r^2$. Its volume is $(4/3)\pi r^3$

3) The area of the sides of a cylinder of radius r and height h is $2\pi rh$. Its volume is $\pi r^2 h$

Integrals that may be useful

$$\int_a^b dr = b - a$$

$$\int_a^b \frac{dr}{r} = \ln(b/a)$$

$$\int_a^b \frac{1}{r^2}dr = \left(\frac{1}{a} - \frac{1}{b}\right)$$

Some potentially useful numbers

$$k_e = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \frac{\text{N m}^2}{\text{C}^2}$$

8.02 Exam One Spring 2010

P	L	A	S	M	E	I	E	R							

FAMILY (last) NAME

M	I	C	H	A	E	L									

GIVEN (first) NAME

9	2	1		6	4		5	2	6	1

Student ID Number

Your Section:

X **L01 MW 9 am** ____ **L02 MW 11 am** ____ **L03 MW 1 pm** ____ **L04 MW 3 pm**
____ **L05 TR 9 am** ____ **L06 TR 11 am** ____ **L07 TR 1 pm** ____ **L08 TR 3 pm**

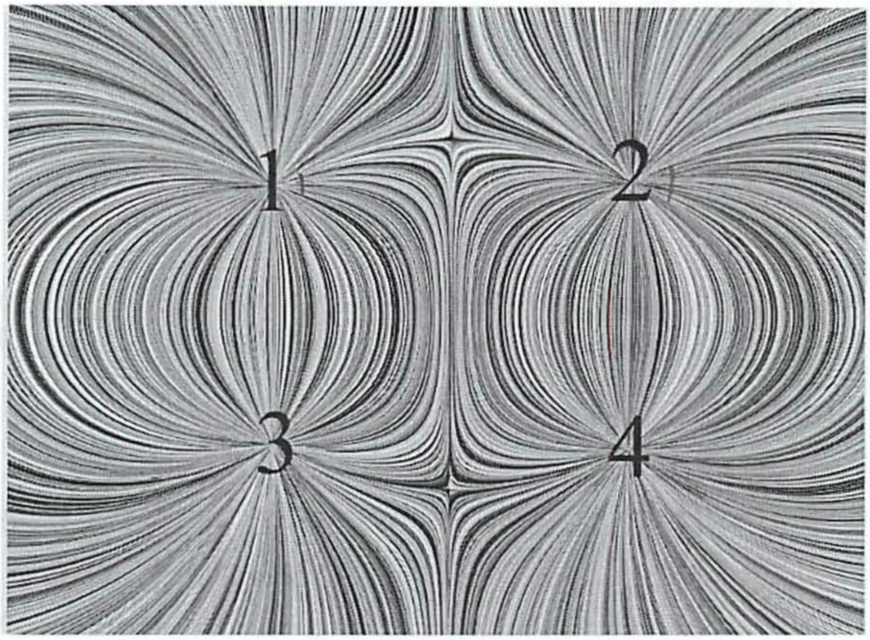
Your Table and Group (e.g. 10A): 11C

	Score	**Grader**
Problem 1 (25 points)	25	PHF
Problem 2 (25 points)	17	Ef
Problem 3 (25 points)	14	AES
Problem 4 (25 points)	15	EF
TOTAL	71	

Problem 1 (25 points)

In this problem you are asked to answer 5 questions, each worth 5 points. You do not have to show your work; in most cases you may simply circle the chosen answer.

Question 1 (5 points)



1. Above we show the grass seeds representation of the field of four point charges, located at the positions indicated by the numbers. Which statement is true about the signs of these charges:

a) All four charges have the same sign.

b) Charges 1 and 2 have the same sign, and that sign is opposite the sign of 3 and 4.

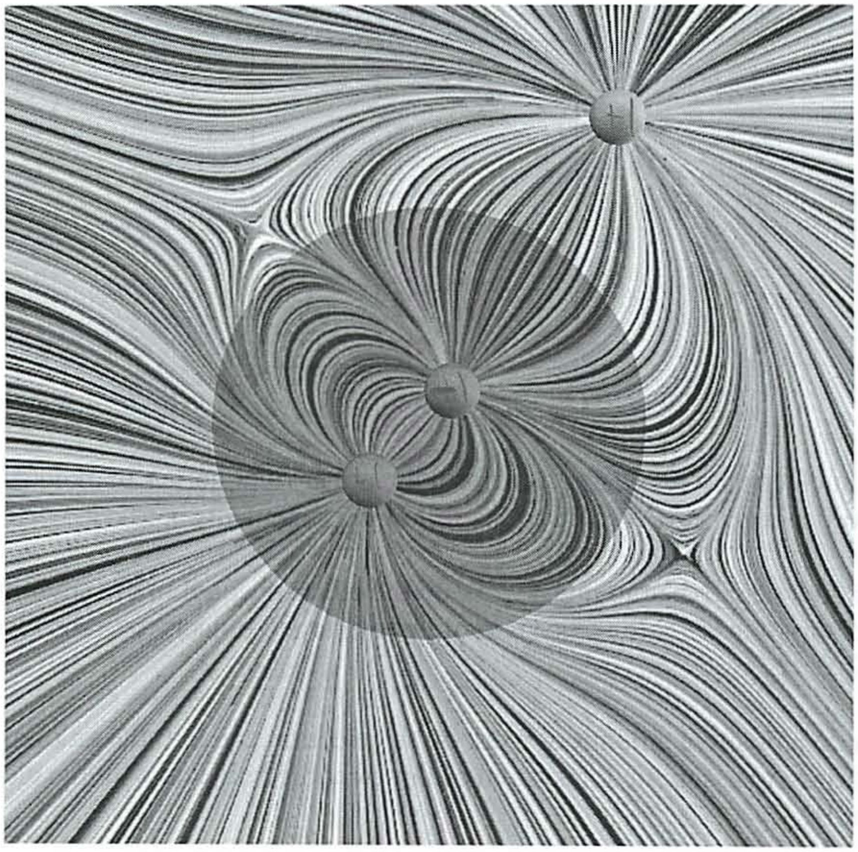
c) Charges 1 and 3 have the same sign, and that sign is opposite the sign of 2 and 4.

d) Charges 1 and 4 have the same sign, and that sign is opposite the sign of 2 and 3.

e) None of the above.

Question 2 (5 points)

The grass seeds figure below shows the electric field of three charges with charges +1, +1, and -1, The Gaussian surface in the figure is a sphere containing two of the charges.



The total electric flux through the spherical Gaussian surface is

a) Positive

b) Negative

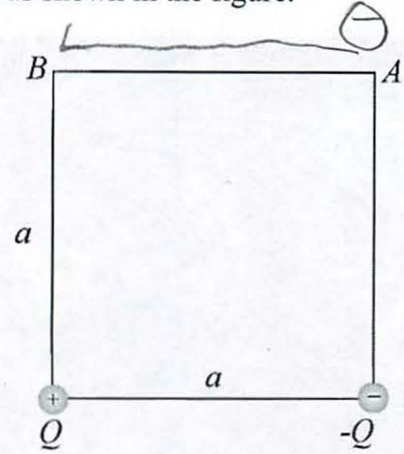
Net $Q_{enc} = 0$

c) Zero

d) Impossible to determine without more information

Question 3 (5 points)

Two point-like charged objects with charges $+Q$ and $-Q$ are placed on the bottom corners of a square of side a, as shown in the figure.



easy to do

you − work

You move an electron with charge $-e$ from the upper right corner marked A to the upper left corner marked B. Which of the following statements is true?

a) You do a negative amount of work on the electron equal to the amount of energy necessary to assemble the system of three charged objects with the electron at point B.

2 diff things

b) You do a positive amount of work on the electron equal to the amount of energy necessary to assemble the system of three charged objects with the electron at point B.

c) You do a positive amount of work on the electron and the potential energy of the system of three charged objects increases.

d) You do a negative amount of work on the electron and the potential energy of the system of three charged objects decreases. always moves naturally to ↓ PE naturally

e) You do a negative amount of work on the electron and the potential energy of the system of three charged objects increases.

f) You do a positive amount of work on the electron and the potential energy of the system of three charged objects decreases.

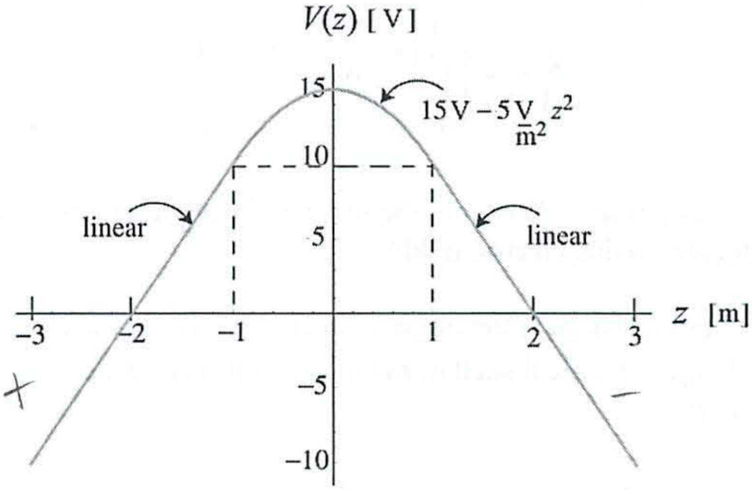
$W = \Delta U$

~~the~~ you − work

System ↓ energy

Question 4 (5 points)

A graph of the electric potential $V(z)$ vs. z is shown in the figure below.



Gradiant

Which of the following statements about the z-component of the electric field E_z is true?

Slope

a) $E_z < 0$ for $-3\,\text{m} < z < 0$ and $E_z < 0$ for $0 < z < 3\,\text{m}$.

b) $E_z < 0$ for $-3\,\text{m} < z < 0$ and $E_z > 0$ for $0 < z < 3\,\text{m}$.

c) $E_z > 0$ for $-3\,\text{m} < z < 0$ and $E_z < 0$ for $0 < z < 3\,\text{m}$.

d) $E_z > 0$ for $-3\,\text{m} < z < 0$ and $E_z > 0$ for $0 < z < 3\,\text{m}$.

e) None of the above because E_z cannot be determined from information in the graph for the regions $-3\,\text{m} < z < 0$ and $0 < z < 3\,\text{m}$.

– no you can by looking graph

$E = -\nabla$

1, 5
2 - 3

Question 5 (5 points)

Careful measurements reveal an electric field

$$\vec{\mathbf{E}}(r)=\begin{cases}\dfrac{a}{r^2}\left(1-\dfrac{r^3}{R^3}\right)\hat{\mathbf{r}}\,; & r\le R\\[2ex] \vec{0}\,; & r\ge R\end{cases}$$

where a and R are constants. Which of the following best describes the charge distribution giving rise to this electric field?

is it a sphere or shell?

a) A negative point charge at the origin with charge $q=4\pi\varepsilon_0 a$ and a uniformly positive charged spherical shell of radius R with surface charge density $\sigma=-q/4\pi R^2$.

b) A positive point charge at the origin with charge $q=4\pi\varepsilon_0 a$ and a uniformly negative charged spherical shell of radius R with surface charge density $\sigma=-q/4\pi R^2$.

c) A positive point charge at the origin with charge $q=4\pi\varepsilon_0 a$ and a uniformly negative charged sphere of radius R with charge density $\rho=-q/(4\pi R^3/3)$.

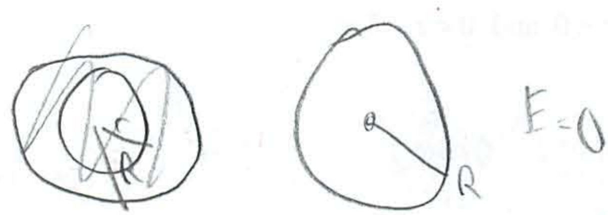
d) A negative point charge at the origin with charge $-q=-4\pi\varepsilon_0 a$ and a uniformly positive charged sphere of radius R with charge density $\rho=q/(4\pi R^3/3)$.

also credit – but does not really work

e) Impossible to determine from the given information.

could it be – don't know which +/–

a is ⊕ did not say so d or e also correct ambigeous



$\hat{r}$ from source to field pt

this is confusing

a is ⊕ so ⊖

⊕ in middle

✓ great performance on part 1

$\sqrt{a^2+a^2} = \sqrt{2a^2} = \sqrt{2}a$

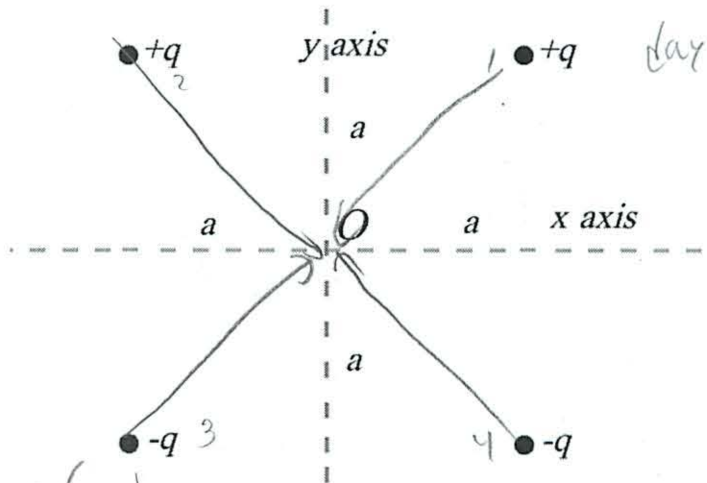
Problem 2 (25 points)

NOTE: YOU MUST SHOW WORK in order to get any credit for this problem. Make it clear to us that you understand what you are doing (use a few words!) .

Coulomb's law

day 2

Four charged point-like objects, two of charge $+q$ and two of charge $-q$, are arranged on the vertices of a square with sides of length $2a$, as shown in the sketch.



a) What is the electric field at point O, which is at the center of the square? Indicate the direction and the magnitude.

$$kq\left(\frac{a}{\sqrt{a^2+a^2}}\hat{i} - \frac{a}{\sqrt{a^2+a^2}}\hat{j}\right) + kq\left(-\frac{a}{\sqrt{a^2+a^2}}\hat{i} - \frac{a}{\sqrt{a^2+a^2}}\hat{j}\right)$$
$$+ kq\left(\frac{a}{\sqrt{a^2+a^2}}\hat{i} + \frac{a}{\sqrt{a^2+a^2}}\hat{j}\right) + kq\left(-\frac{a}{\sqrt{a^2+a^2}}\hat{i} + \frac{a}{\sqrt{a^2+a^2}}\hat{j}\right)$$

top 2 cancle horiz vert ↓

bottom 2 horiz cancle vert ↓

$$\vec{E} = -2\frac{kqa}{\sqrt{2}a\sqrt{2}a\sqrt{2}a}\hat{i} + \frac{2kqa}{\sqrt{2}a\sqrt{2}a\sqrt{2}a}\hat{j} = \frac{2kq}{2\sqrt{2}a^2}\hat{i} - \frac{2kq}{2\sqrt{2}a^2}\hat{j}$$

$\hat{i}$ cancles

$$\vec{E} = \left(\frac{kq}{\sqrt{2}a^2}\right)(\hat{i} - \hat{j})\overset{?1}{}\frac{1}{\sqrt{2}}$$

← direction? since it is 45°
why? - that is the ↓ component
of the $\frac{1}{\sqrt{2}}$ direction

$$|\vec{E}| = \sqrt{\left(\frac{kq}{\sqrt{2}a^2}\right)^2 + \left(\frac{kq}{\sqrt{2}a^2}\right)^2}$$
$$= \sqrt{\frac{k^2q^2}{2a^4} + \frac{k^2q^2}{2a^4}}$$
$$= \sqrt{\frac{2k^2q^2}{2a^4}}$$
$$= -\frac{kq}{a^2}$$ ← why no $\sqrt{2}$

$E = 4|\vec{E}|\sin\theta \; -\hat{j}$

$4k\frac{q}{2a}\frac{1}{\sqrt{2}}\hat{j}$

? so have 4 times pointing ↓

-3

b) What is the electric potential V at point O, the center of the square, taking the potential at infinity to be zero?

$$V(P) - V(\infty) = V(P) - 0 = V(P) = -\int E \cdot ds$$

$$-\int \frac{kq}{\sqrt{2}a^2}\hat{i} - \frac{kq}{\sqrt{2}a^2}\hat{j} \cdot ds$$

$\frac{1}{a^2}$

$$-\frac{kq}{\sqrt{2}}\left(\int \frac{1}{a^2}\hat{i} - \int \frac{1}{a^2}\hat{j}\right)$$

a^{-2}

$\frac{a^{-1}}{-1}$

$$-\frac{kq}{\sqrt{2}}\left(-\frac{1}{a}\hat{i} - -\frac{1}{a}\hat{j}\right)$$

$= -\frac{1}{a}$

$$V(P) = \frac{kq}{\sqrt{2}a}\hat{i} - \frac{kq}{\sqrt{2}a}\hat{j}$$ ← no direction! (scalar)

So I almost had it grrr

looks right

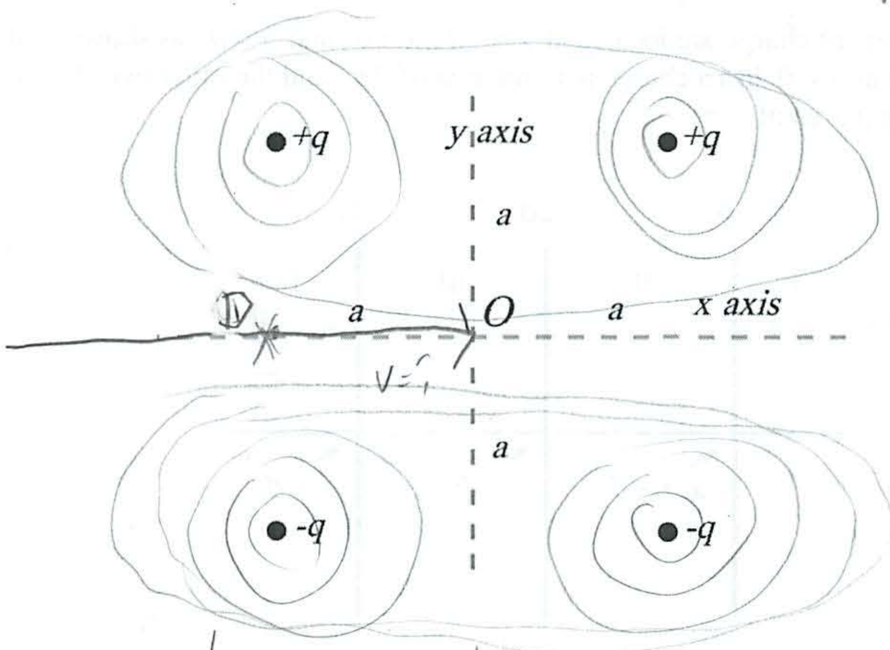
$V(P) = 0$ ✓

$$\frac{kq}{\sqrt{2}a} + \frac{kq}{\sqrt{2}a} + \frac{k\,{-q}}{\sqrt{2}a} + \frac{k\,{-q}}{\sqrt{2}a} = 0$$

So write it out full
(like on pratice test)
and use that

c) Sketch on the figure below one path leading from infinity to the origin at O where the integral $\int_{\infty}^{O} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$ is trivial to do by inspection. Does your answer here agree with your result in b)?

like experiment



0

equipotential curve

$$V = \frac{kq}{\sqrt{2}a} \uparrow - \frac{kq}{\sqrt{2}a} \downarrow$$

↑ sums to 0

Voltage at top near ⊕ = total voltage
bottom near ⊖ = 0

Voltage at pt D is $\frac{1}{2}$ total voltage

$$= \int_{\infty}^{0} \frac{ka}{\sqrt{2}a} \uparrow - \frac{ka}{\sqrt{2}a} \downarrow$$

Should have better studied the plane ones – not just sphere

– the practice test I did was conductors – crap – why did they have to do that

– this is like P-set one

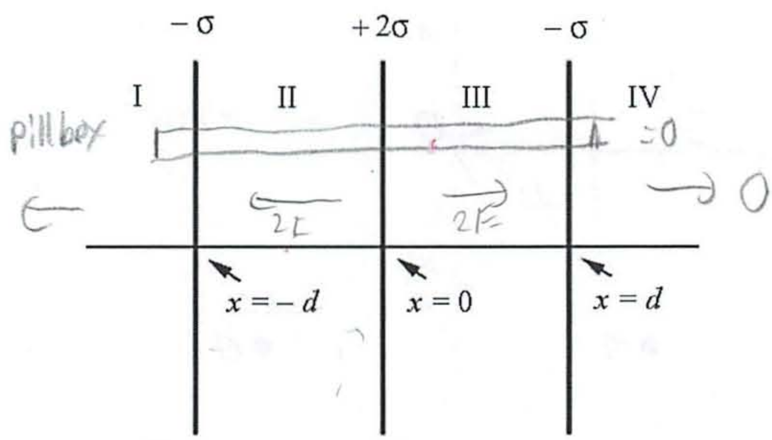
Problem 3 (25 points)

NOTE: YOU MUST SHOW WORK in order to get any credit for this problem. Make it clear to us that you understand what you are doing (use a few words!)

if line would be λ not σ

Three infinite sheets (slabs that are thin) of charge are located at $x=-d$, $x=0$, and $x=d$, as shown in the sketch. The sheet at $x=0$ has a charge per unit area of 2σ, and the other two sheets have charge per unit area of $-\sigma$.

↑ constant



pt $\frac{1}{r^2}$ (?) line $\frac{1}{r}$ plane slab 1

remember linear

+2/2

a) What is the electric field in each of the four regions I-IV labeled in the sketch? Clearly present your reasoning, relevant figures, and any accompanying calculations. Plot the x component of the electric field, E_x, on the graph on the bottom of the next page. Clearly indicate on the vertical axis the values of E_x for the different regions.

E in 1 and 4 is 0 since the charges in the

+5/5

pill box balance out (no net charge). Also with a slab the charge beyond it is constant.

E in regions 2 and 3 on both the right and the left will be positive

$$EA = \frac{\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma A}{\epsilon_0 A} = \frac{\sigma}{\epsilon_0} \quad \cancel{\epsilon 2\sigma}$$

2/4

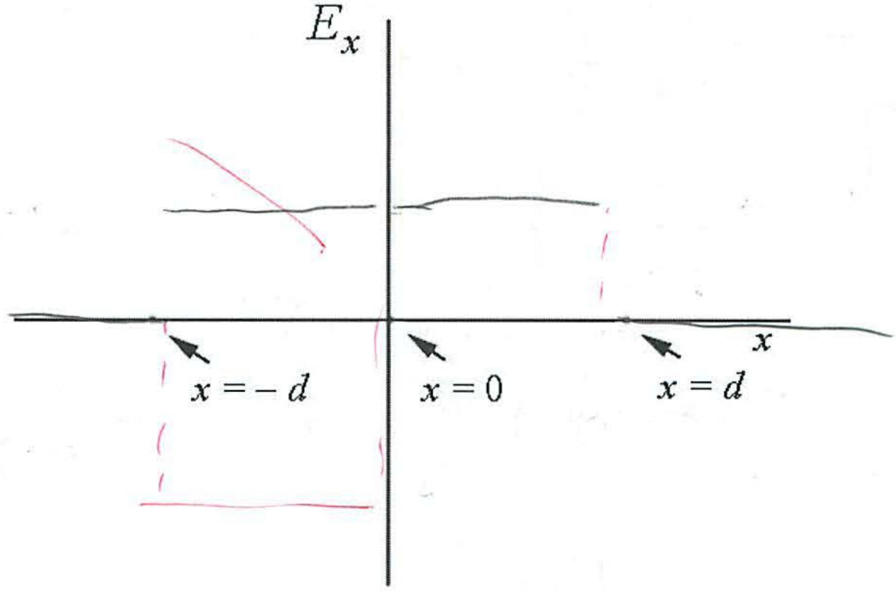
$$\begin{cases} -\frac{\sigma}{\epsilon_0} & -d<x<0 \\ \frac{\sigma}{\epsilon_0} & 0<x<d \end{cases}$$

so I only made a sign mistake. Thats not -3

Also not 2σ

$2EA = \frac{2\sigma A}{\varepsilon_0}$

should have thought more about not just wrote as sidebar



b) Find the electric potential in each of the four regions I-IV labeled above, with the choice that the potential is zero at $x = +\infty$ i.e. $V(+\infty) = 0$. Show your calculations. Plot the electric potential as a function of x on the graph on the bottom of the next page. Indicate units on the vertical axis.

$V(P) = V(P) - V(\infty) = V(P) - 0 = -\int E \cdot ds$

1 $\rightarrow = -\int_{-\infty}^{-d} 0\, ds$ $\quad$ 4 $\rightarrow -\int_{d}^{\infty} 0\, ds$

$= -r \Big|_{-\infty}^{-d}$ $\quad$ $= -r \Big|_{d}^{\infty}$

$- -d - -\infty$ $\quad$ $-\infty - d$

$\cancel{d}$ 0 $\quad$ $\cancel{-d}$ 0

$\int 0 = 0$

grrr math error

2 $\rightarrow -\int_{-d}^{0} \frac{\sigma}{\epsilon_0} ds$ $\quad$ 2σ $\quad$ $-\int_{0}^{x} \frac{\sigma}{\epsilon_0}$ $\quad$ 3 $\rightarrow -\int_{0}^{x} \frac{\sigma}{\epsilon_0} ds$

$\int_{\infty}^{d} \frac{\sigma}{\epsilon_0}$ $\quad$ $-\int_{\infty}^{d} \frac{\sigma}{\epsilon_0}$

$\cancel{-\frac{\sigma}{\epsilon_0} r \Big|_{-d}^{0}}$ $\quad$ $\cancel{-\frac{\sigma}{\epsilon_0} r \Big|_{0}^{d}}$

$-d \quad 0 \quad x$

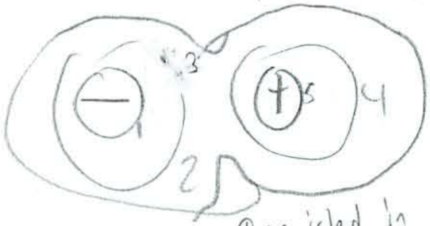
$-\frac{\sigma}{\epsilon_0} \cdot \cancel{d} + \frac{\sigma}{\epsilon_0} \cancel{d}$ x $\quad$ $-\frac{\sigma}{\epsilon_0} \cancel{x} - \frac{\sigma}{\epsilon_0} d$

$\cancel{\infty} \rightarrow \cancel{\frac{\sigma}{\epsilon_0} d}$ $\quad$ $\cancel{-\frac{\sigma}{\epsilon_0} d}$

know what going from to to and what you have to go through to get here

3/6

think conceptually from experiment

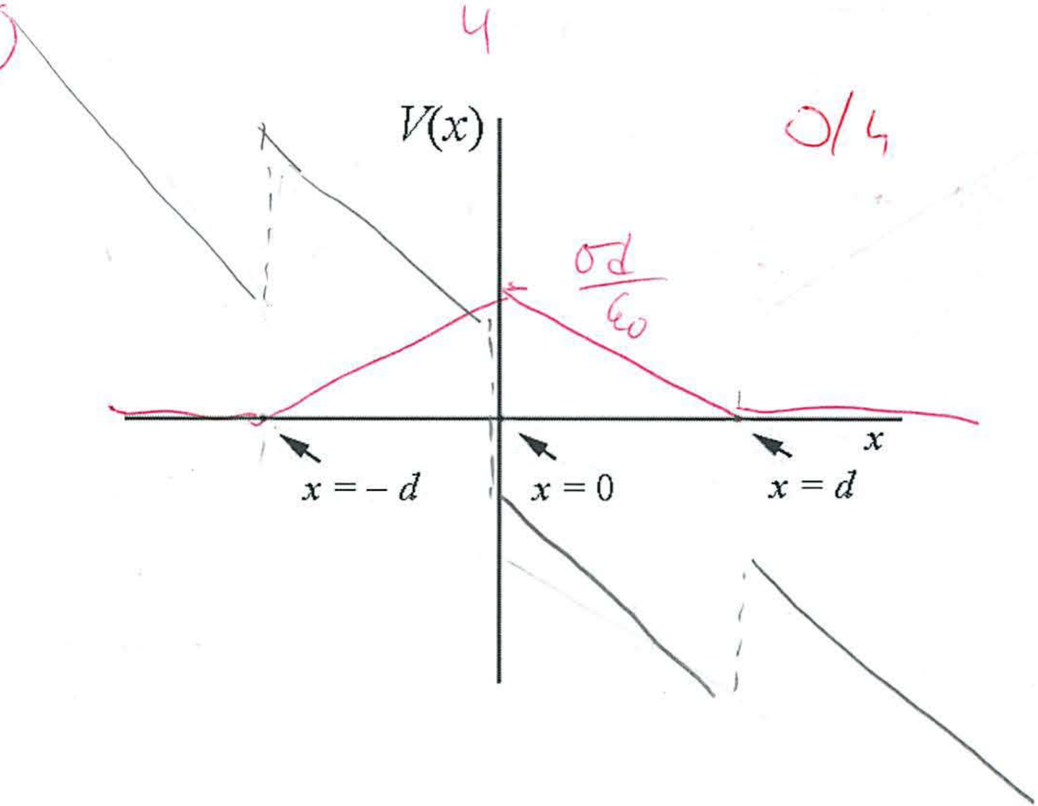


↑ squished in middle b/w largest voltage change

$$\begin{cases} 0 & 1 \\ \dfrac{\sigma}{\varepsilon_0} d + \dfrac{\sigma}{\varepsilon_0} x & 2 \\ \dfrac{\sigma}{\varepsilon_0} d - \dfrac{\sigma}{\varepsilon_0} x & 3 \\ 0 & 4 \end{cases}$$

$$\dfrac{\sigma}{\varepsilon_0} d - \dfrac{\sigma}{\varepsilon_0} |x|$$

0/4



They are not slopes - remember how to graph

note page is not slopes

c) How much work must you do to bring a point like object with charge $+Q$ in from infinity to the origin $x = 0$?

$W = \cancel{qE} = \Delta U = qV = q(V(P) - 0)$

$W = +Q\, V(x=0)$

or

~~The sheet has charge (+) so how can you bring +Q into it - it will repel there is no way you can get it to touch?~~

← my mistake from before - I just jammed it in there w/o thinking

$W = q \frac{\sigma}{\epsilon_0} d$

$W = \frac{q \cancel{\sigma} \sigma d}{\epsilon_0}$

3/5

↑ should be worth more

$\frac{Q\sigma}{\epsilon_0} d$

did not study cylinders too much

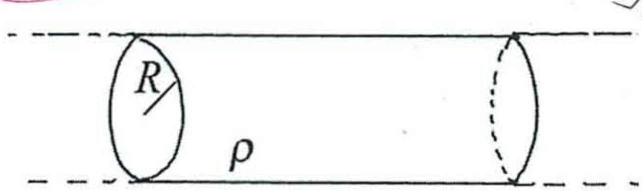
Problem 4 (25 points)

NOTE: YOU MUST SHOW WORK in order to get any credit for this problem. Make it clear to us that you understand what you are doing (use a few words!). You may find the following integrals helpful in this answering this question.

$$\int_{r_a}^{r_b}\frac{dr}{r^2}=-\left(\frac{1}{r_b}-\frac{1}{r_a}\right),\ \int_{r_a}^{r_b}\frac{dr}{r}=\ln(r_b/r_a),\ \int_{r_a}^{r_b}dr=r_b-r_a,\ \int_{r_a}^{r_b}rdr=\frac{1}{2}\left(r_b^2-r_a^2\right).$$

$$\int_{r_a}^{r_b} r^n\,dr = \frac{1}{n+1}\left(r_b^{n+1}-r_a^{n+1}\right)$$

Consider a charged infinite cylinder of radius R.



The charge density is non-uniform and given by

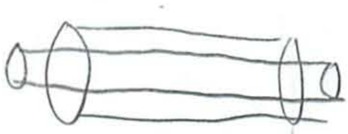
$$\rho(r)=br\ ;\ r<R,$$

where r is the distance from the central axis and b is a constant.

a) Find an expression for the direction and magnitude of the electric field everywhere i.e. inside and outside the cylinder. Clearly present your reasoning, relevant figures, and any accompanying calculations.

Gaussian surface = larger + smaller cylinders

inside



- it is leaking charge on both sides and ends

$$EA = \frac{\rho V}{\varepsilon_0}$$

Since its ∞ long no endcaps! −2

$$E(\cancel{2\pi} r^2 + 2\pi r h) = \int \frac{b r\, 2\pi r h}{\varepsilon_0}\, dr$$

$$\frac{1}{\varepsilon_0}\int_0^{r'} b r^2 2\pi h\, dr$$

$$\frac{2}{\varepsilon_0} b \pi h \int_0^{r'} r^2\, dr$$

prob missing
some small thing

$$\frac{1}{\epsilon_0} b \pi h \left. \frac{r^3}{3} \right|_0^{r'}$$

$$E = \frac{2 b \pi h r^3}{\epsilon_0 3 (\cancel{2\pi r^2} + 2\pi r h)}$$

$$= \frac{b \pi h r^{3\,2}}{\epsilon_0 8 \pi r^2 + \epsilon_0 8 \pi r h}$$

$$= \frac{b h r^2}{\epsilon_0 8 (1 + h)}$$

?

$$\frac{b r^2}{3 \epsilon_0} \hat{r} \quad r < R$$

outside

$$E A = \frac{\rho V}{\epsilon_0}$$

$$\frac{b R^2}{3 \epsilon_0} \frac{1}{r} \quad r > R$$

$$E (\cancel{2\pi r^2} + 2\pi r h) = \frac{\int_0^R \rho 2\pi r h \, dr}{\epsilon_0}$$

-2

$$\int_0^R b r \, 2\pi r h \, dr$$

$$\pi h \left. \frac{b r^2}{2} \right|_0^R = \frac{b R^2 \pi h}{2}$$

$$E = \frac{b R^2 \pi h}{\epsilon_0 2 (2\pi R^2 + 2\pi R h)} = \frac{b R^2 \pi h}{\epsilon_0 2 \pi R (2R + 2h)}$$

-2

endcaps
still don't
matter

$$E = \frac{b R h}{\epsilon_0 4 (R + h)}$$

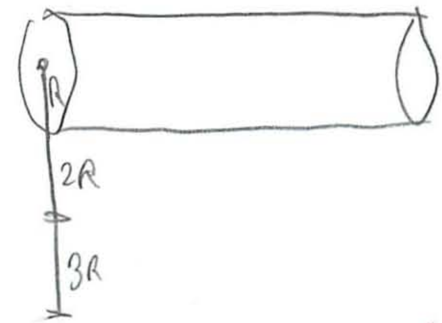
$r \neq R$!

don't screw this up
- its sometimes true sometimes
not be displined
I had it and erased it so
could simplify more

Energy approach

did not look too long at since it was last q.

b) A point-like object with charge $+q$ and mass m is released from rest at the point a distance $2R$ from the central axis of the cylinder. Find the speed of the object when it reaches a distance $3R$ from the central axis of the cylinder



$0 = U + K$

$U = q\Delta V$

do I have to find.... grrr

$K = \frac{1}{2}mv^2$

$q\Delta V = \frac{1}{2}mv^2$

$vel = \sqrt{\frac{2q\Delta V}{m}}$

$\Delta V = -\int_{2R}^{3R} E \cdot ds$ pot diff

$\Delta V = -\int \frac{bR^2 h}{\epsilon_0 4(R+h)} dr$

??

-4

no need to integrate just subtract

$\frac{1}{R}$

R^{-1}

$\ln R$

$\Delta V = -\frac{bh \ln(3R/2R)}{\epsilon_0 4(1+h)(2)}\left((3R)^2-(2R)^2\right)$

$k(3R) - k(2R)$

$= -(U(3R) - U(2R))$

$= -q(V(3R) - V(2R))$

$\Delta V = -\frac{bh \ln(3R/2R)(9R^2-4R^2)}{\epsilon_0 8(1+h)}$

$k(2R) = 0$

$k(3R) = \frac{1}{2}mv_f^2$

plug in

$velocity = \sqrt{\frac{2q\Delta V}{m}}$

ok

$velocity = \sqrt{\frac{2q\left(\frac{-bh\ln(3R/2R)(9R^2-4R^2)}{\epsilon_0 8(1+h)}\right)}{m}}$

$\boxed{v_f = \sqrt{\frac{2qbR^3}{3m\epsilon_0}} \ln(3/2)}$

Oh they just write V

still no end caps

think I major screwed up - sure side and ends?
- but have been messy problems before

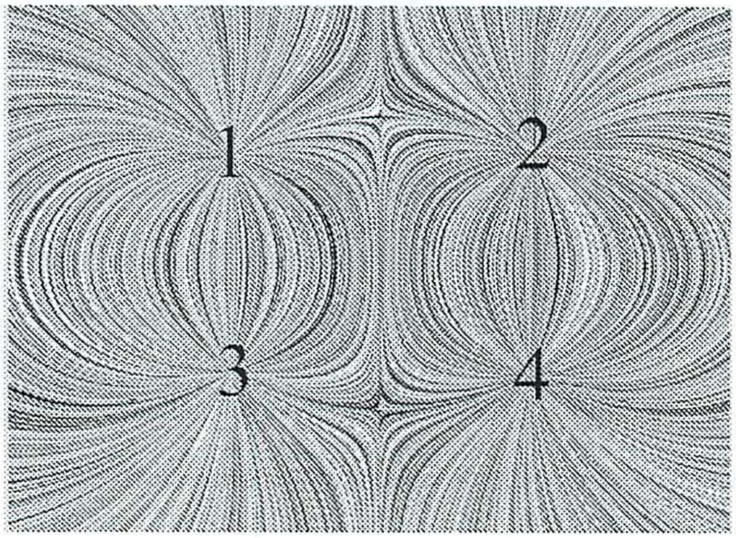
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 **Spring 2010**

8.02 Exam One Solutions Spring 2010

Problem 1 (25 points)

Question 1 (5 points)



1. Above we show the grass seeds representation of the field of four point charges, located at the positions indicated by the numbers. Which statement is true about the signs of these charges:

a) All four charges have the same sign.

b) Charges 1 and 2 have the same sign, and that sign is opposite the sign of 3 and 4.

c) Charges 1 and 3 have the same sign, and that sign is opposite the sign of 2 and 4.

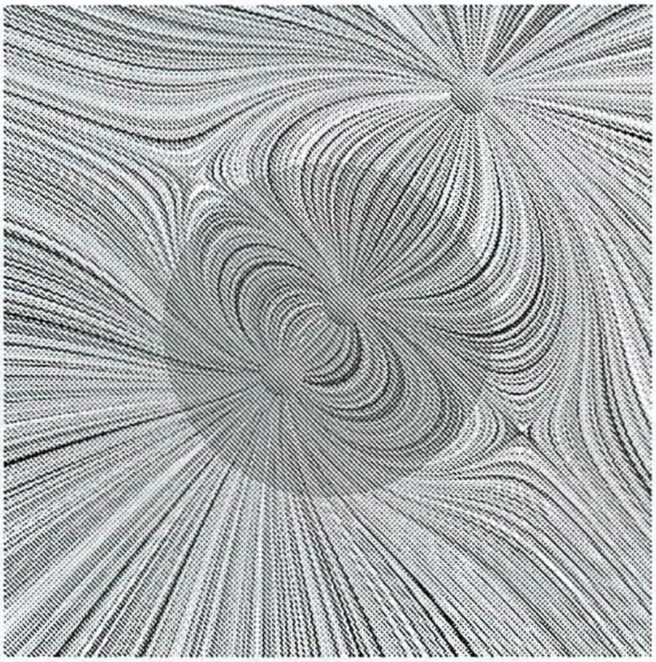
d) Charges 1 and 4 have the same sign, and that sign is opposite the sign of 2 and 3.

e) None of the above.

Solution b. Field lines continuously connect charges 1 and 3, and 2 and 4 respectively, indicating that the charge of those pairs are opposite in sign. The field is a zero between charges 1 and 2 indicating that they repel and hence are of the same sign. A smilar argument holds for charges 3 and 4.

Question 2 (5 points)

The grass seeds figure below shows the electric field of three charges with charges +1, +1, and -1, The Gaussian surface in the figure is a sphere containing two of the charges.



The total electric flux through the spherical Gaussian surface is

a) Positive

b) Negative

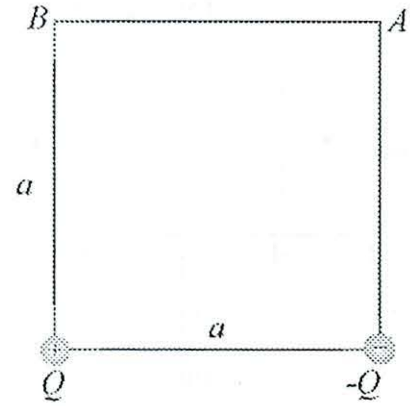
c) Zero

d) Impossible to determine without more information

Solution c. Because the field lines connect the two charges within the Gaussian surface they must have opposite sign. Therefore the charge enclosed in the Gaussian surface is zero. Hence the electric flux through the surface of the Gaussian surface is also zero.

Question 3 (5 points)

Two point-like charged objects with charges $+Q$ and $-Q$ are placed on the bottom corners of a square of side a, as shown in the figure.



You move an electron with charge $-e$ from the upper right corner marked A to the upper left corner marked B. Which of the following statements is true?

a) You do a negative amount of work on the electron equal to the amount of energy necessary to assemble the system of three charged objects with the electron at point B.

b) You do a positive amount of work on the electron equal to the amount of energy necessary to assemble the system of three charged objects with the electron at point B.

c) You do a positive amount of work on the electron and the potential energy of the system of three charged objects increases.

d) You do a negative amount of work on the electron and the potential energy of the system of three charged objects decreases.

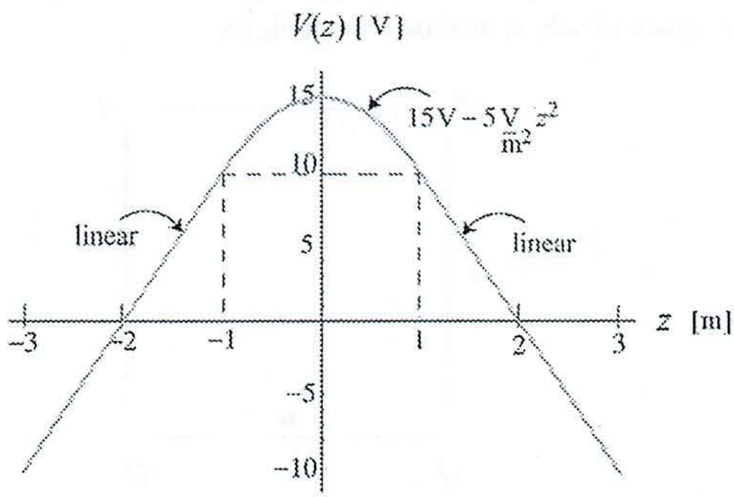
e) You do a negative amount of work on the electron and the potential energy of the system of three charged objects increases.

f) You do a positive amount of work on the electron and the potential energy of the system of three charged objects decreases.

Solution d. Because point B is closer to the positive charge than the point A, the electric potential difference $V(B)-V(A)>0$. When you move an electron with charge $-e$ from the upper right corner marked A to the upper left corner marked B, the potential energy difference is $U(B)-U(A)=-e(V(B)-V(A))<0$. This means that you do a negative amount of work and the potential energy of the system decreases.

Question 4 (5 points)

A graph of the electric potential $V(z)$ vs. z is shown in the figure below.



Which of the following statements about the z-component of the electric field E_z is true?

a) $E_z < 0$ for $-3\,\text{m} < z < 0$ and $E_z < 0$ for $0 < z < 3\,\text{m}$.

b) $E_z < 0$ for $-3\,\text{m} < z < 0$ and $E_z > 0$ for $0 < z < 3\,\text{m}$.

c) $E_z > 0$ for $-3\,\text{m} < z < 0$ and $E_z < 0$ for $0 < z < 3\,\text{m}$.

d) $E_z > 0$ for $-3\,\text{m} < z < 0$ and $E_z > 0$ for $0 < z < 3\,\text{m}$.

e) None of the above because E_z cannot be determined from information in the graph for the regions $-3\,\text{m} < z < 0$ and $0 < z < 3\,\text{m}$.

Solution b. For values of $-3\,\text{m} < z < 0$, the derivative $dV(z)/dz > 0$, and $E_z = -dV(z)/dz < 0$. For values of $0 < z < 3\,\text{m}$, the derivative $dV(z)/dz < 0$, and $E_z = -dV(z)/dz > 0$.

Question 5 (5 points)

Careful measurements reveal an electric field

$$\vec{\mathbf{E}}(r) = \begin{cases} \dfrac{a}{r^2}\left(1 - \dfrac{r^3}{R^3}\right)\hat{\mathbf{r}}\,; & r \le R \\ \vec{0}\,; & r \ge R \end{cases}$$

where a and R are constants. Which of the following best describes the charge distribution giving rise to this electric field?

a) A negative point charge at the origin with charge $q = 4\pi\varepsilon_0 a$ and a uniformly positive charged spherical shell of radius R with surface charge density $\sigma = -q/4\pi R^2$.

b) A positive point charge at the origin with charge $q = 4\pi\varepsilon_0 a$ and a uniformly negative charged spherical shell of radius R with surface charge density $\sigma = -q/4\pi R^2$.

c) A positive point charge at the origin with charge $q = 4\pi\varepsilon_0 a$ and a uniformly negative charged sphere of radius R with charge density $\rho = -q/(4\pi R^3/3)$.

d) A negative point charge at the origin with charge $-q = -4\pi\varepsilon_0 a$ and a uniformly positive charged sphere of radius R with charge density $\rho = q/(4\pi R^3/3)$.

e) Impossible to determine from the given information.

Was Confused On

Solution c. As you shall see below the answer should be c. because the problem does not specify the sign of the constant a. However both description c. and d. do seem plausible so we shall accept answers c., d., and e.

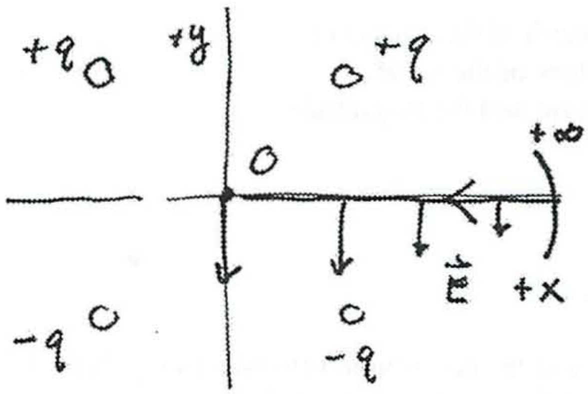
Assume $a > 0$. Then the electric field can be thought of as the superposition of two fields, $\vec{\mathbf{E}}_+(r) = \dfrac{a}{r^2}\hat{\mathbf{r}}$ and $\vec{\mathbf{E}}_-(r) = -\dfrac{ar}{R^3}\hat{\mathbf{r}}$. $\vec{\mathbf{E}}_+(r)$ is the electric field of a positive point charge at the origin with $q = 4\pi\varepsilon_0 a$. $\vec{\mathbf{E}}_-(r)$ is the electric field of a uniformly negative charged sphere of radius R. Because the electric field for radius $r > R$ is zero, the sum of the two charges distributions must be zero. Therefore the charge density must satisfy $\rho = -q/(4\pi R^3/3) = -4\pi\varepsilon_0 a/(4\pi R^3/3) = -3\varepsilon_0 a/R^3$.

Now assume $a<0$. Suppose the electric field can now be thought of as the superposition of two fields, $\vec{\mathbf{E}}_-(r)=\frac{a}{r^2}\hat{\mathbf{r}}$ and $\vec{\mathbf{E}}_+(r)=-\frac{ar}{R^3}\hat{\mathbf{r}}$. $\vec{\mathbf{E}}_-(r)$ is the electric field of a negative point charge at the origin with $-q=4\pi\varepsilon_0 a>0$, hence $q<0$. $\vec{\mathbf{E}}_+(r)$ is the electric field of a uniformly positively charged sphere of radius R. Because the electric field for radius $r>R$ is zero, the sum of the two charges distributions must be zero. Therefore the charge density must satisfy $\rho=q/(4\pi R^3/3)<0$. Therefore when $a<0$ the only possible answer d. cannot be correct.

sum

$$V(O) - V(\infty) = V(O) = k\frac{q}{(2a^2)^{1/2}} + k\frac{q}{(2a^2)^{1/2}} + k\frac{(-q)}{(2a^2)^{1/2}} + k\frac{(-q)}{(2a^2)^{1/2}} = 0.$$

c) Sketch on the figure below one path leading from infinity to the origin at O where the integral $\int_{\infty}^{O} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$ is trivial to do by inspection. Does your answer here agree with your result in b)?



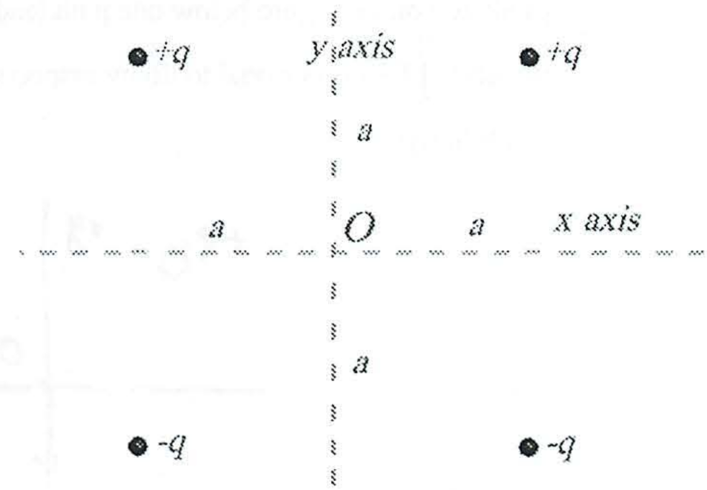
Solution: The electric field at any point along the x-axis is points in the –y-direction. Therefore for a path from infinity to the origin at O along the x-axis, the dot product $\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = 0$ and hence the integral $\int_{\infty}^{O} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = 0$. Because by definition

$\int_{\infty}^{O} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -(V(O) - V(\infty)) = 0$, and the integral is path independent, our answer for the above path along the x-axis sagrees with our result in part b.

Problem 2 (25 points)

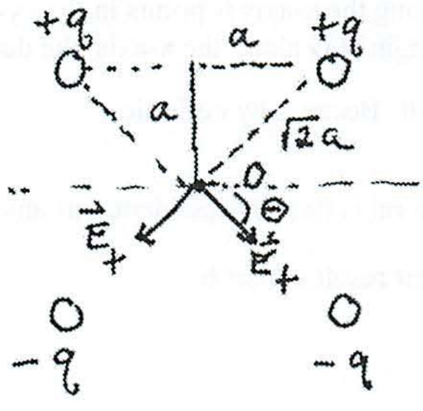
NOTE: YOU MUST SHOW WORK in order to get any credit for this problem. Make it clear to us that you understand what you are doing (use a few words!) .

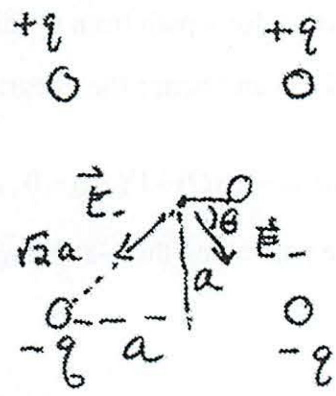
Four charged point-like objects, two of charge $+q$ and two of charge $-q$, are arranged on the vertices of a square with sides of length $2a$, as shown in the sketch.



a) What is the electric field at point O, which is at the center of the square? Indicate the direction and the magnitude.

Solution: When I add the contributions to the electric field at the origin from the two positive charges on the upper corners of the square, the horizontal component cancels and the vertical component points down.





A similar argument holds for the contributions to the electric field at the origin from the two negative charges on the lower corners of the square. Therefore the electric field at the origin is

$$\vec{\mathbf{E}}_O = 4\left|\vec{\mathbf{E}}_{+q}\right|\sin\theta(-\hat{\mathbf{j}}) = 4k\frac{q}{2a^2}\left(\frac{1}{\sqrt{2}}\right)(-\hat{\mathbf{j}}) = \frac{1}{4\pi\varepsilon_0}\frac{\sqrt{2}q}{a^2}(-\hat{\mathbf{j}})$$

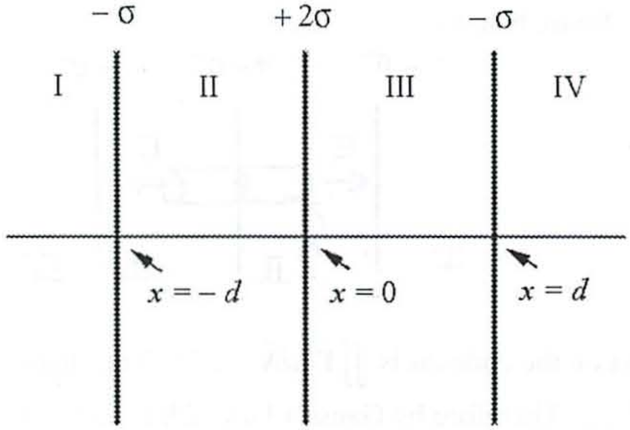
b) What is the electric potential V at point O, the center of the square, taking the potential at infinity to be zero?

Solution zero. The electric potential difference between infinity and the origin is just the

Problem 3 (25 points)

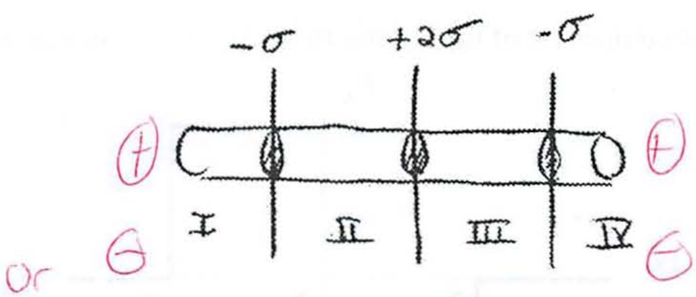
NOTE: YOU MUST SHOW WORK in order to get any credit for this problem. Make it clear to us that you understand what you are doing (use a few words!)

Three infinite sheets of charge are located at $x=-d$, $x=0$, and $x=d$, as shown in the sketch. The sheet at $x=0$ has a charge per unit area of 2σ, and the other two sheets have charge per unit area of $-\sigma$.



a) What is the electric field in each of the four regions I-IV labeled in the sketch? Clearly present your reasoning, relevant figures, and any accompanying calculations. Plot the x component of the electric field, E_x, on the graph on the bottom of the next page. Clearly indicate on the vertical axis the values of E_x for the different regions.

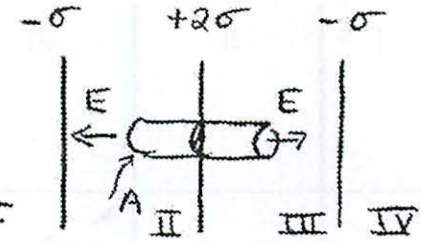
Solution: We begin by choosing a Gaussian cylinder with end caps in regions I and IV as shown in the figure below. The total charge enclosed is zero and hence the electric flux on the endcaps must be zero. Thus the electric field must be zero in regions I and IV.



This turns out to be correct but the conclusion depends on an additional argument based on symmetry. If the electric field is non-zero on the endcaps it must point either in the +x-direction in both regions I and IV or in the –x-direction in both regions I and IV. Neither is possible due to the symmetry of the charge distribution. For example, if the electric field pointed in the +x-direction in both regions I and IV. Then if we looked at the charge distribution from the other side of the plane of the paper, the field should point in the –x-direction. However the charge distribution is identical when looking from the other side of the paper. Therefore the field must point in the +x-direction according to our original assertion. Therefore by symmetry the only possibility is for the fields in regions I and IV to point toward $x = 0$ or away from $x = 0$. In the first case the flux would be non-zero on our Gaussian surface but it must be zero because the charge enclosed is zero. Hence the electric field in regions I and IV is zero. (A similar argument holds if we assume that the field points in the –x-direction in both regions I and IV.)

??? !!! Don't get at ALL

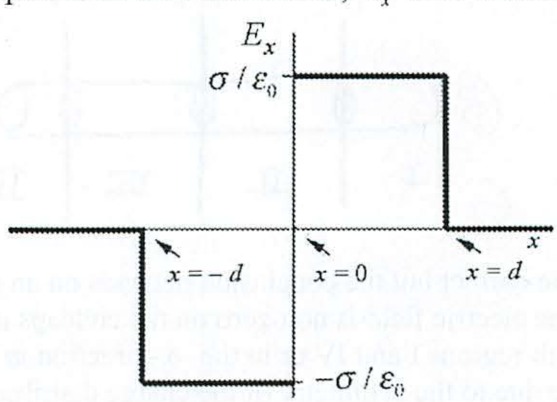
For regions II and III, we choose a Gaussian cylinder with end caps in regions II and III as shown in the figure below.



The electric flux on the endcaps is $\iint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = 2EA$. The charge enclosed divided by ε_0 is $Q_{enc} / \varepsilon_0 = 2\sigma A / \varepsilon_0$. Therefore by Gauss's Law, $2EA = 2\sigma A / \varepsilon_0$ which implies that the magnitude of the electric field is $E = \sigma / \varepsilon_0$. Thus the electric field is given by

$$\vec{\mathbf{E}} = \begin{cases} \vec{\mathbf{0}}; & x < -d \\ -\dfrac{\sigma}{\varepsilon_0}\hat{\mathbf{i}}; & -d < x < 0 \\ \dfrac{\sigma}{\varepsilon_0}\hat{\mathbf{i}}; & 0 < x < +d \\ \vec{\mathbf{0}}; & d < x \end{cases}$$

The graph of the x component of the electric field, E_x vs x is shown on the graph below.



b) Find the electric potential in each of the four regions I-IV labeled above, with the choice that the potential is zero at $x = +\infty$ i.e. $V(+\infty) = 0$. Show your calculations. Plot the electric potential as a function of x on the graph on the bottom of the next page. Indicate units on the vertical axis.

Solution: The electric potential difference between infinity and a point P located at x, is given by

$$V(x) - V(\infty) = -\int_{\infty}^{P} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}\,.$$

We shall evaluate this integral for points in each region. We start with P anywhere in region IV, $d < x$. Because the electric field in region IV is zero, the integral is zero,

$$V(x) - V(\infty) = -\int_{\infty}^{x} \vec{\mathbf{E}}_{IV} \cdot d\vec{\mathbf{s}} = 0\,.$$

If P is anywhere in region III, $0 < x < +d$ then

$$\begin{aligned} V(x) - V(\infty) &= -\int_{\infty}^{d} \vec{\mathbf{E}}_{IV} \cdot d\vec{\mathbf{s}} - \int_{d}^{x} \vec{\mathbf{E}}_{III} \cdot d\vec{\mathbf{s}} \\ &= 0 - \int_{d}^{x} E_x dx = -\int_{d}^{x} \frac{\sigma}{\varepsilon_0} dx = -\frac{\sigma}{\varepsilon_0}(x - d) = \frac{\sigma}{\varepsilon_0} d - \frac{\sigma}{\varepsilon_0} x \end{aligned}$$

If P is anywhere in region II, $-d < x < 0$ then

$$\begin{aligned} V(x) - V(\infty) &= -\int_{\infty}^{d} \vec{\mathbf{E}}_{IV} \cdot d\vec{\mathbf{s}} - \int_{d}^{0} \vec{\mathbf{E}}_{III} \cdot d\vec{\mathbf{s}} - \int_{0}^{x} \vec{\mathbf{E}}_{II} \cdot d\vec{\mathbf{s}} \\ &= 0 \ - \int_{d}^{0} \frac{\sigma}{\varepsilon_0} dx - \int_{0}^{x} -\frac{\sigma}{\varepsilon_0} dx = \frac{\sigma}{\varepsilon_0} d + \frac{\sigma}{\varepsilon_0} x \end{aligned}$$

If P is anywhere in region I, $x < -d$ then

$$\begin{aligned} V(x) - V(\infty) &= -\int_{\infty}^{d} \vec{\mathbf{E}}_{IV} \cdot d\vec{\mathbf{s}} - \int_{d}^{0} \vec{\mathbf{E}}_{III} \cdot d\vec{\mathbf{s}} - \int_{0}^{-d} \vec{\mathbf{E}}_{II} \cdot d\vec{\mathbf{s}} - \int_{-d}^{x} \vec{\mathbf{E}}_{I} \cdot d\vec{\mathbf{s}} \\ &= 0 - \int_{d}^{0} \frac{\sigma}{\varepsilon_0} dx - \int_{0}^{-d} -\frac{\sigma}{\varepsilon_0} dx - 0 = \frac{\sigma}{\varepsilon_0} d - \frac{\sigma}{\varepsilon_0} d = 0 \end{aligned}$$

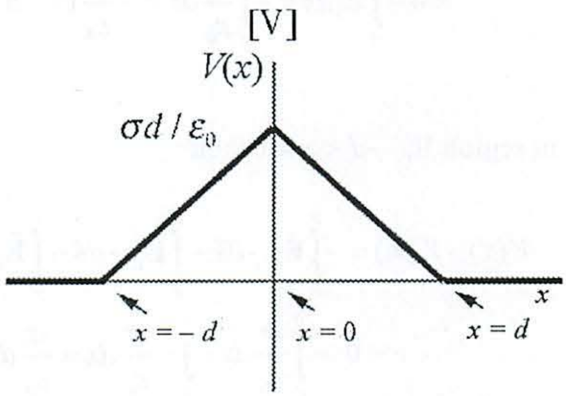
Because the electric field is continuous we can write our result as

$$V(x)-V(\infty)=\begin{cases} 0\ ; & x\le -d \\ \dfrac{\sigma}{\varepsilon_0}d+\dfrac{\sigma}{\varepsilon_0}x\ ; & -d\le x\le 0 \\ \dfrac{\sigma}{\varepsilon_0}d-\dfrac{\sigma}{\varepsilon_0}x\ ; & 0\le x\le +d \\ 0\ ; & d\le x \end{cases}.$$

Note this can be written as

$$V(x)-V(\infty)=\begin{cases} 0\ ; & x\le -d \\ \dfrac{\sigma}{\varepsilon_0}d-\dfrac{\sigma}{\varepsilon_0}\left|x\right|\ ; & -d\le x\le d \\ 0\ ; & d\le x \end{cases}.$$

This result looks good because the area under the graph of the x component of the electric field, E_x vs x for the region $-d<x<d$ is zero. The plot of the electric potential as a function of x on the graph is shown below with units of [V] on the vertical axis.



c) How much work must you do to bring a point-like object with charge $+Q$ in from infinity to the origin $x=0$?

Solution. The work you must do is equal to the change in potential energy (assuming the point-like object begins and ends at rest). Therefore

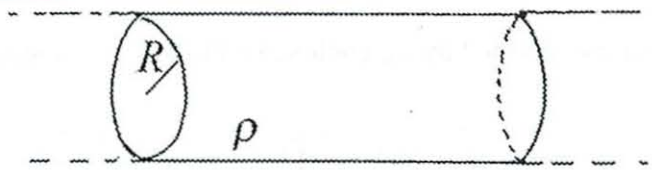
$$W=U(0)-U(\infty))=+Q(V(0)-V(\infty))=+\frac{Q\sigma}{\varepsilon_0}d\,.$$

Problem 4 (25 points)

NOTE: YOU MUST SHOW WORK in order to get any credit for this problem. Make it clear to us that you understand what you are doing (use a few words!). You may find the following integrals helpful in this answering this question.

$$\int_{r_a}^{r_b} r^n dr = \frac{1}{n+1}\left(r_b^{\,n+1} - r_a^{\,n+1}\right); n \neq 1 \,, \qquad \int_{r_a}^{r_b} \frac{dr}{r} = \ln(r_b / r_a) \,.$$

Consider a charged infinite cylinder of radius R.



The charge density is non-uniform and given by

$$\rho(r) = br \,; \; r < R \,,$$

where r is the distance from the central axis and b is a constant.

a) Find an expression for the direction and magnitude of the electric field everywhere i.e. inside and outside the cylinder. Clearly present your reasoning, relevant figures, and any accompanying calculations.

Solution. Because the charge distribution defines two distinct regions of space, region I defined by $r < R$ and region II defined by $r > R$, we must apply Gauss's Law twice to find the electric field everywhere.

In region I, where $r < R$, we choose a Gaussian cylinder of radius r and length l.

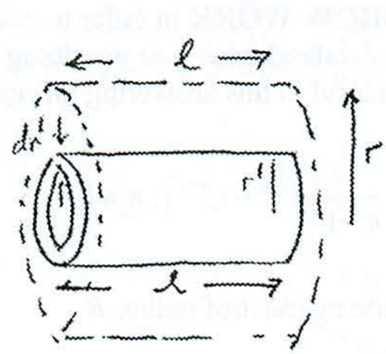
* infinitly long
-So no endcaps

Because the electric field points away from the central axis, the electric flux on our Gaussian surface is

grr

$$\iint \vec{\mathbf{E}}_I \cdot d\vec{\mathbf{A}} = E_I 2\pi r l \,.$$

Because the charge density is non-uniform, we must integrate the charge density. We choose as our integration volume a cylindrical shell of radius r', length l and thickness dr'. The integration volume is then $dV' = 2\pi r' l dr'$.



Therefore the charge divided by ε_0 enclosed within our Gaussian surface is

$$Q_{enc} / \varepsilon_0 = \frac{1}{\varepsilon_0}\int_0^r \rho 2\pi r' l dr' = \frac{1}{\varepsilon_0}\int_0^r b r' 2\pi r' l dr' = \frac{2\pi l b}{\varepsilon_0}\int_0^r r'^2 dr' = \frac{2\pi l b r^3}{3\varepsilon_0}.$$

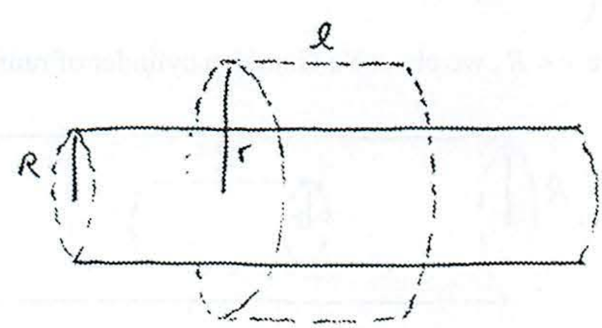
Therefore Gauss's Law becomes

$$E_I 2\pi r l = 2\pi l b r^3 / 3.$$

We can now solve for the direction and magnitude of the electric field when $r < R$,

$$\vec{\mathbf{E}}_I = \frac{b r^2}{3\varepsilon_0}\hat{\mathbf{r}}.$$

In region II where $r > R$, we choose a Gaussian cylinder of radius r and length l.



Because the electric field points away from the central axis, the electric flux on our Gaussian surface is

$$\iint \vec{\mathbf{E}}_{II} \cdot d\vec{\mathbf{A}} = E_{II} 2\pi r l.$$

We again must integrate the charge density but this time taking our endpoints as $r = 0$ and $r = R$. Therefore the charge divided by ε_0 enclosed within our Gaussian surface is

$$Q_{enc} / \varepsilon_0 = \frac{1}{\varepsilon_0}\int_0^r \rho 2\pi r' l dr' = \frac{1}{\varepsilon_0}\int_0^R br' 2\pi r' l dr' = \frac{2\pi lb}{\varepsilon_0}\int_0^R r'^2 dr' = \frac{2\pi lbR^3}{3\varepsilon_0}.$$

Therefore Gauss's Law becomes

$$E_{II} 2\pi rl = 2\pi lbR^3 / 3.$$

We can now solve for the direction and magnitude of the electric field when $r > R$,

$$\vec{\mathbf{E}}_{II} = \frac{bR^3}{3\varepsilon_0}\frac{1}{r}\hat{\mathbf{r}}.$$

Collected our results we have that

$$\vec{\mathbf{E}} = \begin{cases} \dfrac{br^2}{3\varepsilon_0}\hat{\mathbf{r}}; & r < R \\ \dfrac{bR^3}{3\varepsilon_0}\dfrac{1}{r}\hat{\mathbf{r}}; & r > R \end{cases}$$

b) A point-like object with charge $+q$ and mass m is released from rest at the point a distance $2R$ from the central axis of the cylinder. Find the speed of the object when it reaches a distance $3R$ from the central axis of the cylinder.

Solution: The change in kinetic energy when the object moves from a distance $2R$ from the central axis of the cylinder to a distance $3R$ is given by

$$K(3R) - K(2R) = -(U(3R) - U(2R)) = -q(V(3R) - V(2R)).$$

Because the particle was released at rest, $K(2R) = 0$, and $K(3R) = (1/2)mv_f^2$, the final speed of the object is

$$v_f = \sqrt{-\frac{2q}{m}(V(3R) - V(2R))}.$$

The electric potential difference between two points in region II is given by

$$V(3R)-V(2R) = -\int_{2R}^{3R} \vec{\mathbf{E}}_{II} \cdot d\vec{\mathbf{s}} = -\int_{2R}^{3R} \frac{bR^3}{3\varepsilon_0} \frac{1}{r} \hat{\mathbf{r}} \cdot d\vec{\mathbf{s}}$$

$$= -\int_{2R}^{3R} \frac{bR^3}{3\varepsilon_0} \frac{1}{r} dr = -\frac{bR^3}{3\varepsilon_0} \ln \frac{3R}{2R} = -\frac{bR^3}{3\varepsilon_0} \ln(3/2)$$

Therefore the speed of the object when it reaches a distance $3R$ from the central axis of the cylinder is

$$v_f = \sqrt{\frac{2qbR^3}{3m\varepsilon_0} \ln(3/2)} \, .$$